

# Program Analysis

Metrics, Soundness, Lattices, AI

YEGOR BUGAYENKO

Lecture #6 out of 10

90 minutes

All visual and text materials presented in this slidedeck are either originally made by the author or taken from public Internet sources, such as website. Copyright belongs to their respected authors.



Basics

Quality of Analysis

Lattice

Abstract Interpretation

## Chapter #1: Basics

## Syntactic & Semantic Properties

Semantic property can be completely defined with respect to the set of executions of a program, while a syntactic property can be decided directly based on the program text.

```
if (x) { printf("大家好"); }
```

Which properties are dynamic?

- A program may print a text to the console
- A program may call `printf()` C library function
- A program prints to the console
- A program consists of one line of code

## Rice's Theorem

Rice's theorem states that all non-trivial semantic properties of programs are undecidable.

A property is non-trivial if it is neither true for every partial computable function, nor false for every partial computable function.

Halting problem is the problem of determining, from 1) a description of an arbitrary computer program and 2) an input, whether the program will finish running, or continue to run forever. A general algorithm to solve the halting problem for all possible program–input pairs **cannot exist**.

## Non-trivial Properties

Examples of a non-trivial properties:

- A program exits
- A program prints “Hello”
- A program finishes in less than 5 seconds
- A program dies with “Segmentation Fault”
- A program prints user password to the console

Suggest a few properties.

## Static Analysis

Consider two C++ programs given to a static analyzer (e.g. Clang Tidy):

```
int f() {  
    int x = 0;  
    return 42 / x;  
}
```

```
int f(int x) {  
    return 42 / x;  
}
```

Expected answers from Clang Tidy:

**Yes! :)**

**No :(**

## Style Checking

Consider two C++ programs given to a style checker (e.g. cpplint):

```
int f (int x)
{
    return 42 / x;
}
```

```
int foo(int x) {
    return 42 / x;
}
```

Expected answers from cpplint:

Extra space before ( in  
function call ; { should  
almost always be at the end  
of the previous line

**No :(**



## Dynamic Analysis

Consider this C++ program (Clang Tidy finds no issues) given to a dynamic analyzer (AddressSanitizer):

```
int foo(int i) {
    int a[5];
    return a[i];
}

int main() {
    return foo(6);
}
```

```
$ gcc -fsanitize=address -g a.cpp
$ ./a.out
```

```
=====
==76375==ERROR: AddressSanitizer: stack-buffer-overflow on address 0x00016babf0d8
READ of size 4 at 0x00016babf0d8 thread T0
#0 0x104343e54 in foo(int) a.cpp:9
#1 0x104343f38 in main a.cpp:12
#2 0x1a07c7e4c (<unknown module>)

Address 0x00016babf0d8 is located in stack of thread T0 at offset 56 in frame
#0 0x104343cf0 in foo(int) a.cpp:7

This frame has 1 object(s):
[32, 52) 'a' (line 8) <== Memory access at offset 56 overflows this variable
```

Dynamic analysis == testing.

Chapter #2:

# Quality of Analysis

## Sound & Complete



## Precision & Recall

Precision is the fraction of relevant instances among the retrieved instances (100% precision means soundness).

Recall is the fraction of relevant instances that were retrieved (100% recall means completeness).

$$\text{Precision} = \frac{TP}{TP + FP} \quad \text{Recall} = \frac{TP}{TP + FN} \quad \text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$
$$F1 = \frac{2 \times TP}{2 \times TP + FP + FN}$$

# Experiment

Say, we give a few programs to a static analyzer:

|     |     |    |     |    |     |     |    |
|-----|-----|----|-----|----|-----|-----|----|
| a   | b   | c  | d   | e  | f   | g   | h  |
| Yes | Yes | No | Yes | No | Yes | Yes | No |

$$TP = \underline{\hspace{1cm}} \quad FP = \underline{\hspace{1cm}} \quad TN = \underline{\hspace{1cm}} \quad FN = \underline{\hspace{1cm}}$$

$$\text{Precision} = \frac{TP}{TP + FP} = \underline{\hspace{2cm}} \quad \text{Recall} = \frac{TP}{TP + FN} = \underline{\hspace{2cm}}$$

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN} = \underline{\hspace{2cm}} \quad F1 = \frac{2 \times TP}{2 \times TP + FP + FN} = \underline{\hspace{2cm}}$$

## Flip of Terminology

Soundness and Completeness: With Precision by Prof. Bertrand Meyer, in Blog@CACM: “It is very easy to obtain soundness if we forsake completeness: reject every case.”

## Chapter #3:

# Lattice

## Total Order

Total order is a binary relation  $\leq$  (strict total order is  $<$ ).

Lineary ordered set (loset) is a set equipped with a total order.

Which of them are losets?:

$\{1, -5, 2, 0, 42\}$

$\{3, 5, -9, 5, 12\}$

$\{3, 5, \text{"Hello"}, 12, 5.0\}$

$\{x, y, z\}$

$\emptyset$



## Partially Ordered Set

Partial order is total order but only between some elements.

Partially ordered set (poset) is a set equipped with a partial order.

Which of them are posets?:

$\{1, \text{"apple"}, 2, -7, \text{"orange"}\}$

$\{3, 5, -9, 5, 12\}$

$\{0, 1, 2, 3, \dots\}$

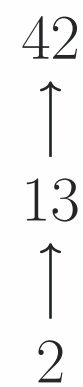
$\{x, y, z\}$

$\emptyset$

Lattice

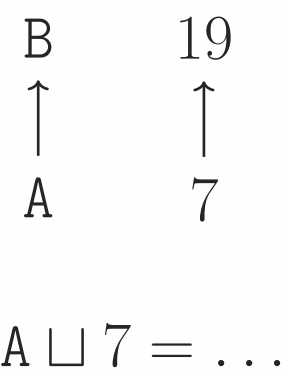
Lattice is a poset where each two elements  $(x, y)$  have least upper bound (join operator  $x \sqcup y$ ) and greatest lower bound (meet operator  $x \sqcap y$ ).

$\{42, 2, 13\}$

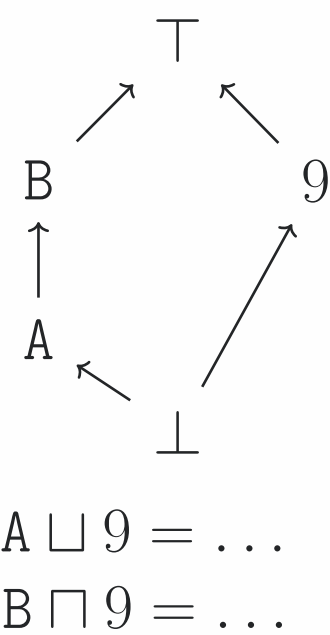


$42 \sqcup 2 = \dots$

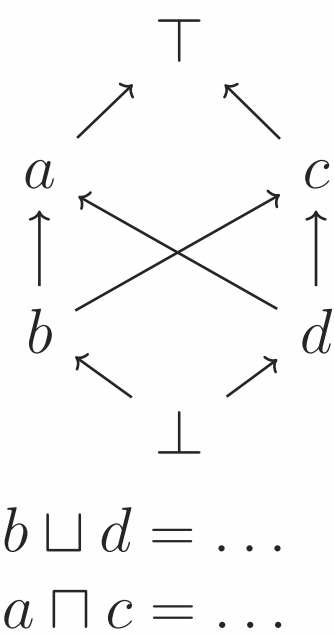
$\{A, 7, 19, B\}$



$\{A, \top, 9, B, \perp\}$



$\{\top, \perp, a, b, c, d\}$



## Intervals

A lattice may be used to represent intervals in a set of values, e.g. in  $\mathbb{Z}$ :



Partial order is  $\in$ .

Chapter #4:

# Abstract Interpretation

## What is it about?

There is a compromise to be made between the precision of the analysis and its decidability (computability), or tractability (computational cost).

## Over and Under Approximation

What is the square of this oval?



1) **over**-approximation



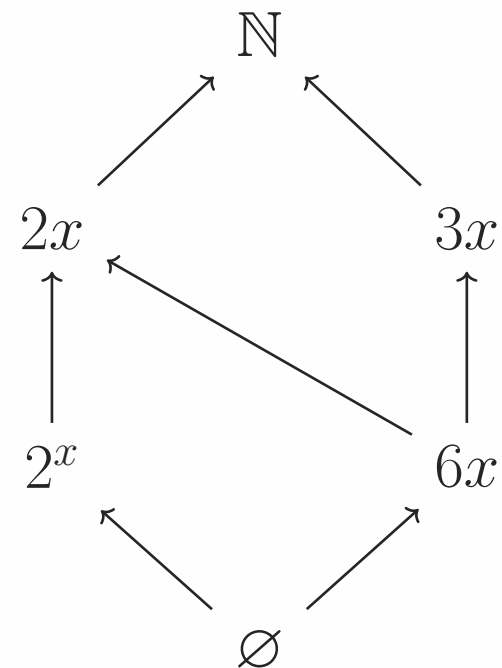
2) **under**-approximation

## Abstraction and Concretization

Concrete domain  $C$   
 $(\mathbb{Z}, <)$

12  
 $\uparrow$   
 7  
 $\uparrow$   
 4  
 $\uparrow$   
 0  
 $\uparrow$   
 -9

Abstract domain  $A$   
 $(\{s | \{n | n \in \mathbb{N}\}\}, \in)$



Abstraction function:  
 $\alpha(c) \rightarrow a$

Concretization function:  
 $\gamma(a) \rightarrow c$

Domains must be related  
 through Galois connection:  
 $\forall c \in C, \forall a \in A$   
 $\alpha(c) \sqsubseteq a \iff c \sqsubseteq \gamma(a)$

Are they?

## Abstract Semantics (Transformers)

Abstract domain:



Transformers:

$$\mathbb{N} + \mathbb{N} = \dots$$

$$2x + 2x = \dots$$

$$2x + 3x = \dots$$

$$2x \times 3x = \dots$$

$$\emptyset + 2x = \dots$$

Concrete counterparts:

$$1024 + 1 = \dots$$

$$46 + 4 = \dots$$

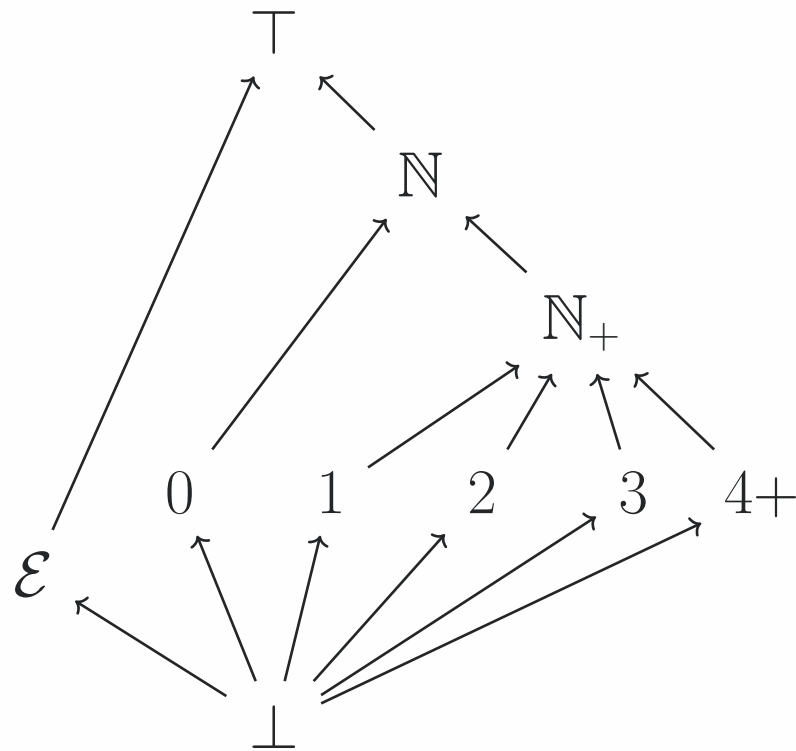
$$8 + 9 = \dots$$

$$6 \times 12 = \dots$$

$$-1 + 4 = \dots$$



# Widening and Narrowing



$$0 \nabla 1 = \dots$$

$$1 \triangleright \mathbb{N}_+ = \dots$$

$$0 \triangleright \mathbb{N}_+ = \dots$$

$$1 \triangle \mathbb{N}_+ = \dots$$

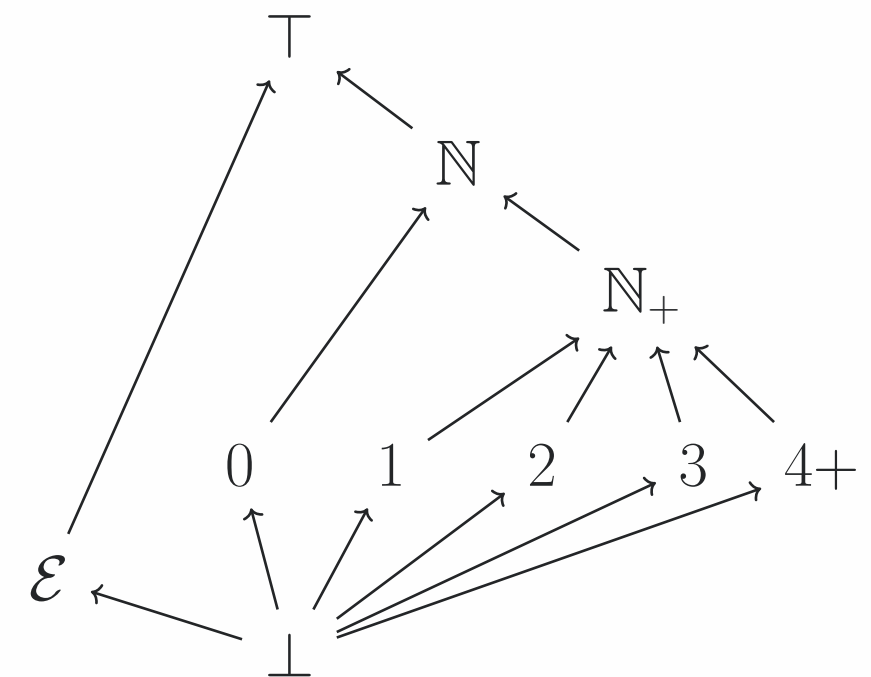
$$0 \triangle 1 = \dots$$

$$3 \triangle 4+ = \dots$$

## Fixed-Point Computation

Fixed-Point Computation is a repeated symbolic execution of the program using abstract semantics until approximation reaches an equilibrium.

```
int f(int x) {
  while x > 0 {
    x = x - 1;
    x = 2 / x;
  }
  return x;
}
```



## Further Reading/Watching

Lecture by Patrick Cousot, on [YouTube](#)

Mozilla wiki page on [Abstract Interpretation](#).

[Slide deck](#) of Işıl Dillig.

[Introduction to Abstract Interpretation](#) by Bruno Blanchet.