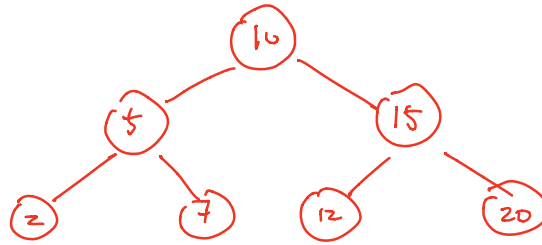


Binary Tree: at most 2 children node. (left & right)



Tree Traversal:

pre-order: self  $\rightarrow$  left  $\rightarrow$  right.

10, 5, 2, 7, 15, 12, 20

in-order: left  $\rightarrow$  self  $\rightarrow$  right.

2, 5, 7, 10, 12, 15, 20

post-order: left - right - self

2, 7, 5, 12, 20, 15, 10.

Balanced: height of the left & right subtrees of every node differ by  $\leq 1$ .

if a tree has  $n$  nodes, & is balanced.

then its height =  $O(\log_2 n)$ .

if not balanced, height =  $O(n)$ .

Completed: Every level (except the last) is completely filled.

as far as left possible.

if complete, must be balanced.

Intuition: want to get value for a problem w/ size  $n$ .

recursion: solve problem of size  $n-1$ .

example: `getHeight()`.

Q1: Is a tree balanced? subtrees height differ by 1 or less.

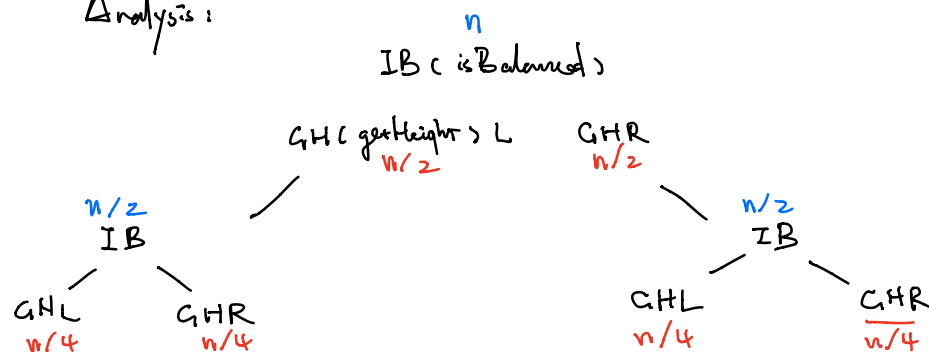
Soln:

Base: null - true.

recursion: if diff  $> 1$ , false.

return `isBalanced(left) && isBalanced(right)`

Analysis:



each level:  $O(n)$

level:  $O(\log n)$ .

Total:  $O(n \log n)$ .

Q2. Judge if a BST is symmetric.

check both left & right. `isSymmetric(L, R)`.

Base: #1.  $L = \text{null} \ \&\& \ R = \text{null} \rightarrow \text{true}$

#2.  $L = \text{null} \ \vee \ R = \text{null} \rightarrow \text{false}$ .

#3.  $L.\text{val} \neq R.\text{val} \rightarrow \text{false}$ .

recursion: return `isSymmetric(L.L, R.R)`.

IS (L.R, R.L).

analysis : Time:  $O(n/2) \rightarrow O(n)$ .

Space:  $O(\text{height})$

Iterative way: Queue. ★

add 2 roots. & while (!Q.isEmpty()).

poll 2 nodes.

check base cases.

add n1.left & n2.right.

add n1.right & n2.left.

Q3, if same Structure (omitted). Todo.

Q4. Determine if a BST is valid.

valid: left only contains nodes w/ keys less than itself.

right -- greater than

both left & right subtrees also BST

Soln 0: #1 inorder traversal and store

#2 iterate & check  $Arr[i] \leq Arr[i+1]$ .

Primitive but bad in space consumption.

Soln 0 better: inorder traversal. when print new.

compare w/ previous & check.

Soln 1: Set min & max bounds for each level (level)

whenever

Assumption:  
NO Duplicates

Go down to the Left: update MAX;

Go down to the right: update MIN

Base:  $\text{root} == \text{null} \rightarrow \text{true}$ .

Reursion: if  $\text{root.val} \notin (\text{min}, \text{max}) \rightarrow \text{false}$ .

return recursion( $\text{root.left}$ ,  $\text{min}$ ,  $\text{root.val}$ )

&& recursion( $\text{root.right}$ ,  $\text{root.val}$ ,  $\text{max}$ )

Note: function signature (int min, int max).

assume Integer.MIN or MAX\_VALUE is not key.

IF NO SUCH ASSUMPTION B.

use (Integer min, Integer max)

and pass (null, null) to handle edge cases.

Note: Recursion in Tree.

# 1. pass value from top to bottom

and then pass back from bottom to top

e.g: validate BST.

# 2. Only pass values from bottom to top.

e.g: getHeight()

isBalanced()

isSymmetric()

Assign the value of each node to be the total

# of nodes that belong to its left subtree.

Q5. Print BST keys in the given range (Unique keys).

AKA: Trim a binary Tree. Given low & high.

Keep the relative order of the subtree.

Notes: instead of returning boolean. need to cut off nodes that are not [Low, high].

BST might NOT be valid. Strictly.

Determine which direction to go.

if  $root > lower$ : go down left.

within range. print

if  $root < upper$ : go down right

If we want to Trim the tree.

First, need to locate a node that's within the range.

if  $root > high$ :  $root \rightarrow root.left$

if  $root < low$ :  $root \rightarrow root.right$

return root (within range)

Cut off Situation:

if  $root == null$ . return null

if  $root < low$ . return recursion( $root.right$ )

if  $root > high$ . return recursion( $root.left$ ).

Divide into small problems: (once we found a node in range)

then determine directions to go. ✓

~~Go left: if root > low~~

root.left = recursion(root.left, low, high)

~~Go right: if root < high~~

root.right = recursion(root.right, low, high)

At last return root (restructured)