EECS 442 – Computer vision

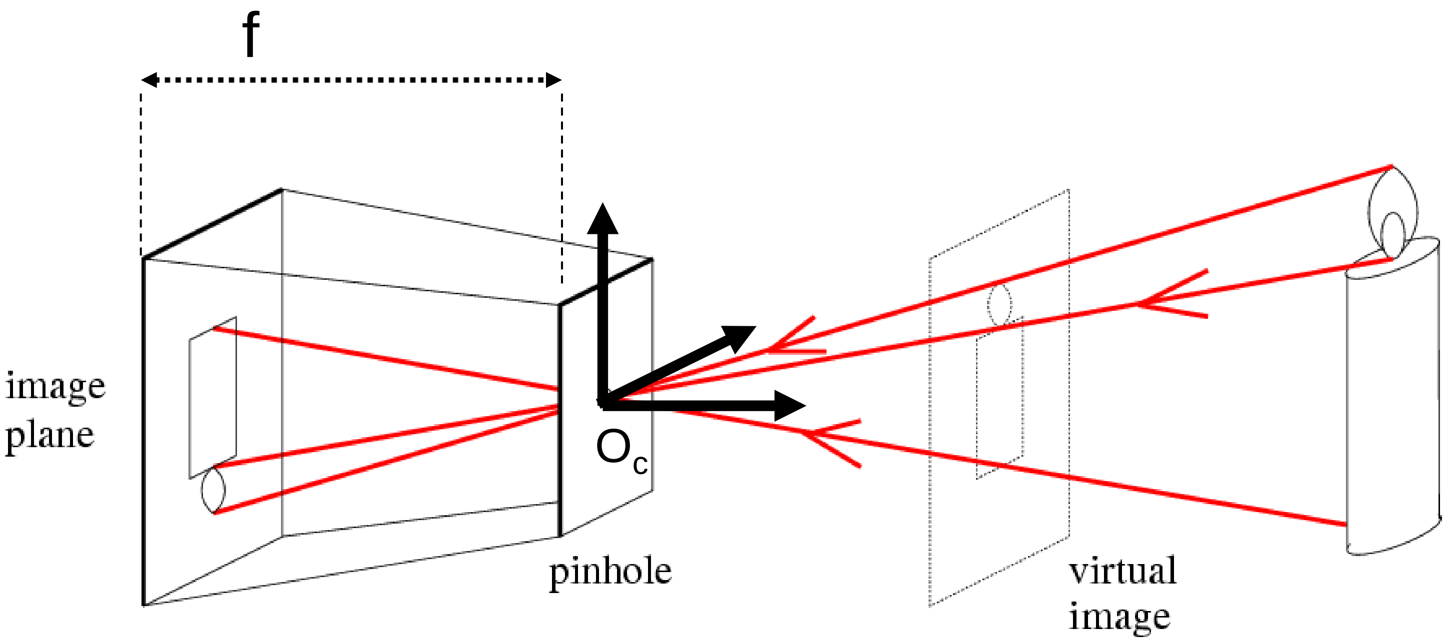
Camera Calibration

- Review camera parameters
- Camera calibration problem
- Example

Reading: [FP] Chapter 3
[HZ] Chapter 7

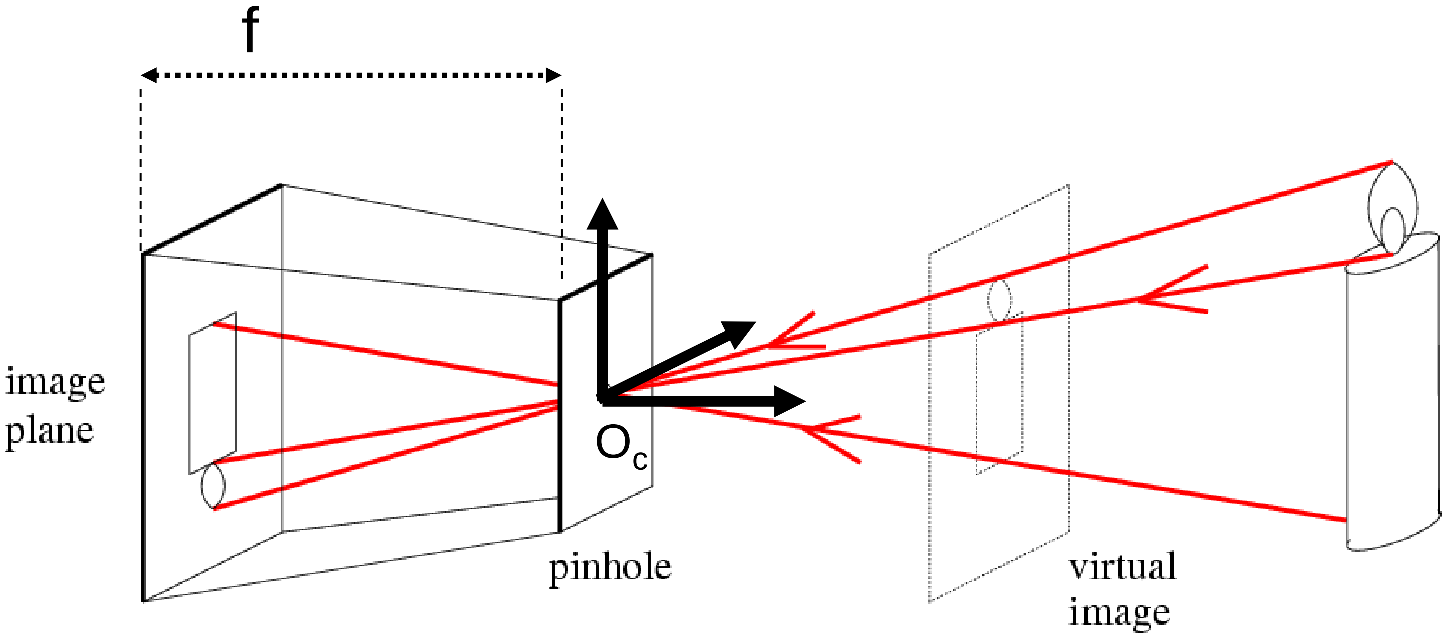
Some slides in this lecture are courtesy to Profs. J. Ponce & F-F Li

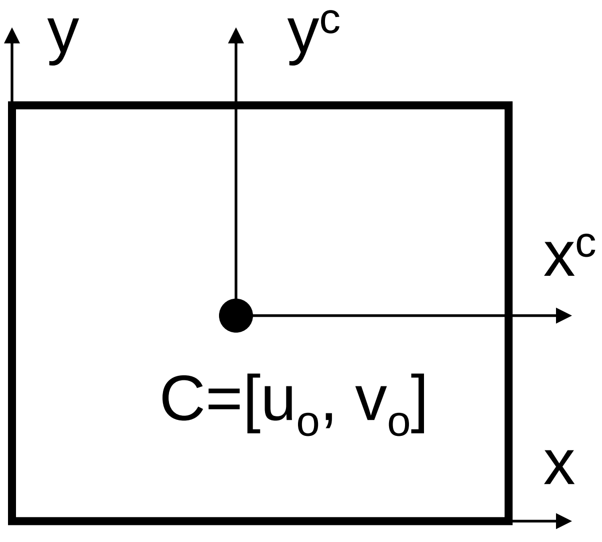
Projective camera



f = focal length

Projective camera

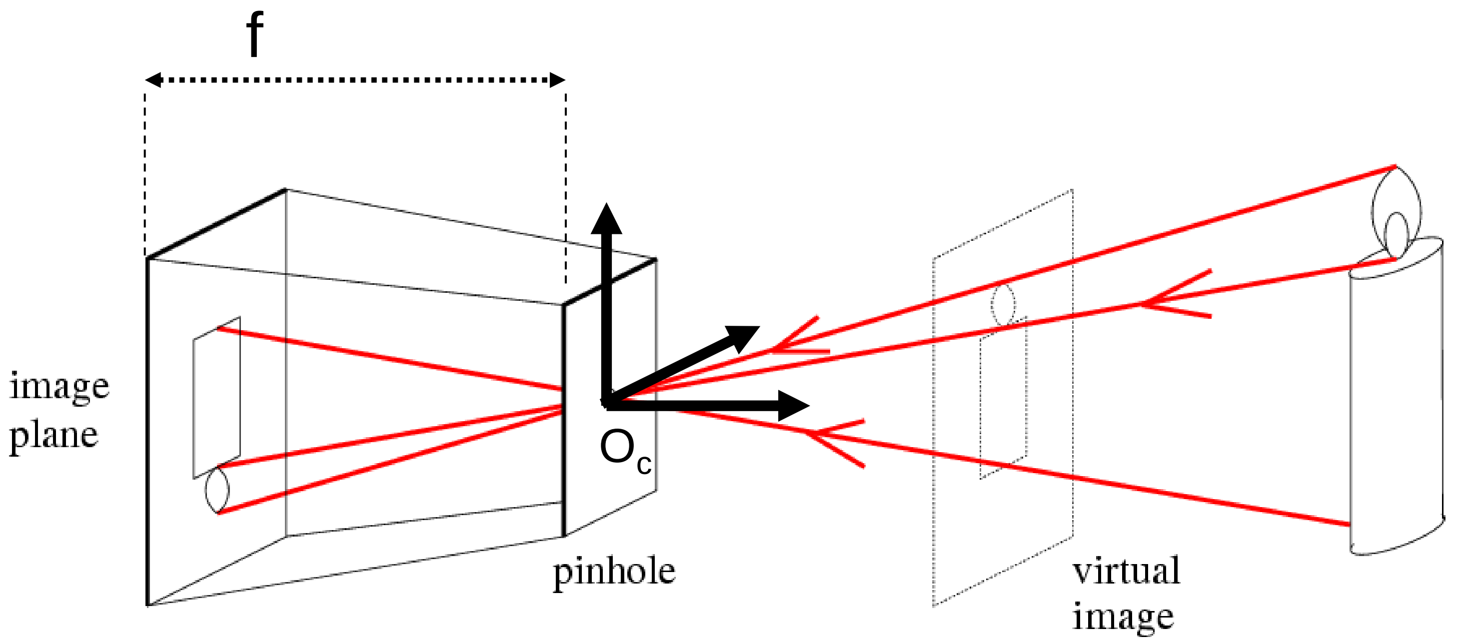




f = focal length

u_o, v_o = offset

Projective camera



Units: k,l [pixel/m]

f [m]

Non-square pixels

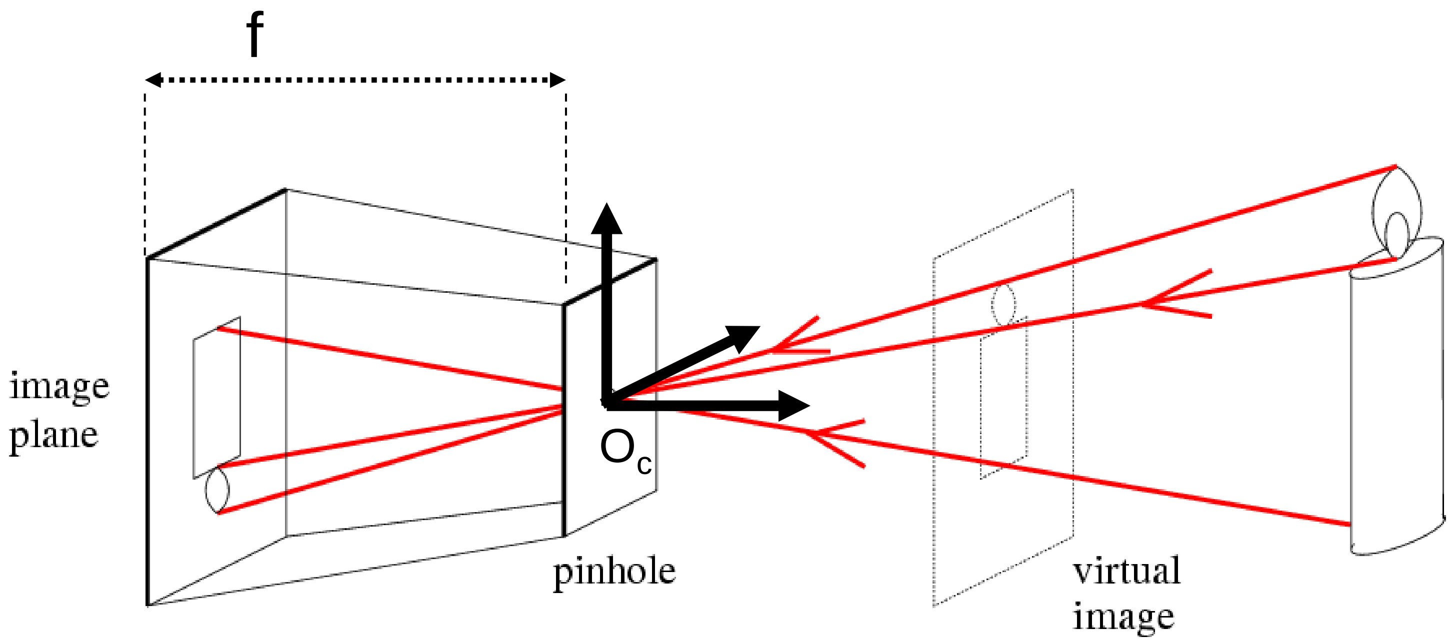
α, β [pixel]

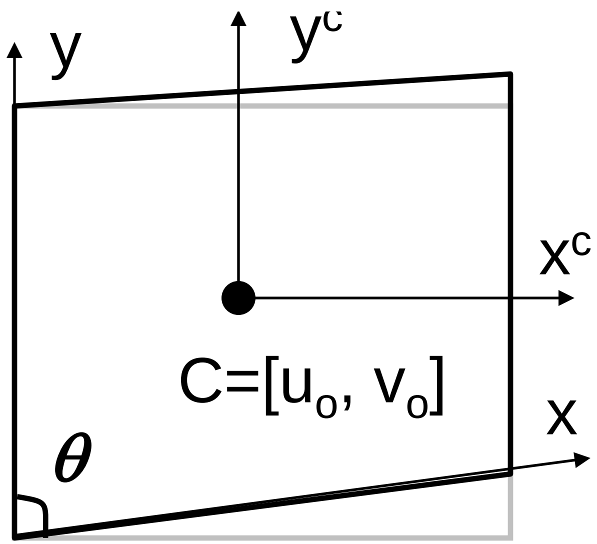
f = focal length

u_o, v_o = offset

α, β $\rightarrow$ non-square pixels

Projective camera





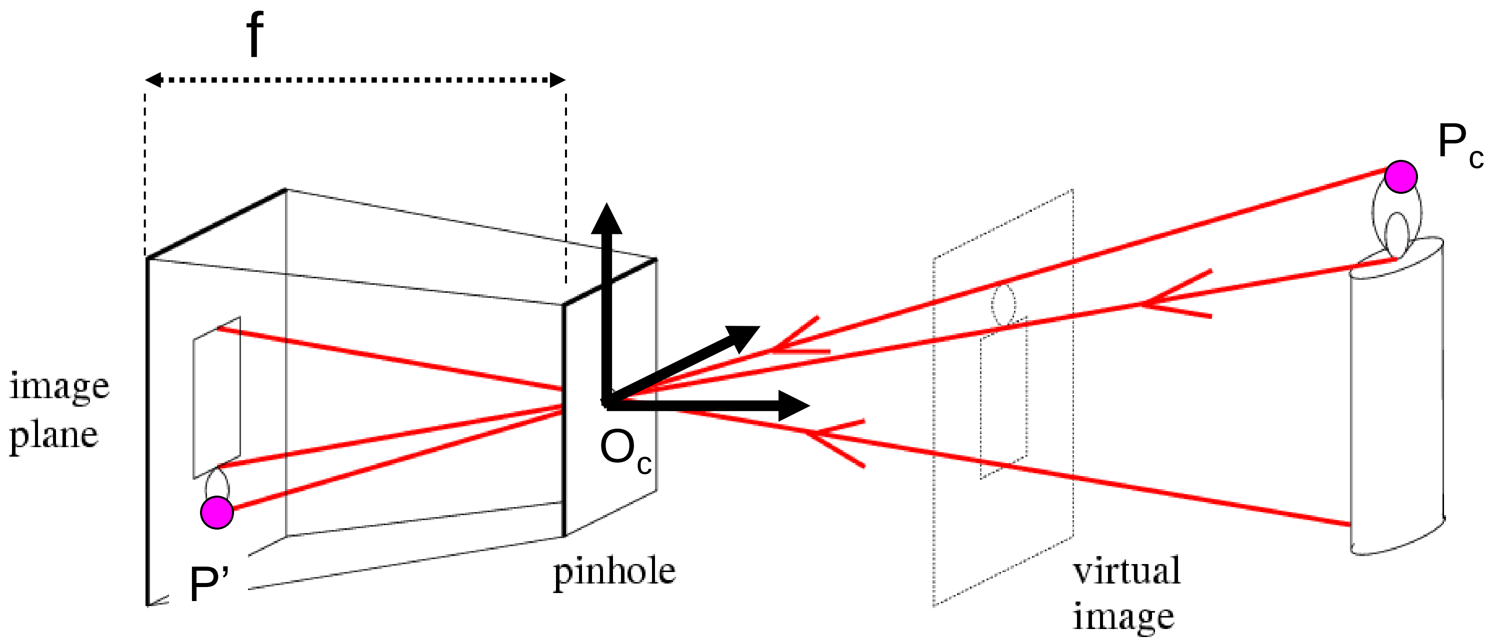
f = focal length

u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

Projective camera



$$P' = \begin{bmatrix} \alpha & s & u_o & 0 \\ 0 & \beta & v_o & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

f = focal length

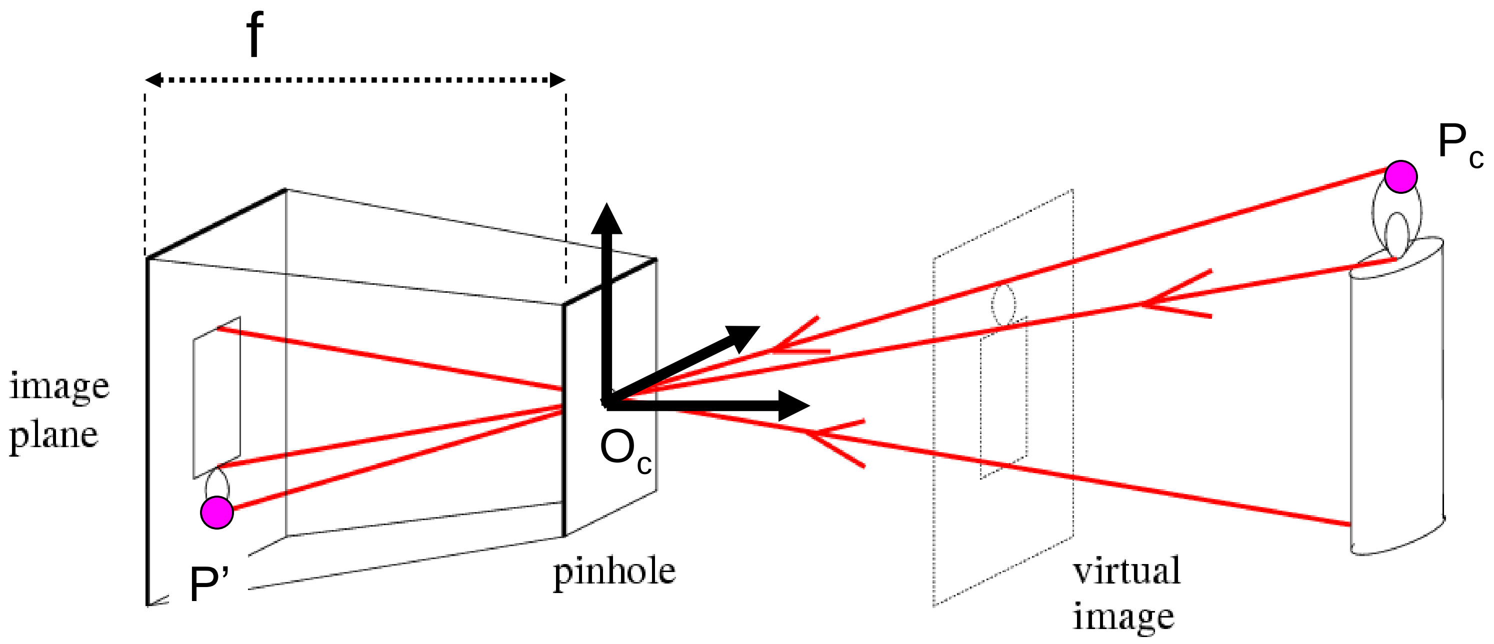
u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

K has 5 degrees of freedom!

Projective camera



$$P' = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o & 0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_o & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

f = focal length

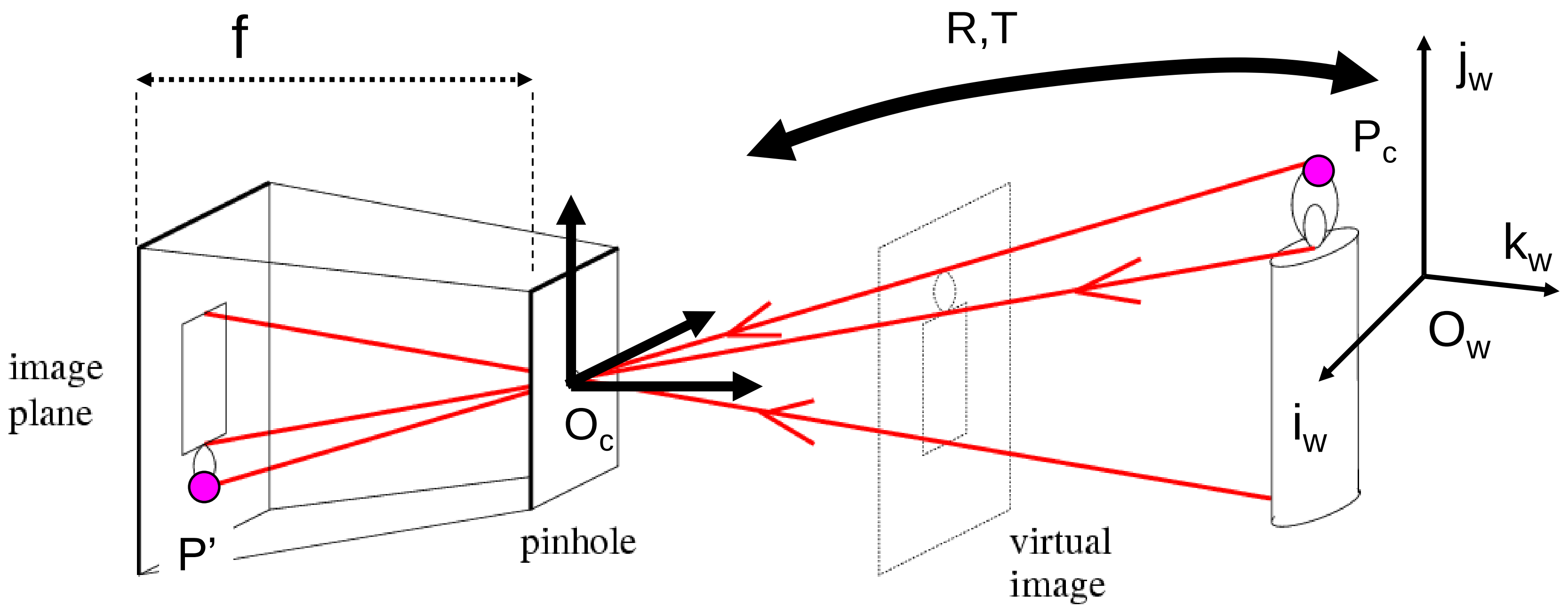
u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

K has 5 degrees of freedom!

Projective camera



$$P = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times 4} P_w$$

$$T = -R\tilde{O}_c$$

f = focal length

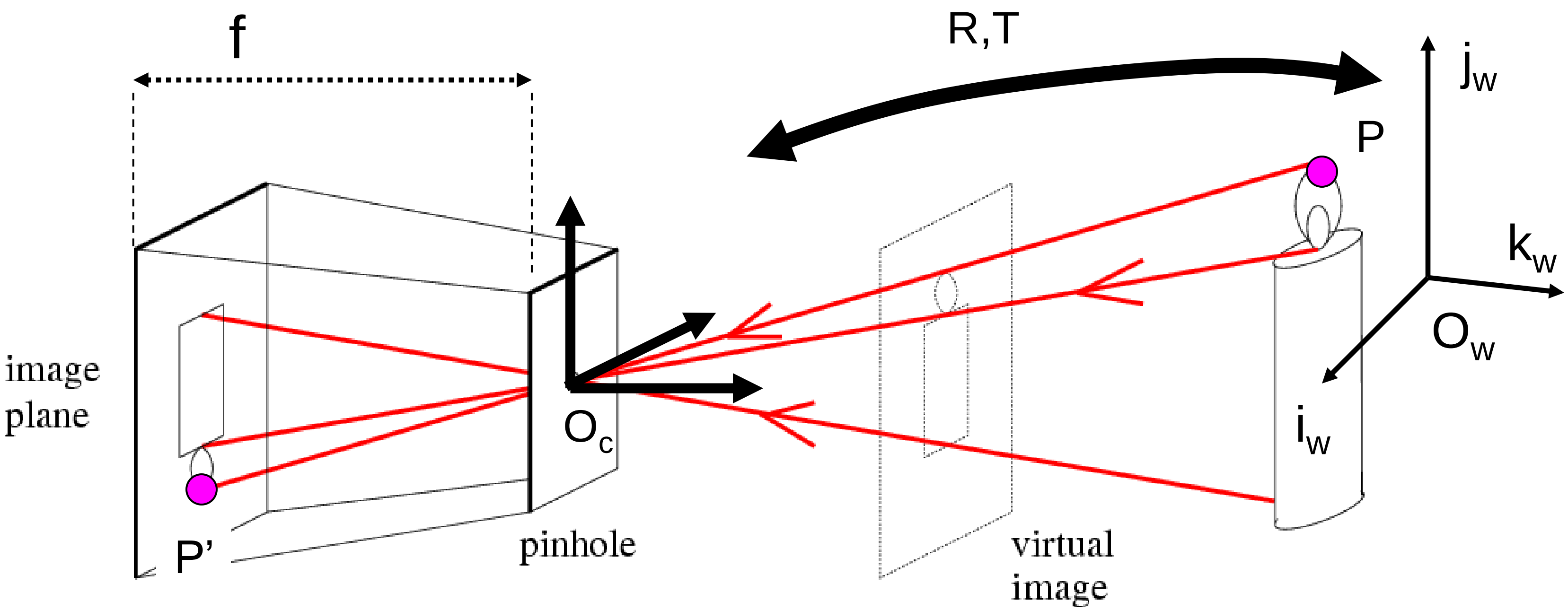
u_o, v_o = offset

$\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ $\rightarrow$ non-square pixels

θ = skew angle

R,T = rotation, translation

Projective camera



$$P' = M\, P_w$$

$$= \underbrace{\boxed{K}}_{\text{Internal parameters}} \underbrace{\boxed{[R \quad T]}}_{\text{External parameters}} P_w$$

f = focal length

u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

R,T = rotation, translation

Properties of Projection

- Points project to points
- Lines project to lines
- Distant objects look smaller

Properties of Projection

- Angles are not preserved
- Parallel lines meet!

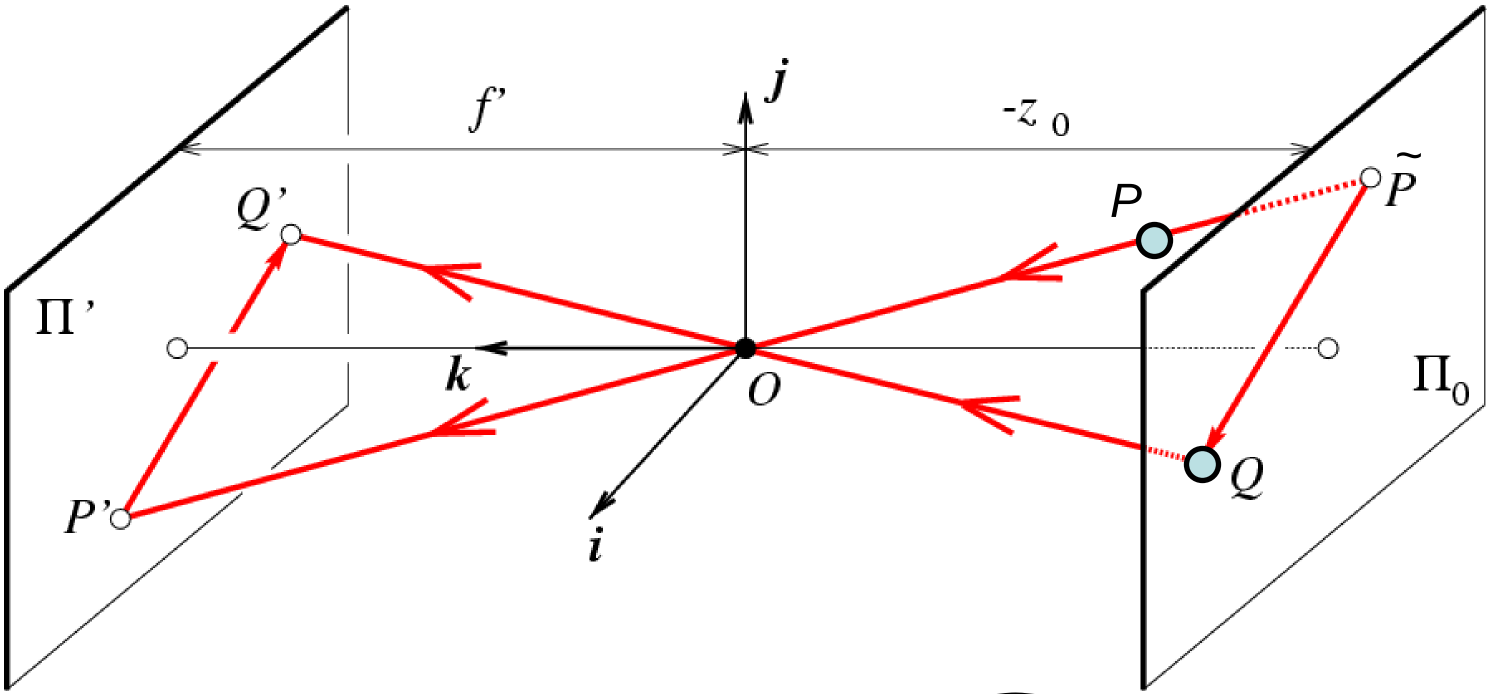
Vanishing point

Cameras

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Weak perspective projection

When the relative scene depth is small compared to its distance from the camera

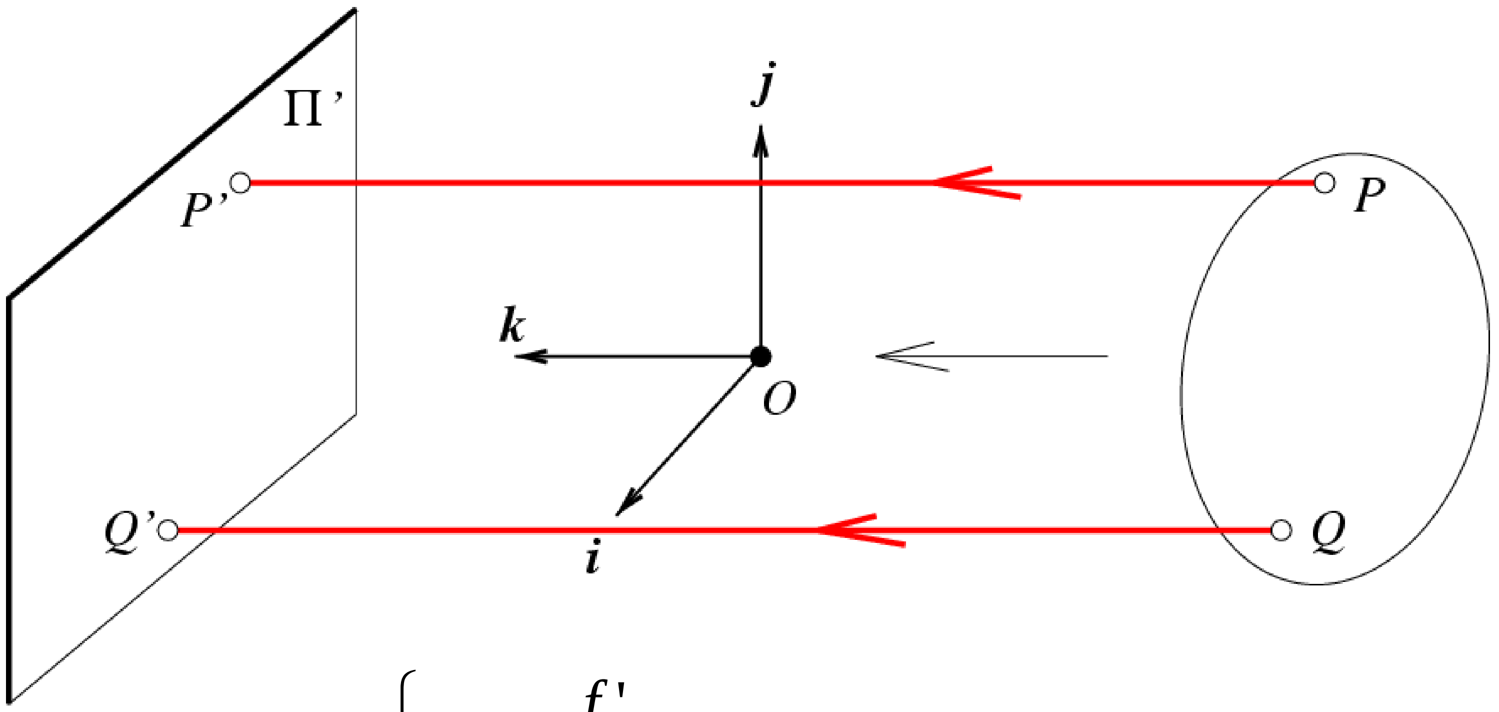


$$\begin{cases} x' = -\dfrac{f'}{z}x \\ y' = -\dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = -\boxed{\dfrac{f'}{z_0}}x \\ y' = -\dfrac{f'}{z_0}y \end{cases}$$

Magnification m

Orthographic (affine) projection

Distance from center of projection to image plane is infinite



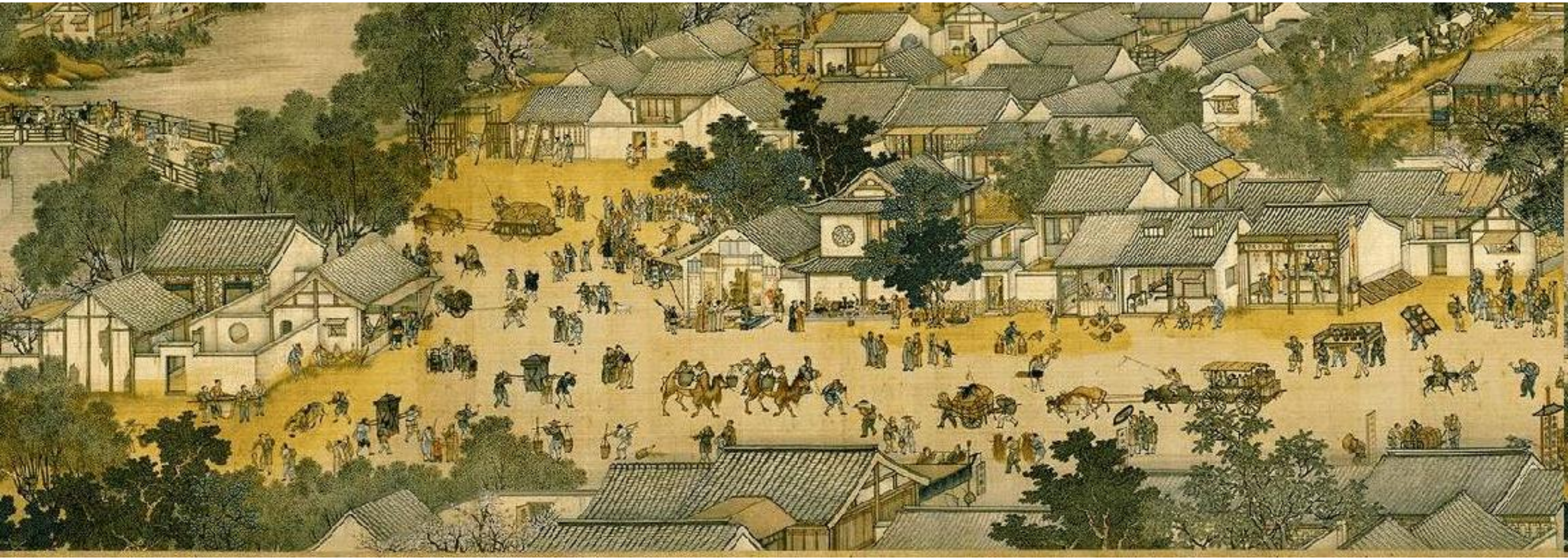
$$\begin{cases} x' = -\dfrac{f'}{z}x \\ \\ y' = -\dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = -x \\ y' = -y \end{cases}$$

Pros and Cons of These Models

- Weak perspective much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.

- Pinhole perspective much more accurate for scenes.
  - Used in structure from motion.

Weak perspective projection



Qingming Festival by the Riverside Zhang Zeduan ~900 AD

The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui

Projective camera

$$P' = M\,P_w \quad = \boxed{K}\left[\boxed{R \quad T}\right]P_w$$

Internal parameters

External parameters

Projective camera

$$P' = M\, P_w \quad = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

$$\boxed{\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}}$$

$$K = \begin{bmatrix} \boldsymbol{\alpha} & -\boldsymbol{\alpha}\cot\boldsymbol{\theta} & u_o \\ 0 & \dfrac{\boldsymbol{\beta}}{\sin\boldsymbol{\theta}} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Goal of calibration

Estimate intrinsic and extrinsic parameters from 1 or multiple images

$$P' = M\,P_w \quad = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

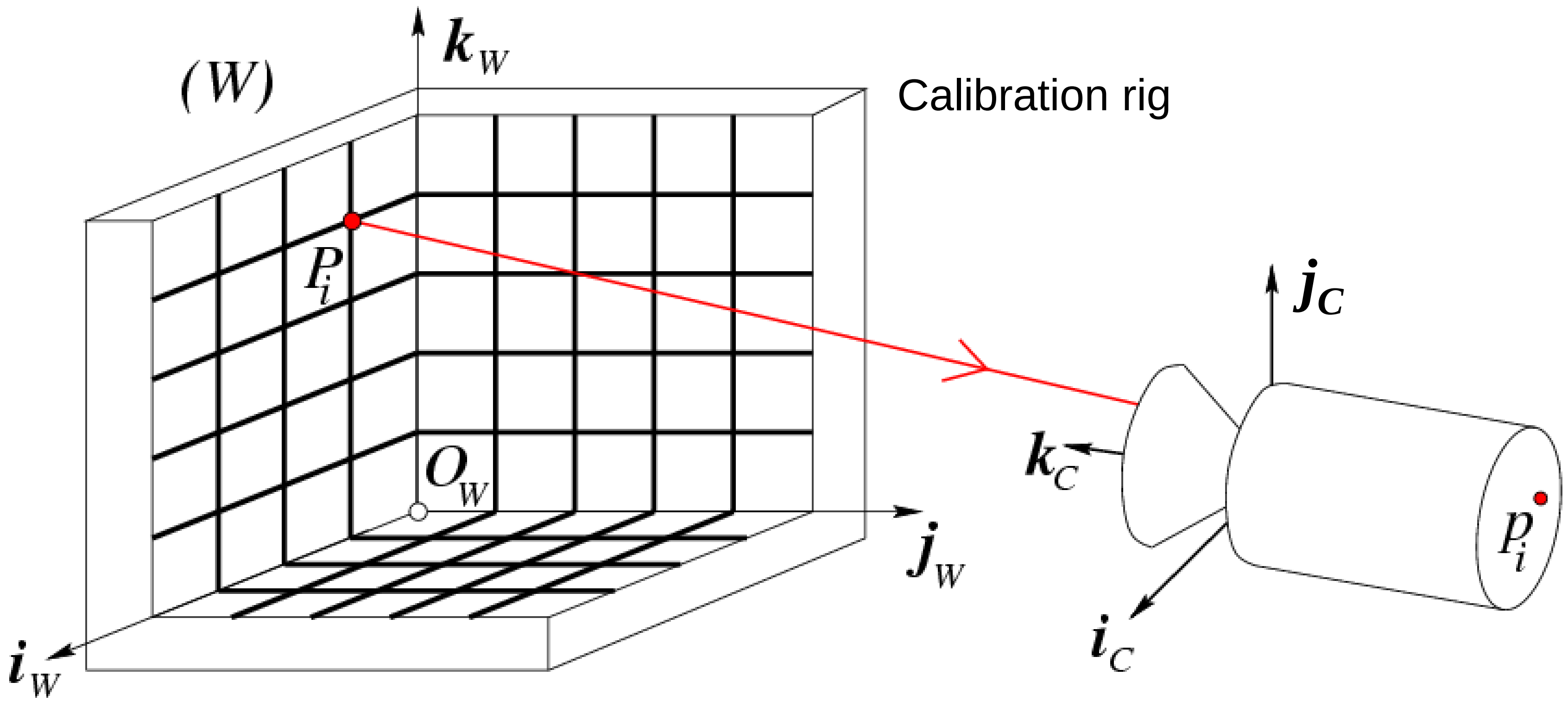
$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha\cot\theta\,\boldsymbol{r}_2^T + u_0\boldsymbol{r}_3^T & \alpha t_x - \alpha\cot\theta\, t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0\boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Change notation:
$P = P_w$

$p = P'$

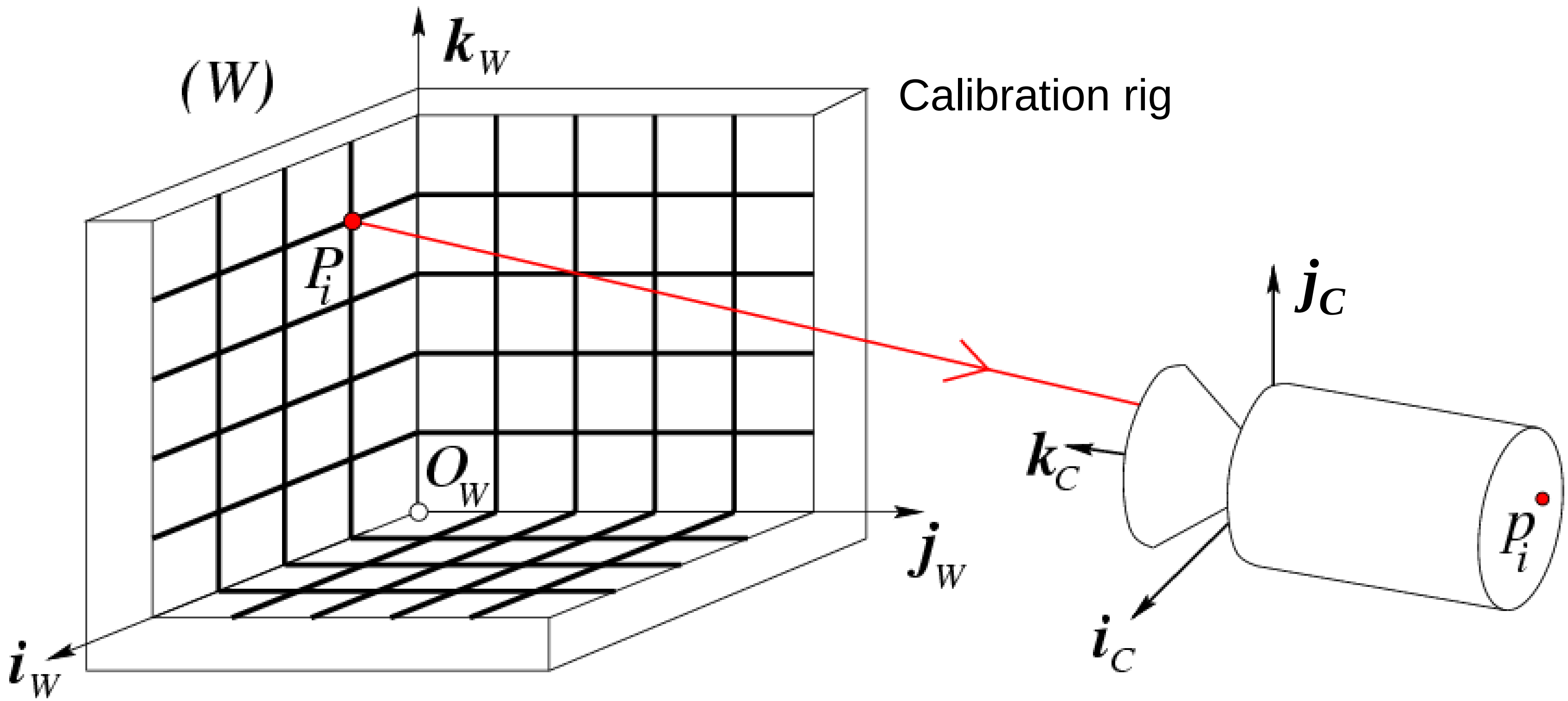
Calibration Problem



- $P_1 \ldots P_n$ with known positions in $[O_W, i_W, j_W, k_W]$
- $p_1, \ldots p_n$ known positions in the image

Goal: compute intrinsic and extrinsic parameters

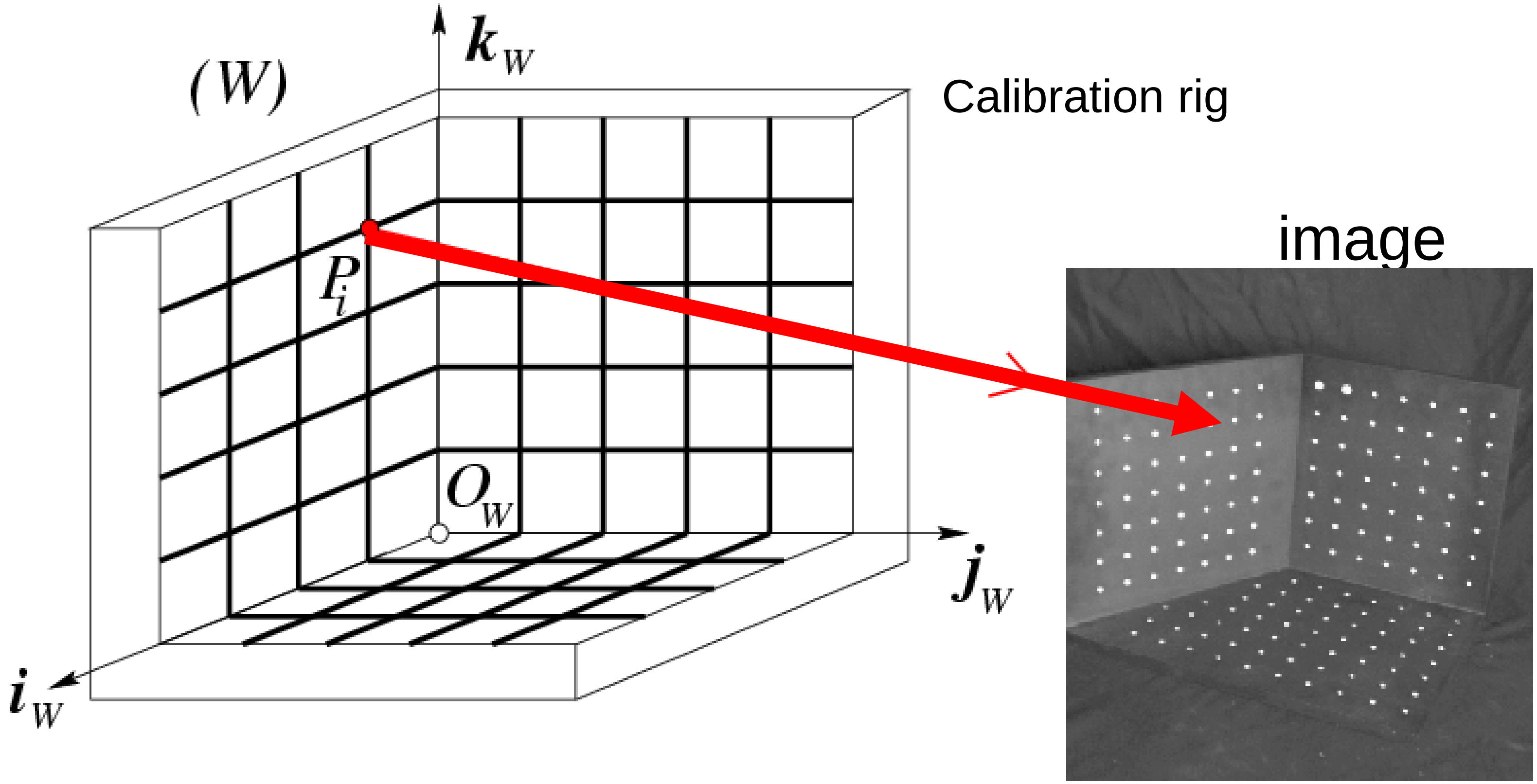
Calibration Problem



How many correspondences do we need?

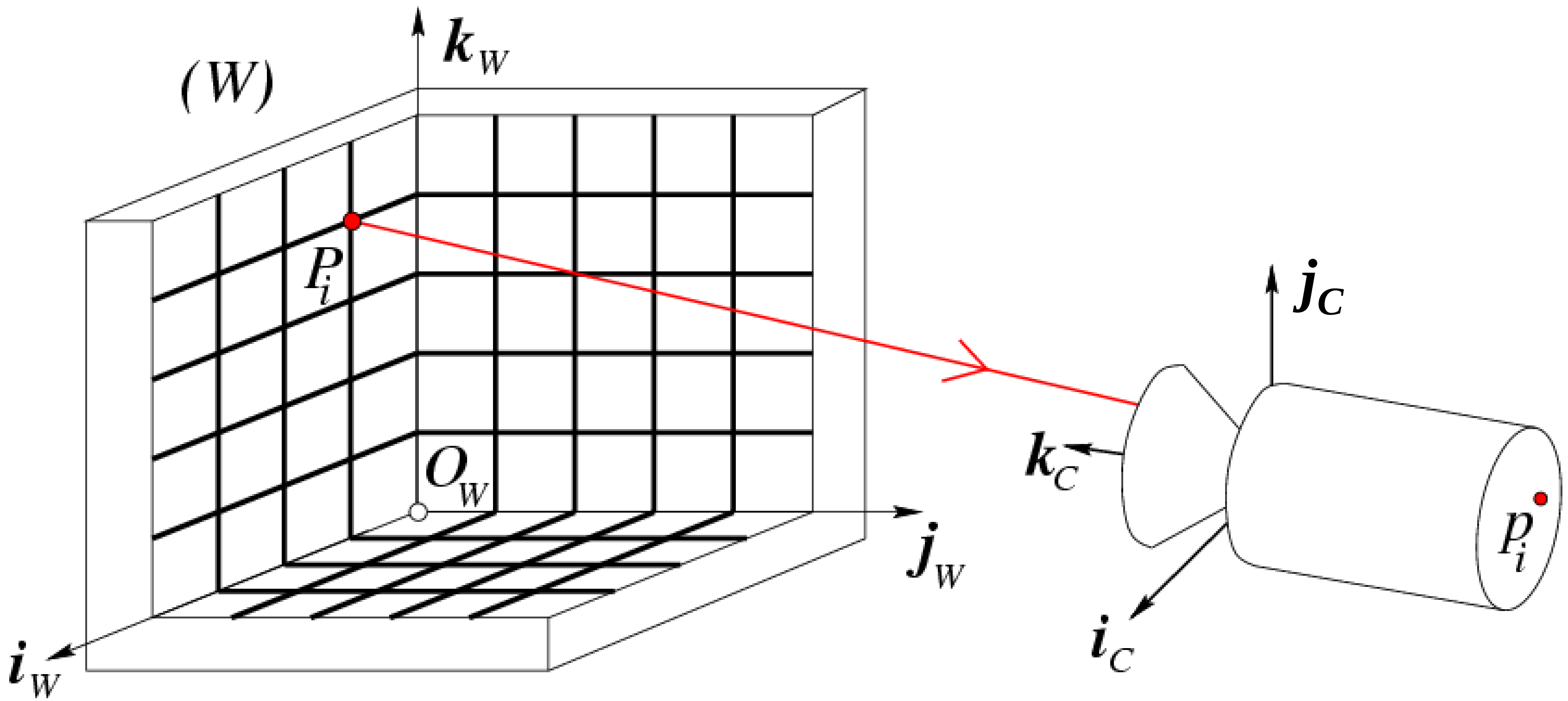
- P has 11 unknown
- We need 11 equations
- 6 correspondences would do it

Calibration Problem



In practice: user may need to look at the image and select the n>=6 correspondences

Calibration Problem



$$\mathrm{P_i} \rightarrow \mathrm{M\,P_i} \rightarrow \mathrm{p_i} = \begin{bmatrix} \mathrm{u_i} \\ \mathrm{v_i} \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1\,\mathrm{P_i}}{\mathbf{m}_3\,\mathrm{P_i}} \\ \dfrac{\mathbf{m}_2\,\mathrm{P_i}}{\mathbf{m}_3\,\mathrm{P_i}} \end{bmatrix} \qquad \mathrm{M} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1\ P_i}{\mathbf{m}_3\ P_i} \\ \dfrac{\mathbf{m}_2\ P_i}{\mathbf{m}_3\ P_i} \end{bmatrix}$$

$$u_i = \frac{\mathbf{m}_1\ P_i}{\mathbf{m}_3\ P_i} \rightarrow u_i(\mathbf{m}_3\ P_i) = \mathbf{m}_1\ P_i \rightarrow u_i(\mathbf{m}_3\ P_i) - \mathbf{m}_1\ P_i = 0$$

$$v_i = \frac{\mathbf{m}_2\ P_i}{\mathbf{m}_3\ P_i} \rightarrow v_i(\mathbf{m}_3\ P_i) = \mathbf{m}_2\ P_i \rightarrow v_i(\mathbf{m}_3\ P_i) - \mathbf{m}_2\ P_i = 0$$

Calibration Problem

$$\begin{cases} u_1(\mathbf{m}_3 \, P_1) - \mathbf{m}_1 \, P_1 = 0 \\ v_1(\mathbf{m}_3 \, P_1) - \mathbf{m}_2 \, P_1 = 0 \\ \vdots \\ u_i(\mathbf{m}_3 \, P_i) - \mathbf{m}_1 \, P_i = 0 \\ v_i(\mathbf{m}_3 \, P_i) - \mathbf{m}_2 \, P_i = 0 \\ \vdots \\ u_n(\mathbf{m}_3 \, P_n) - \mathbf{m}_1 \, P_n = 0 \\ v_n(\mathbf{m}_3 \, P_n) - \mathbf{m}_2 \, P_n = 0 \end{cases}$$

Block Matrix Multiplication

$$\mathrm{A} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} \\ \mathrm{A}_{21} & \mathrm{A}_{22} \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} \\ \mathrm{B}_{21} & \mathrm{B}_{22} \end{bmatrix}$$

What is *AB* ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Calibration Problem

$$\begin{cases} -u_1(\mathbf{m}_3\ P_1) + \mathbf{m}_1\ P_1 = 0 \\ -v_1(\mathbf{m}_3\ P_1) + \mathbf{m}_2\ P_1 = 0 \\ \vdots \\ -u_n(\mathbf{m}_3\ P_n) + \mathbf{m}_1\ P_n = 0 \\ -v_n(\mathbf{m}_3\ P_n) + \mathbf{m}_2\ P_n = 0 \end{cases} \longrightarrow \boxed{\mathcal{P}\boldsymbol{m} = 0}$$

known

unknown

Homogenous linear system

1x4

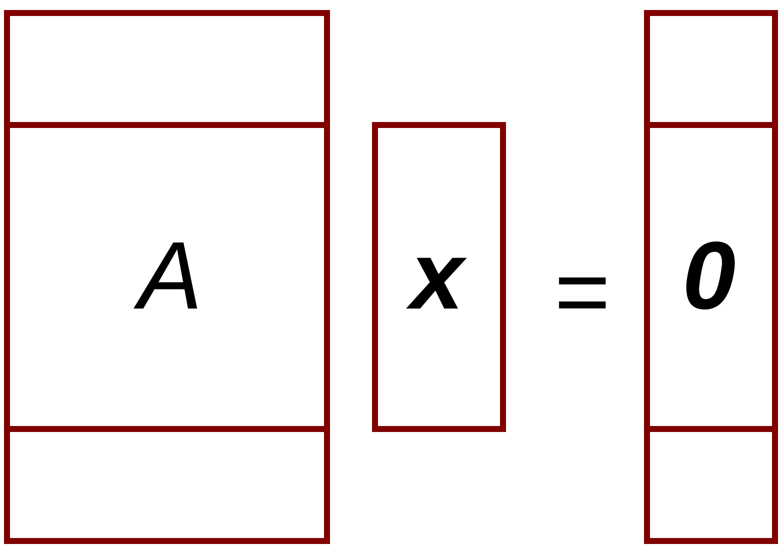
$$\mathcal{P} \stackrel{\text{def}}{=} \begin{pmatrix} \boldsymbol{P}_1^T & \mathbf{0}^T & -u_1 \boldsymbol{P}_1^T \\ \mathbf{0}^T & \boldsymbol{P}_1^T & -v_1 \boldsymbol{P}_1^T \\ \dots & \dots & \dots \\ \boldsymbol{P}_n^T & \mathbf{0}^T & -u_n \boldsymbol{P}_n^T \\ \mathbf{0}^T & \boldsymbol{P}_n^T & -v_n \boldsymbol{P}_n^T \end{pmatrix}_{2n \times 12}$$

4x1

$$\boldsymbol{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^{\mathrm{T}} \\ \mathbf{m}_2^{\mathrm{T}} \\ \mathbf{m}_3^{\mathrm{T}} \end{pmatrix}_{12 \times 1}$$

Homogeneous M x N Linear Systems

M=number of equations = 2n
N=number of unknown = 11



Rectangular system (M>N)

- 0 is always a solution
- To find non-zero solution

  Minimize $|Ax|^2$

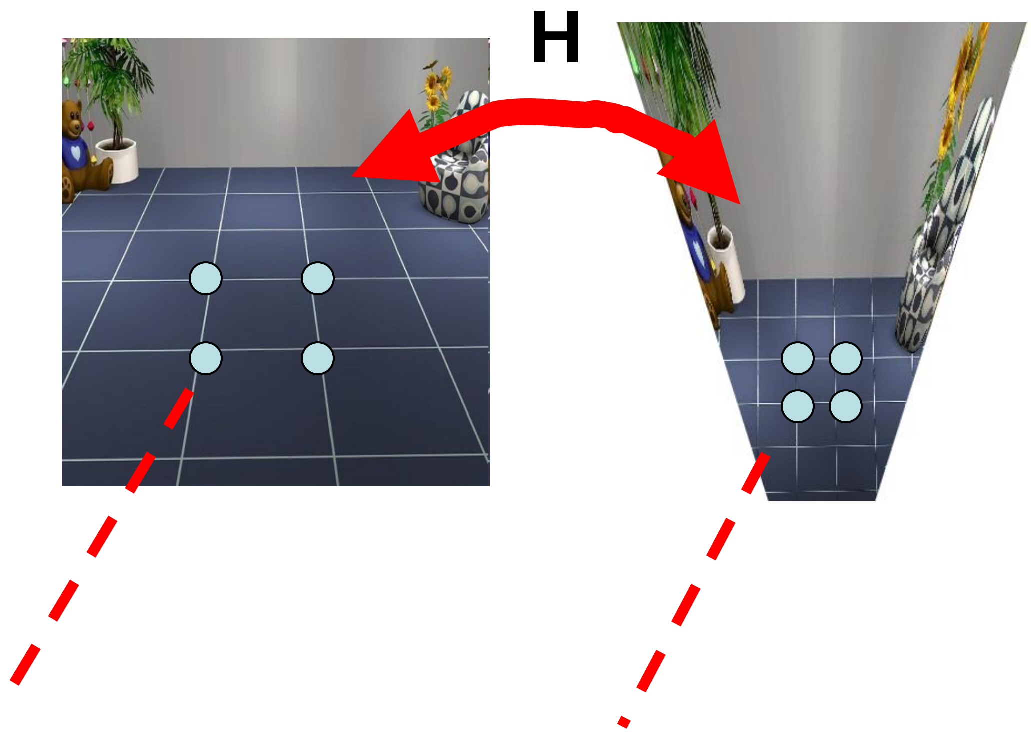
  under the constraint $|x|^2 = 1$

Calibration Problem

$$\mathcal{P}\boldsymbol{m} = 0 \, .$$

How do we solve this homogenous linear system?

DLT algorithm (Direct Linear Transformation)



$\mathbf{x}_i$ $\mathbf{x}'_i$

$$\mathbf{x}'_i = \mathbf{H}\,\mathbf{x}_i \longrightarrow \underbrace{\mathbf{A}_i}_{\text{Function of measurements}} \overbrace{\mathbf{h}}^{\text{unknown}} = 0$$

General Calibration Problem

$$\boxed{\mathcal{P}}\boldsymbol{m} = 0 .$$

Compute SVD decomposition of P

$$\boxed{\mathrm{U}_{2n\times 12}\ \mathrm{D}_{\ 12\times 12}\mathrm{V}^{\mathrm{T}}{}_{12\times 12}}$$

Last column of V gives $\boldsymbol{m}$

$$M$$

$$\mathrm{M}\ \mathrm{P}_i \rightarrow \mathrm{p}_i$$

Why? See pag 593 of AZ

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \left(\begin{array}{c|c} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{array}\right) = \mathrm{K}\begin{bmatrix}\mathrm{R} & \mathrm{T}\end{bmatrix}$$

$$\underbrace{\qquad}_{A} \qquad \underbrace{\qquad}_{\mathbf{b}}$$

$$\mathrm{K} = \begin{bmatrix} \boldsymbol{\alpha} & -\boldsymbol{\alpha}\cot\boldsymbol{\theta} & \mathrm{u_o} \\ 0 & \dfrac{\boldsymbol{\beta}}{\sin\boldsymbol{\theta}} & \mathrm{v_o} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\boldsymbol{\rho} = \frac{\pm 1}{|\mathbf{a}_3|} \qquad \mathrm{u_o} = \boldsymbol{\rho}^2(\mathbf{a}_1 \cdot \mathbf{a}_2)$$

$$\mathrm{v_o} = \boldsymbol{\rho}^2(\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos\boldsymbol{\theta} = \frac{(\mathbf{a}_1 \times \mathbf{a}_3)\cdot(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

Theorem (Faugeras, 1993)

Let $\mathcal{M} = (\mathcal{A} \quad \boldsymbol{b})$ be a 3×4 matrix and let $\boldsymbol{a}_i^T$ $(i = 1, 2, 3)$ denote the rows of the matrix $\mathcal{A}$ formed by the three leftmost columns of $\mathcal{M}$.

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for $\mathcal{M}$ to be a zero-skew perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$(\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0.$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0, \\ (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_1 \times \boldsymbol{a}_3) = (\boldsymbol{a}_2 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3). \end{cases}$$

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \left(\underbrace{\begin{matrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T \\ \boldsymbol{r}_3^T \end{matrix}}_{A} \quad \underbrace{\begin{matrix} \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ t_z \end{matrix}}_{\mathbf{b}} \right) = \mathrm{K}[\mathrm{R} \quad \mathrm{T}]$$

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\boldsymbol{\alpha} = \boldsymbol{\rho}^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin\boldsymbol{\theta}$$

$$\boldsymbol{\beta} = \boldsymbol{\rho}^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin\boldsymbol{\theta} \quad \rightarrow \quad \mathrm{f}$$

Extracting camera parameters

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = \mathrm{K}[\mathrm{R} \quad \mathrm{T}]$$

A $\qquad$ **b**

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

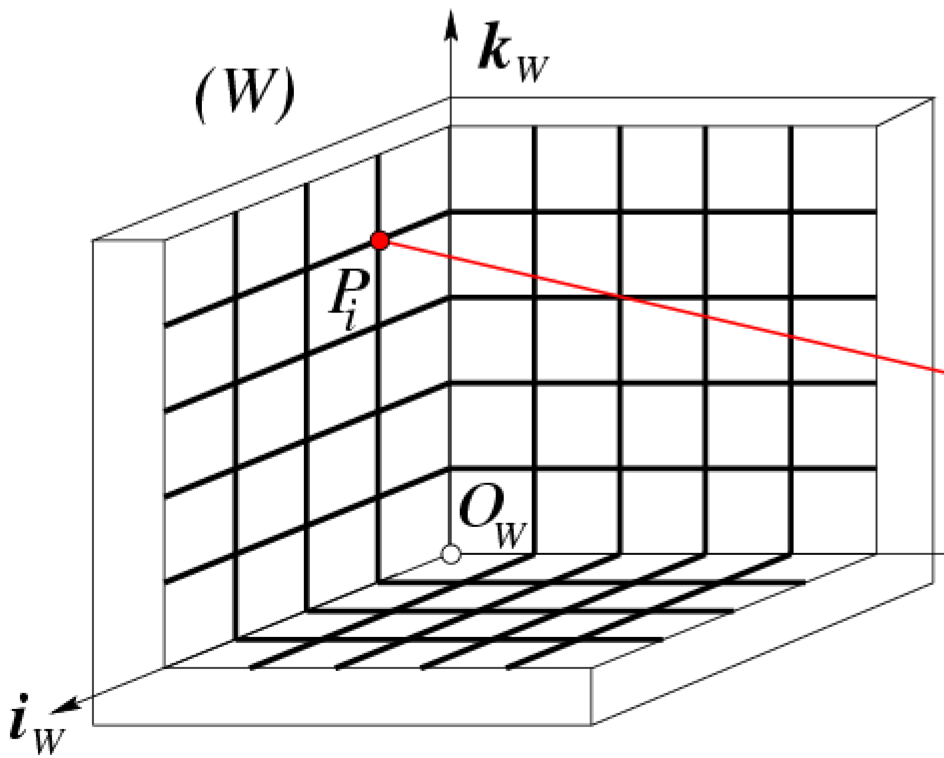
Estimated values

Extrinsic

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \qquad \mathbf{r}_3 = \frac{\pm 1}{|\mathbf{a}_3|}$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \qquad \mathrm{T} = \rho\, \mathrm{K}^{-1}\mathbf{b}$$

Degenerate cases



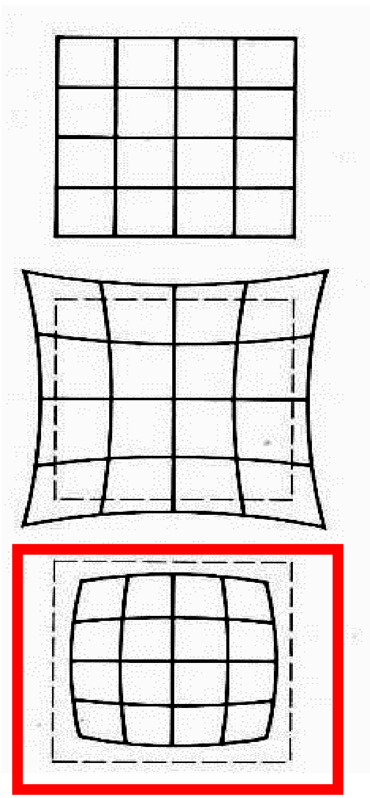
$$
\mathcal{P} \stackrel{\text{def}}{=}
\begin{array}{c}
\boldsymbol{i} \\
\begin{pmatrix}
\boldsymbol{P}_1^T & \boldsymbol{0}^T & -u_1\boldsymbol{P}_1^T \\
\boldsymbol{0}^T & \boldsymbol{P}_1^T & -v_1\boldsymbol{P}_1^T \\
\cdots & \cdots & \cdots \\
\boldsymbol{P}_n^T & \boldsymbol{0}^T & -u_n\boldsymbol{P}_n^T \\
\boldsymbol{0}^T & \boldsymbol{P}_n^T & -v_n\boldsymbol{P}_n^T
\end{pmatrix}
\end{array}
$$

- P_i's cannot lie on the same plane!
- Points cannot lie on the intersection curve of two quadric surfaces

Radial Distortion

– Caused by imperfect lenses

– Deviations are most noticeable for rays that pass through the edge of the lens

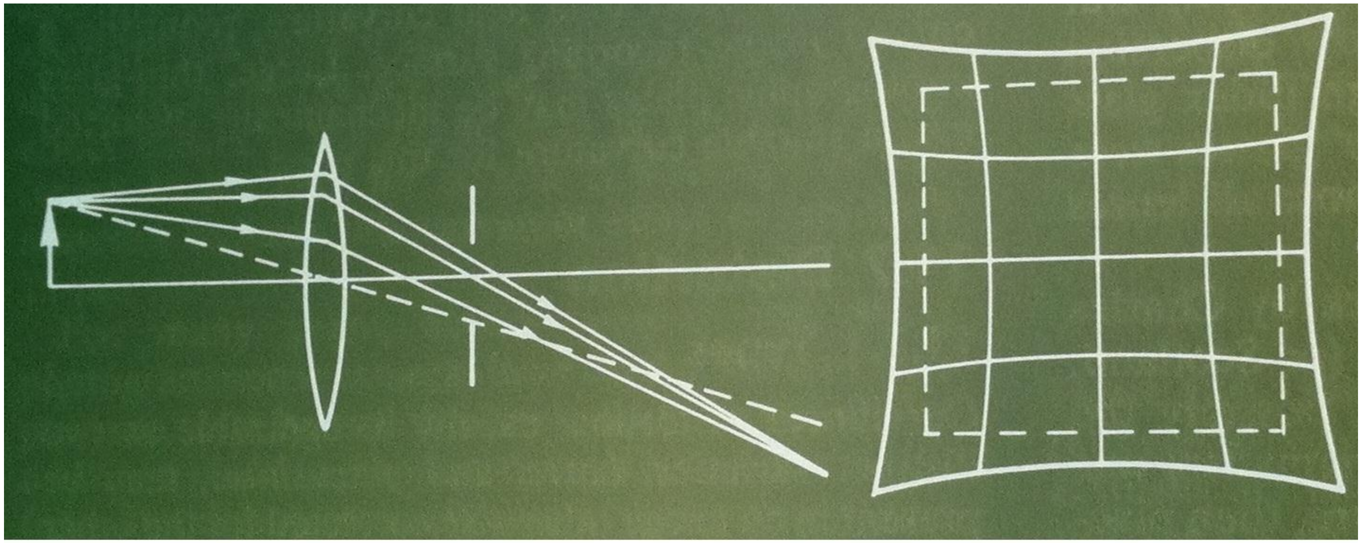


No distortion

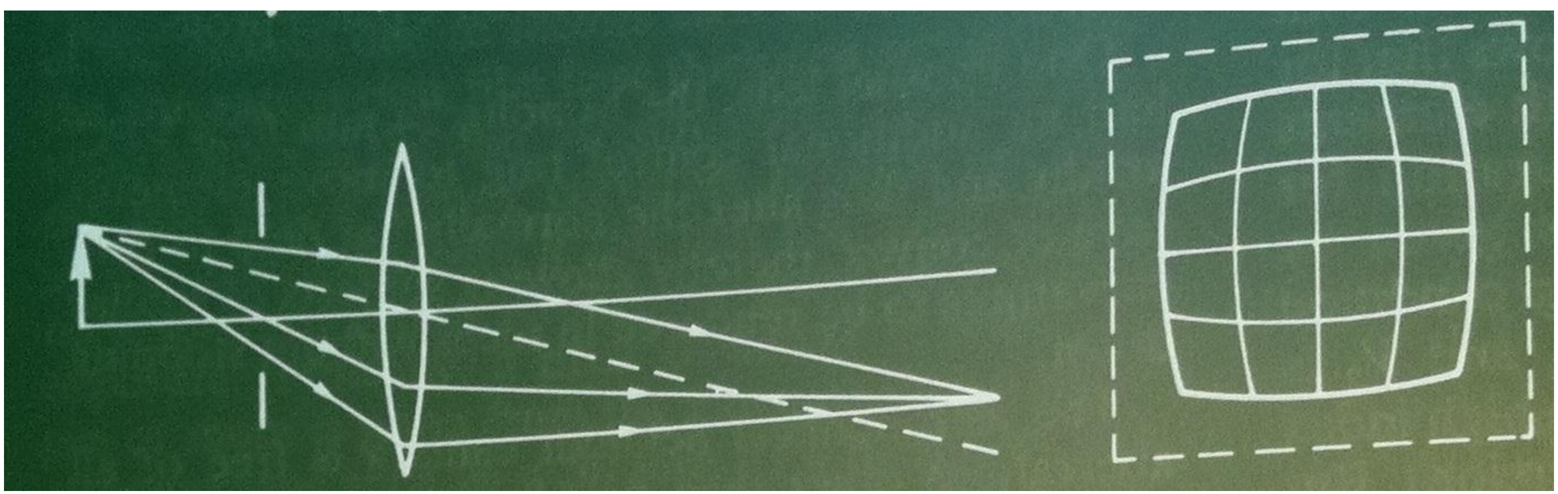
Pin cushion

Barrel

Issues with lenses: Radial Distortion



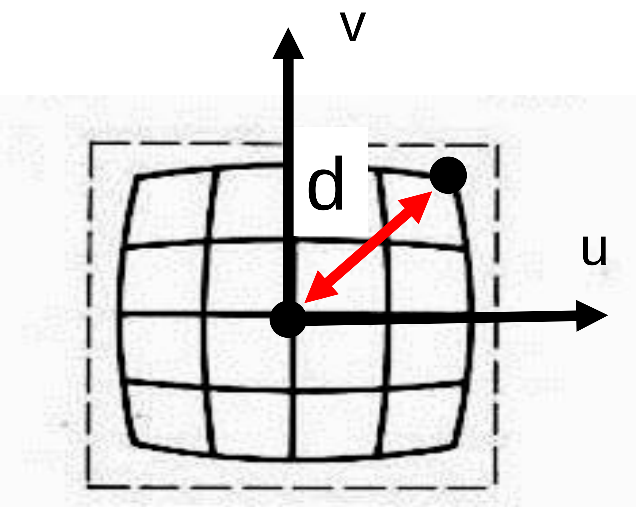
Pin cushion



Barrel (fisheye lens)

Radial Distortion

Image magnification decreases with distance from the optical center



$$\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M\, P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

$$d^2 = a\,u^2 + b\,v^2 + c\,u\,v$$

To model radial behavior

Distortion coefficient

$$\lambda = \underbrace{1 \pm \sum_{p=1}^{3} \boxed{\kappa_p} d^{2p}}_{\text{Polynomial function}}$$

Radial Distortion

$$\underbrace{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M}_{Q} P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i \qquad Q = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}$$

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 P_i}{\mathbf{q}_3 P_i} \\ \dfrac{\mathbf{q}_2 P_i}{\mathbf{q}_3 P_i} \end{bmatrix} \rightarrow \begin{cases} u_i \mathbf{q}_3 P_i = \mathbf{q}_1 P_i \\ v_i \mathbf{q}_3 P_i = \mathbf{q}_2 P \end{cases}$$

Is this a linear system of equations?

No! why?

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \mathrm{P_i}}{\mathbf{q}_3 \mathrm{P_i}} \\ \dfrac{\mathbf{q}_2 \mathrm{P_i}}{\mathbf{q}_3 \mathrm{P_i}} \end{bmatrix} \longrightarrow X = f(P)$$

measurement

parameter

f( ) is nonlinear

-Newton Method

-Levenberg-Marquardt Algorithm

- Iterative, starts from initial solution
- May be slow if initial solution far from real solution
- Estimated solution may be function of the initial solution
- Newton requires the computation of J, H
- Levenberg-Marquardt doesn't require the computation of H

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, \mathrm{P_i}}{\mathbf{q}_3 \, \mathrm{P_i}} \\ \dfrac{\mathbf{q}_2 \, \mathrm{P_i}}{\mathbf{q}_3 \, \mathrm{P_i}} \end{bmatrix} \longrightarrow X = f(P)$$

measurement

parameter

f( ) is nonlinear

A possible algorithm

1. Solve linear part of the system to find approximated solution
2. Use this solution as initial condition for the full system
3. Solve full system using Newton or L.M.

General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \mathrm{P_i}}{\mathbf{q}_3 \mathrm{P_i}} \\ \dfrac{\mathbf{q}_2 \mathrm{P_i}}{\mathbf{q}_3 \mathrm{P_i}} \end{bmatrix} \longrightarrow X = f(P)$$

measurement

parameter

f( ) is nonlinear

Typical assumptions:

- zero-skew, square pixel
- u_o, v_o = known center of the image
- no distortion

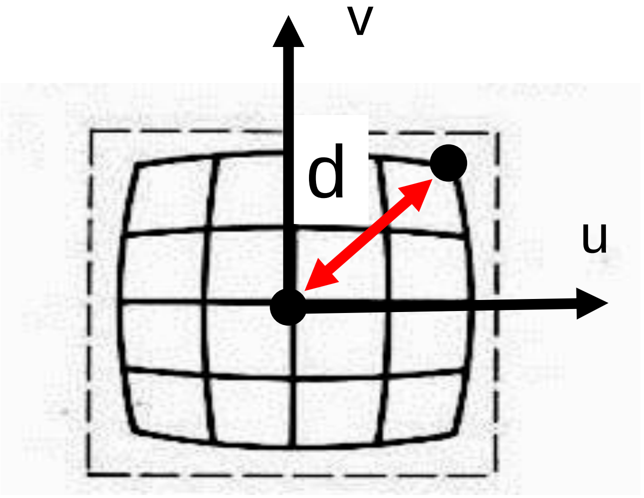
$\longrightarrow$ Just estimate f and R, T

Radial Distortion

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix} \longrightarrow p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

Can estimate m_1 and m_2 and ignore the radial distortion?

Hint:



$$\frac{u_i}{v_i} = \text{slope}$$

Radial Distortion

Estimating m_1 and m_2…

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix}$$

$$\frac{u_i}{v_i} = \frac{\dfrac{(\mathbf{m}_1 \, P_i)}{(\mathbf{m}_3 \, P_i)}}{\dfrac{(\mathbf{m}_2 \, P_i)}{(\mathbf{m}_3 \, P_i)}} = \frac{\mathbf{m}_1 \, P_i}{\mathbf{m}_2 \, P_i}$$

$$\begin{cases} v_1(\mathbf{m}_1 \, P_1) - u_1(\mathbf{m}_2 \, P_1) = 0 \\ v_i(\mathbf{m}_1 \, P_i) - u_i(\mathbf{m}_2 \, P_i) = 0 \\ \vdots \\ v_n(\mathbf{m}_1 \, P_n) - u_n(\mathbf{m}_2 \, P_n) = 0 \end{cases}$$

$$\mathrm{Q}\,\mathbf{n} = 0 \qquad \mathbf{n} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix}$$

Tsai technique [87]

Radial Distortion

Once that $\mathbf{m}_1$ and $\mathbf{m}_2$ are estimated…

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1\ P_i}{\mathbf{m}_3\ P_i} \\ \dfrac{\mathbf{m}_2\ P_i}{\mathbf{m}_3\ P_i} \end{bmatrix}$$

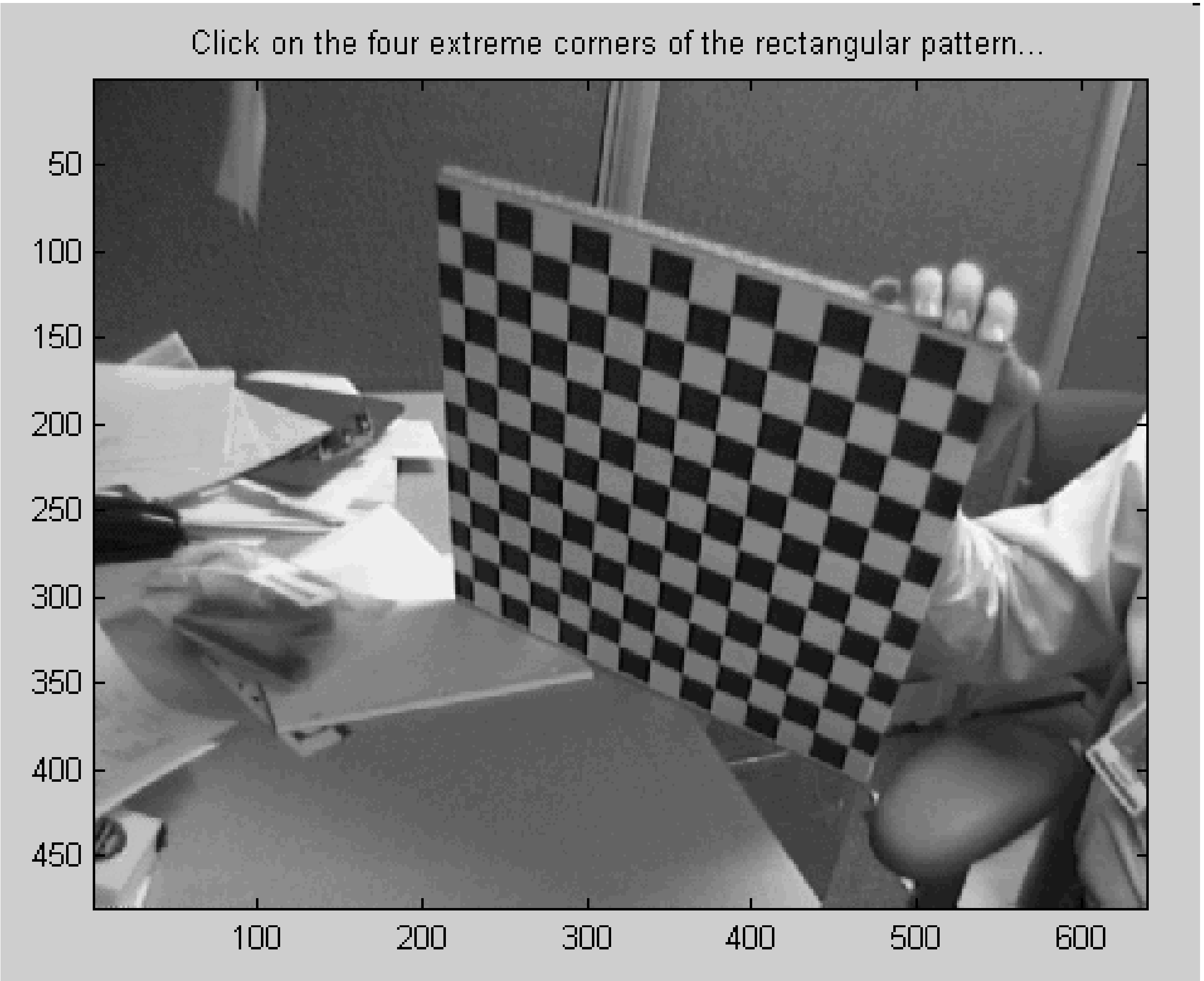
$\mathbf{m}_3$ is non linear function of $\mathbf{m}_1$, $\mathbf{m}_2$, λ

There are some degenerate configurations for which m_1 and m_2 cannot be computed

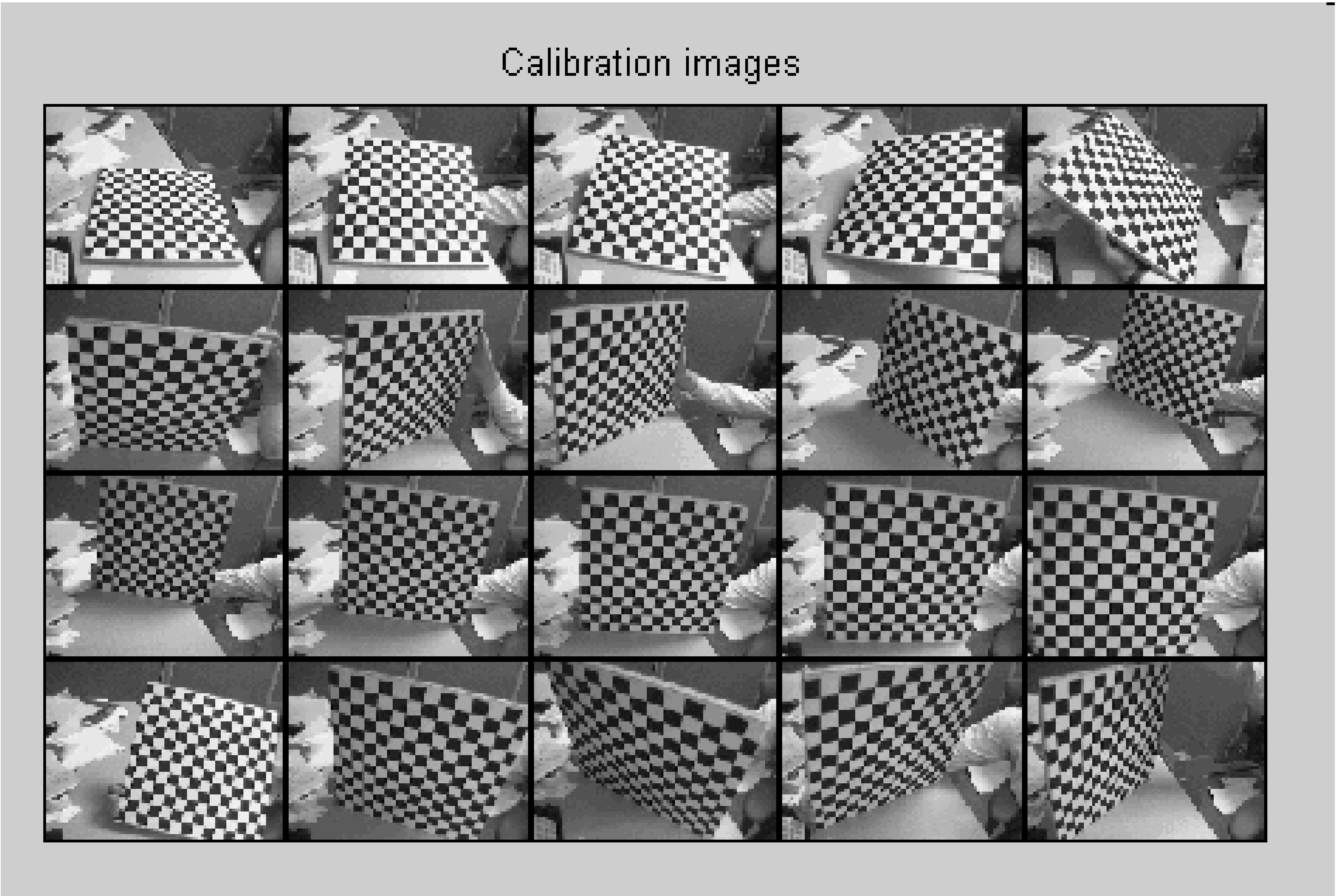
Calibration Procedure

Camera Calibration Toolbox for Matlab
J. Bouguet – [1998-2000]

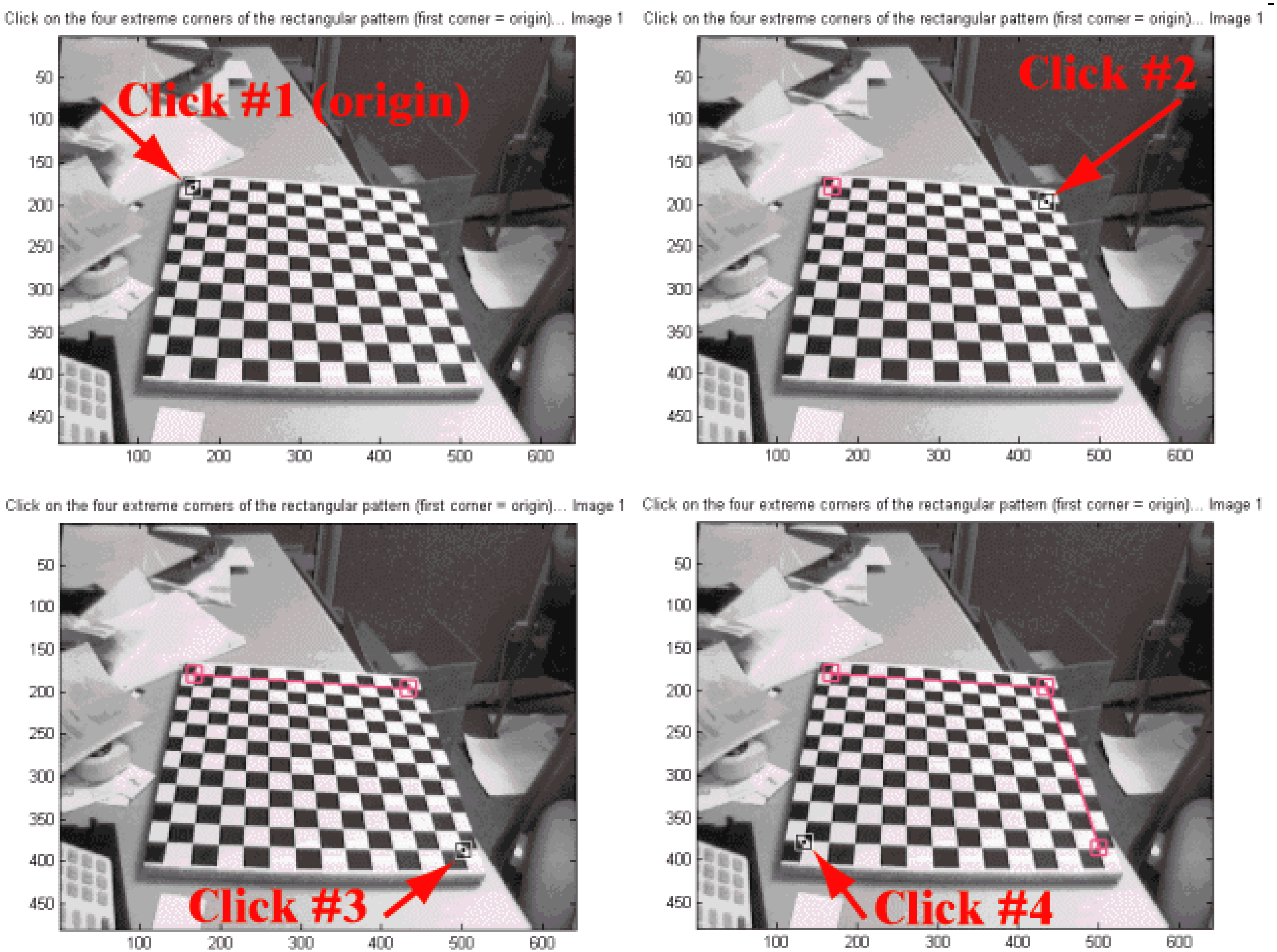
http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples



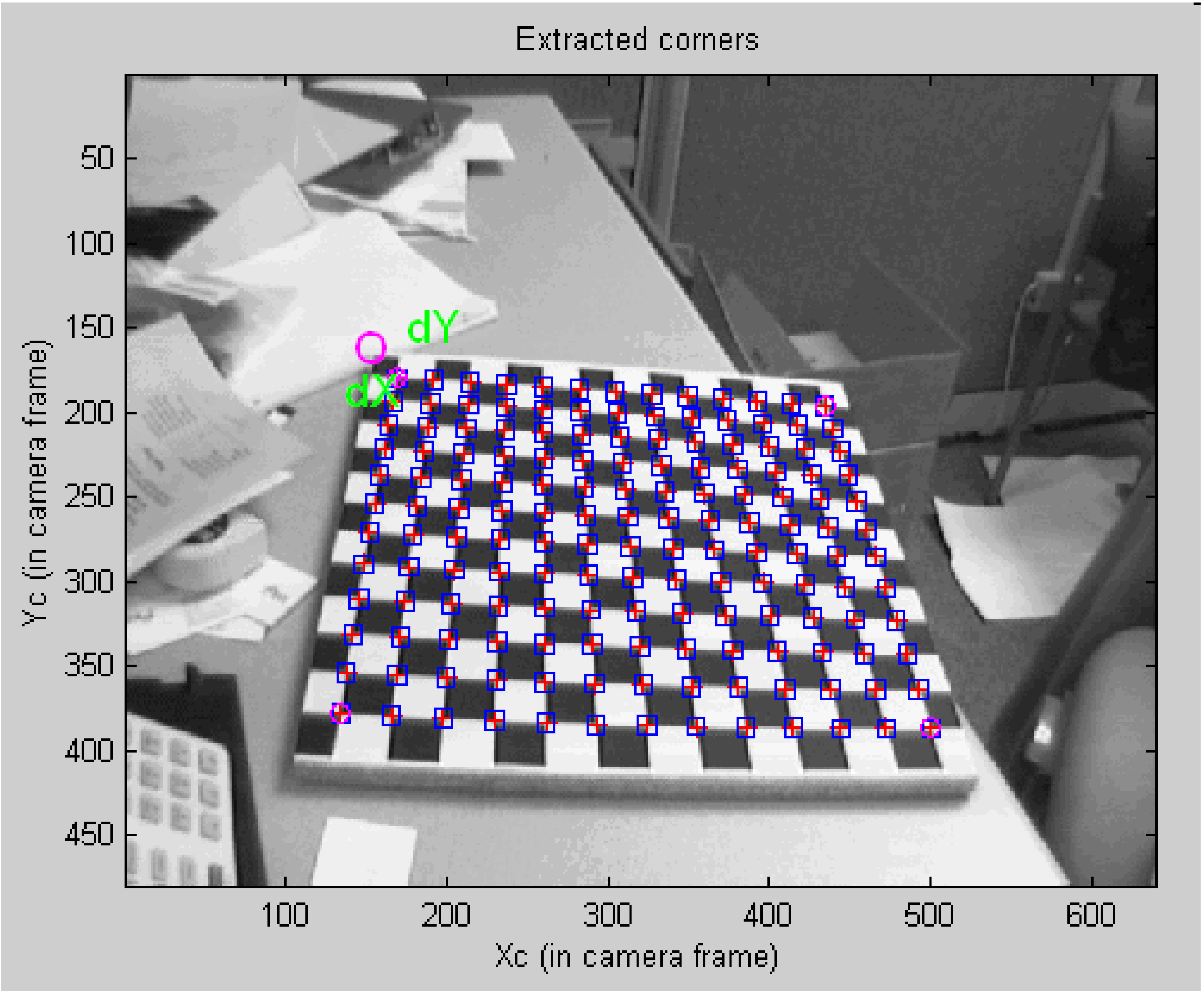
Calibration Procedure



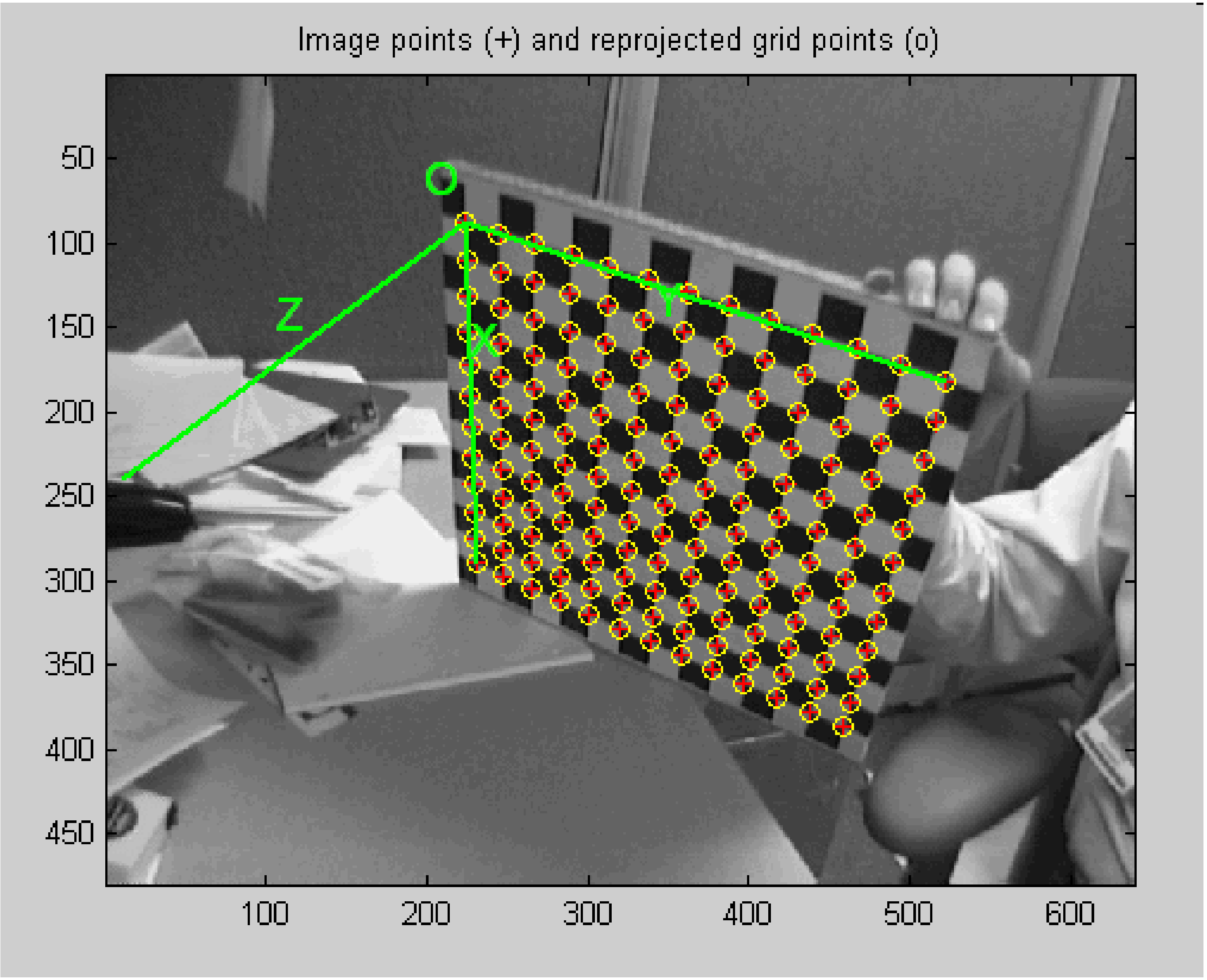
Calibration Procedure



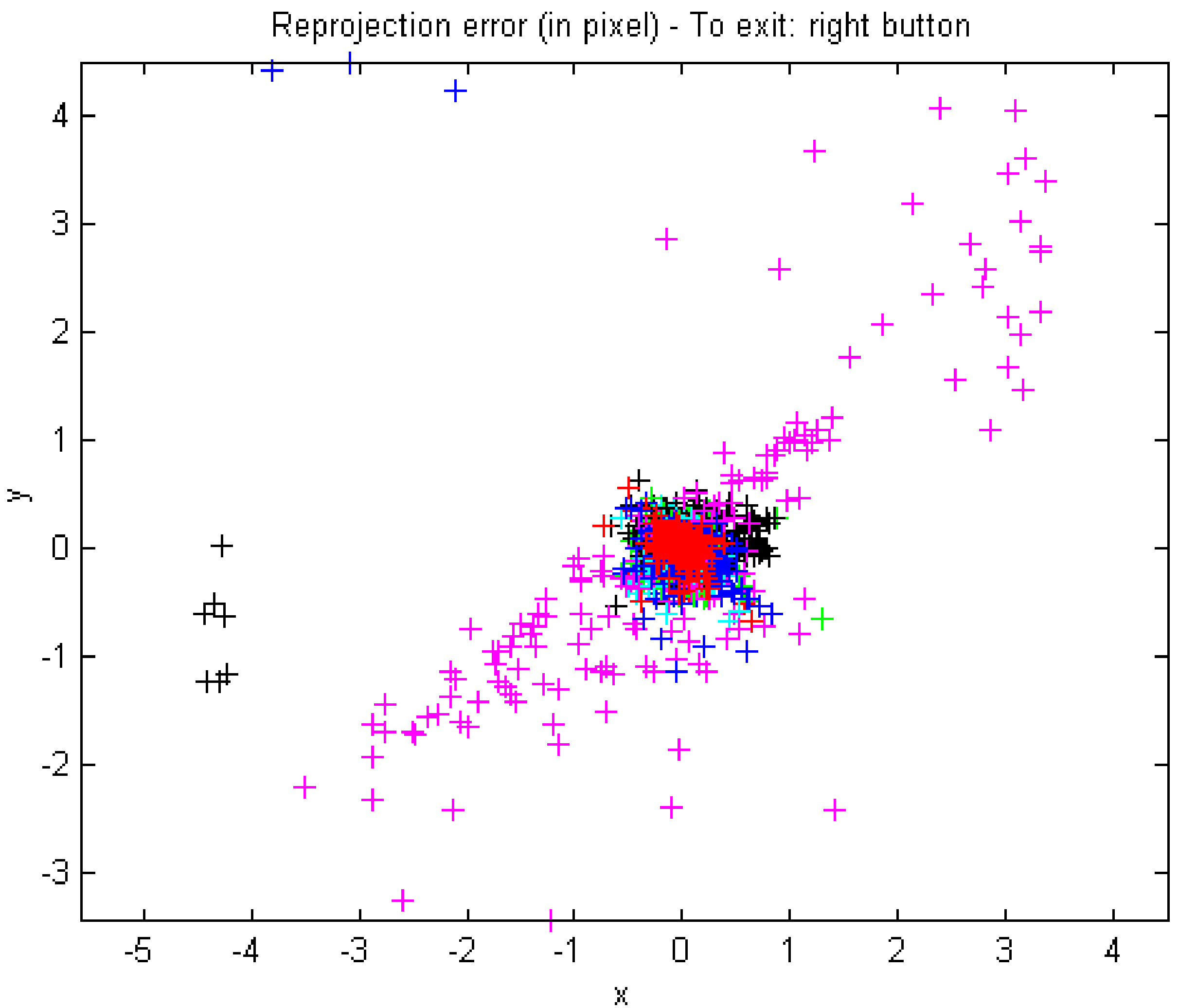
Calibration Procedure



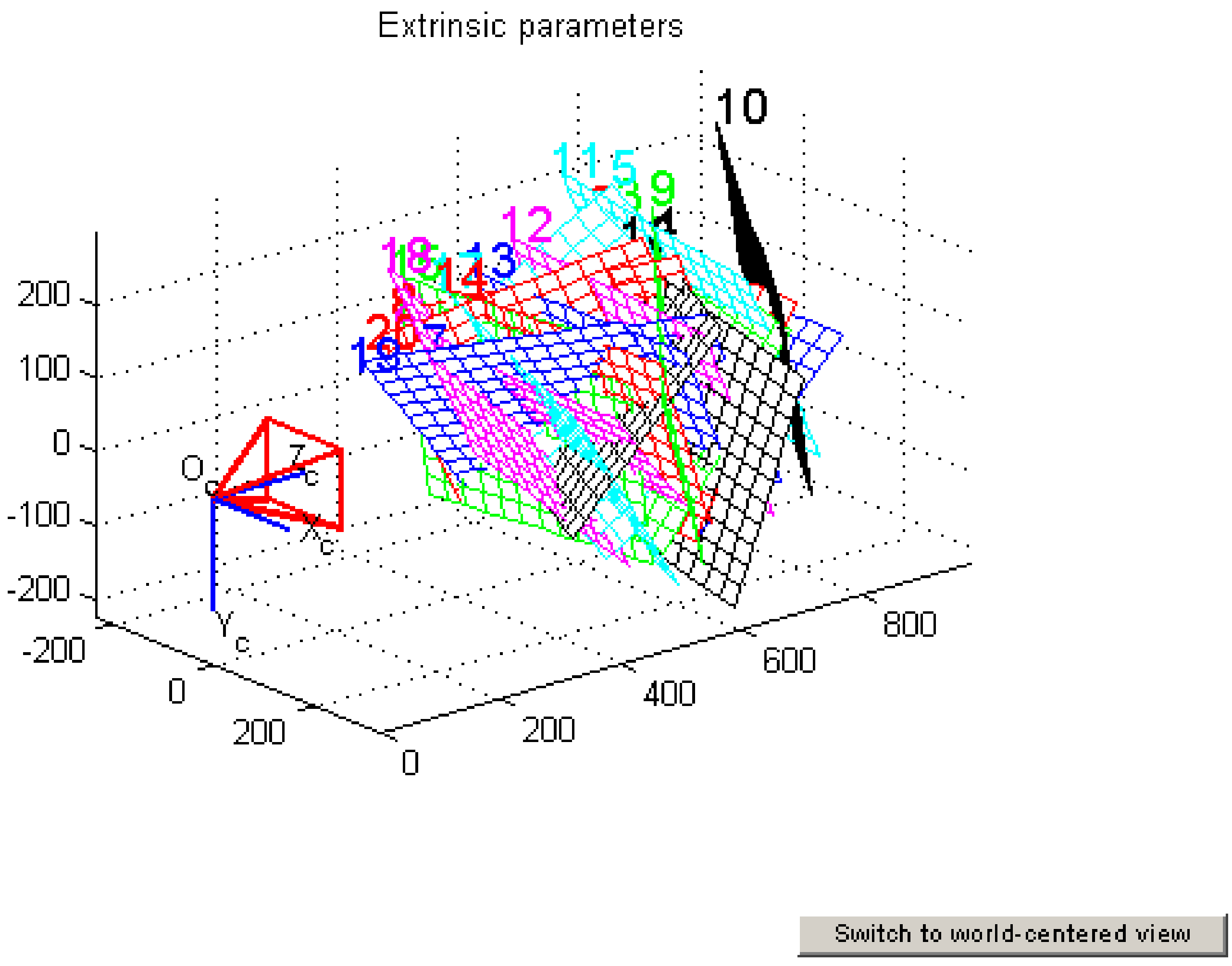
Calibration Procedure



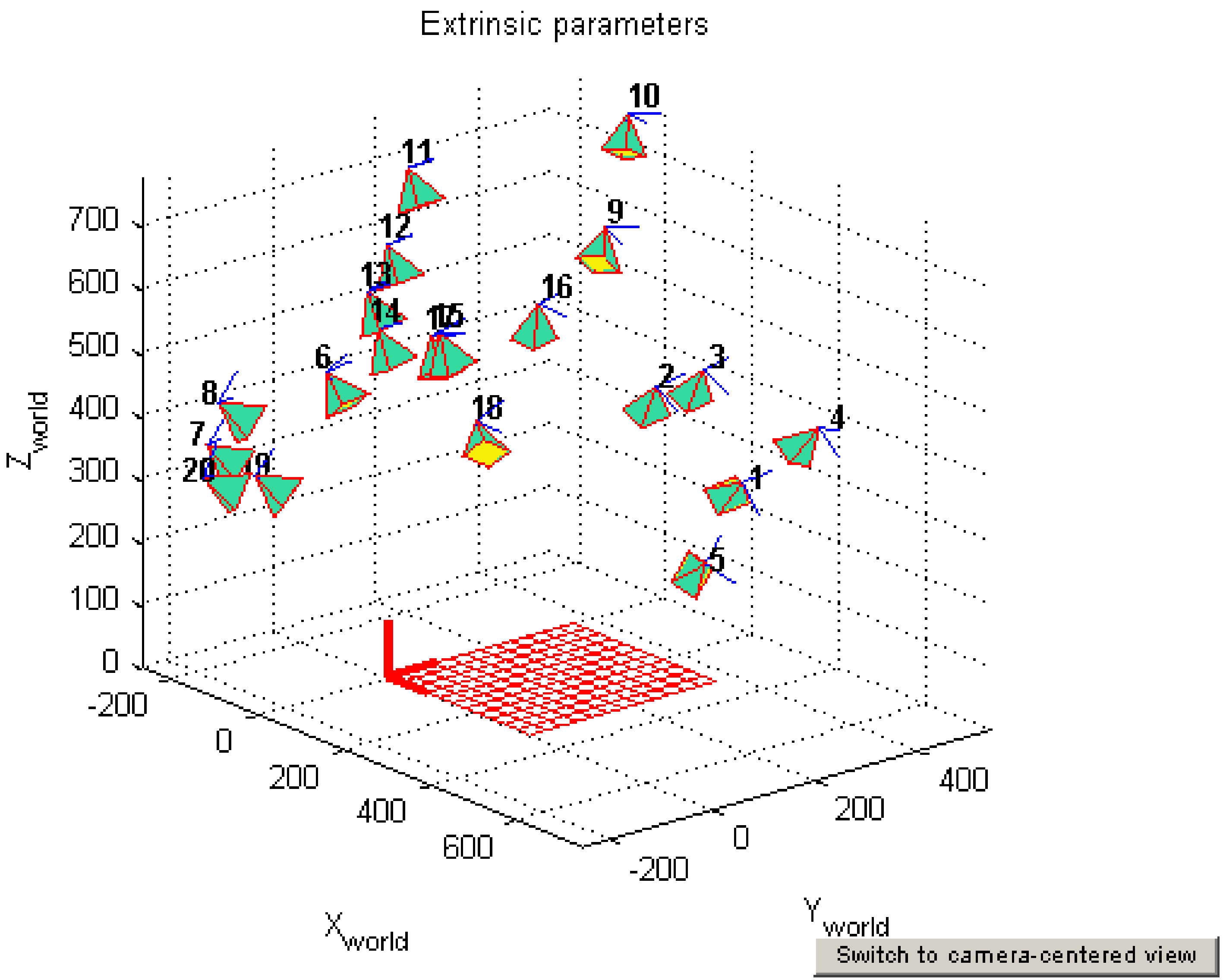
Calibration Procedure



Calibration Procedure



Calibration Procedure



Next lecture

- Single view reconstruction