Homework 5

ECE-GY 6223 – System Optimization Methods

Fall 2021, New York University

Due Date: Dec. 10, 2021

Problem 1. Find the stationary curves for the functional

$$J(y(x)) = \int_{x_0}^{x_1} F(x, y, y')dx; \;\; y(x_0) = y_0, y(x_1) = y_1,$$

when $F(x, y, y')$ equals

(a) $\frac{\sqrt{1+(y')^2}}{y}$

(b) $\sqrt{y(1 + (y')^2)}$

(c) $y'(1 + x^2 y')$

Problem 2 (Maximum Entropy Principle)**.** Let X be an absolutely continuous random variable defined on Ω. The maximum entropy principle attempts to find the probability density function $\phi(x)$ that maximizes the entropy

$$H[X] = -\int_{\Omega} \phi(x) \ln(\phi(x))dx,$$

Let $\Omega = [a, b]$. Use Euler-Lagrange equation to show that the uniform distribution

$$\phi^*(x) = \frac{1}{b-a}, \;\; a \leq x \leq b$$

is a candidate probability density function that maximizes

$$H[X] = -\int_a^b \phi(x) \ln(\phi(x))dx,$$

subject to the natural constraint

$$\int_a^b \phi(x) = 1.$$

The associated entropy is

$$H[X^*] = \ln(b - a).$$

Problem 3. This problem illustrates the fact that in stochastic control, closed-loop control generally out-performs open-loop control. Consider the linear stochastic system

$$x_{k+1} = x_k + u_k + w_k,$$

with cost criterion $J = \mathbb{E}\sum_{k=0}^{N} x_k^2$, where $N \geq 1, \mathbb{E}[x_0] = 0, \mathbb{E}[x_0^2] = 1, \mathbb{E}[w_k] = 0, \mathbb{E}[w_k^2] = 1$, and w_k is an independent sequence, also independent of x_0.

(a) Let u_k be any deterministic sequence (corresponding to open-loop control). Determine the cost criterion in terms of N and u_k.

(b) Let u_k be given by the closed-loop control law $u_k = -x_k$. Determine the cost criterion associated with this policy and show that it is strictly less that the cost criterion determined in (a), regardless of the open-loop control sequence used in (a).