STAT545 Midterm1 Bonus

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11/16/2018

Dynamic programming

Implement the overall algorithm to compute the cost of $A_1 \cdot A_2 \cdot A_3 \cdots A_N$.

```r
# FORWARD FUNCTION TO CALCULATE THE COMPLEXITY OF CHAINE MULTIPLICATION OF MATRIX
# mdim is the matrix dimension array
Mult_Matrix_Chain <- function(mdim){
  n <- length(mdim)
  cost <- matrix(rep(0,n^2), nrow = n)
  ind_interrupt <- matrix(rep(0,n^2), nrow = n)

  for (clip in seq(2,n-1)){
    for (i in seq(1,n-clip)){
      k <- i + clip - 1
      cost[i,k] <- Inf
      for (j in seq(i,k-1)){
        temp <- cost[i,j] + cost[j+1,k] + mdim[i]*mdim[j+1]*mdim[k+1]
        if(temp < cost[i,k]){
          cost[i,k] <- temp
          ind_interrupt[i,k] <- j
        }
      }
    }
  }
  return(list(Optimal_total_cost=cost[1,n-1], index_interrupt=ind_interrupt))
}

# helper function to print out the multiplicative order
printOrder <- function(i, k, Ind_interrupt){
  if (i == k){
    cat(paste0('A',i))
  }
  else {
    cat('(')
    printOrder(i, Ind_interrupt[i,k], Ind_interrupt)
    printOrder(Ind_interrupt[i,k]+1, k, Ind_interrupt)
    cat(')')
  }
}
```

Let's test this function as follows

```r
mdim <- c(2,2,2)
rslt <- Mult_Matrix_Chain(mdim)
rslt
```

```
## $Optimal_total_cost
## [1] 8
##
```

```
## $index_interrupt
##      [,1] [,2] [,3]
## [1,]    0    1    0
## [2,]    0    0    0
## [3,]    0    0    0
```

```r
printOrder(1,length(mdim)-1,rslt$index_interrupt)
```

```
## (A1A2)
```

This is the multiplication of two 2×2 matrix.

```r
mdim <- c(1,2,3,4,5)
rslt <- Mult_Matrix_Chain(mdim)
rslt
```

```
## $Optimal_total_cost
## [1] 38
##
## $index_interrupt
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    0
## [2,]    0    0    2    3    0
## [3,]    0    0    0    3    0
## [4,]    0    0    0    0    0
## [5,]    0    0    0    0    0
```

```r
printOrder(1,length(mdim)-1,rslt$index_interrupt)
```

```
## (((A1A2)A3)A4)
```

This is the multiplication of four matrices: $A_1 \in \mathcal{R}^{1\times 2}, A_2 \in \mathcal{R}^{2\times 3}, A_3 \in \mathcal{R}^{3\times 4}$ and $A_4 \in \mathcal{R}^{4\times 5}$.