HW3

Problem 1

A convex problem in which strong duality fails. Consider the optimization problem

$$\begin{array}{ll} \text{minimize} & e^{-x} \\ \text{subject to} & x^2/y \leq 0 \end{array}$$

with variables x and y, and domain $\mathcal{D} = \{(x, y) | y > 0\}$.

(a) Verify that this is a convex optimization problem. Find the optimal value.

(b) Give the Lagrange dual problem, and find the optimal solution λ^* and optimal value d^* of the dual problem. What is the optimal duality gap?

(c) Does Slater's condition hold for this problem?

Problem 2

Consider the QCQP

$$\begin{array}{ll} \text{minimize} & x_1^2 + x_2^2 \\ \text{subject to} & (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1 \\ & (x_1 - 1)^2 + (x_2 + 1)^2 \leq 1 \end{array}$$

with variable $x \in \mathbf{R}^2$.

(a) Sketch the feasible set and level sets of the objective. Find the optimal point x^* and optimal value p^*.

(b) Give the KKT conditions. Do there exist Lagrange multipliers λ_1^* and λ_2^* that prove that x^* is optimal?

(c) Derive and solve the Lagrange dual problem. Does strong duality hold?

Problem 3

The problem

$$\begin{array}{ll} \text{minimize} & -3x_1^2 + x_2^2 + 2x_3^2 + 2(x_1 + x_2 + x_3) \\ \text{subject to} & x_1^2 + x_2^2 + x_3^2 = 1 \end{array}$$

is a special case of (5.32), so strong duality holds even though the problem is not convex. Derive the KKT conditions. Find all solutions x, v that satisfy the KKT conditions. Which pair corresponds to the optimum?