Homework2

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1. 使用 log likelihood estimator 來計算 Bernoulli distribution estimator p_ML:

   (a.) p = 0.25 N=1000, 使用 binornd function 來生成 sample data :A。

   假設今天 p 為未知數，利用生成 A(已知)來求 p。

   使用 log likelihood function，並對 p 做偏微分求極值

   式子如下:

$$L(p) = \sum_{i=1}^{N} \ln p(x_i|p) = \sum_{i=1}^{N} (x_i \ln p + (1 - x_i) \ln(1 - p))$$
$$= \ln p \sum_{i=1}^{N} x_i + \ln(1 - p)\,(N - \sum_{k=1}^{N} x_i)$$

$$\frac{\partial L(p)}{\partial p} = 0 \quad \hat{p}_{ML} = \frac{\sum_{i=1}^{N} x_i}{N}$$

   程式最終結果(每次結果可能不一樣(random 生成 data))

   P_ML:

```
p_ml_A =

    0.2600
```

   (b.) p=0.5 N=1000 運算方式如上題

   P_ML =

```
p_ml_B =

    0.4730
```

2. 此題為 Multivariate Normal Distribution，來求算 mean vector 與 covariance matrix 的 estimator:

使用 log maximum likelihood estimator:來依序解未知數 mean, covariance matrix:

$$L(\boldsymbol{\mu}, \Sigma) = \sum_{i=1}^{N} \ln p(\mathbf{x}_i|\boldsymbol{\mu}, \Sigma)$$
$$= -\frac{Nl}{2}\ln(2\pi) - \frac{N}{2}\ln|\Sigma| - \sum_{i=1}^{N}\left\{\frac{1}{2}(\mathbf{x}_i - \boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}_i - \boldsymbol{\mu})\right\}$$

求得 mean_ML:

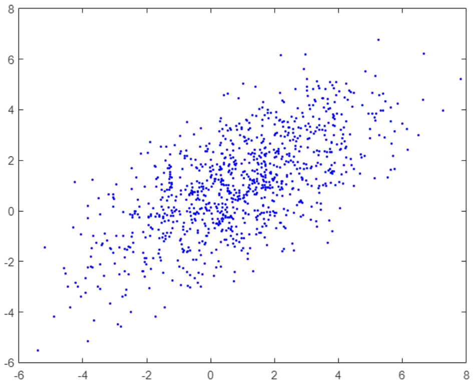
Let $\nabla_{\mu} L = 0$

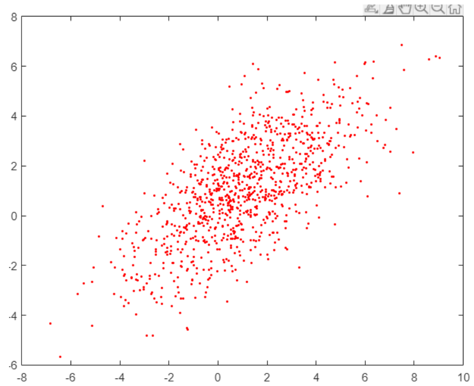
- $\Rightarrow \hat{\boldsymbol{\mu}}_{ML} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i$

並利用求來的 mean_ML 來計算出 covariance_ML:

$$\Rightarrow \hat{\Sigma}_{ML} = \frac{1}{N}\sum_{i=1}^{N}(\mathbf{x}_i - \hat{\boldsymbol{\mu}})(\mathbf{x}_i - \hat{\boldsymbol{\mu}})^T$$

(a.) N=1000 ,使用 m=[1;1],S= [5 3;3 4] 來生成 sample data(藍點)，並用估算出來的 mean,S 生成 data(紅點)



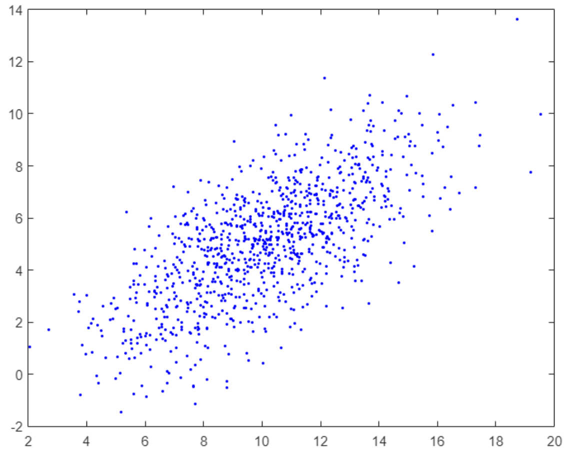


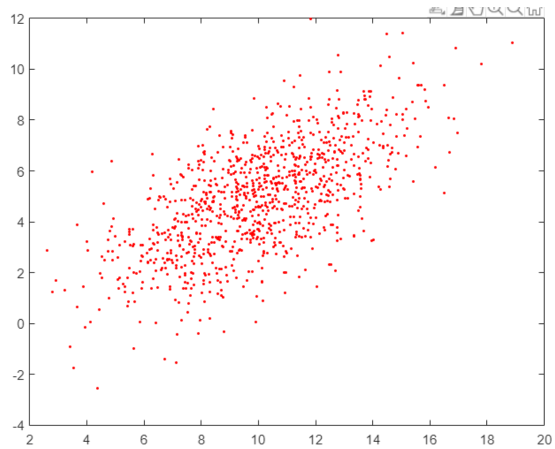
mean_ML: covariance_ML:

```
A_m =

    1.1172
    1.0464
```

```
A_S =

    5.4799    3.2192
    3.2192    3.9569
```

(b) N=1000 ,使用 m=[10;5],S= [7 4;4 5] 來生成 sample data(藍點)，並用估算出來的 mean, S 生成 data(紅點





mean_ML: covariance_ML:

```
B_m =

    9.9998
    4.9378
```

```
B_S =

    7.0998    4.3173
    4.3173    5.3475
```