〈Matrix multiplication〉

① Dot product between vectors (vector X vector)

$\vec{a} = [1 \quad 2]$ $\vec{w} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $\vec{z} = \vec{a} \cdot \vec{w} = [1 \quad 2] \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = [(1\times3)+(2\times4)] = [11]$

(1×2) (2×1)

*transpose

$\vec{b} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ $\vec{b}^T = [5 \quad 6]$ $\vec{z} = \vec{b}^T \cdot \vec{w} = [5 \quad 6] \begin{bmatrix} 3 \\ 4 \end{bmatrix} = [(5\times3)+(6\times4)] = [39]$

② Vector-Matrix multiplication

$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{a}^T = [1 \quad 2]$ $W = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ $Z = \vec{a}^T W = [\leftarrow \vec{a}^T \rightarrow] \cdot \begin{bmatrix} \uparrow & \uparrow \\ \vec{w}_1 & \vec{w}_2 \\ \downarrow & \downarrow \end{bmatrix}$

(1×2) (2×2)

$= [\vec{a}^T\vec{w}_1 \quad \vec{a}^T\vec{w}_2] = [11 \quad 17]$

③ Matrix-Matrix multiplication

$A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ $W = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

row1, row2, col1, col2

$Z = A^T W = \begin{bmatrix} \leftarrow \vec{a}_1^T \rightarrow \\ \leftarrow \vec{a}_2^T \rightarrow \end{bmatrix} \begin{bmatrix} \uparrow & \uparrow \\ \vec{w}_1 & \vec{w}_2 \\ \downarrow & \downarrow \end{bmatrix} = \begin{bmatrix} \vec{a}_1^T \cdot \vec{w}_1 & \vec{a}_1^T \cdot \vec{w}_2 \\ \vec{a}_2^T \cdot \vec{w}_1 & \vec{a}_2^T \cdot \vec{w}_2 \end{bmatrix} = \begin{bmatrix} 11 & 17 \\ -11 & -17 \end{bmatrix}$

* Matrix multiplication rule

$A = \begin{bmatrix} 1 & -1 & 0.1 \\ 2 & -2 & 0.2 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 0.1 & 0.2 \end{bmatrix}$ $W = \begin{bmatrix} 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 0 \end{bmatrix}$

3 x 2 2 x 4

can only take dot products of vectors that are same length

row length of result of dot product

column length of result of dot product

$Z = A^T W = \begin{bmatrix} 11 & 17 & 23 & 9 \\ -11 & -17 & -23 & -9 \\ 1.1 & 1.7 & 2.3 & 0.9 \end{bmatrix} \Rightarrow 3 \times 4$ matrix