EECS 442 Homework #1

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Problem 1

(a)

Suppose X_c and X_w are the same coordinates represented in Camera and World coordinates. Hence, we have the relation $X_c = PX_w$, P is the 4×4 combined rotation and translation matrix from homogeneous world coordinates to homogeneous camera coordinates. It is easy to get following relations:

$$W_x = -\cos(45^\circ) \times C_x + 0 \times C_y - \cos(45^\circ) \times C_z$$

$$W_y = C_y$$

$$W_z = \cos(45^\circ) \times C_x + 0 \times C_y - \cos(45^\circ) \times C_z$$

Hence, the transformation matrix:

$$P = \begin{bmatrix} -\cos(45^\circ) & 0 & \cos(45^\circ) & 0 \\ 0 & 1 & 0 & 0 \\ -\cos(45^\circ) & 0 & -\cos(45^\circ) & d \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

Consider the unit square in the world coordinate and transform it into camera coordinate, suppose $a^w = [0\ p\ q\ 1]^T$, $b^w = [0\ p+1\ q\ 1]^T$, $c^w = [0\ p+1\ q+1\ 1]^T$ and $d = [0\ p\ q+1\ 1]^T$. After transformation, the new coordinates for four points of the square are: $a^c = [\frac{\sqrt{2}}{2}q,\ p,\ -\frac{\sqrt{2}}{2}q,\ 1]^T$, $b^c = [\frac{\sqrt{2}}{2}q,\ p+1,\ -\frac{\sqrt{2}}{2}q,\ 1]^T$, $c^c = [\frac{\sqrt{2}}{2}(q+1),\ p+1,\ -\frac{\sqrt{2}}{2}(q+1),\ 1]^T$ and $d^c = [\frac{\sqrt{2}}{2}(q+1),\ p,\ -\frac{\sqrt{2}}{2}(q+1),\ 1]^T$. From these coordinates we see the transformation will not change the form of the square, thereby the area of the the square will remain the same.

(c)

Assume two end point of one line is $a^w = [x_1,\ y_1,\ z_1,\ 1]$, $b^w = [x_1 + p,\ y_1 + q,\ z_1 + r,\ 1]$ and the end points of the parallel line are $c^w = [x_2,\ y_2,\ z_2,\ 1]$, $d^w = [x_2 + pt,\ y_2 + qt,\ z_2 + rt,\ 1]$. After the transformation, we get the new coordinates for new end points of these two parallel lines:

$$a^c = [-\frac{\sqrt{2}}{2}x_1 + \frac{\sqrt{2}}{2}z_1,\ y_1,\ -\frac{\sqrt{2}}{2}x_1 - \frac{\sqrt{2}}{2}z_1 + d,\ 1]^T$$

$$b^c = [-\frac{\sqrt{2}}{2}(x_1 + p) + \frac{\sqrt{2}}{2}(z_1 + r),\ y_1 + q,\ -\frac{\sqrt{2}}{2}(x_1 + p) - \frac{\sqrt{2}}{2}(z_1 + r) + d,\ 1]^T$$

$$c^c = [-\frac{\sqrt{2}}{2}x_2 + \frac{\sqrt{2}}{2}z_2,\ y_2,\ -\frac{\sqrt{2}}{2}x_2 - \frac{\sqrt{2}}{2}z_2 + d,\ 1]^T$$

$$b^c = [-\frac{\sqrt{2}}{2}(x_2+pt) + \frac{\sqrt{2}}{2}(z_2+rt),\ y_2+qt,\ -\frac{\sqrt{2}}{2}(x_2+pt) - \frac{\sqrt{2}}{2}(z_2+rt) + d,\ 1]^T$$

We can easily get the following relations:

$$\overrightarrow{a^cb^c} = [\frac{\sqrt{2}}{2}(r-p),\ q,\ -\frac{\sqrt{2}}{2}(r+p), 0]^T$$

$$\overrightarrow{c^cd^c} = [\frac{\sqrt{2}}{2}(r-p)t,\ qt,\ -\frac{\sqrt{2}}{2}(r+p)t, 0]^T$$

Hence, we can conclude that parallel lines in the world reference system still parallel in the camera reference system as proven above.

(d)

$$\overrightarrow{a^wb^w} = [0,\ 1,\ 0,\ 0]^T$$

$$\overrightarrow{a^cb^c} = [0,\ 1,\ 0,\ 0]^T$$

we can easily conclude that vector defined by a and c have the same orientation in both reference system.

Problem 2

The projection point $P' = [x',\ y']^T$ can be calculated by:

$$x' = f'\frac{x}{z}$$

$$y' = f'\frac{y}{z}$$

Hence, the projection of points on line are given by $x' = f'\frac{1}{t}$ and $y' = f'\frac{1}{t}$, we see the end points of the original line requires t equal to $-\infty$ and -1, so we can get the end points of the projection are $[-f',\ -f']^T$ and $[0,\ 0]^T$.

Problem 3

(a)

This problem will introduce projective geometry in 2D techniques to us. We will have the following propositions:

- We are in a plane P and want to describe points and lines on P
- We consider a third dimension to make things easier when dealing with infinity
- To each point of P, we associate it with a ray $X = [x_1,\ x_2,\ x_3]^T$
- To each line of P, we associate it with a plane $(a,\ b,\ c)^T$

Hence, we can conclude that the line passes through two points is:

$$l = x_1 \times x_2 = (2,\ 2,\ -8)^T$$

(b)

With the same method, we can get the line passes through x'_1 and x'_2 is:

$$l' = (Hx_1) \times (Hx_2) = [137.1000,\ 62.5400,\ -396.7632]^T$$

The Matlab code is listed below:

```
%% EECS442 HW1 codes
% by Yi Yang
% Date: 9/20/2016
H = [1.52 -1.902 1; 3.3 23.49 3; 1 3 1];
x_1 = [1 3 1]';
x_2 = [3 1 1]';
cross(H*x_1, H*x_2)
```

(c)

To derive the relations between l, l' and H, we have to use the formula introduced in discussion:

$$l' = (Hx_1) \times (Hx_2) = \det(H)H^{-T}x_1 \times x_2 = \det(H)H^{-T}l$$

Hence, the relations between l', l and H is:

$$l' = \det(H)H^{-T}l$$