Homework 4

ECE-GY 6223 – System Optimization Methods

Fall 2021, New York University

Due Date: Nov. 11, 2021 5:00 PM

Please submit a PDF report and 3 modified Jupyter notebook files.

Problem 1 (Optimization)**.** Consider the entropy maximization problem,

$$
\begin{aligned}
\text{maximize} \quad & f_0(x) = -\sum_{i=1}^{n} x_i \log x_i \\
\text{s.t.} \quad & Ax \preceq b \\
& \mathbf{1}^T x = 1
\end{aligned}
$$

where $x \in \mathbf{R}^n$. $A \in \mathbf{R}^{p \times n}$ and $b \in \mathbf{R}^p$ are provided in the code.

Find the optimal solution to the problem using CVXPY. Please complete the python code in the *Entropy.ipynb* file. (The objective function can be formulated in CVXPY using **entr**.)

In your pdf report, you should answer the following questions:

(1) What is the optimal value of the objective function $f_0(x^*)$?

(2) What is the optimal solution x^*?

Problem 2 (Gradient Descent)**.** ([BV] P.464/P.466)
Consider the following unconstrained minimization problem

$$
\min_{x \in R^3} f(x) = \frac{1}{2} x^T A x + x^T b
$$

where A and b are defined as

$$
A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
$$

Please complete the python code in the *GD.ipynb* file.

In your pdf report, you should answer the following questions:

(1) Determine whether the matrix A is positive definite and find the minimum of f if exists.

(2) Implement the gradient descent method with backtracking line search starting with $x_0 = [1,1,1]^T$. Let the maximum iteration number be 50, $\alpha = 0.25$ and $\beta = 0.1$. Record the function value of each iteration and plot it (x-axis: number of iterations / y-axis: function value of that iteration).

Attach a screenshot of your plot. Compare the solution with the optimal value you solved in (1).

(3) Change the learning rate to $\beta = 0.3$ and $\beta = 0.8$, repeat the process in (2) and plot the function value each iteration. Attach the screenshot of your plots.

(4) Comment on how the learning rate β will affect the gradient descent results.

Problem 3 (Linear Regression)**.** In this problem, you will analyze data using linear regression. The goal of the problem is to predict the miles per gallon a car will get using six quantities (features) of that car. The data is broken into training and testing sets. Each row in both "X" files contains 6 features for a single car (plus a 1 in the last dimension) and the same row in the corresponding "y" file contains the miles per gallon for that car.

See https://archive.ics.uci.edu/ml/datasets/Auto+MPG for more details on this dataset. We will use the 2nd to 7th features to predict the 1st attribute in the data set. The data has been preprocessed, thus you must use the data provided with this homework.

- **Training Part** Using the training data only, complete the python code in the *LR.ipynb* file to solve the linear regression problem.

$$\begin{aligned}\min_{\omega} \mathcal{L} = &\sum_{i=1}^{N_{train}} \|y_i - x_i\omega\|^2 \\ =&(Y - X\omega)^T(Y - X\omega)\end{aligned}$$

In your pdf report, you should answer the following questions:

(1) Suppose the data is well-defined, derive the matrix form solution of this problem.

(2) Based on the matrix form solution in (1), learn the vector ω using the training data. Print your result.

- **Testing Part**

By learning the vector ω, we an predict the value y_i in the testing set using x_i, as

$$y_i^{pred} = \omega_1 x_{i1} + \omega_2 x_{i2} + \cdots + \omega_6 x_{i6} + \omega_0 = x_i\omega$$

Using only the testing data. Compute the root mean squared error (RMSE) between the predicted value y^{pred} and the real value y^{test}.

$$RMSE = \sqrt{\frac{1}{N_{test}} \sum_{i=1}^{N_{test}} (y_i^{pred} - y_i^{test})}$$

In your pdf report, you should answer the following question:

(3) What is the RMSE value of your code?