09.21.2016

Complete, Compact, closure, open, closure set, open set.

closure under a collection of operations: Subspace must be closed under Addition and Scalar Multiplication

closure in topology.

Subspace must be a convex set, but it may not be closed in topology.

A simple example: Suppose a metric space: infinite sequences with finite number of nonzero elements. Consider $A_k: \{1, \frac{1}{2}, \frac{1}{2^2}, \cdots, \frac{1}{2^k}, 0, 0, \cdots\}$, $k=1,2,\cdots$

we see the limit of this sequence $\{A_k\}_{k=1}^{\infty}$ lies out of subspace which means it's not closed.

For a normed vector space, if it is complete, then it must also be closed in topology, since it contains all of the limit points. (convergent sequences must be Cauchy and for a complete space, Cauchy sequences must converge in the space.)

The subspace of a complete vector space is complete iff it is closed in topology. (Since closed.) $\Rightarrow$ every Cauchy sequence in subspace will converge to the limit point inside subspace.

A finite dimensional vector space must be complete, closed, compact.

Finite dimensional space: compact = closed + bounded

General space: compact = complete + totally bounded.

Take care of conditions for "Orthogonality Principle" & "Projection Theorem"

- inner product space $\downarrow$ Hilbert space
- inner product space $\downarrow$ Hilbert space

Subspace: can not promise existance

$\downarrow$

closed subspace: can promise existance and uniqueness

nonempty convex subset: ...

$\downarrow$

closed convex subset: ...