Say whether the following is true or false and support your answer by a proof: For any integer n, the number $n^2 + n + 1$ is odd.

Answer: This proposition is true and we will prove it by **induction**.

Proof by induction:

Base case: First we will show that the statement $P(n) = n^2 + n + 1$ is odd holds true for smallest integer $n = 0$.

$P(0) = 0^2 + 0 + 1 = \mathbf{1}$ which is clearly an odd number. So $P(0)$ is proven.

Inductive step: We will show that if *P(n)* holds true, then *P(n+1)* also holds true for any integer *n*.

Assume that for a particular *n*, *P(n)* is true. Then

$$P(n+1) = (n+1)^2 + (n+1) + 1$$
$$= n^2 + 2n + 1 + n + 1 + 1$$
$$= (n^2 + n + 1) + (2n + 2)$$

Here, the left hand part $(n^2 + n + 1)$ is odd as we assumed that P(n) is odd, while right hand part $(2n + 2)$ is clearly an even number for any integer *n*.

Since it is clear that the sum of an odd and an even number is odd *P*(*n*+*1*) is also odd.

This holds true for *P(n+1)* establishing the inductive step.

Conclusion: Since both the base case and inductive step have been proven true, by mathematical induction the statement "$P(n) = n^2 + n + 1$ is odd" holds true for every integer n.