Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Proof: This proposition can be stated as the following

let $\boldsymbol{\varepsilon} > 0$ be given $(\forall \varepsilon > 0)(\exists n \in \mathbb{N})(\forall m > n)(|Ma_m - ML| < \varepsilon)$

Since $M > 0$, $|Ma_m - ML| = M|a_m - L| < \varepsilon$

Then $|a_m - L| < \frac{\varepsilon}{M}$

Now let $\boldsymbol{k} = \frac{\varepsilon}{M}$, since both ε and M are greater than 0, $\boldsymbol{k}$ is also greater than 0.

From the definition of the limit of a sequence $\{a_y\}_{y=1}^{\infty}$ tends to limit L as $y \to \infty$, means that **<u>for every</u>** $\boldsymbol{k} > 0$, there exists an integer x such that if $x > y$ then $|a_y - L| < \boldsymbol{k}$ then
$|Ma_y - ML| < M\boldsymbol{k}$

$|Ma_y - ML| < \boldsymbol{M}\frac{\varepsilon}{\boldsymbol{M}}$

$|Ma_y - ML| < \varepsilon$

This concludes our proof.