Mathematics for Computer Science
Recitation 1

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0.1 1 Team Problem: A Myster

A certain cabal within the 6.042 course staff is plotting to make the final exam ridiculously hard. ("Problem 1. Prove that the axioms of mathematics are complete and consistent. Express your answer in Mayan hieroglyphics.") The only way to stop their evil plan is to determine exactly who is in the cabal. The course staff consists of nine people:

$$\{Oscar, Stav, Darren, Patrice, David, Nick, Martyna, Marten, Tom\}$$

The cabal is a subset of these nine. A membership roster has been found and appears below, but it is deviously encrypted in logic notation. The predicate *incabal* indicates who is in the cabal; that is, $incabal(x)$ is true if and only if x is a member. Translate each statement below into English and deduce who is in the cabal.

(i) $\exists x \exists y \exists z (x \neq y \wedge x \neq z \wedge y \neq z \wedge incabal(x) \wedge incabal(y) \wedge incabal(z))$

There are at least three people in the cabal

(ii) $\neg(incabal(Stav) \wedge incabal(David))$

Stav	David	Stav ∧ David	¬(Stav ∧ David)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Stav or David or neither are in the cabal

(iii) $(incabal(Martyna) \vee incabal(Patrice)) \rightarrow \forall x \, incabal(x)$

If Martyna or Patrice or both are in Cabal then everyone is in the cabal

(iv) $incabal(Stav) \rightarrow incabal(David)$

If Stav is in the cabal then David is also in the cabal

(v) $incabal(Darren) \rightarrow incabal(Martyna)$

If Darren is in the cabal then Martyna is also in the cabal

(vi) $(incabal(Oscar) \vee incabal(Nick)) \rightarrow \neg incabal(Tom)$

If Oscar or Nick or both are in the cabal then Tom is not in the cabal

(vii) $(incabal(Oscar) \vee incabal(David)) \rightarrow \neg incabal(Marten)$

If Oscar or David or both are in the cabal then Marten is not in the cabal

Solution: From (i) we know that there are at least three people in the cabal. From (ii) we know that Stav and David cannot both be in the cabal. From (iii) we know if either Martyna or Patrice are in the cabal then everyone is in the cabal which is impossible since from (ii) we know that Stav and David cannot both be in the cabal. Therefore, Martyna and Patrice are not in the cabal. From (iv) we know if Stav is in the cabal then David is also in the cabal which is impossible for Stav to be in the cabal.

At this point we have eliminated three people from being in the cabal: {Martyna, Patrice and Stav.}

From (v) we know if Darren is in the cabal then Martyna is also in the cabal which is impossible since Martyna is not in the cabal. Therefore, Darren is not in the cabal.

Current potential members of the cabal are: {Oscar, David, Nick, Marten, Tom.}

From (vi) we know if Oscar or Nick or both are in the cabal then Tom is not in the cabal. From (vii) we know if Oscar or David or both are in the cabal then Marten is not in the cabal. Since we need at least three people in the cabal from (i) the only way to satisfy both (vi) and (vii) is by having Oscar, David and Nick in the cabal.

Final members of the cabal are: {Oscar, David, Nick.}