Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).

Answer: This statement is true and we will prove it again by a **direct proof**.

Direct proof: This proposition can also be stated as

$$(\forall n \in \mathbb{Z})[5|\big(n + (n+1) + (n+2) + (n+3) + (n+4)\big)]$$

More simply

$$(\forall n \in \mathbb{Z})[5|(5n + 10)]$$

Then we can simplify the equation inside the parenthesis as

$$(5n + 10) = 5(n + 2)$$

If we divide $5(n + 2)$ by 5 the result is $(n + 2)$ which is clearly an integer as $n \in \mathbb{Z}$

Then it is clear that $(\mathbf{5n + 10})$ or $\mathbf{5(n + 2)}$is divisible by 5 and this proposition is true.