A classic unsolved problem in number theory asks if there are infinitely many pairs of 'twin primes', pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.

Actually, the solution of the previous question provides some clues on this one.

We will prove this proposition by a **proof by contradiction**.

Proof by contradiction:

This statement can also be stated as

$$(n > 3)(\forall n \in \mathbb{N})(n, n+2 \text{ and } n+4 \text{ can not all be prime numbers})$$

Consider the reverse is true, then

$\neg[(n > 3)(\forall n \in \mathbb{N})(n, n+2 \text{ and } n+4 \text{ can not all be prime numbers})] =$

$(n > 3)(\exists n \in \mathbb{N})(n, n+2 \text{ and } n+4 \text{ are all prime numbers})$

Now if n is a prime number it should not be divisible by 3 since $n > 3$

Then it is either the of the form $3m + 1$ *or* $3m + 2$ *where* $m \in \mathbb{Z}$

But it is also clear that if n is of the form $3m + 1$ then $n + 2 = 3m + 3$ which is divisible by 3.

Else if n is of the form $3m + 2$ then $n + 4 = 3m + 6$ which is divisible by 3. Then if $n > 3$ at least one of these: $n, n + 2$ *or* $n + 4$ is divisible by 3. But this can not be true as the only prime that is divisible by 3 is 3.

As the converse of the statement can not be true, then this statement is true. This concludes the proof.