

Mathematics for Computer Science

Lecture 1 Notes

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11/25/2025

Lecture Notes:

What is a Proof? :

- Its a method of ascertaining truth.
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Examples:

- Experiments
 - Sampling
 - Legal proceedings
 - Investigations
 - Questions
 - Authority
 - Religion
 - Inner Conviction
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Def: A mathematical proof is a verification of a proposition by a chain of logical deductions from a base set of axioms.

Def: A proposition is a statement that is true or false.

Examples:

- $2 + 3 = 5$
 - $2 + 3 = 4$
 - $\forall n \in \mathbb{N} : n^2 + n + 41$ is a prime.
 - P is prime (not a proposition) this is a predicate
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Def: A predicate is a proposition whose truth depends on variables

Goldbach's Conjecture: Every even number ≥ 2 is the sum of 2 primes.

Example: $- 12 = 7 + 5$

Def: Conjecture not sure if it true or not

A, B are propositions

not A : $\neg A$: means a is false

A and B : $A \wedge B$

A or B : $A \vee B$

exclusive Or, XOR : $A \oplus B$: One or the Other, not neither or both

Not And, NAND: $A \bar{\wedge} B : \neg(A \wedge B)$: One or the Other or Neither not both

Implication: A implies B or $A \rightarrow B$: “A implies B” aka “if A then B”

$$A \implies B$$

- counterpositive $\neg B \implies \neg A$

- converse $B \implies A$

- inverse $\neg A \implies \neg B$

the converse and inverse are counterpositive of each other.

Def: A set is a collection of objects. Order doesn't matter and no repeats.

Example:

- $6, 1, 2, 0 ::= \{2, 1, 6, 0\} ::= \{6, 1, 2, 0, 0\}$

- $\mathbb{N} ::= \{0, 1, 2, 3, \dots\}$

- $\mathbb{Z} ::= \{0, 1, 2, 3, -1, -2, -3, \dots\}$

- $\mathbb{Q} ::= \text{rationals}$

- $\mathbb{R} ::= \text{reals}$

- $\mathbb{C} ::= \text{complex}$

- Empty Set $\emptyset, \{\}$

- $\mathbb{B} ::= \{2, 3, 4, \emptyset\}$

$x \in A$ means x is an element of A

$A \subset B$ “A is a subset or equal to B”

$A \cup B$: “union”

$A \cap B$: “intersection”

$A - B$: “set difference”

Set Builder Notation:

$$n \in \mathbb{N} \mid \text{isprime}(n) = \{2, 3, 5, 7, 11, \dots\}$$

Tuples: ordered list of elements, and repeats are ok, uses () instead {}

$$(6, 1, 2, 0) \neq (6, 1, 2, 0, 0) \neq (2, 1, 6, 0)$$

Def: an Axiom is a proposition we assume is true.

Examples:

- Euclid's Parallel Postulate: For every point p and line l with $p \notin l$, \exists a unique line l' through p parallel to l
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A set of Axioms is consistent when you can't prove that False is True.

... is complete when every true proposition can be proved from the axioms.

The Godel incompleteness theorem: Can't have both. There are true statements that cannot be proved.