

圖形識別(程式作業四)  
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1. (a.)

Distribution 1: # of data: 250,mean:[5;5],Cov: [1,-0.6;-0.6,1]

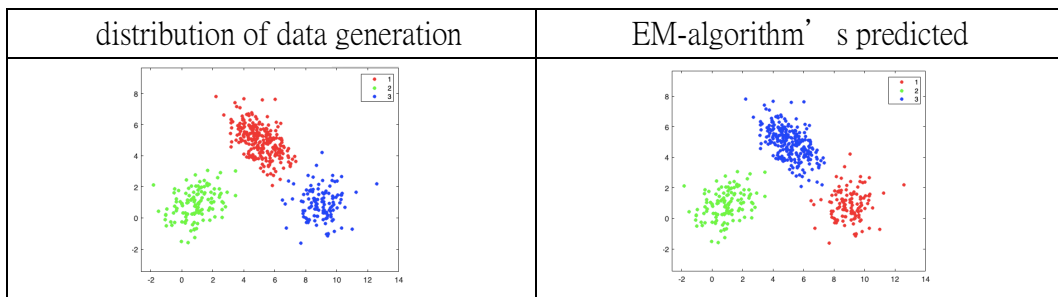
Distribution 2: # of data 125,mean:[1;1] ,Cov:[1,0.4;0.4,1]

Distribution 3: # of data 125,mean:[9;1] ,Cov:[1,0;0,1]

**Initialization:** use random function to generate the initial mean vector and covariance matrix. (Due to using the random function to generate, the E-step could produce the singular matrix for covariance matrix. )

**Stop criterion:** when  $e = \text{sum}(\text{sum}(\text{abs}(m - m_{\text{old}}))) + \text{sum}(\text{sum}(\text{abs}(S - S_{\text{old}}))) + \text{sum}(\text{abs}(P - P_{\text{old}}))$ ;  $< 0.0001$  , max\_iter=1000

Suppose equal prior( $P_1=P_2=P_3=1/3$ )



Data generation' s prior	EM-algorithm' s estimate prior
[0.5,0.25,0.25]	<b>[0.5038,0.2465,0.2498]</b>

Data generation' s mean vector	EM-algorithm' s estimate mean
[5;5]	[5.0941;4.9393]
[1;1]	[0.9150;0.9023]
[9;1]	[9.0068;0.8717]

Data generation' s covariance	EM-algorithm' s estimate covariance
[1,-0.6;-0.6,1]	<b>[0.9296,-0.5436;-0.5436,1.0685]</b>
[1,0.4;0.4,1]	<b>[1.0599,0.3368;0.3368,0.8952]</b>
[1,0;0,1]	<b>[0.8259,0.0742;0.0742,0.9547]</b>

Confuse matrix:

	C1	C2	C3
Distribute 1	250	0	0
Distribute 2	0	125	0
Distribute 3	2	0	123

1. (b.)

Distribution 1: # of data: 250,mean:[3.5;3.5],Cov: [1,-0.6;-0.6,1]

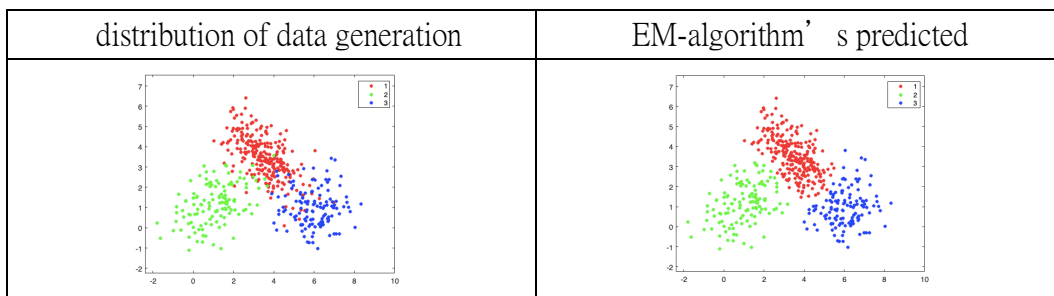
Distribution 2: # of data 125,mean:[1;1] ,Cov:[1,0.4;0.4,1]

Distribution 3: # of data 125,mea:[6;1] ,Cov:[1,0;0,1]

**Initialization:** use random function to generate the initial mean vector and covariance matrix. (Due to using the random function to generate, the E-step could produce the singular matrix for covariance matrix. )

**Stop criterion:** when  $e = \text{sum}(\text{sum}(\text{abs}(m - m_{\text{old}}))) + \text{sum}(\text{sum}(\text{abs}(S - S_{\text{old}}))) + \text{sum}(\text{abs}(P - P_{\text{old}}))$ ;  $< 0.0001$  max\_iter=1000

Suppose equal prior( $P_1 = P_2 = P_3 = 1/3$ )



Data generation' s prior	EM-algorithm' s estimate prior
[0.5,0.25,0.25]	[0.4961,0.2533,0.2506]

Data generation' s mean vector	EM-algorithm' s estimate mean
[3.5;3.5]	[3.5202;3.4689]
[1;1]	[1.0702;1.2488]
[6;1]	[6.0341;0.9709]

Data generation' s covariance	EM-algorithm' s estimate covariance
[1,-0.6;-0.6,1]	[0.8319,-0.5899;-0.5899,1.0967]
[1,0.4;0.4,1]	[1.1299,0.4751;0.4751,1.0843]
[1,0;0,1]	[0.7480,0.0664;0.0664,0.8488]

Confuse matrix:

	C1	C2	C3
Distribute 1	231	5	14
Distribute 2	7	118	0
Distribute 3	11	0	114

1. (c.)

Distribution 1: # of data: 250,mean:[2;2],Cov: [1,-0.6;-0.6,1]

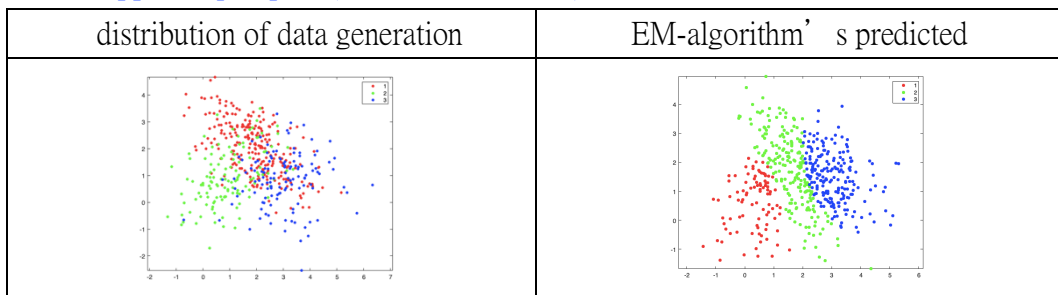
Distribution 2: # of data 125,mean:[1;1] ,Cov:[1,0.4;0.4,1]

Distribution 3: # of data 125,mea:[3;1] ,Cov:[1,0;0,1]

**Initialization:** use random function to generate the initial mean vector and covariance matrix. (Due to using the random function to generate, the E-step could produce the singular matrix for covariance matrix. )

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**Suppose equal prior**( $P_1 == P_2 == P_3 == 1/3$ )



Data generation' s prior	EM-algorithm' s estimate prior
[0.5,0.25,0.25]	[0.2008,0.3740,0.4252]

Data generation' s mean vector	EM-algorithm' s estimate mean
[2;2]	[0.4904;0.8117]
[1;1]	[1.7952;1.6908]
[3;1]	[2.7800;1.5166]

Data generation' s covariance	EM-algorithm' s estimate covariance
[1,-0.6;-0.6,1]	[0.8319,-0.5899;-0.5899,1.0967]
[1,0.4;0.4,1]	[1.1299,0.4751;0.4751,1.0843]
[1,0;0,1]	[0.7480,0.0664;0.0664,0.8488]

Confuse matrix:

	C1	C2	C3
Distribute 1	10	126	114
Distribute 2	78	39	9
Distribute 3	3	41	81

(d.)根據(a.) (b.) (c.) confuse matrix and 2-D visualization 結果我們可以發現，每個 distribution data 越靠近，Gaussian mixture data 越緊密，clustering 的結果會越難以區分正確（跟 ground truth 相差越遠）

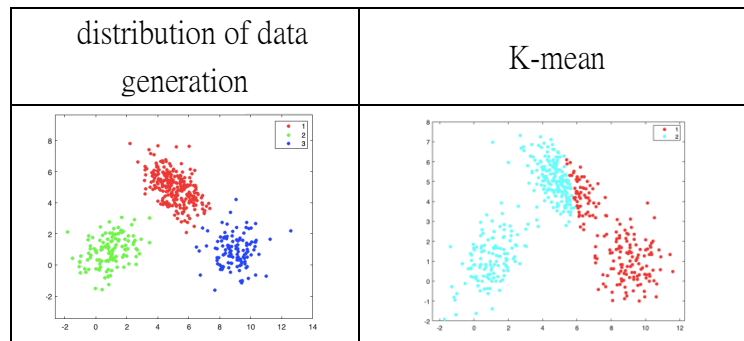
2. (a.) Distribution 1: # of data: 250,mean:[5;5],Cov: [1,-0.6;-0.6,1]  
 Distribution 2: # of data 125,mean:[1;1] ,Cov:[1,0.4;0.4,1]  
 Distribution 3: # of data 125,mean:[9;1] ,Cov:[1,0;0,1]

Initialization:

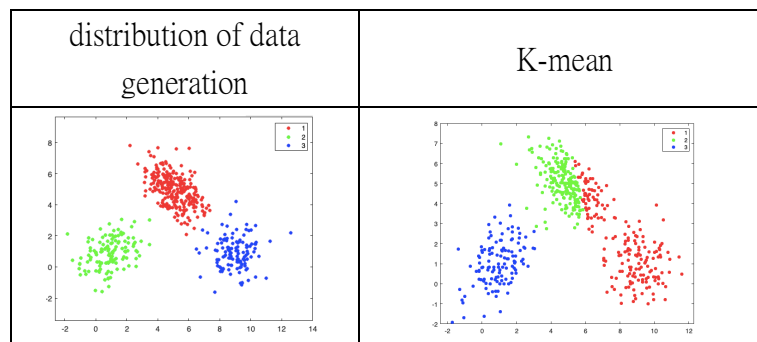
randn(2,1) for four mean vector

Stop criterion: when  $\text{sum}(\text{sum}(|\text{mean\_new} - \text{mean\_old}|)) < 0.0001$  ,stop

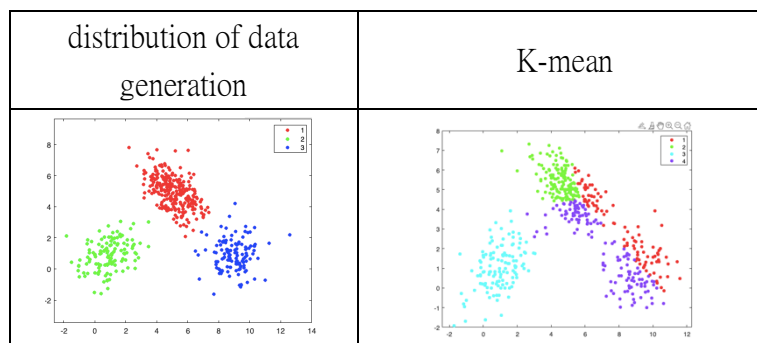
K = 2



K = 3



K = 4



2. (b.)

Distribution 1: # of data: 250,mean:[3.5;3.5],Cov: [1,-0.6;-0.6,1]

Distribution 2: # of data 125,mean:[1;1] ,Cov:[1,0.4;0.4,1]

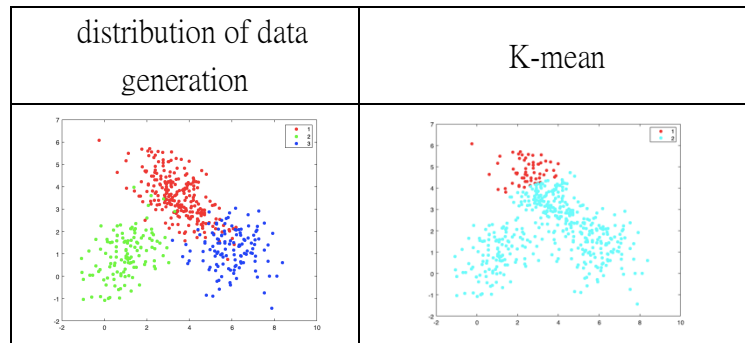
Distribution 3: # of data 125, mea:[6;1] ,Cov:[1,0;0,1]

Initialization:

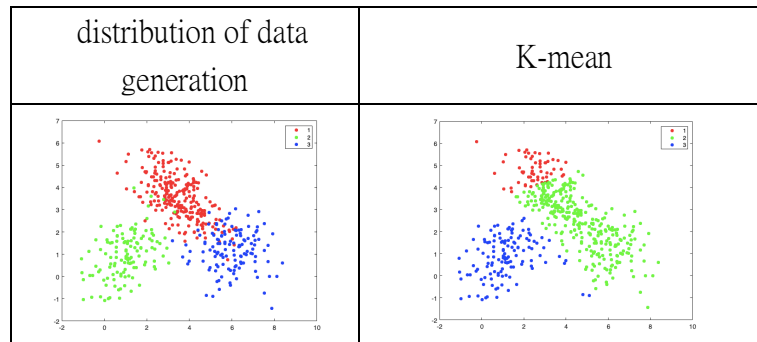
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Stop criterion: when  $\text{sum}(\text{sum}(|\text{mean\_new} - \text{mean\_old}|)) < 0.0001$  ,stop

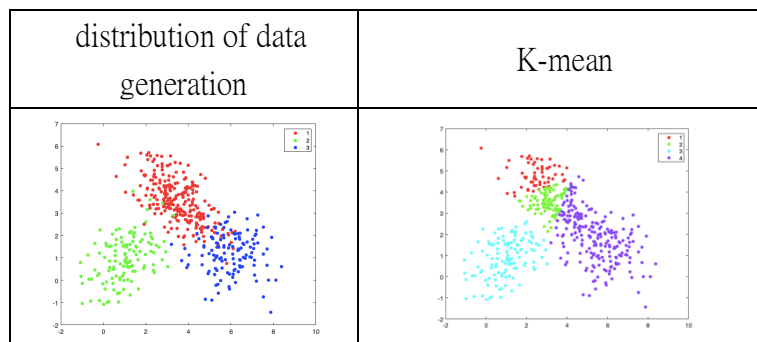
K = 2



K = 3



K = 4



2. (c.)

Distribution 1: # of data: 250,mean:[2;2],Cov: [1,-0.6;-0.6,1]

Distribution 2: # of data 125,mean:[1;1] ,Cov:[1,0.4;0.4,1]

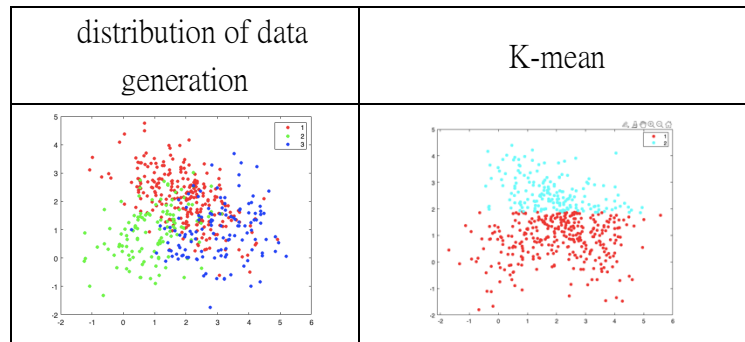
Distribution 3: # of data 125,mea:[3;1] ,Cov:[1,0;0,1]

Initialization:

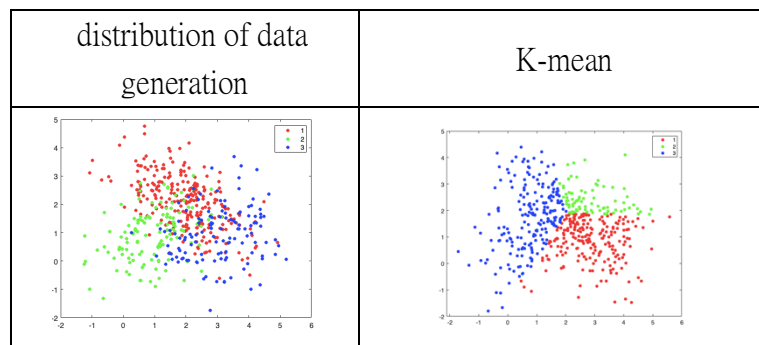
randn(2,1) for four mean vector

Stop criterion: when  $\text{sum}(\text{sum}(|\text{mean\_new} - \text{mean\_old}|)) < 0.0001$ , stop

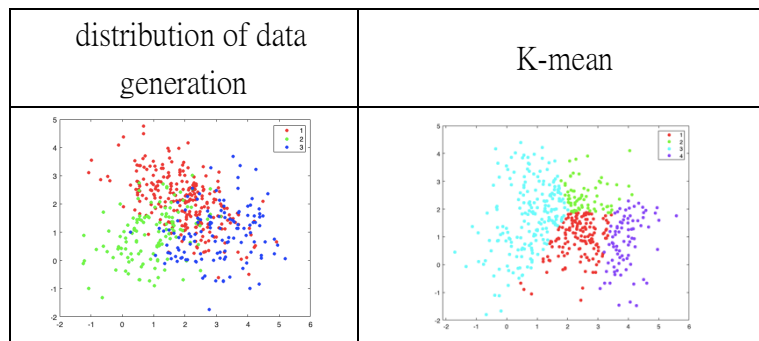
K = 2



K = 3



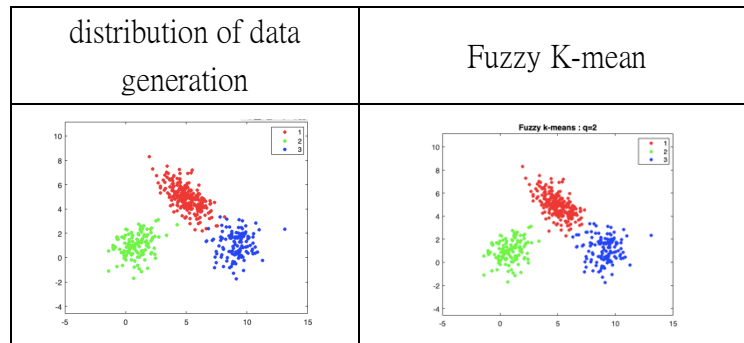
K = 4



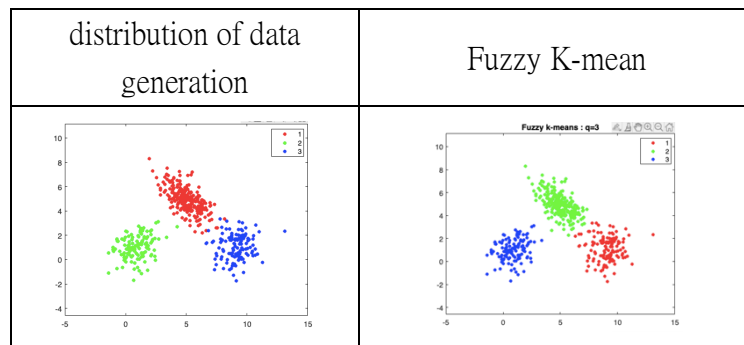
- 2 (d.) Conclusion: 我們可以發現，與 E-M algorithm 相比，K-mean 預測果較不穩定。initial mean vector 會影響 k-mean clustering 預測結果，不同 distribute dataset (a),(b),(c) 使用 random mean vector，預測的最終分群結果會不一樣

- 3 (a.) Distribution 1: # of data: 250,mean:[5;5],Cov: [1,-0.6;-0.6,1]  
 Distribution 2: # of data 125,mean:[1;1] ,Cov:[1,0.4;0.4,1]  
 Distribution 3: # of data 125,mean:[9;1] ,Cov:[1,0;0,1]  
 Stop criterion: error<0.00001 ,max\_iter =100 stop

q=2



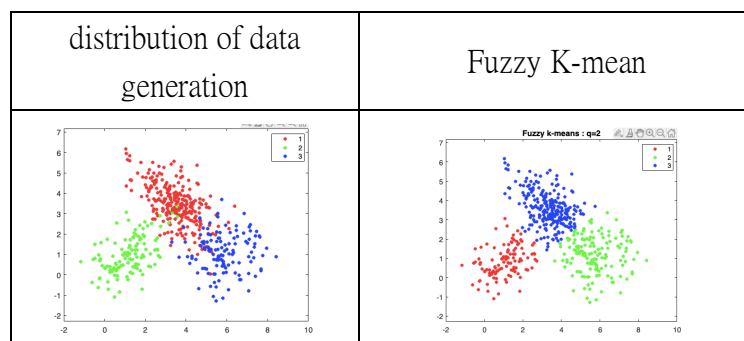
q=3



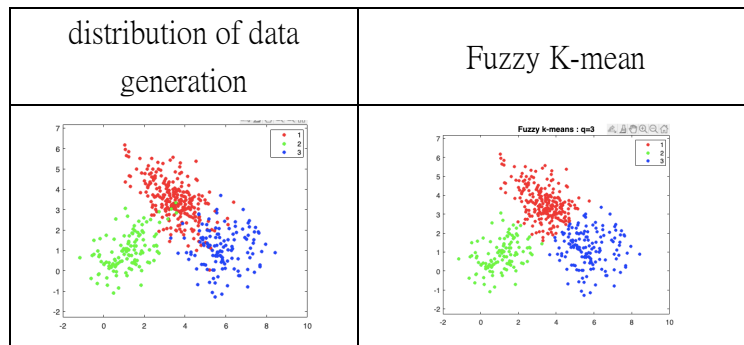
3. (b.)

Distribution 1: # of data: 250,mean:[3.5;3.5],Cov: [1,-0.6;-0.6,1]  
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 Distribution 3: # of data 125,mea:[6;1] ,Cov:[1,0;0,1]  
 Stop criterion: error<0.00001 ,max\_iter =100 stop

q=2



q=3



3. (c.)

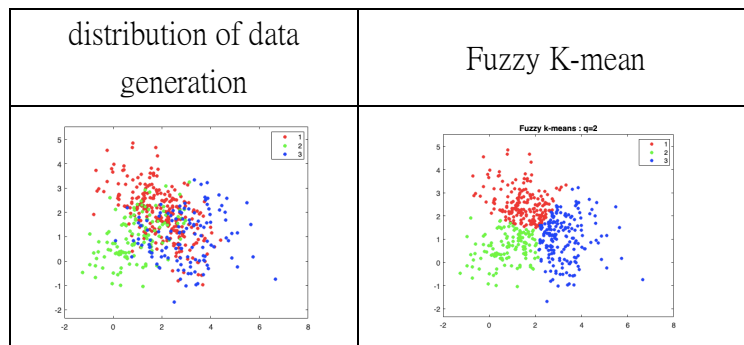
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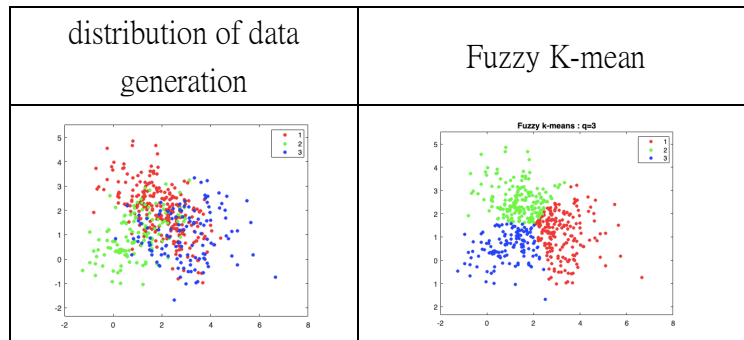
Distribution 3: # of data 125, mea: [3;1], Cov: [1,0;0,1]

Stop criterion: error<0.00001, max\_iter =100 stop

q=2



q=3



(d.) Conclusion: q=2 與 q=3 (模糊) 所生成的中心點緊密程度不同，雖然此題只問 q=2,q=3 的差異，但當 q 值越大，中心點彼此靠的越近; q 越小，中心點彼此靠的越稀疏