

# Homework 1

ECE-GY 6223 – System Optimization Methods

Fall 2021, New York University

Due Date: Sept. 24, 2021

**Problem 1.** Find the characteristic polynomial of  $A$  and use it to find all the eigenvalues and eigenvectors of  $A$ . Determine whether the matrix is positive definite.

$$A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Problem 2.** Consider the following unconstrained minimization problem

$$\min_{\mathbf{x} \in \mathbb{R}^2} f(\mathbf{x}) := x_1^2 - x_1x_2 + x_2^2 - 3x_2$$

Find a local minimum. Is the local minimum also a global minimum?

**Problem 3.** Let  $f(\mathbf{x}) = -2x_1^2 - 2x_1x_2 - 2x_2^2 + x_1 + x_2 + 3$ .

- (a) Find the minimum and the maximum of  $f$  over  $\mathbb{R}^2$  if they exist.
- (b) Show that  $f(\mathbf{x})$  can be made to approach  $-\infty$  by choosing  $\mathbf{x}$  appropriately.