Program Analysis

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Lecture #6 out of 10
80 minutes

The slidedeck was presented by the author in this YouTube Video

Basics

Quality of Analysis

Lattice

Abstract Interpretation

Further Reading/Watching

Chapter #1:
Basics

[Property Rice Non-trivial Static Style Dynamic]

Syntactic & Semantic Properties

Semantic property can be completely defined with respect to the set of executions of a program, while a *syntactic* property can be decided directly based on the program text.

```
if (x) { printf("大家好"); }
```

Which properties are dynamic?

- A program may print a text to the console

- A program may call `printf()` C library function

- A program prints to the console

- A program consists of one line of code

[Property Rice Non-trivial Static Style Dynamic]

Rice's Theorem

Rice's theorem states that all non-trivial semantic properties of programs are undecidable.

A property is *non-trivial* if it is neither true for every partial computable function, nor false for every partial computable function.

Halting problem is the problem of determining, from 1) a description of an arbitrary computer program and 2) an input, whether the program will finish running, or continue to run forever. A general algorithm to solve the halting problem for all possible program–input pairs **cannot exist**.

Non-trivial Properties

Examples of a non-trivial properties:

- A program exits

- A program prints "Hello"

- A program finishes in less than 5 seconds

- A program dies with "Segmentation Fault"

- A program prints user password to the console

Suggest a few properties.

Static Analysis

Consider two C++ programs given to a *static analyzer* (e.g. Clang Tidy):

```
int f() {                              int f(int x) {
  int x = 0;                             return 42 / x;
  return 42 / x;                       }
}
```

Expected answers from Clang Tidy:

Yes! :)   **No :(**

[Property Rice Non-trivial Static Style Dynamic]

Style Checking

Consider two C++ programs given to a *style checker* (e.g. cpplint):

```
int f (int x)
{
  return 42 / x;
}
```

```
int foo(int x) {
    return 42 / x;
}
```

Expected answers from cpplint:

Extra space before (in   **No :(**
function call ; { should
almost always be at the end
of the previous line

[Property Rice Non-trivial Static Style Dynamic]

Dynamic Analysis

Consider this C++ program (Clang Tidy finds no issues) given to a *dynamic analyzer* (AddressSanitizer):

```cpp
int foo(int i) {
  int a[5];
  return a[i];
}
int main() {
  return foo(6);
}
```

```
$ gcc -fsanitize=address -g a.cpp
$ ./a.out
```

```
================================================================
==76375==ERROR: AddressSanitizer: stack-buffer-overflow on address 0x00016babf0d8
READ of size 4 at 0x00016babf0d8 thread T0
    #0 0x104343e54 in foo(int) a.cpp:9
    #1 0x104343f38 in main a.cpp:12
    #2 0x1a07c7e4c  (<unknown module>)

Address 0x00016babf0d8 is located in stack of thread T0 at offset 56 in frame
    #0 0x104343cf0 in foo(int) a.cpp:7

  This frame has 1 object(s):
    [32, 52) 'a' (line 8) <== Memory access at offset 56 overflows this variable
```

Dynamic analysis == testing.

Chapter #2:
Quality of Analysis

[Sound Metrics Experiment Flip]

Sound & Complete

Precision & Recall

Precision is the fraction of relevant instances among the retrieved instances (100% precision means *soundness*).

Recall is the fraction of relevant instances that were retrieved (100% recall means *completeness*).

$$\text{Precision} = \frac{TP}{TP + FP} \qquad \text{Recall} = \frac{TP}{TP + FN} \qquad \text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

$$\text{F1} = \frac{2 \times TP}{2 \times TP + FP + FN}$$

[Sound Metrics Experiment Flip]

Experiment

Say, we give a few programs to a static analyzer:

a	b	c	d	e	f	g	h
Yes	Yes	No	Yes	No	Yes	Yes	No

$$TP = \underline{\quad} \qquad FP = \underline{\quad} \qquad TN = \underline{\quad} \qquad FN = \underline{\quad}$$

$$\text{Precision} = \frac{TP}{TP + FP} = \underline{\qquad\qquad} \qquad \text{Recall} = \frac{TP}{TP + FN} = \underline{\qquad\qquad}$$

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN} = \underline{\qquad} \qquad \text{F1} = \frac{2 \times TP}{2 \times TP + FP + FN} = \underline{\qquad}$$

Flip of Terminology

Soundness and Completeness: With Precision by Prof. Bertrand Meyer, in Blog@CACM: "It is very easy to obtain soundness if we forsake completeness: *reject* every case."

Chapter #3:
Lattice

Total Order

Total order is a binary relation $\leq$ (*strict total order* is $<$).

Lineary ordered set (loset) is a set equipped with a total order.

Which of them are losets?:

$$\{1, -5, 2, 0, 42\}$$

$$\{3, 5, -9, 5, 12\}$$

$$\{3, 5, \text{"Hello"}, 12, 5.0\}$$

$$\{x, y, z\}$$

$$\varnothing$$

Partially Ordered Set

Partial order is total order but only between some elements.

Partially ordered set (poset) is a set equipped with a partial order.

Which of them are posets?:

$$\{1, \texttt{"apple"}, 2, -7, \texttt{"orange"}\}$$

$$\{3, 5, -9, 5, 12\}$$

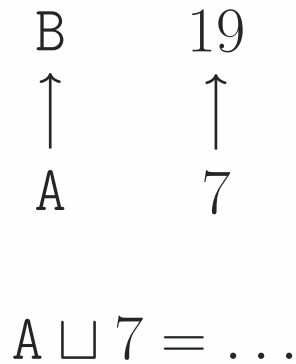$$\{0, 1, 2, 3, \dots\}$$

$$\{x, y, z\}$$

$$\varnothing$$

Lattice

Lattice is a poset where each two elements (x, y) have *least upper bound* (*join* operator $x \sqcup y$) and *greatest lower bound* (*meet* operator $x \sqcap y$).
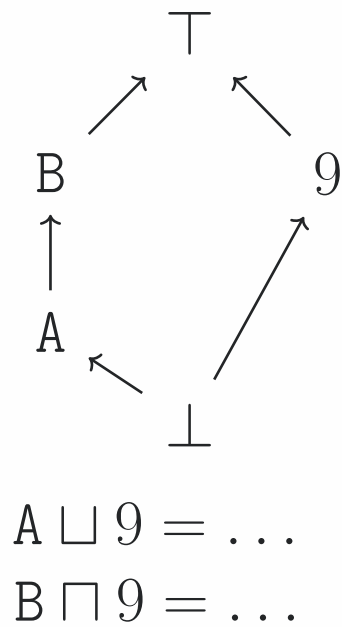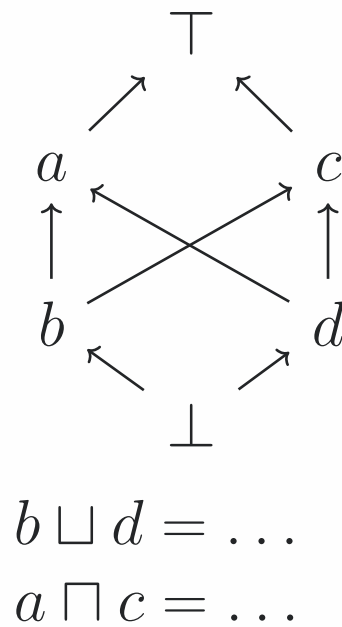
$\{42, 2, 13\}$

42
$\uparrow$
13
$\uparrow$
2

$42 \sqcup 2 = \ldots$

$\{A, 7, 19, B\}$

$B \qquad 19$
$\uparrow \qquad \uparrow$
$A \qquad 7$

$A \sqcup 7 = \ldots$

$\{A, \top, 9, B, \bot\}$

$\top$

$B \qquad 9$

A

$\bot$

$A \sqcup 9 = \ldots$
$B \sqcap 9 = \ldots$

$\{\top, \bot, a, b, c, d\}$

$\top$

$a \qquad c$

$b \qquad d$

$\bot$

$b \sqcup d = \ldots$
$a \sqcap c = \ldots$

Intervals

A lattice may be used to represent *intervals* in a set of values, e.g. in $\mathbb{Z}$:

$$(-\infty, +\infty)$$

$$(-\infty, 0) \qquad \{0\} \qquad\qquad (0, +\infty)$$

$$\{1, 3, 5, \dots\} \qquad\qquad \{2, 4, 6, \dots\}$$

$$\{2, 4, 8, 16, \dots\}$$

$$\varnothing$$
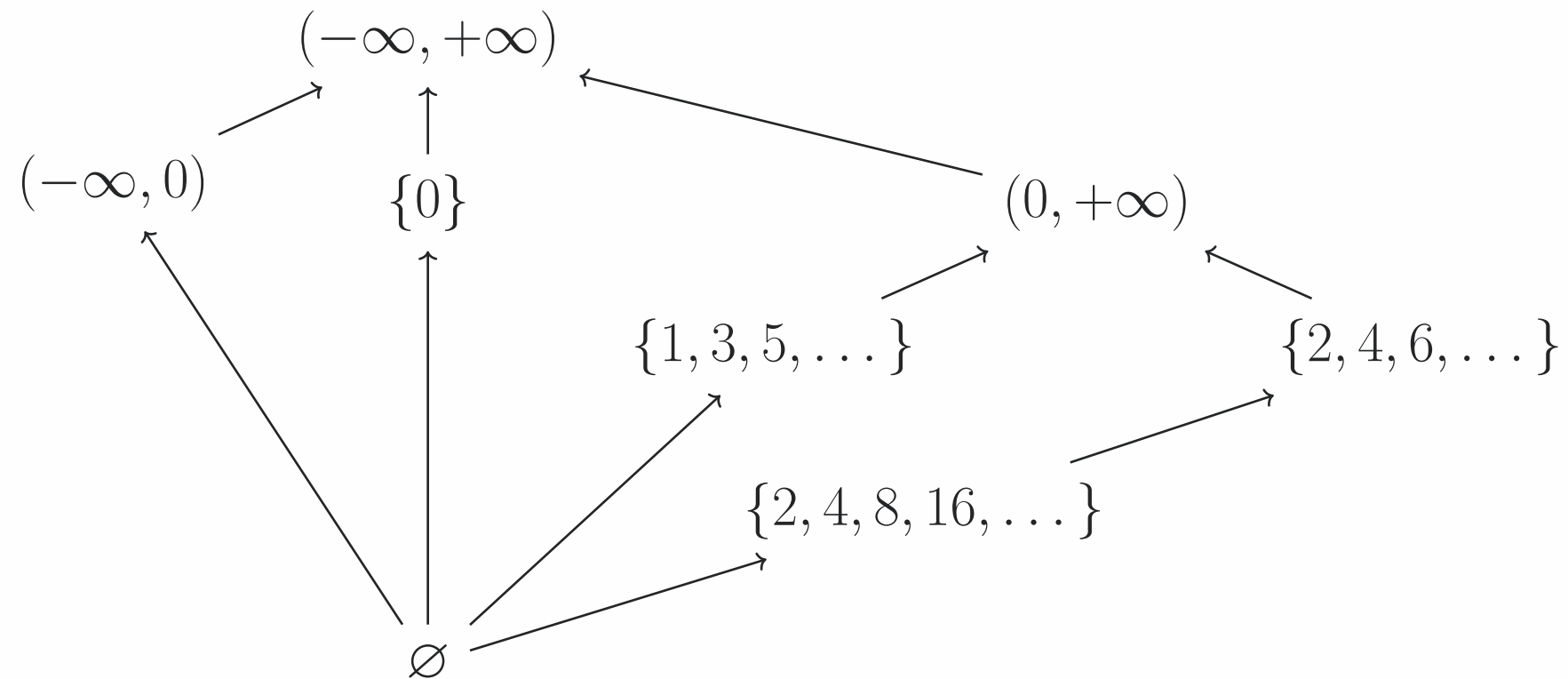
Partial order is $\in$.

Chapter #4:
Abstract Interpretation

[WTF Approximation Functions Transformers Widening Fixed-Point]

What is it about?

There is a compromise to be made between the precision of the analysis and its decidability (computability), or tractability (computational cost).

Over and Under Approximation

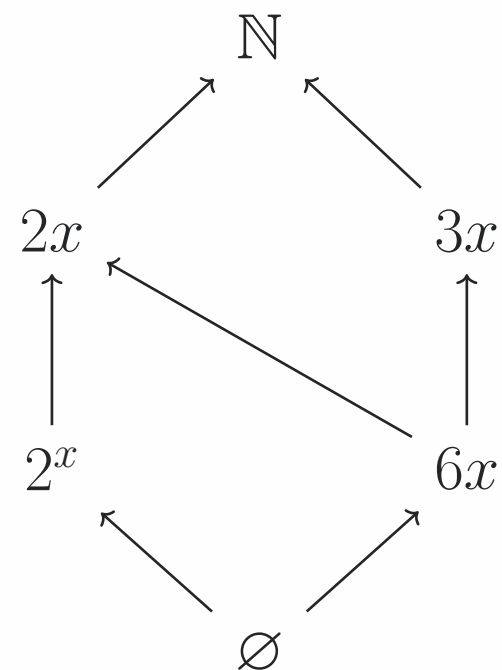What is the square of this oval?

1) **over**-approximation

2) **under**-approximation

Abstraction and Concretization

Concrete domain C
$(\mathbb{Z}, <)$

Abstract domain A
$(\{s|\{n|n \in \mathbb{N}\}\}, \in)$
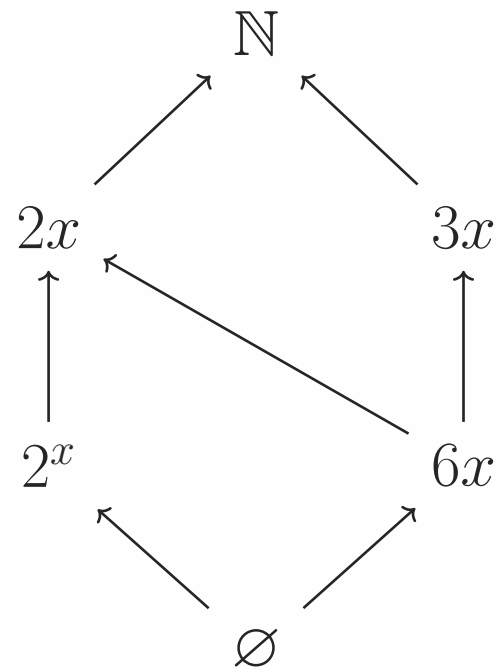
Abstraction function:
$\alpha(c) \rightarrow a$

Concretization function:
$\gamma(a) \rightarrow c$

Domains must be related through *Galois connection*:
$\forall c \in C, \forall a \in A$
$\alpha(c) \sqsubseteq a \iff c \sqsubseteq \gamma(a)$

Are they?

12
$\uparrow$
7
$\uparrow$
4
$\uparrow$
0
$\uparrow$
-9

$\mathbb{N}$

$2x$ $3x$

2^x $6x$

$\varnothing$

Abstract Semantics (Transformers)

Abstract domain:

Transformers:

$$\mathbb{N} + \mathbb{N} = \ldots$$

$$2x + 2x = \ldots$$

$$2x + 3x = \ldots$$

$$2x \times 3x = \ldots$$

$$\varnothing + 2x = \ldots$$

Concrete counterparts:

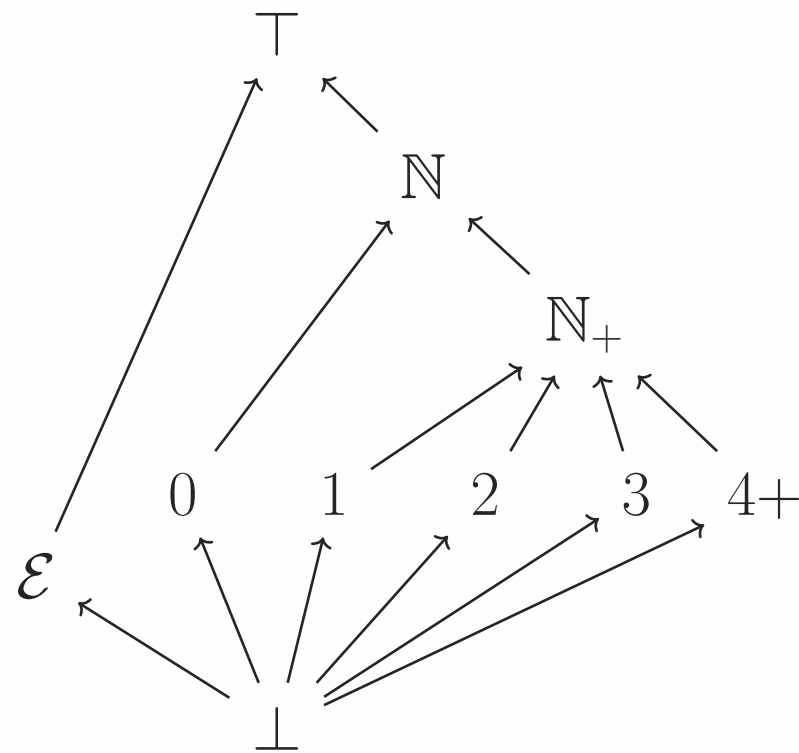$$1024 + 1 = \ldots$$

$$46 + 4 = \ldots$$

$$8 + 9 = \ldots$$

$$6 \times 12 = \ldots$$

$$-1 + 4 = \ldots$$

[WTF Approximation Functions Transformers Widening Fixed-Point]

Widening and Narrowing

$$0 \triangledown 1 = \ldots$$

$$1 \triangledown \mathbb{N}_+ = \ldots$$

$$0 \triangledown \mathbb{N}_+ = \ldots$$

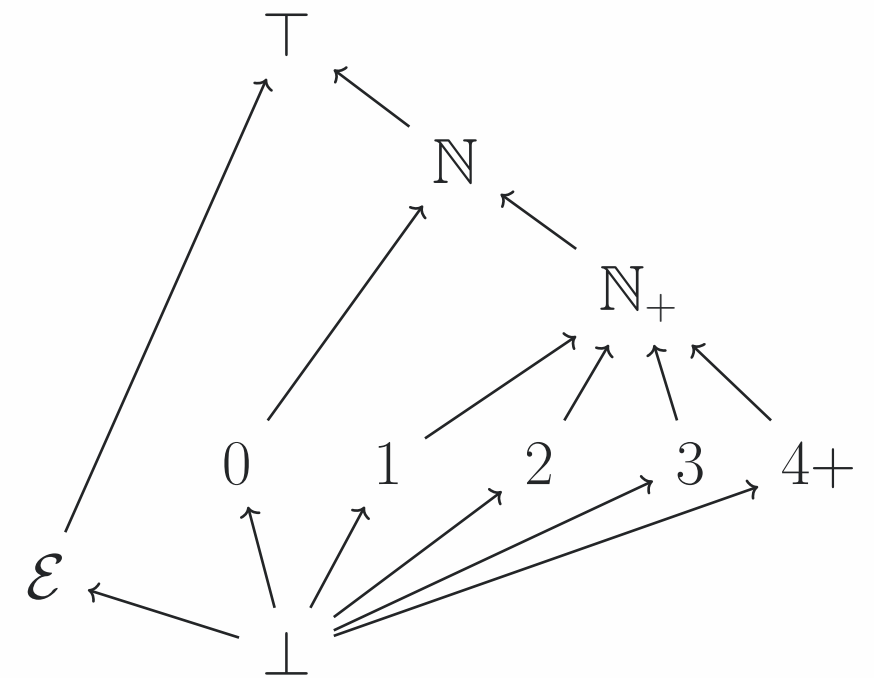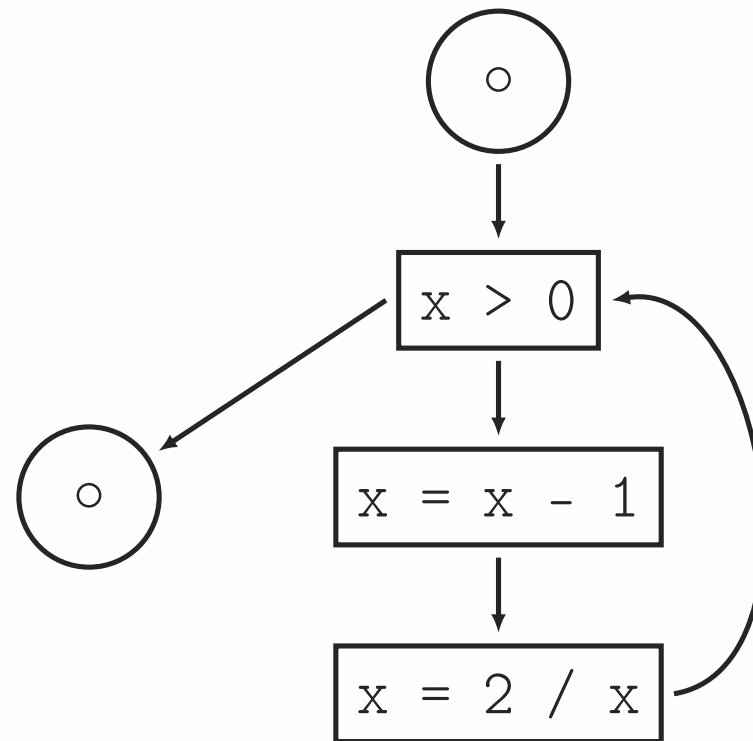$$1 \triangle \mathbb{N}_+ = \ldots$$

$$0 \triangle 1 = \ldots$$

$$3 \triangle 4+ = \ldots$$

Fixed-Point Computation

Fixed-Point Computation is a repeated symbolic execution of the program using abstract semantics until approximation reaches an *equilibrium.*

```
int f(int x) {
  while x > 0 {
    x = x - 1;
    x = 2 / x;
  }
  return x;
}
```

Chapter #5:
Further Reading/Watching

Lecture by Patrick Cousot, on YouTube

Mozilla wiki page on Abstract Interpretation.

Slide deck of Işil Dillig.

Introduction to Abstract Interpretation by Bruno Blanchet.

References