2013 IEEE Intelligent Vehicles Symposium (IV)
June 23-26, 2013, Gold Coast, Australia

Robust Predictive Control for Semi-Autonomous Vehicles with an Uncertain Driver Model

Andrew Gray, Yiqi Gao, J. Karl Hedrick, Francesco Borrelli
University of California, Berkeley
Berkeley, CA

***Abstract*—A robust control design is proposed for the lane-keeping and obstacle avoidance of semiautonomous ground vehicles. A robust Model Predictive Controller (MPC) is used in order to enforce safety constraints with minimal control intervention. An uncertain driver model is used to obtain sets of predicted vehicle trajectories in closed-loop with the predicted driver's behavior. The robust MPC computes the smallest corrective steering action needed to keep the driver safe for all predicted trajectories in the set. Simulations of a driver approaching multiple obstacles, with uncertainty obtained from measured data, show the effect of the proposed framework.**

I. Introduction

Advances in sensing technologies have enabled the introduction and commercialization of several automated driving features over the last two decades. Examples of such applications are threat assessment Warning Strategies [1], Adaptive Cruise Control (ACC) [2], Rear-end Collision Avoidance systems [3], as well as Lane Keeping systems [4]. In safety applications, autonomous interventions are activated automatically. Over-activation of automated safety interventions might be felt as intrusive by the driver, while on the other hand, a missed or delayed intervention might lead to a collision. In the literature, a large variety of threat assessment and decision making approaches can be found [3], [5], [6], [7]. In the simplest approaches, used in production vehicles, automated steering or braking interventions are issued when the *time to collision* [3] or *time to line crossing* [5] pass certain thresholds. More sophisticated approaches must both determine a safe trajectory for the vehicle as well as coordinate the vehicle actuators. The literature on vehicle path planning and control is rich, see, e.g. [6], [7], [8], [9], [10], [11]. The approach in [6] includes the computation of Bayesian collision probabilities and [7] calculates sets of safe states from which the vehicle can safely evolve. Because of its capability to systematically handle system nonlinearities and constraints, work in a wide operating region and close to the set of admissible states and inputs, Model Predictive Control (MPC) has been shown to be an attractive method for solving the path planning and control problem [8], [12]. However, previous approaches to lane departure prevention using predictive control, as in [13], do not incorporate any driver model and therefore fail to capture the predicted driver's behavior.

This material is based upon work supported by the National Science Foundation under Grant No. 1239323. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.
{ajgray@, yiqigao@, khedrick@me, fborrelli@me.}berkeley.edu

In previous work [14], [15] the authors of this paper proposed an active safety system for prevention of unintended roadway departures with a human-in-the-loop. Rather than separately solving the threat assessment, decision making, and intervention problems, we reformulate them as a single combined optimization problem. In particular, a predictive optimal control problem is formulated which simultaneously uses predicted drivers behavior and determines the least intrusive intervention to keep the vehicle in a region of the state space where the driver is deemed safe. This work assumed a perfect driver model and did not model the uncertainty in the prediction.

In this paper we extend the work presented in [14], [15] and propose an uncertain driver model to provide robust guarantees of constraint satisfaction in the presence of the driver's uncertain behavior. The uncertainty in the driver model is handled at the design stage by the computation of a robust invariant set that captures the spread of the vehicle's future trajectories given the uncertainty in the driver model. By tightening the constraints of the original nominal system we solve the optimization problem to yield the optimal corrective action needed to augment the driver's steering to ensure satisfaction of the safety constraints in the presence of the uncertain driver behavior [16]. The proposed controller is always active, which avoids the design of switching logic or the tuning of a sliding scale. In addition, since the proposed controller is designed to apply only the correcting control action necessary to avoid violation of the safety constraints, the intrusiveness of the safety application is kept minimal. In this paper we detail the proposed framework and show its effectiveness through simulations.

This paper is structured as follows. In section II definitions of invariant sets are presented and the Robust MPC framework is outlined. In section III the vehicle and uncertain driver models are developed. Section IV details the robust invariant set computation. Section V presents the safety constraints and section VI formulates the Robust MPC problem. Finally, in section VII we present the simulation results showing the behavior of the proposed controller.

II. Invariant Sets and Robust MPC

A. Background on Set Invariance Theory

In this section several definitions are provided that will be important in developing the Robust MPC later in this paper. We follow the notation used in [17].

Denote by f_a the constrained, discrete time linear autonomous system perturbed by a bounded, additive disturbance. The system dynamics are

$$x(t+1) = f_a(x(t), w(t)) = \mathbf{A}x(t) + w(t) \quad (1)$$

where $x(t)$ and $w(t)$ denote the state and the disturbance vectors, respectively. System (1) is subject to the constraints

$$x(t) \in \mathcal{X} \subseteq \mathbb{R}^n, \quad w(t) \in \mathcal{W} \subseteq \mathbb{R}^d, \quad (2)$$

where $\mathcal{X}$ and $\mathcal{W}$ are polyhedra that contain the origin in their interiors.

Definition 1: (Reachable set for autonomous systems) *we define the one-step robust reachable set for initial states x contained in the set $\mathcal{S}$ as*

$$\begin{aligned} \operatorname{Reach}_{f_a}(\mathcal{S}, \mathcal{W}) \triangleq \{&x \in \mathbb{R}^n | \\ &\exists\, x(0) \in \mathcal{S},\ \exists\, w \in \mathcal{W} : x = f_a(x(0), w)\}. \end{aligned} \quad (3)$$

For the nominal system, i.e., with $w(t) = 0$, $\forall t$, the one-step reachable set is defined as

$$\operatorname{Reach}_{f_a}(\mathcal{S}) \triangleq \{x \in \mathbb{R}^n | \exists\, x(0) \in \mathcal{S} : x = f_a(x(0))\}. \quad (4)$$

Similarly, for systems with inputs

$$x(t+1) = f(x(t), u(t), w(t)) = \mathbf{A}x(t) + \mathbf{B}u(t) + w(t), \quad (5)$$

subject to the constraints

$$x(t) \in \mathcal{X}, \quad u(t) \in \mathcal{U} \subseteq \mathbb{R}^m, \quad w(t) \in \mathcal{W}, \quad (6)$$

the one-step robust reachable set is defined as follows.

Definition 2: (Reachable set for systems with external inputs) *the one-step robust reachable set for initial states x contained in the set $\mathcal{S}$ is*

$$\begin{aligned} &\operatorname{Reach}_{f}(\mathcal{S}, \mathcal{W}) \triangleq \{x \in \mathbb{R}^n | \\ \exists\, x(0) \in \mathcal{S},\ &\exists\, u \in \mathcal{U},\ \exists\, w \in \mathcal{W} : x = f_a(x(0), u, w)\}. \end{aligned} \quad (7)$$

Therefore, all states contained in $\mathcal{S}$ are mapped into the reach set $\operatorname{Reach}_{f_a}$ under the map f_a for all disturbances $w \in \mathcal{W}$, and under the map f for all inputs $u \in \mathcal{U}$ and all disturbances $w \in \mathcal{W}$. We will next define *robust invariant sets*. Robust invariant sets are computed for the autonomous system (1)-(2). We define the robust invariant set as follows:

Definition 3: (Robust Positive Invariant Set) *A set $\mathcal{Z} \subseteq \mathcal{X}$ is said to be a robust invariant set for the autonomous system* (1) *subject to the constraints in* (2)*, if*

$$x(0) \in \mathcal{Z} \ \Rightarrow\ x(t) \in \mathcal{Z}, \quad \forall w(t) \in \mathcal{W}, \quad \forall t \in \mathbb{N}^+ \quad (8)$$

Definition 4: (Maximal Robust Positive Invariant Set) *The set $\mathcal{Z}_\infty \subseteq \mathcal{X}$ is the maximal robust invariant set for the autonomous system* (1) *subject to the constraints in* (2)*, if $\mathcal{Z}_\infty$ is a robust invariant set and $\mathcal{Z}_\infty$ contains all positive invariant sets contained in $\mathcal{X}$ that contain the origin.*

Two important operations for the discussion to follow are the Pontryagin difference and the Monkowski sum. The Pontryagin difference of two polytopes $\mathcal{P}$ and $\mathcal{Q}$ is a polytope

$$\mathcal{P} \ominus \mathcal{Q} := \{x \in \mathbb{R}^n \ :\ x + q \in \mathcal{P},\ \forall q \in \mathcal{Q}\}, \quad (9)$$

and the Minkowski sum of $\mathcal{P}$ and $\mathcal{Q}$ is a polytope

$$\mathcal{P} \oplus \mathcal{Q} := \{x + q \in \mathbb{R}^n \ :\ x \in \mathcal{P},\ q \in \mathcal{Q}\}. \quad (10)$$

B. Background on Robust MPC

In this section we outline the framework used to develop the robust model predictive controller in section VI. We follow a notation similar to [18]. The control problem is divided into two components: (1) a feedforward control input computed for the nominal system and (2) a linear state feedback controller that acts on the error between the actual state and the predicted nominal state. We denote the control sequence and the disturbance sequence for system (5)-(6) as $\mathbf{u} = \{u_0, u_1, ..., u_{N-1}\}$ and $\mathbf{w} = \{w_0, w_1, ..., w_{N-1}\}$ for $t = 0...N-1$. Let $\Phi(t; x, \mathbf{u}, \mathbf{w})$ denote the solution of (5) at time t controlled by $\mathbf{u}$ when $x(0) = x$. Furthermore, let $\bar{\Phi}(t, x, \bar{\mathbf{u}})$ denote the solution of the *nominal* system

$$\bar{x}(t+1) = \mathbf{A}\bar{x}(t) + \mathbf{B}\bar{u}(t) \quad (11)$$

at time t controlled by the nominal control sequence $\bar{\mathbf{u}} = \{\bar{u}_0, \bar{u}_1, ..., \bar{u}_{N-1}\}$ when $x(0) = x$. Denote the predicted nominal state trajectory by $\bar{\mathbf{x}} = \{\bar{x}_0, \bar{x}_1, ..., \bar{x}_{N-1}\}$. We write the controller as

$$v(t) = \bar{u}(t) + K(x(t) - \bar{x}(t)) \quad (12)$$

where $\bar{u}(t)$ is the feedforward component for the nominal system and $K(x(t) - \bar{x}(t))$ is the linear state feedback component acting on the error between the actual state and the predicted nominal state. We make use of the following assumption.

Assumption 1: (Stabilizing Disturbance Rejection Controller) The linear state feedback gain $K \in \mathbb{R}^{m \times n}$ in (12) is chosen such that $A_K = A + BK$ is Hurwitz.

Using the above definitions we can formulate the following Proposition. The details can be found in [18].

Proposition 1: Suppose that Assumption 1 is satisfied and that $\mathcal{Z}$ is a robust positively invariant set for the perturbed system (5)-(6) with control law (12). If $x \in \{\bar{\mathbf{x}}\} \oplus \mathcal{Z}$, then $x(t+1) \in \{\bar{\mathbf{x}}(t+1)\} \oplus \mathcal{Z}$ for all admissible disturbance sequences $w(t) \in \mathcal{W}$.

Proposition 1 states that if the control law (12) is used it will keep the states $x(t) = \Phi(t; x, \mathbf{u}, \mathbf{w})$ of the uncertain system (5) within the robust positive invariant set $\mathcal{Z}$ centered on the predicted state trajectory $\bar{\Phi}(t, x, \bar{\mathbf{u}})$ of the nominal system (11) for all admissible disturbance sequences $\mathbf{w}$:

$$x(0) \in \{\bar{x}_0\} \oplus \mathcal{Z} \ \Rightarrow\ x(t) \in \{\bar{x}_t\} \oplus \mathcal{Z} \quad \forall w(t) \in \mathcal{W},\ \forall t \geq 0, \quad (13)$$

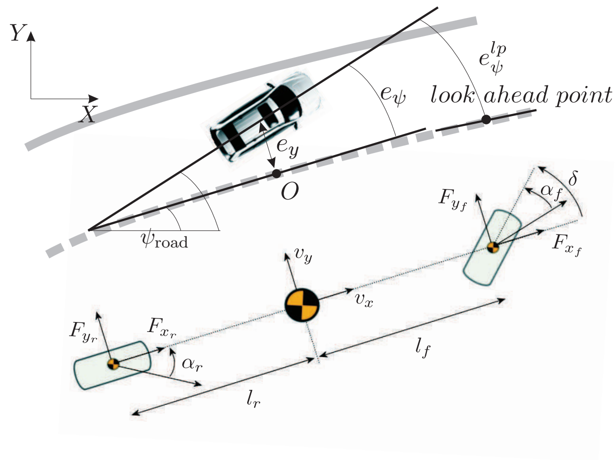

Fig. 1. Modeling notation.

where $x(0)$ and $\bar{x}_0$ are the initial states of (5) and (11). Proposition 1 suggests that if the optimal control problem for the *nominal* system (11) is solved for the tightened constraints

$$\bar{\mathcal{X}} = \mathcal{X} \ominus \mathcal{Z}, \qquad \bar{\mathcal{U}} = \mathcal{U} \ominus K\mathcal{Z}, \tag{14}$$

then the use of the control law (12) will ensure persistent constraint satisfaction for the controlled *uncertain* system (5) [18].

III. Modeling

In this section the models utilized for control are introduced. The dynamic equations of the vehicle are presented in III-A and the driver model used for prediction of driver behavior is introduced in III-B. In III-C the vehicle and driver models are combined to form a closed-loop model the incorporates both the vehicle and driver behavior.

A. Vehicle Model

The error dynamics of a vehicle are linear with respect to the lateral motion within the lane by assuming a constant velocity, V_x, and constraining the slip angles, α_i, to operate in the linear region of the tire forces. The differential equations describing the motion are compactly written as,

$$\dot{x}(t) = \mathbf{A}\, x(t) + \mathbf{B}\, u(t) + \mathbf{E}\dot{\psi}_{\text{road}}(t) \tag{15}$$

where the state is $x(t) = [e_y,\, \dot{e}_y,\, e_\psi,\, \dot{e}_\psi]^T \in \mathbb{R}^{4\times 1}$ at time t and $x_0 = x(0)$, the control input $u = \delta$ is the steering angle command and the system matrices $\mathbf{A} \in \mathbb{R}^{4\times 4}$, $\mathbf{B} \in \mathbb{R}^{4\times 1}$, and $\mathbf{E} \in \mathbb{R}^{4\times 1}$ are detailed in [2]. we denote by m and I_z the vehicle mass and yaw inertia. e_ψ and e_y denote the vehicle orientation and lateral position, respectively, in a road aligned coordinate frame. The tire cornering stiffness is denoted $C_{\alpha f}$ and $C_{\alpha r}$ for the front and rear tires, respectively. The lateral tire force components in the vehicle body frame are modeled as,

$$F_{yi} = -C_{\alpha i}\alpha_i, \qquad i \in \{f, r\} \tag{16}$$

where α_i is the slip angle at wheel i. We assume only the steering angles of the front wheels can be controlled, i.e., $\delta_f = \delta$ and $\delta_r = 0$. In addition, an actuator which corrects the driver commanded steering angle, such that $\delta = \delta_d + \delta_c$, is available, where δ_d is the driver commanded steering angle and δ_c is the correcting steering angle component. See the reference in [2] for more details of the vehicle model.

B. Uncertain Driver Model

We utilize a model of the driver's steering behavior. In general, an accurate description of the driver's behavior requires complex models accounting for a large amount of exogenous signals [19], [20]. We are interested in very simple model structures, enabling the design of a low complexity model-based threat assessment and control design algorithm. In this paper the driver's steering behavior is described by a model, where the vehicle state and the road geometry information are exogenous signals, the steering angle is the model output and the steering model parameters are estimated based on the observed behavior of the driver. The modeling and estimation of the driver behavior considered in this paper was presented in [7]. In this paper we extend the model to include the uncertain characteristics.

Define the orientation error e_ψ^{lp}, w.r.t. a look-ahead point as in Figure 1,

$$e_\psi^{lp} = \psi - \psi_{\text{road}}^{lp} = e_\psi + \Delta\psi_{\text{road}}, \tag{17}$$

where ψ_d^{lp} is the desired orientation at time $t + t_{lp}$, with t the current time, $\Delta\psi_d = \psi_d - \psi_d^{lp}$ and t_{lp} the preview time that can be mapped into the preview distance d_{lp} under the assumption of constant speed v_x. We compute an estimate of the driver commanded steering angle $\hat{\delta}_d$ as,

$$\hat{\delta}_d = K_y e_y + K_\psi e_\psi^{lp} = K_y e_y + K_\psi e_\psi + K_\psi \Delta\psi_{\text{road}}, \tag{18}$$

with K_y and K_ψ as gains that are, in general, time varying and are updated online. Clearly, $\Delta\psi_{\text{road}}$ in (17) depends on the preview time t_{lp} that, in our modeling framework, is considered as a parameter of the driver model. We also remark that the steering model (18) is velocity dependant since $\Delta\psi_{\text{road}}$ also depends on the vehicle speed v_x.

Estimation results of the driver model parameters in (17)-(18), obtained using a nonlinear recursive least squares algorithm, are presented in [7] for both normal and aggressive driving styles. We use the value of $\hat{\delta}_d$ obtained in (18) as a linear state-dependent estimate of the driver's steering input. The actual value of δ_d is assumed to lie in an interval centered at $\hat{\delta}_d$. Then,

$$\mathcal{W}(x) = \left\{ \delta_d : \|\delta_d - \hat{\delta}_d\| \le \epsilon > 0,\ \|\delta_d\| \le \delta_{d,\max} \right\}, \tag{19}$$

where ϵ is a parameter that must be chosen. The constraint $\delta_d \in \mathcal{W}(x)$ can also be expressed in terms of a polytopic contraint in $\mathbb{R}$, independent of x, by using a conservative approximation. That is,

$$\delta_d \in \mathcal{W}(x) \Rightarrow \delta_d \in \{\hat{\delta}_d \oplus \mathcal{W}\} \subseteq \mathbb{R} \tag{20}$$

The feedback equation for the driver model is compactly written as

$$u(t) = \mathbf{F}x(t) + \mathbf{G}\Delta\psi_{\text{road}} + w(t) + v(t) \qquad (21)$$

where $w(t) \in \mathcal{W}$, $\mathbf{F} = [K_y,\, 0,\, K_\psi,\, 0] \in \mathbb{R}^{1\times 4}$ and $\mathbf{G} = [K_\psi] \in \mathbb{R}$. Clearly, $u(t) = \hat{\delta}_d(t) + w(t) + v(t)$ where $v(t)$ has been introduced as an exogenous input signal and will be determined by the robust control law (12).

C. Driver-in-the-Loop Vehicle Model

We write the model (15), in closed-loop with the uncertain driver model (21), as

$$\dot{x}(t) = \mathbf{A}_{dm}\, x(t) + \mathbf{B}\, v(t) + \mathbf{E}_{dm} p(t) + \mathbf{B} w(t) \qquad (22)$$

where $\mathbf{A}_{dm} = (\mathbf{A} + \mathbf{B}\mathbf{F}) \in \mathbb{R}^{4\times 4}$ is the closed-loop system matrix, $\mathbf{E}_{dm} \in \mathbb{R}^{4\times 2}$ is the augmented parameter matrix where $\mathbf{E}_{dm} = [\mathbf{E} \;\; \mathbf{B}\mathbf{G}]$, $p(t) = [\dot{\psi}_{\text{road}} \;\; \Delta\psi_{\text{road}}]^T \in \mathbb{R}^{2\times 1}$, and $w(t)$ is the bounded additive disturbance vector. By propagating the state according (22) a prediction that incorporates both the vehicle dynamics and the driver's behavior is obtained. By using the definitions provided in section II-A and the framework introduced in section II-B, $v(t)$ is chosen to provide robust guarantees on constraint satisfaction in the presence of uncertain driver behavior.

IV. Robust Invariant Set Computation

The objective of the robust model predictive controller is to determine a corrective steering action to keep the driver safe in the presence of uncertain driver input (20). In this section we derive the robust control law for the uncertain system defined in (22). The robust analysis is done off-line and the notion of robust invariant sets is important for the discussion to follow.

Recall control law (12). The choice of the stabilizing state feedback gain matrix K will determine the size of the robust invariant set. In this paper we choose K as the optimal infinite horizon LQR solution K^{∞}_{LQR}. Then,

$$v(t) = \bar{u}(t) + K^{\infty}_{\text{LQR}}(x(t) - \bar{x}(t)). \qquad (23)$$

We can then compute the robust positive invariant set $\mathcal{Z}$ needed to calculate the tightened constraints $\bar{\mathcal{X}}$ and $\bar{\mathcal{U}}$ defined in (14). Note that $\mathcal{Z}$ is dependent upon the choice of K.

The robust invariant set $\mathcal{Z}$ is used to determine the tightened constraints for the nominal system. Algorithm 1 will calculate the reachable set $\mathcal{Z}$ (definition 3) if it converges in a finite number of steps. For the problem to be well-posed we make the assumption that the tightened constraints $\bar{\mathcal{X}}$ and $\bar{\mathcal{U}}$ exist and contain the origin. For this assumption to hold it is required that $\mathcal{W}$ is sufficiently small. Clearly there is a design trade-off between disturbance rejection properties (large K) and the size of the tightened constraints.

Algorithm 1 Computation of $\mathcal{Z}$

Input: $f_a,\ \mathcal{X}_0,\ \mathcal{W}$
Output: $\mathcal{Z}$
1: Let $\Omega_0 = \mathcal{X}_0$
2: Let $\Omega_{k+1} = \text{Reach}_{f_a}(\Omega_k, \mathcal{W}) \cup \Omega_k$
3: **if** $\Omega_{k+1} = \Omega_k$ **then**
4: $\quad\mathcal{Z} \leftarrow \Omega_{k+1}$
5: **else**
6: $\quad$*GOTO* 2.
7: **end if**

V. System Constraints

We recall that the overall aim of the safety system proposed in this paper is to keep the vehicle in the lane while maintaining a stable vehicle motion. In this section, we express the requirements that the vehicle stays in the lane while operating in a stable operating region as constraints on the vehicle state and input variables.

Let e_{y_i} be the distances of the four vehicle corners from the lane centerline. The requirement that the vehicle stays in the lane is then expressed as,

$$\bar{e}_{y_{\min}} \le e_{y_i} \le \bar{e}_{y_{\max}},\ \ i \in \{1, 2, 3, 4\}. \qquad (24)$$

where $\bar{e}_{y_{\min}}$ and $\bar{e}_{y_{\max}}$ are derived from the tightened constraint set $\bar{\mathcal{X}}$ projected onto the state e_y, $[\bar{e}_{y_{\min}},\ \bar{e}_{y_{\max}}] = \text{Proj}_{e_y}(\bar{\mathcal{X}})$. In addition to staying in the lane, we require that the vehicle operates in a region of the state space where the vehicle is easily maneuverable by a normally skilled driver. By limiting the slip angles to the linear region of the lateral tire force characteristics the vehicle behavior is predictable by most drivers and Electronic Stability Control (ESC) systems are inactive [21], [22]. The requirement that the vehicle operates in stable operating conditions is thus ensured by limiting the tire slip angles α_i,

$$\alpha_{i_{\min}} \le \alpha_i \le \alpha_{i_{\max}},\ \ \forall i. \qquad (25)$$

The constraints (24)-(25) are compactly written as,

$$h(x, v) \le \mathbf{0}, \qquad (26)$$

where $\mathbf{0}$ is the zero vector with appropriate dimension.

VI. Robust Predictive Control Problem

In this section we formulate the threat assessment and control problems as a Model Predictive Control Problem (MPC). At each sampling time instant an optimal input sequence is calculated by solving a constrained finite time optimal control problem. The computed optimal control input sequence is only applied to the plant during the following sampling interval. At the next time step the optimal control problem is solved again, using new measurements.

We discretize the closed-loop driver controlled system (22) with a fixed sampling time T_s and formulate the nominal optimization problem with tightened constraints, to be solved at each time instant, as

$$\min_{\bar{\mathbf{u}},\varepsilon} \sum_{k=0}^{N-1} ||\bar{u}_{t+k,t}||_Q^2 + ||\Delta\bar{u}_{t+k,t}||_R^2 + \rho\varepsilon \tag{27a}$$

$$s.t.\ \bar{x}_{t+k+1,t} = f_{\text{dm}}(\bar{x}_{t+k,t}, \bar{u}_{t+k,t}), \tag{27b}$$

$$h_t(\bar{x}_{t+k,t}, \bar{u}_{t+k,t}) \leq \mathbf{1}\varepsilon, \quad \varepsilon \geq 0, \tag{27c}$$

$$\bar{u}_{t+k,t} = \Delta\bar{u}_{t+k,t} + \bar{u}_{t+k-1,t}, \tag{27d}$$

$$\bar{u}_{t+k,t} \in \bar{\mathcal{U}}, \tag{27e}$$

$$\Delta\bar{u}_{t+k,t} \in \Delta\bar{\mathcal{U}}, \tag{27f}$$

$$\bar{u}_{t-1,t} = \bar{u}(t-1), \tag{27g}$$

$$\bar{x}_{t,t} = \bar{x}(t), \tag{27h}$$

where t denotes the current time instant and $\bar{x}_{t+k,t}$ denotes the predicted state at time $t+k$ obtained by applying the control sequence $\bar{\mathbf{u}} = \{\bar{u}_{t,t}, \ldots, \bar{u}_{t+k,t}\}$ to system (27d) with $\bar{x}_{t,t} = \bar{x}(t)$. N denotes the prediction horizon. The safety constraints (26) have been imposed as soft constraints, by introducing the slack variable ε in (27a) and (27c). Q, R and ρ are weights of appropriate dimension penalizing control action, change rate of control, and violation of the soft constraints, respectively.

We note that no penalty on deviation from a tracking reference is imposed in the cost function (27a). The objective here is to ensure that the safety constraints (26) are not violated, while utilizing minimal control action. If the driver steering model (18) predicts the vehicle will not violate the safety constraints (26), no control action will be applied and the optimal cost will thus be zero.

VII. Results

In this section the results from simulations are presented. The model predictive control problem is solved using Tomlab/NPSOL at each time step. The off-line analyses to determine the robust invariant sets and solve for the tightened constraints was done by running Algorithm 1 in Matlab using MPT Toolbox. Table I lists the parameters used in the simulations. Two scenarios are considered, (1) where the driver approaches an obstacle on the right of the roadway, and (2) where the driver navigates between two obstacles obstructing the lane.

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Units	Parameter	Value	Units
m	2050	kg	N	15	-
I_z	3344	kg.m^2	e_y	[0,-5]	m
μ	1	-	$\bar{e}_y$	[-0.36,-4.63]	m
C_α	80,000	-	u	[0.2,-0.2]	rad
T_s	50	ms	$\bar{u}$	[0.16,-0.16]	rad
Q, R	1,1	-	α	[4°,-4°]	deg
ρ	10^4	-	$\mathcal{W}$	[0.1,-0.1]	rad

A. Single Obstacle

Figure 2 captures a snapshot of the moment the model predictive controller must add corrective steering action. Two trajectories are shown. Trajectory 2 is the one predicted by the

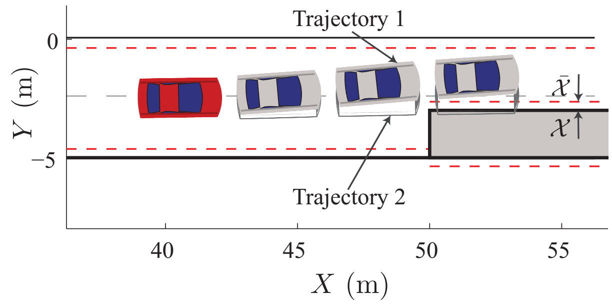

Fig. 2. Trajectory 1 is assisted by the controller to keep the driver safe. Trajectory 2 is the expected driver input and collides with the obstacle.

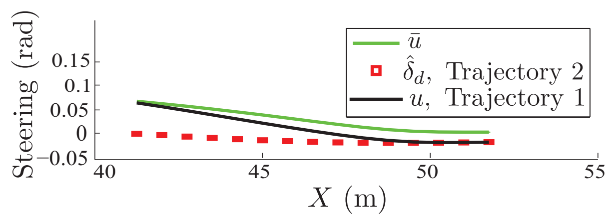

Fig. 3. The inputs in Trajectory 1 and 2 as well as the corrective action $\bar{u}$ determined by the controller.

nominal driver model only, in Equation (18), and is depicted by the vehicles in outline. This is the expected steering input predicted for the driver. Clearly, the vehicle is predicted to collide with the obstacle, denoted by the constraint $\mathcal{X}$. Trajectory 1 is the corrected trajectory and ensures satisfaction for the tightened constraints $\bar{\mathcal{X}}$ for any disturbance in the predicted driver model, $w \in \mathcal{W}$. The tightened constraints are shown in dashes and the drawn vehicles show the predicted trajectory with the corrective action. Figure 3 plots the inputs for Trajectory 1 and 2. $\bar{u}$ is the added corrective action from the solution to the nominal optimization problem, $\hat{\delta}_d$ is the nominal driver model from Trajectory 2, and u is the final augmented steering from Trajectory 1.

B. Multiple Obstacles

In this section we simulate the proposed controller during a scenario where the vehicle approaches two obstacles. Similar to the scenario detailed in section VII-A, the vehicle encounters an obstacle in the road and the controller must intervene to keep the driver safe. Immediately after the first obstacle the vehicle encounters a second, an intervention is again needed to ensure the safety constraints are satisfied.

In Figure 4 the nominal trajectory, $\bar{x}$, as well as the disturbed trajectory, x, are shown. Boxes are plotted along the trajectory to show the geometry of the vehicle at various points in time. In addition, a sketch $\mathcal{Z}^{e_y} = \text{proj}_{e_y}(\mathcal{Z})$ is plotted to illustrate the size of the robust invariant set in the e_y dimension. The tightened constraints $\bar{\mathcal{X}}$ are shown in dashed lines. In Figure 5 the calculated inputs are shown for the scenario presented in Figure 4. $\hat{\delta}_d$ is the nominal steering angle determined by the driver model, $\bar{u}$ is the corrective action calculated by the nominal MPC problem, and u_{nom} is the final augmented input for the nominal trajectory, $\bar{x}$. Further, the disturbed input,

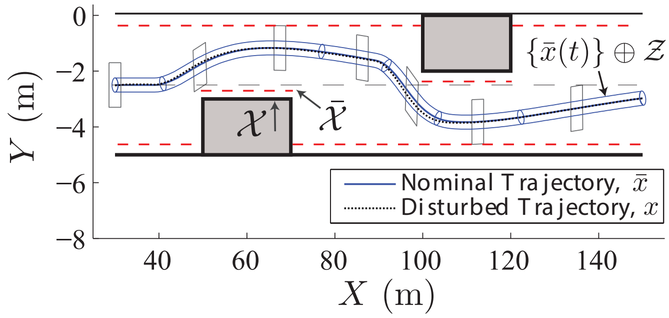

Fig. 4. A plot of the nominal trajectory, $\bar{x}$, the disturbed trajectory, x, and a projection of the robust invariant set along the nominal trajectory.

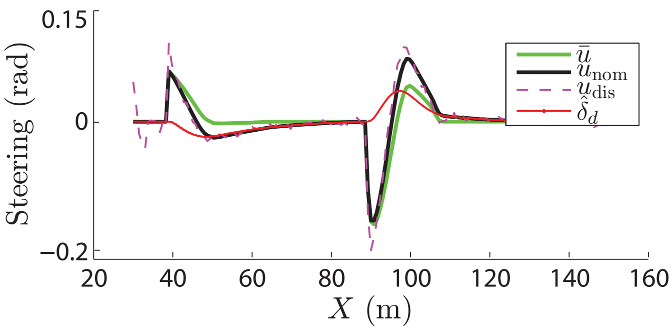

Fig. 5. The inputs showing the corrective action from the controller, $\bar{u}$, the input from the driver model augmented with the controller action, u_{nom}, the input with additive disturbance, u_{dis}, and the steering from the driver model, $\hat{\delta}_d$.

u_{dis}, is plotted. It is clear to see the disturbed trajectory is contained within the robust invariant set around the nominal state trajectory.

VIII. Conclusion

In this paper a robust control framework is proposed for lane departure avoidance and obstacle collision avoidance for semi-autonomous ground vehicles. The framework formulates this problem as a model predictive control problem. A vehicle model is simulated in closed-loop with an uncertain driver model to obtain a prediction of the driver's future trajectory. A robust positive invariant set is found for a given control law and Robust MPC is used to tighten the original input and state constraints to ensure constraint satisfaction even in the presence of uncertain driver behavior. Various scenarios are simulated where the driver approaches multiple obstacles in the roadway. An optimization problem is solved to find the minimal amount of corrective action to keep the driver safe while still satisfying the safety constraints in the presence of uncertainty. The promising results of the simulations presented motivate an effort to study the behavior of the proposed controller in an experimental setting.

References

[1] F. Biral, M. D. Lio, and R. Lot, "Vehicle and driver modeling and threat assessment for driving support functions," *Atti XX Congresso dell'Associazione Italiana di Meccanica Teorica e Applicata*, 2011.

[2] R. Rajamani, "Vehicle dynamics and control, 1st ed." *Springer US*, 2005.

[3] M. Distner, M. Bengtsson, T. Broberg, and L. Jakobsson, "City Safety - A System Addressing Rear-End Collisions at Low Speeds," *In Proc. 21st International Technical Conference on the Enhanced Safety of Vehicles*, 2009.

[4] J.-F. Liu, J.-H. Wu, and Y.-F. Su, "Development of an interactive lane keeping control system for vehicle," *Vehicle Power and Propulsion Conference*, 2007.

[5] S. Mammar, S. Glaser, and M. Netto, "Time to Line Crossing for Lane Departure Avoidance: A Theoretical Study and an Experimental Setting," *IEEE Transactions on Intelligent Transportation Systems*, vol. 7, no. 2, pp. 226–241, 2006.

[6] J. Jansson and F. Gustafsson, "A framework and automotive application of collision avoidance decision making," *Automatica*, vol. 44, no. 9, pp. 2347–2351, 2008.

[7] P. Falcone, M. Ali, and J. Sjöberg, "Predictive Threat Assessment via Reachability Analysis and Set Invariance Theory," *IEEE Transactions on Intelligent Transportation Systems*, vol. 12, no. 4, pp. 1352–1361, 2011.

[8] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat, "Predictive Active Steering Control for Autonomous Vehicle Systems," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 3, pp. 566–580, 2007.

[9] Y. Kuwata, J. Teo, G. Fiore, S. Karaman, E. Frazolli, and J. P. How, "Real-time Motion Planning with Applications to Autonomous Urban Driving," *IEEE Transactions on Control Systems Technology*, vol. 17, no. 5, pp. 1105–1118, 2009.

[10] C. Zhao, W. Xiang, and P. Richardson, "Vehicle Lateral Control and Yaw Stability Control through Differential Braking," *In Proc. IEEE Industrial Electronics Symposium*, 2006.

[11] J. C. Gerdes, E. J. Rossetter, and U. Saur, "Combining Lanekeeping and Vehicle Following with Hazard Maps," *Vehicle System Dynamics*, vol. 36, no. 4-5, pp. 391–411, 2001.

[12] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat, "Low complexity mpc schemes for integrated vehicle dynamics control problems," 9^{th} *International Symposium on Advanced Vehicle Control*, 2008.

[13] S. J. Anderson, S. C. Peters, T. E. Pilutti, and K. Iagnemma, "An optimal-control-based framework for trajectory planning , threat assessment , and semi-autonomous control of passenger vehicles in hazard avoidance scenarios," *Int. J. Vehicle Autonomous Systems*, vol. 8, pp. 190–216, 2010.

[14] A. Gray, M. Ali, Y. Gao, J. K. Hedrick, and F. Borrelli, "Integrated threat assessment and control design for roadway departure avoidance," *Intelligent Transportation Systems Conference*, 2012.

[15] A. Gray, M. Ali, Y. Gao, J. Hedrick, and F. Borrelli, "Semi-autonomous vehicle control for road departure and obstacle avoidance," *IFAC Control of Transportation Systems*, 2012.

[16] W. Langson, I. Chryssochoos, S. V. Rakovic, and D. Mayne, "Robust model predictive control using tubes," *Automatica*, vol. 40(1), pp. 125–133, 2004.

[17] E. C. Kerrigan, "Robust Constraint Satisfaction: Invariant Sets and Predictive Control," 2000.

[18] D. Q. Mayne, M. M. Seron, and S. V. Rakovic, "Robust model predictive control of constrained linear systems with bounded disturbances," *Automatica*, vol. 41(2), pp. 219–224, 2005.

[19] C. Cacciabue, *Modelling Driver Behaviour in Automotive Environments*. Springer, 2007.

[20] R. Vasudevan, V. Shia, Y. Gao, R. Cervera-Navarro, R. Bajcsy, and F. Borrelli, "Safe semi-autonomous control with enhanced driver modeling," *American Controls Conference*, 2012.

[21] T. D. Gillespie, *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers, Inc, 1992.

[22] H. E. Tseng, B. Ashrafi, D. Madau, T. A. Brown, and D. Recker, "The development of vehicle stability control at Ford," *IEEE/ASME Transactions on Mechatronics*, vol. 4, no. 3, pp. 223–234, Sept. 1999.