

Virtual Lead Wall: A Monte Carlo Study of Gamma-Ray Attenuation

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1. Problem Statement and Objective

Gamma-ray shielding is often introduced using the macroscopic exponential attenuation law

$$I(x) = I_0 e^{-\mu x},$$

which predicts how intensity decreases with material thickness. While convenient, this model hides the underlying stochastic physics: individual photons undergo discrete interaction events (absorption or scattering), and scattering changes photon energy, which in turn changes subsequent interaction probabilities.

The objectives of this project are:

- To simulate gamma-ray attenuation in matter using a Monte Carlo (stochastic) photon-transport model.
- To connect microscopic interaction processes (photoelectric absorption and Compton scattering) with macroscopic attenuation behavior.
- To incorporate Compton scattering energy loss and quantify its influence on predicted shielding performance.
- To extract the Half-Value Layer (HVL) from simulated survival data and compare it with a theoretical estimate.
- To discuss statistical confidence, model limitations, and possible extensions for improved realism.

2. Introduction and Theory

2.1 Exponential Attenuation Law

Consider a narrow beam traversing a homogeneous material. If the probability of interaction in an infinitesimal thickness dx is proportional to dx , then

$$dI = -\mu I dx,$$

where μ is the linear attenuation coefficient (units: cm^{-1}). Solving,

$$I(x) = I_0 e^{-\mu x}.$$

This can also be interpreted as a Poisson survival process: the probability that a photon experiences *no* interaction over path length x is $e^{-\mu x}$.

2.2 Microscopic Processes in This Model

In the energy regime around a few hundred keV (e.g. Cs-137 at 662 keV), two dominant photon interactions are:

- **Photoelectric absorption:** photon is absorbed and removed from the beam.
- **Compton scattering:** photon scatters off an electron and loses energy.

The simulation models these processes probabilistically. The total interaction coefficient is written as

$$\mu_{\text{tot}}(E) = \mu_{\text{photo}}(E) + \mu_{\text{Compton}}(E).$$

2.3 Half-Value Layer (HVL)

The Half-Value Layer is defined as the thickness needed to reduce intensity to half:

$$\frac{I(\text{HVL})}{I_0} = \frac{1}{2}.$$

For the exponential model with constant μ ,

$$\text{HVL}_{\text{theory}} = \frac{\ln 2}{\mu}.$$

In a Monte Carlo transport model, HVL is obtained numerically from the simulated curve $I(x)/I_0$.

3. Methodology

3.1 Assumptions and Scope

To keep the simulation simple and transparent (“Monte Carlo Light”), the following assumptions are used:

- Photons are monoenergetic at emission with energy E_0 .
- The shielding slab is homogeneous and planar.
- Only photoelectric absorption and Compton scattering are considered; pair production is neglected.
- Transport is treated in one dimension (forward propagation).
- Compton-scattered photons continue forward and are counted as “survivors” unless absorbed later.

These assumptions are appropriate for demonstrating how exponential attenuation emerges statistically, but they are not a full clinical radiation-transport model.

3.2 Step-wise Transport and Interaction Sampling

A photon moves through the slab in steps of thickness Δx . At each step, with current photon energy E , the simulation computes

$$p_{\text{int}} = 1 - e^{-\mu_{\text{tot}}(E)\Delta x},$$

the probability that at least one interaction occurs within that step. If an interaction occurs, the type is selected by relative rates:

$$P(\text{photo} \mid \text{int}) = \frac{\mu_{\text{photo}}}{\mu_{\text{tot}}}, \quad P(\text{Compton} \mid \text{int}) = \frac{\mu_{\text{Compton}}}{\mu_{\text{tot}}}.$$

3.3 Energy Dependence and Compton Energy Loss

To represent the strong energy dependence of photoelectric absorption, a simple power-law model is used:

$$\mu_{\text{photo}}(E) \propto E^{-p},$$

with user-adjustable exponent p (default $p = 3$). For Compton, a constant baseline μ_{Compton} is assumed.

After a Compton event, energy is updated using the Compton formula:

$$E' = \frac{E}{1 + \frac{E}{m_e c^2} (1 - \cos \theta)}.$$

In the code, $\cos \theta$ is sampled (rejection sampling) using the Klein–Nishina angular distribution (shape only). This produces physically plausible energy loss histories and prevents “unphysically penetrating” photons.

3.4 Outputs and What the Plots Mean

The code generates three main plots:

- $I(x)/I_0$ vs x (linear scale): shows attenuation curve and HVL crossing.
- Semi-log plot of $I(x)/I_0$: tests whether attenuation is approximately exponential (straight line).
- Histogram of absorption depths: shows the stochastic distribution of where photons are removed.

Additionally, the program prints summary statistics, including theoretical HVL, Monte Carlo HVL, and transmitted fractions.

3.5 Interactivity and Computational Cost

A practical advantage of this implementation is interactivity: the user can adjust N , E_0 , thickness T , Δx , and the reference coefficients. The main computational cost scales approximately as

$$\mathcal{O}\left(N \frac{T}{\Delta x}\right),$$

so increasing N improves statistical smoothness (roughly as $1/\sqrt{N}$) but increases runtime, while increasing thickness T or decreasing Δx increases the number of simulation steps per photon.

4. Results

4.1 Input Parameters (Baseline Run)

The baseline run uses:

$$E_0 = 662 \text{ keV}, \quad \mu_{\text{photo,ref}} = 0.40 \text{ cm}^{-1}, \quad \mu_{\text{Compton,ref}} = 0.10 \text{ cm}^{-1}, \quad N = 1000, \quad T = 5 \text{ cm}, \quad \Delta x = 0.01 \text{ cm}.$$

4.2 Numerical Output (Baseline Run)

```
Reference mu_total at E0: 0.5000 1/cm
Theory HVL (ln2/mu_ref_total): 1.3863 cm
MC HVL (energy loss ON):      1.5400 cm
MC HVL (energy loss OFF):     1.7250 cm
Transmitted fraction (ON):     0.0840
Transmitted fraction (OFF):    0.1190
```

4.3 Graphical Results (Baseline Run)

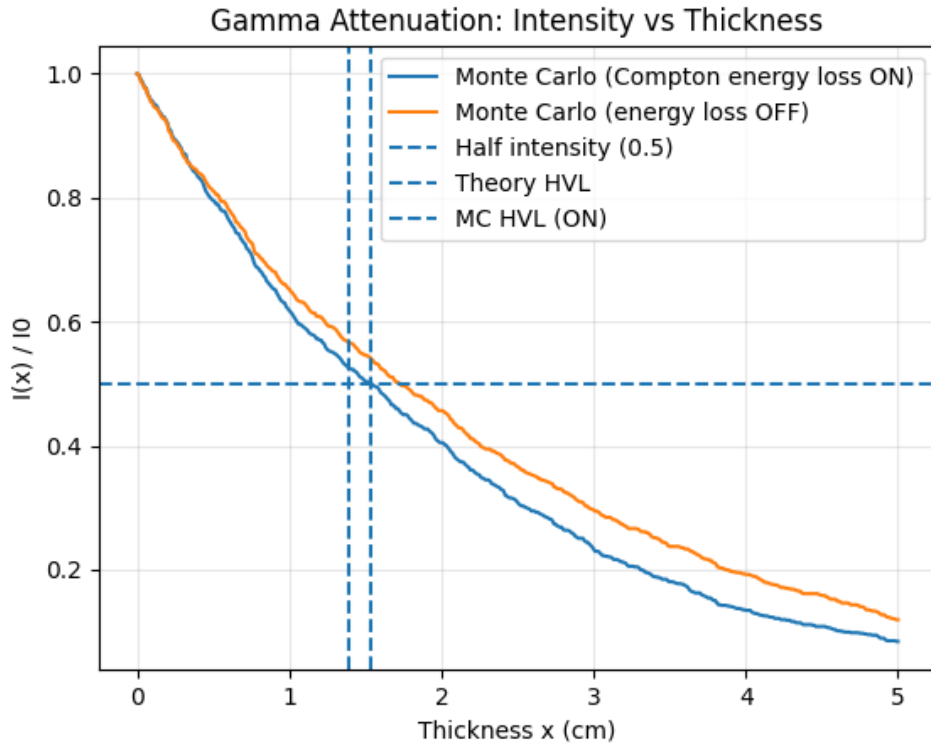


Figure 1: Normalized intensity $I(x)/I_0$ vs thickness for two cases: Compton energy loss ON (physical) and OFF (diagnostic). Horizontal dashed line marks half intensity. Vertical dashed lines show theoretical HVL and simulated HVL (ON).

Figure 1 shows that energy loss ON produces stronger attenuation (smaller transmission) than the energy loss OFF case. This is expected: when Compton energy loss is included, the photon spectrum softens as photons scatter, increasing the probability of subsequent absorption and reducing penetration.

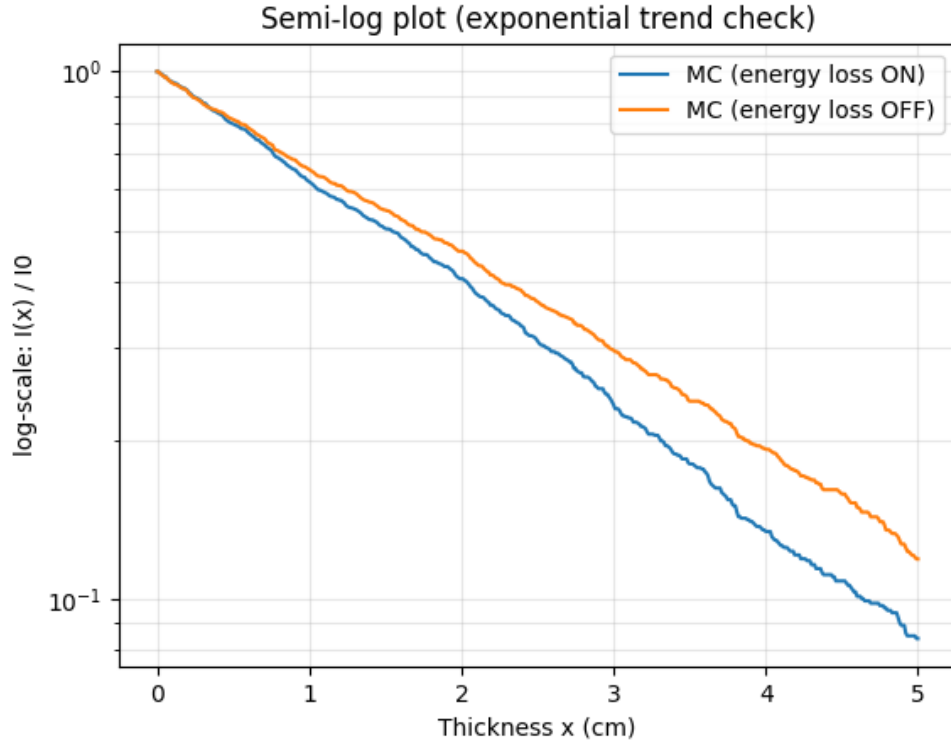


Figure 2: Semi-log plot of $I(x)/I_0$ vs thickness. Approximately linear behavior supports exponential attenuation emerging from stochastic interaction sampling.

Figure 2 provides a clearer test of exponential attenuation: a straight line on semi-log axes indicates $I(x)/I_0 \propto e^{-\mu x}$. Small deviations are expected because the simulation includes finite statistics and energy-dependent coefficients.

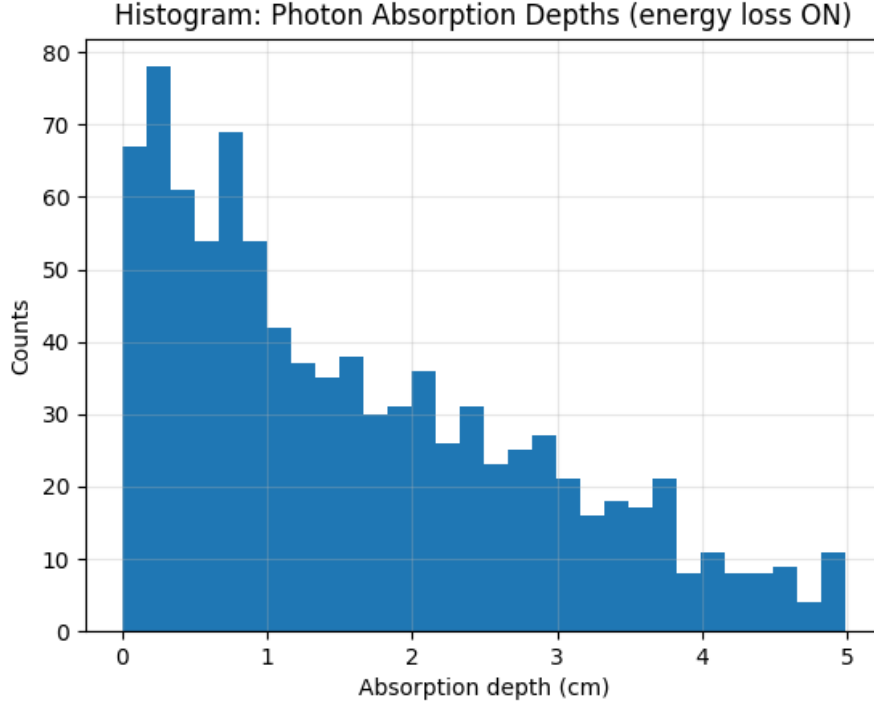


Figure 3: Histogram of photon absorption depths (energy loss ON). The peak near small depths indicates many photons are removed early; the long tail represents rare deep-penetrating histories.

5. Data Analysis

5.1 HVL Accuracy and Error

Using $\mu_{\text{tot}}(E_0) = 0.50 \text{ cm}^{-1}$,

$$\text{HVL}_{\text{theory}} = \frac{\ln 2}{0.50} = 1.386 \text{ cm.}$$

The Monte Carlo value (energy loss ON) is $\text{HVL}_{\text{MC}} = 1.540 \text{ cm}$. The relative deviation is

$$\delta = \frac{1.540 - 1.386}{1.386} \times 100\% \approx 11.1\%.$$

5.2 Statistical Confidence (Counting Uncertainty)

Near the HVL point, approximately half the photons survive. For $N = 1000$, counting statistics give a characteristic relative uncertainty

$$\frac{\sqrt{500}}{500} \approx 4.5\%.$$

This provides a baseline estimate for the expected fluctuations in I/I_0 around 0.5. Increasing N should reduce fluctuations roughly as $1/\sqrt{N}$.

5.3 Why Simulation Can Differ from the Simple Theory

The theoretical HVL formula assumes a *single constant* attenuation coefficient and a narrow-beam “removal” model. In this simulation:

- $\mu_{\text{photo}}(E)$ is energy-dependent, and E changes after Compton scattering.
- Compton-scattered photons continue forward and are still counted as survivors, increasing apparent transmission.
- The step-wise method introduces small discretization effects when Δx is not extremely small.

Therefore, a modest systematic deviation from $\ln 2/\mu$ is expected even with large N .

5.4 Diagnostic Comparison: Energy Loss ON vs OFF

Disabling Compton energy loss increases HVL to 1.725 cm and increases transmitted fraction from 0.084 to 0.119:

$$\frac{0.119 - 0.084}{0.084} \times 100\% \approx 41.7\%.$$

This shows that ignoring energy degradation can significantly underestimate shielding effectiveness.

6. Discussion

6.1 Limitations of the Current Model

- **One-dimensional transport:** angular scattering is not tracked in 3D; all scattered photons continue forward.
- **Simplified coefficients:** μ_{photo} uses a power law and μ_{Compton} is constant; real materials require tabulated data.
- **No pair production:** becomes relevant at higher energies (MeV range).
- **No detector geometry:** real “narrow beam” measurements exclude scattered photons from the detector acceptance.

6.2 Future Improvements

Possible extensions include:

- Sampling full scattering angle and tracking photon direction in 3D.
- Adding a detector acceptance cone to distinguish transmitted primary beam from scattered contributions.
- Using energy-tabulated $\mu(E)$ data for specific materials (lead, concrete) instead of placeholders.
- Running multiple independent trials to estimate confidence intervals on HVL.

7. Conclusion

A Monte Carlo photon-transport simulation was implemented to study gamma-ray attenuation through shielding material. The simulation reproduces approximately exponential attenuation and yields an HVL of 1.540 cm compared with the theoretical estimate 1.386 cm, giving an $\approx 11.1\%$ deviation. This discrepancy is consistent with finite sampling and model differences from the ideal constant- μ narrow-beam approximation.

The diagnostic comparison confirms that including Compton energy loss is essential: disabling energy loss increases the transmitted fraction by $\approx 41.7\%$, leading to physically incorrect shielding predictions. Overall, the results support the objective that macroscopic attenuation laws emerge from microscopic stochastic interactions.