

Program Analysis

Metrics, Soundness, Lattices, AI

YEGOR BUGAYENKO

Lecture #6 out of 10

90 minutes

All videos are in [this YouTube playlist](#).

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Basics

Quality of Analysis

Lattice

Abstract Interpretation

Chapter #1: Basics

Syntactic & Semantic Properties

Semantic property can be completely defined with respect to the set of executions of a program, while a syntactic property can be decided directly based on the program text.

```
if (x) { printf("大家好"); }
```

Which properties are dynamic?

- A program may print a text to the console
- A program may call `printf()` C library function
- A program prints to the console
- A program consists of one line of code

Rice's Theorem

Rice's theorem states that all non-trivial semantic properties of programs are undecidable.

A property is non-trivial if it is neither true for every partial computable function, nor false for every partial computable function.

Halting problem is the problem of determining, from 1) a description of an arbitrary computer program and 2) an input, whether the program will finish running, or continue to run forever. A general algorithm to solve the halting problem for all possible program–input pairs **cannot exist**.

Non-trivial Properties

Examples of a non-trivial properties:

- A program exits
- A program prints “Hello”
- A program finishes in less than 5 seconds
- A program dies with “Segmentation Fault”
- A program prints user password to the console

Suggest a few properties.

Static Analysis

Consider two C++ programs given to a static analyzer (e.g. Clang Tidy):

```
int f() {  
    int x = 0;  
    return 42 / x;  
}
```

```
int f(int x) {  
    return 42 / x;  
}
```

Expected answers from Clang Tidy:

Yes! :)

No :(

Style Checking

Consider two C++ programs given to a style checker (e.g. cpplint):

```
int f (int x)
{
    return 42 / x;
}
```

```
int foo(int x) {
    return 42 / x;
}
```

Expected answers from cpplint:

Extra space before (in
function call ; { should
almost always be at the end
of the previous line

No :(

Dynamic Analysis

Consider this C++ program (Clang Tidy finds no issues) given to a dynamic analyzer (AddressSanitizer):

```
int foo(int i) {
    int a[5];
    return a[i];
}

int main() {
    return foo(6);
}
```

```
$ gcc -fsanitize=address -g a.cpp
$ ./a.out
```

```
=====
==76375==ERROR: AddressSanitizer: stack-buffer-overflow on address 0x00016babf0d8
READ of size 4 at 0x00016babf0d8 thread T0
#0 0x104343e54 in foo(int) a.cpp:9
#1 0x104343f38 in main a.cpp:12
#2 0x1a07c7e4c (<unknown module>)

Address 0x00016babf0d8 is located in stack of thread T0 at offset 56 in frame
#0 0x104343cf0 in foo(int) a.cpp:7

This frame has 1 object(s):
[32, 52) 'a' (line 8) <== Memory access at offset 56 overflows this variable
```

Dynamic analysis == testing.

Chapter #2:

Quality of Analysis

Sound & Complete



Precision & Recall

Precision is the fraction of relevant instances among the retrieved instances (100% precision means soundness).

Recall is the fraction of relevant instances that were retrieved (100% recall means completeness).

$$\text{Precision} = \frac{TP}{TP + FP} \quad \text{Recall} = \frac{TP}{TP + FN} \quad \text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$
$$F1 = \frac{2 \times TP}{2 \times TP + FP + FN}$$

Experiment

Say, we give a few programs to a static analyzer:

a	b	c	d	e	f	g	h
Yes	Yes	No	Yes	No	Yes	Yes	No

$$TP = \underline{\hspace{1cm}} \quad FP = \underline{\hspace{1cm}} \quad TN = \underline{\hspace{1cm}} \quad FN = \underline{\hspace{1cm}}$$

$$\text{Precision} = \frac{TP}{TP + FP} = \underline{\hspace{2cm}} \quad \text{Recall} = \frac{TP}{TP + FN} = \underline{\hspace{2cm}}$$

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN} = \underline{\hspace{2cm}} \quad F1 = \frac{2 \times TP}{2 \times TP + FP + FN} = \underline{\hspace{2cm}}$$

Flip of Terminology

Soundness and Completeness: With Precision by Prof. Bertrand Meyer, in Blog@CACM: “It is very easy to obtain soundness if we forsake completeness: reject every case.”

Chapter #3:

Lattice

Total Order

Total order is a binary relation \leq (strict total order is $<$).

Lineary ordered set (loset) is a set equipped with a total order.

Which of them are losets?:

$\{1, -5, 2, 0, 42\}$

$\{3, 5, -9, 5, 12\}$

$\{3, 5, \text{"Hello"}, 12, 5.0\}$

$\{x, y, z\}$

\emptyset

Partially Ordered Set

Partial order is total order but only between some elements.

Partially ordered set (poset) is a set equipped with a partial order.

Which of them are posets?:

$\{1, \text{"apple"}, 2, -7, \text{"orange"}\}$

$\{3, 5, -9, 5, 12\}$

$\{0, 1, 2, 3, \dots\}$

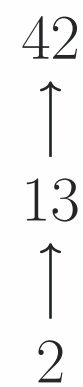
$\{x, y, z\}$

\emptyset

Lattice

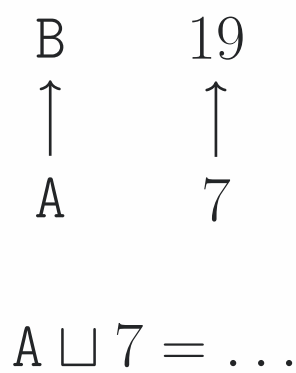
Lattice is a poset where each two elements (x, y) have least upper bound (join operator $x \sqcup y$) and greatest lower bound (meet operator $x \sqcap y$).

$\{42, 2, 13\}$

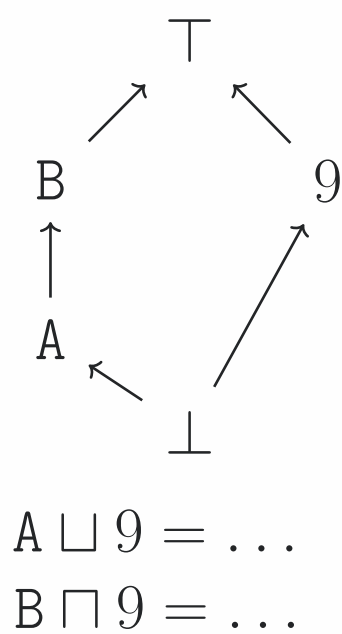


$42 \sqcup 2 = \dots$

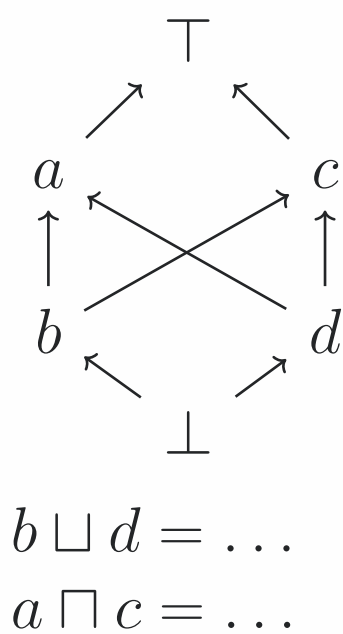
$\{A, 7, 19, B\}$



$\{A, \top, 9, B, \perp\}$



$\{\top, \perp, a, b, c, d\}$



Intervals

A lattice may be used to represent intervals in a set of values, e.g. in \mathbb{Z} :



Partial order is \in .

Chapter #4:

Abstract Interpretation

What is it about?

There is a compromise to be made between the precision of the analysis and its decidability (computability), or tractability (computational cost).

Over and Under Approximation

What is the square of this oval?



1) **over**-approximation



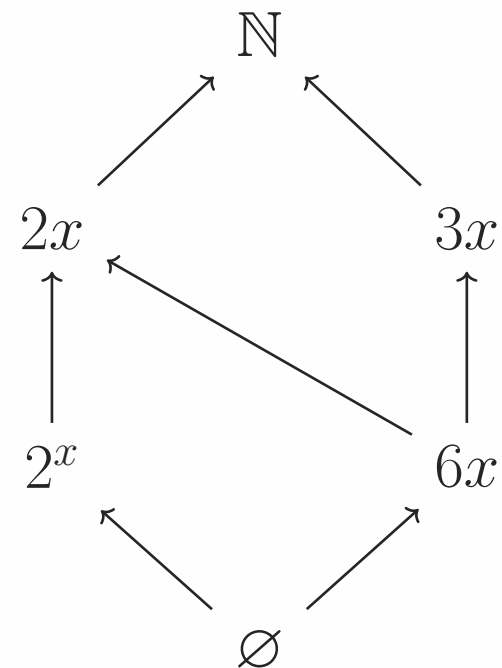
2) **under**-approximation

Abstraction and Concretization

Concrete domain C
($\mathbb{Z}, <$)

12
↑
7
↑
4
↑
0
↑
-9

Abstract domain A
($\{s | \{n | n \in \mathbb{N}\}\}, \in$)



Abstraction function:
 $\alpha(c) \rightarrow a$

Concretization function:
 $\gamma(a) \rightarrow c$

Domains must be related
through Galois connection:
 $\forall c \in C, \forall a \in A$
 $\alpha(c) \sqsubseteq a \iff c \sqsubseteq \gamma(a)$

Are they?

Abstract Semantics (Transformers)

Abstract domain:



Transformers:

$$\mathbb{N} + \mathbb{N} = \dots$$

$$2x + 2x = \dots$$

$$2x + 3x = \dots$$

$$2x \times 3x = \dots$$

$$\emptyset + 2x = \dots$$

Concrete counterparts:

$$1024 + 1 = \dots$$

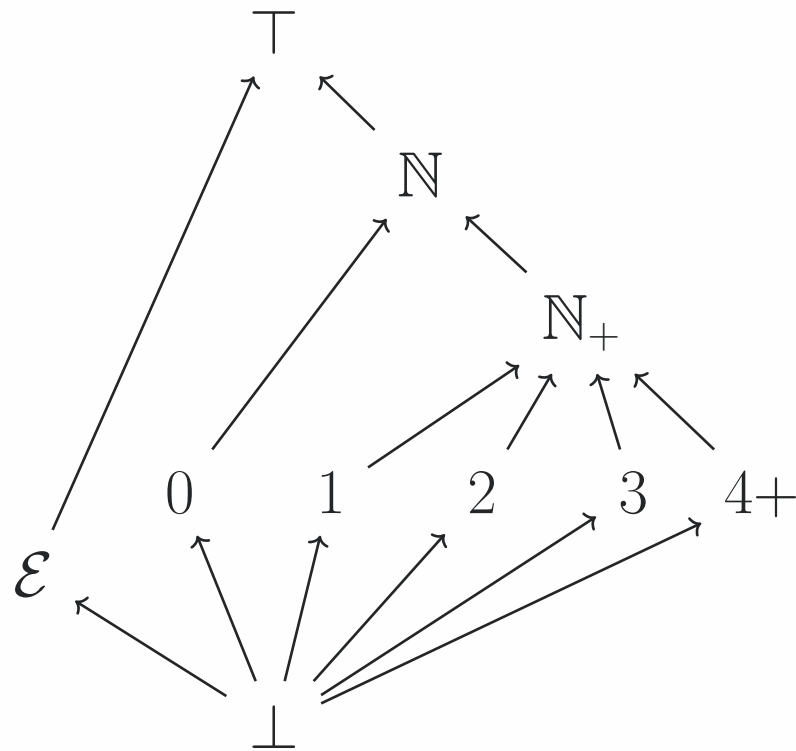
$$46 + 4 = \dots$$

$$8 + 9 = \dots$$

$$6 \times 12 = \dots$$

$$-1 + 4 = \dots$$

Widening and Narrowing



$$0 \nabla 1 = \dots$$

$$1 \triangleright \mathbb{N}_+ = \dots$$

$$0 \triangleright \mathbb{N}_+ = \dots$$

$$1 \triangle \mathbb{N}_+ = \dots$$

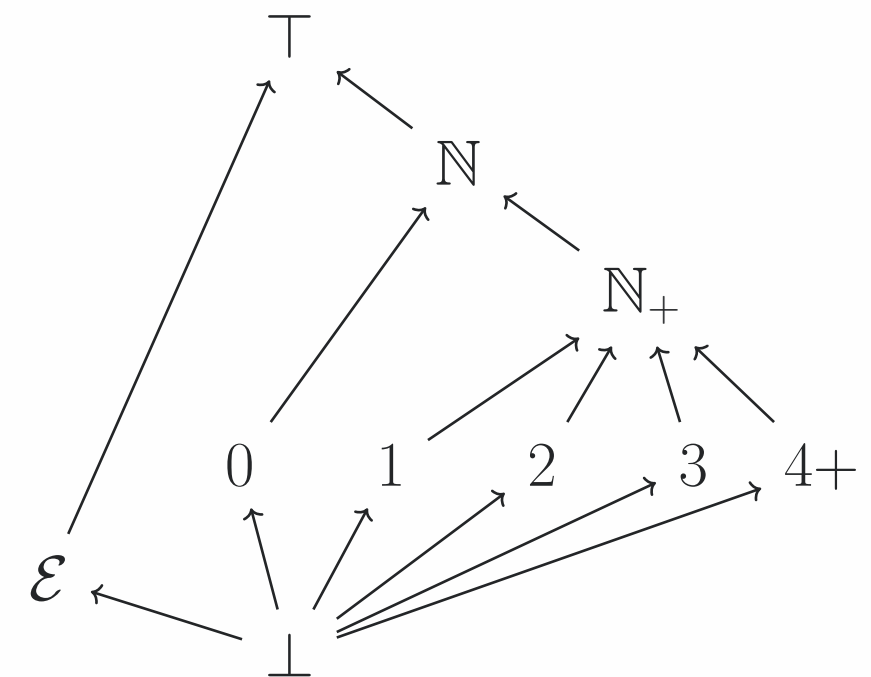
$$0 \triangle 1 = \dots$$

$$3 \triangle 4+ = \dots$$

Fixed-Point Computation

Fixed-Point Computation is a repeated symbolic execution of the program using abstract semantics until approximation reaches an equilibrium.

```
int f(int x) {
  while x > 0 {
    x = x - 1;
    x = 2 / x;
  }
  return x;
}
```



Further Reading/Watching

Lecture by Patrick Cousot, on [YouTube](#)

Mozilla wiki page on [Abstract Interpretation](#).

[Slide deck](#) of Işıl Dillig.

[Introduction to Abstract Interpretation](#) by Bruno Blanchet.