

1. Say whether the following is true or false and support your answer by a proof.

$$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$$

Opening: We will prove that this proposition is false by a **direct proof**.

$$3m + 5n = 12$$

$$5n = 12 - 3m$$

$$5n = 3(4 - m)$$

Then it is clear that  $0 < m < 4$  as both  $m$  and  $n$  should be natural numbers.

Pick  $m$  starting from 1 to 3.

If  $m = 1$  then  $5n = 3(4 - 1)$ , then  $n = 9/5$  but this can not be true as  $9/5$  is not a natural number.

Else if  $m = 2$  then  $5n = 3(4 - 2)$ ,  $n = 6/5$  this also can not be true as  $6/5$  is not a natural number also.

Lastly if  $m = 3$  then  $5n = 3(4 - 3)$ ,  $n = 3/5$  like above  $3/5$  is not a natural number.

So it is clear that there is no  $m \in \mathbb{N}$  and  $n \in \mathbb{N}$  such that  $3m + 5n = 12$

**Then this statement is false**