Title: GNSS Lab Report

Lab 1: GNSS Positioning

Course:	AAE4203 — Guidance and Navigation
Group:	Group 5
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Abstract

This report shows how to structure a PolyU lab report using the hkpolyu-labreport class. It includes examples of figures, tables, equations, code listings, and citations, while conforming to page and submission requirements.

Keywords: GNSS, SPP, least squares, figures, tables, citations, code

1. Data Collection Process

1.1 The used tools:

- U-blox GNSS Receiver
- U-center Software
- RTKLIB

1.2 Environmental conditions:

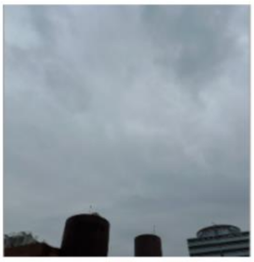


Figure 1

- Cloudy (27/10/2025, 01:29am, UTC)
- Temp: 23 °C , Relative humidity: 68%

1.3 Flowchart of our Data Collection Process

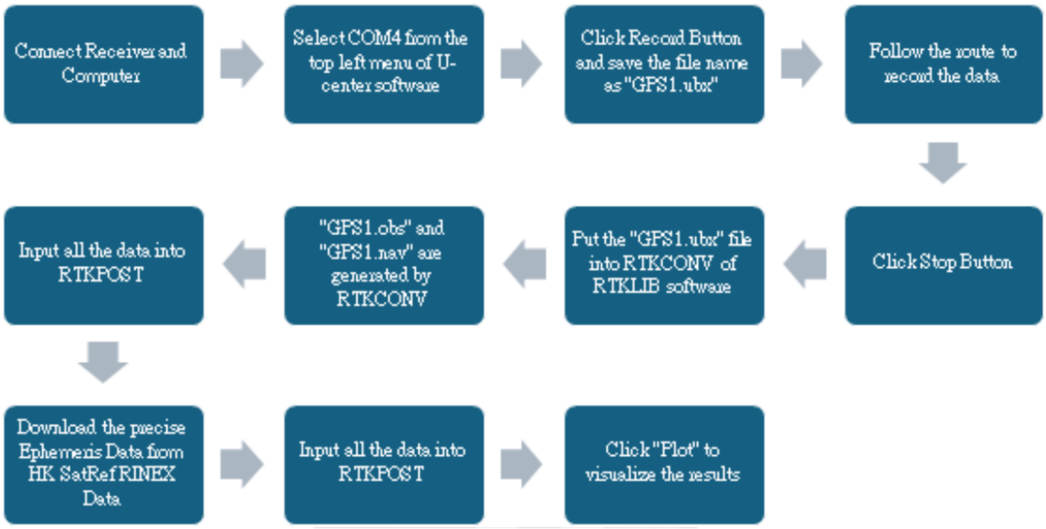


Figure 2

1.4 Our route in Campus map

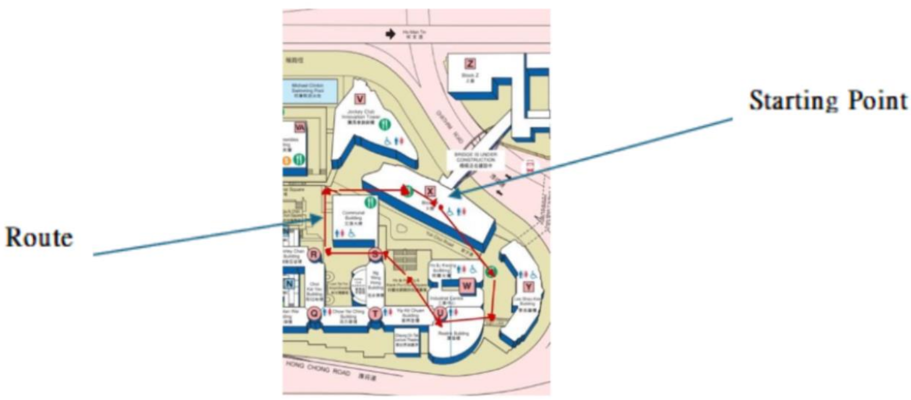


Figure 3

1.5 Our route

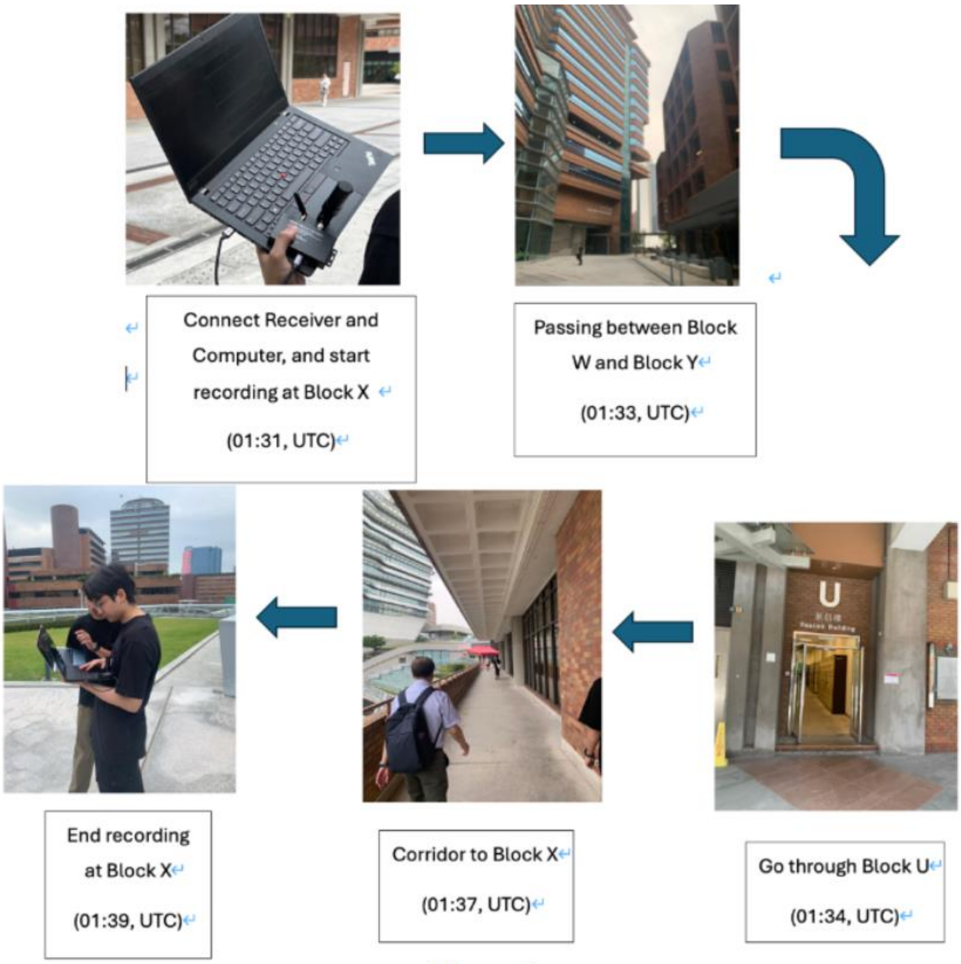


Figure 4

2. Raw Data Analysis

2.1 Overview

Purpose: Assess the quality of the raw GNSS data before filtering or smoothing.

Analysis Areas:

- Satellite visibility, Signal-to-Noise Ratio (SNR), Sky distribution and DOP (GDOP, PDOP, HDOP, VDOP, TDOP)

2.1 Visible Satellite counts:

- Numbers of visible satellites remained high for around 14-26 visible satellites, (Figure 6)
- Two noticeable fluctuations are spotted at around 01:35:00 to 01:35:30 and 01:37:10 to 01:37:30 (Figure 6)

2.2 Signal-to-Noise Ratio (SNR):

- The SNR plot (Figure 5) shows most satellites sit in 20–40 dB-Hz (usable), with occasional very strong (45–50 dB-Hz) and weak (<25 dB-Hz) signals.
- Two noticeable fluctuations are spotted at around 01:34:00 to 01:35:30 and 01:36:30 to 01:37:30
- Simultaneous gap at around 01:35:00

Figure 5

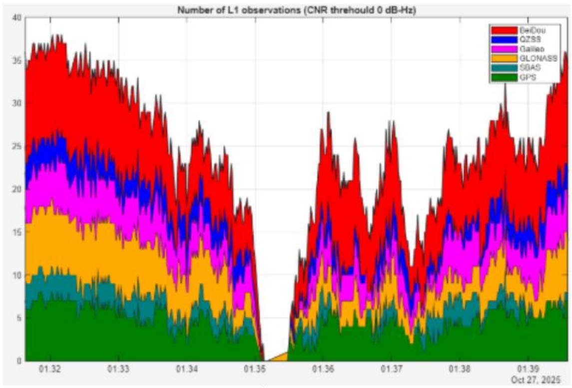


Figure 6

The contribution of different GNSS constellations according to satellite visibility analysis throughout the session (table 1):

Table 1: Multi-constellation satellite visibility statistics

Constellation	Max Satellites	Avg Contribution	Color Code
BeiDou	14 - 16	40 %	Red
GLONASS	4 - 9	20 %	Orange
GPS	3 - 8	20 %	Green
Galileo	2 - 5	10 %	Magenta
SBAS	1 - 3	5 %	Teal
QZSS	1 - 3	5 %	Blue

2.3 Sky distribution:

- Sky plot (Figure 8) shows the visible satellites are distributed between 20°–80° elevation.
- All visible BeiDou satellites are distributed between 40°–80° elevation, provides dominant coverage in the session.(noted in red dot in the Figure 8)

2.4 DOP Analysis

- According to the DOP Metrics over time (Figure 4), mean DOP values (GDOP ≈ 2.46, PDOP ≈ 2.04, HDOP ≈ 1.83, VDOP ≈ 0.88, TDOP ≈ 1.38) indicate good overall geometry with especially strong vertical dilution. (Figure 7)
- Three noticeable peaks are spotted at around 01:34:00 to 01:35:30, 01:36:30 and 01:37:30.

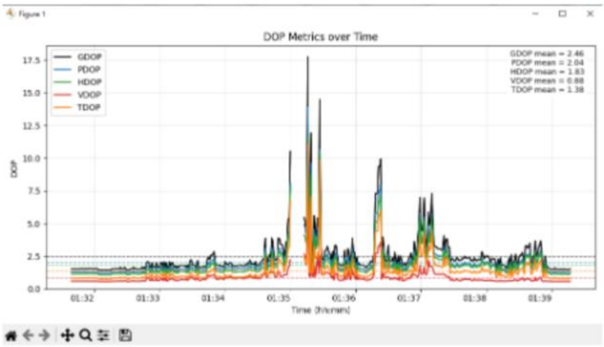


Figure 7

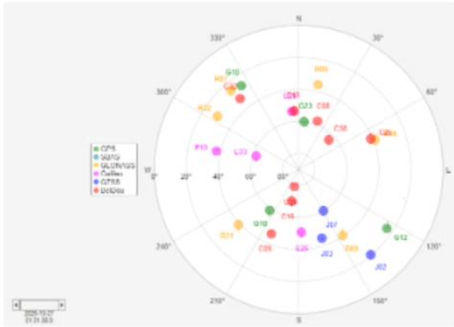


Figure 8

2.5 Raw data summary

Satellite visibility remained adequate, ranging from 14 to 26 satellites most of the time, with BeiDou providing dominant high-elevation coverage.

Signal strength was generally usable, while the mean PDOP value of approximately 2.04 demonstrates favorable satellite geometry and robust positioning accuracy.

Fluctuations observed in the raw data around 01:35:00 to 01:35:30 and 01:37:10 to 01:37:30 are typical of urban GNSS environments, where temporary multipath effect obstructions can occur.

3. SPP Algorithm

Single Point Positioning (SPP) refers to the process of determining the position of a receiver (such as a GPS receiver) using signals from multiple satellites.
For each satellite (ρ_i), the equation is:

$$\rho_i = \sqrt{(x_i^s - x_u)^2 + (y_i^s - y_u)^2 + (z_i^s - z_u)^2} + c \cdot dt_u + I_i + T_i + \epsilon_i$$

In Python, load_estimated_solution() function is used to insert the initial value of the satellite position. Where:

- satellite_positions.csv: Per-satellite ECEF positions(x, y, z)
- pseudoranges_meas.csv: pseudoranges (rows = sat, columns = epoch)
- satellite_clock_bias.csv: satellite clock biases (dt_u)
- ionospheric_delay.csv: ionospheric delay corrections (I_i)
- tropospheric_delay.csv: tropospheric delay corrections (T_i)

<u>Least Square Estimation</u>
The least squares estimation is a mathematical method used to find the best solution to a set of equations
The equation is:

$$\Delta\rho_i = \frac{\partial \rho_i}{\partial x_u}\Delta x + \frac{\partial \rho_i}{\partial y_u}\Delta y + \frac{\partial \rho_i}{\partial z_u}\Delta z + c \cdot \Delta dt_u$$

The partial derivatives (direction cosines) are:

$$a_{xi} = \frac{\partial \rho_i}{\partial x_u} = -\frac{x_i^s - x_0}{r_i^0}$$
$$a_{yi} = \frac{\partial \rho_i}{\partial y_u} = -\frac{y_i^s - y_0}{r_i^0}$$
$$a_{zi} = \frac{\partial \rho_i}{\partial z_u} = -\frac{z_i^s - z_0}{r_i^0}$$

After the input of initial values in Python, lsq(...) function inside load_estimated_solution(...) is used to insert the initial value of the satellite position.

Weight Square Estimation

After the equations have been linearized, the partial derivatives matrix (H) is set up to estimate the most likely position by minimizing the sum of squared differences between measured and calculated pseudoranges.
The observation vector (y) consists of the differences between the measured pseudoranges and the calculated pseudoranges ($\Delta\rho_i$).

$$\mathbf{H} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{xn} & a_{yn} & a_{zn} & 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \vdots \\ \Delta\rho_n \end{bmatrix}$$

In Python, lsq(...) function is used it builds the the partial derivatives matrix G
and calls np.linalg.lstsq(...) to compute the receiver position and receiver clock bias per epoch.

Solution and Covariance

The weighted least squares solution provides the best estimation of the position and the covariance matrix tells the accuracy of the solution.

The weighted least squares solution is:

$$\hat{\mathbf{x}} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{W}\mathbf{y}$$

The covariance matrix of the estimated parameters:

$$\mathbf{C}_{\hat{x}} = \sigma_0^2(\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}$$

where σ_0^2 is the variance of unit weight.

In Python, np.linalg.inv(H.T @ H) inside _dop_from_geometry function is used to get the geometry inverse and tells the accuracy of the solution by calculate the covariance/DOP.

4. Results and Discussion

4.1 Positioning Performance Summary

The SPP algorithm processed 450 epochs of GNSS data. The estimated receiver coordinates varied within the following ranges:

```
COORDINATE RANGES
  Latitude  : 22.303784° to 22.316807°
  Longitude : 114.179571° to 114.190144°
  Altitude  : -1575.702 m to 317.789 m
```

The large elevation range indicates the presence of significant outliers, which are likely due to multipath effects in urban environments or poor satellite geometric configurations at certain epochs.

Error Metrics:

```
2D Error (m):
          Mean:     48.112
           RMS:     88.770
           Std:     74.601
           Max:   1528.533
           Min:      2.199
           P95:     73.123
3D Error (m):
          Mean:     75.537
           RMS:    134.124
           Std:    110.830
           Max:   2196.918
           Min:      6.552
           P95:    170.763
```

○

4.2 Error Analysis

- The large maximum errors (over 1.5 km in 2D and 2.1 km in 3D) indicate severe outliers, likely caused by:
- Multipath effects from buildings and other structures.
- Cycle slips or temporary signal loss in urban canyons.
- Low-elevation satellites with higher atmospheric and multipath errors。
- The 95th percentile errors (73.123 m for 2D,170.763m for 3D) suggest that most of the time, positioning accuracy is within a few tens of meters, which is typical for SPP in urban settings.

4.3 Trajectory Analysis：

4.3.1 2D Trajectory (Horizontal)

- The maximum 2D error of 1528.533 m (Figure 9) indicates that while most points are within reasonable urban SPP accuracy, severe outliers exist—likely during the tunnel passage across block U at around 01:34:00 to 01:35:30 (Figure 10).

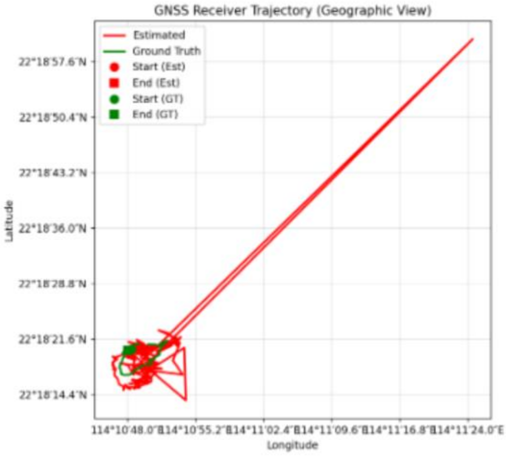


Figure 9 Figure 10

4.3.2 3D Trajectory (Vertical Included）

- The altitude varies wildly: **from –1575.702 m to 317.789 m.** (Figure 11)
- This is physically implausible and points to severe multipath, signal blockage, or loss of lock—especially during the tunnel traversal.
- The 3D max error of **2196.918 m** further confirms the impact of signal degradation in urban canyons and tunnels. (Consistent with the situation described in 2D Trajectory)

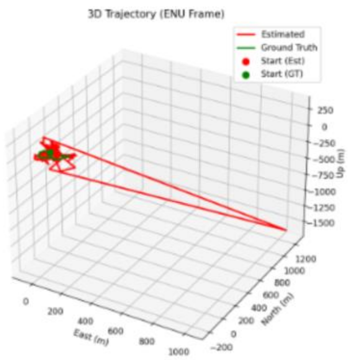


Figure 11

4.4 Tunnel Passage analysis (around 01:35:00 to 01:35:30):

4.4.1 Signal Loss and Reacquisition：

the receiver entered a tunnel, causing:

- Complete loss of GNSS signals.
- Cycle slips and loss of lock on all satellites.
- No valid pseudo range measurements during this period.

4.4.2 Impact on Positioning：

- The SPP algorithm extrapolates or diverges due to lack of measurements.
- This leads to large position jumps and outliers in both 2D and 3D trajectories. (Figure 12)
- The DOP values spike, indicating poor geometry—but in this case, it's due to no usable satellites. (Figure 13)

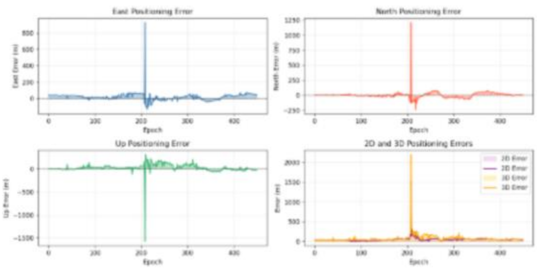


Figure 12

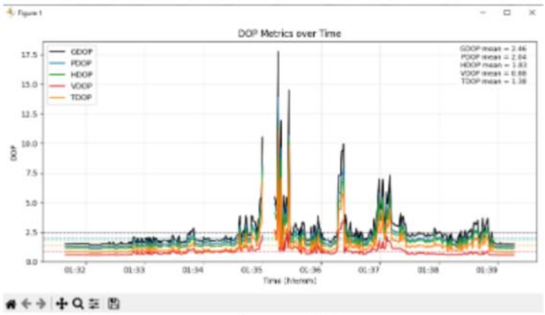
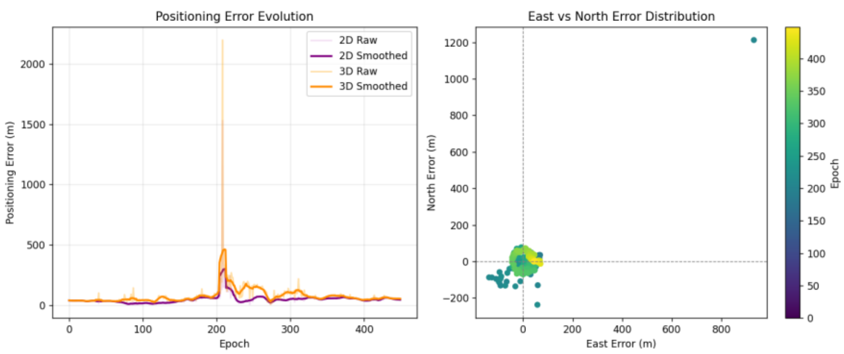


Figure 13

4.5 **Results and Discussion summary**：

The positioning results align with the expected performance of standard SPP in challenging urban environments:

- For 95% of the epochs, the 2D error remained below 73.123 m and the 3D error below 170.763 m, which is characteristic for urban SPP applications.



- The multi-constellation support, particularly the dominant contribution from BeiDou satellites at high elevations, improved satellite availability and geometry (as indicated by the average PDOP of 2.04). However, these advantages could not fully mitigate the fundamental challenges posed by severe multipath and signal obstructions.
- These recommendations provide a roadmap for advancing urban GNSS positioning capabilities, potentially bridging the gap between current Single Point Positioning

performance and the requirements of emerging applications in smart transportation, autonomous navigation, and location-based services. Future work following these directions could significantly enhance the reliability and accuracy of GNSS positioning in challenging urban environments while maintaining practical implementation of feasibility.