Mathematics for Computer Science
Week 2 Notes

Jose Miguel de Lima

11/24/2025

Lecture Notes

Indirect proof or proof by contradiction: To prove a proposition P is true, we assume P is false — (i.e., $\neg P$ is true) and then use hypothesis to derive a falsehood or contradiction.

if $\neg P \Rightarrow F$ is true, then P must be true or $\neg P$ is false.

Example: Prove: $\sqrt{2}$ is irrational.
Proof by contradiction: Assume for the purpose of contradiction that $\sqrt{2}$ is rational.
$\Rightarrow \sqrt{2} = a/b$ (fraction in lowest terms)
$\Rightarrow 2b^2 = a^2$
$\Rightarrow a^2$ is even $(2|a)$
$\Rightarrow a$ is even (since the square of an odd number is odd)
$\Rightarrow 4|a^2 \Rightarrow 4|2b^2 \Rightarrow 2|b^2 \Rightarrow b$ is even
$\Rightarrow a$ and b are both even, which contradicts the assumption that a/b is in lowest terms. Therefore, $\sqrt{2}$ is irrational.

Induction axiom: Let $P(n)$ be a predicate, if $P(0)$ is true, and $\forall n \in \mathbb{N}$ $(P(n) \Rightarrow P(n+1))$ is true, then $\forall n \in \mathbb{N}, P(n)$ is true.

if $P(0)$ is true (base case) and $P(n) \Rightarrow P(n+1)$ is true (inductive step), then $P(n)$ is true for all $n \in \mathbb{N}$.

Example: Prove: $\forall n \geq 0$, $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$. Other ways to write this:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

,

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

(since adding 0 doesn't change the sum),

$$\sum_{i=1}^{n} i = \frac{n^2+n}{2}$$

Special case:

If $n = 1$ $1 + 2 + \cdots + n = 1$ (base case)
If $n \leq 0$ $1 + 2 + \cdots + n = 0$ (base case)

Example:

$$n = 4: \quad 1 + 2 + 3 + 4 = 10 = \frac{4(4+1)}{2} = \frac{20}{2} = 10$$

Proof by induction: Let $P(n)$ be the predicate

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Base Case: $P(0)$ is true

$$\sum_{i=1}^{0} i = 0 = \frac{0(0+1)}{2} = 0$$

Inductive Step: For $n \geq 0$, assume $P(n) \Rightarrow P(n+1)$ is true, i.e.,
Assume $P(n)$ is true for purposes of induction.
(i.e., assume $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$)
need to show $1 + 2 + \cdots + (n+1) = \frac{(n+1)(n+2)}{2}$

$$\begin{aligned} 1 + 2 + \cdots + (n+1) &= (1 + 2 + \cdots + n) + (n+1) \\ &= \frac{n(n+1)}{2} + (n+1) \quad \text{(by inductive hypothesis)} \\ &= \frac{n(n+1) + 2(n+1)}{2} \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$

Definitions

...: figure out the pattern and complete the definition.