# EECS 442 Computer Vision, Fall 2016
# Midterm Exam

This exam is released on Tuesday, November 1, 2016 at 4:30pm, and is due on Thursday, November 3, 2016 at 3:00pm. You should submit your midterm to Canvas.

Please include all your source code submit it along with your final outputs and brief descriptions of your approach towards each question.

## 1 Transformations [25pt]

We have seen that a 3D rotation can be formed as the product of three matrices

$$R = R_x(\alpha)R_y(\beta)R_z(\gamma),$$

where

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \quad (1)$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad (2)$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

are rotations around the x, y, and z axis, respectively. This is called the X-Y-Z Euler angle representation, indicating the order of matrix multiplication

However, representing rotations in this form can lead to ambiguities, since the result depends on the order in which the transforms are performed. Let

$p'$ be the point obtained by rotating a point $p$ with a rotation matrix $R$, so $p' = Rp$. Give an expression for $p'$ obtained by rotating $p$ in the following two ways:

- First rotate $\beta$ around y axis, then rotate $\gamma$ around z axis.
- First rotate $\gamma$ around z axis, then rotate $\beta$ around y axis.

**[10pt]** Show that these rotations produce different values of $p'$.

**[15pt]** To avoid the representation ambiguity introduced in part a, we can fix the rotation order. However, even if the order of rotation is fixed, this rotation system may still result in a degenerate representation. For instance, let $\alpha$, $\beta$, and $\gamma$ be three different angles. We can represent an arbitrary rotation with respect to the X-Y-X axes as the product $R_x(\alpha)R_y(\beta)R_x(\gamma)$, (note that the axes here are X-Y-X, not X-Y-Z). This representation of a rotation matrix should have three degrees of freedom. However if we set $\beta = 0$, something unusual happens. How many degrees of freedom are left? Hint: some trigonometric identities may be useful.

## 2 Panoramic Imaging Theory [15pt]

Assume that we have two cameras with camera matrices $M_1$ and $M_2$

$$M_1 = K_1[R_1 T_1] \text{ and } M_2 = K_2[R_2 T_2],$$

where

$$\begin{aligned} K_1 = K_2 &= K \\ R_1 = I &= \text{Identity matrix} \\ R_2 &= R \\ T_1 = T_2 &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \end{aligned}$$

Suppose that the cameras generate image 1 and 2, respectively, as shown in the figure below, where $p'_1, p'_2, p'_3, p'_4$ and $p_1, p_2, p_3, p_4$ are corresponding points across the two images.

Prove that the homographic transformation $H$ defined by $p'_1, p'_2, p'_3, p'_4$ and $p_1, p_2, p_3, p_4$ can be expressed as $H = KRK^{-1}$.

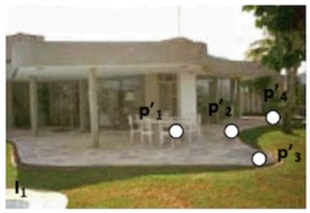

Image 1

Image 2

## Computing H [30pt]

Write a function `Homography` that takes a set of at least 4 image point correspondences between two images (manually selected) and returns an estimate of the homographic transformation between the images. Use the DTL algorithm to compute the homography.

What to return in this problem:

- Your code for the function `Homography` with comments.
- Numerical values for $H$ given the point correspondences in file `4points.txt` (`4points.txt` file can be read using `readPoints.m` function).

## Convolution [30pt]

In class we discussed Gaussian blurring and convolution. Implement in MATLAB **WITHOUT** any built in functions (i.e. conv2, imconv, fspecial, etc.). The construction of a Gaussian kernel with a variable $\sigma$ parameter and the code to apply that kernel to an image through convolution again **WITHOUT** using the built in convolution functions. Apply your Gaussian kernel to one of the images in the midterm folder (garden1,garden2) with 3 $\sigma$ values and discuss the results.

You must return:

- The source code.
- The visual results (images) after blurring
- Comments about your results.