Prove that for any natural number n,

$$\mathbf{2 + 2^2 + 2^3 + \cdots . + 2^n = 2^{n+1} - 2}$$

We will prove this proposition by **induction**,

<u>Base case:</u>

Consider this case as $\boldsymbol{P(n) = 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2}$

Since natural numbers start from 1, we will begin by 1 for P(n) and we will show that P(1) holds true.

$$\boldsymbol{P(1) = 2^1 = 2^{1+1} - 2 = 4 - 2 = 2}$$

Base case is proven to be true for P(1).

<u>Induction step:</u> We will show that if $\boldsymbol{P(n)}$ holds true it will also hold true for $\boldsymbol{P(n+1)}$.

$$\boldsymbol{P(n+1) = 2 + 2^2 + 2^3 + \cdots + 2^{n+1} = 2^{n+2} - 2}$$

$$\boldsymbol{P(n+1) = \boxed{2 + 2^2 + 2^3 + \cdots + 2^n} + 2^{n+1}}$$

This part is equal to $\boldsymbol{P(n) = 2^{n+1} - 2}$

Then $\boldsymbol{P(n+1) = 2^{n+1} - 2 + 2^{n+1} = 2(2^{n+1}) - 2}$

$$\boldsymbol{= 2^{n+2} - 2}$$

This concludes the inductive step.

Since both the base case and the inductive step are proven to be true this concludes the induction. **This proposition is true.**