HW3

Friday, February 9, 2024 4:25 PM

Q1.

1) $\sum P_x() = 1 \rightarrow \frac{c}{3} + \frac{c}{6} + \frac{c}{3} = 1 \rightarrow c = \frac{6}{5}$

2) $P(x>2) = \frac{c}{3} \rightarrow \frac{\frac{6}{5}}{3} \rightarrow P(x>2) = \frac{2}{5}$

3) $E[x] = \sum_x x P_x(x) = 1 \cdot \frac{c}{3} + 2 \cdot \frac{c}{6} + 5 \cdot \frac{c}{3}$ $E[x] = \frac{14}{5}$

$Var[x] = E[x^2] \cdot (E[x])^2 \rightarrow 1 \cdot \frac{c}{3} + 4 \cdot \frac{c}{6} + 25 \cdot \frac{c}{3} \rightarrow E[x^2]^2 = \frac{56}{5}$

$Var[x] = \frac{56}{5} - \left(\frac{14}{5}\right)^2 \rightarrow Var[x] = 3.36$

4) $P_Y = P(y) = P(x^2+1 = y) = P_x(x = -\sqrt{y-1}) + P_x(x = -\sqrt{y-1})$

$y = \{2, 5, 26\}$ because $x = 1, 2, 5.$

$P_y(2) = P_x(\sqrt{2-1}) + P_x(-\sqrt{2-1}) = P_x(1) = \frac{2}{5}$

$P_y(5) = P_x(\sqrt{5-1}) + P_x(-\sqrt{5-1}) = P_y(2) = \frac{1}{5}$

$P_y(26) = P_x(\sqrt{25-1}) + P_x(-\sqrt{25-1}) = P_x(5) = \frac{2}{5}$

$$P(Y=y) : \begin{cases} \frac{c}{3} = \frac{2}{5} & \text{for } y=2 \\ \frac{c}{6} = \frac{1}{5} & \text{for } y=5 \\ \frac{c}{3} = \frac{2}{5} & \text{for } y=26 \end{cases}$$

5) $E[y] = \sum_y y P(y) = 2 \cdot \left(\frac{2}{5}\right) + 5 \cdot \left(\frac{1}{5}\right) + 26 \cdot \left(\frac{2}{5}\right) \rightarrow E[Y] = \frac{61}{5}$

$Var[y] = E[y^2] - (E[y])^2 \rightarrow 4 \cdot \left(\frac{2}{3}\right) + 25\left(\frac{1}{5}\right) + 676 \cdot \left(\frac{2}{5}\right) \rightarrow E[y^2] = 277$

$Var[Y] = 277 - \left(\frac{61}{5}\right)^2 \rightarrow Var[Y] = 128.16$

Q2.

1) $X \sim$ Binomial $\rightarrow P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

where $n = 10$, $x = \#$ of success, p = probability of success

2) $P(x \geq 9) = P(x=9) + P(x=10)$

$\binom{10}{9} \cdot (.7)^9 \cdot (.3)^1 + \binom{10}{10} \cdot (.7)^{10} \cdot (.3)^0 \rightarrow P(X \geq 9) = 0.149$

Q3.

$X \sim$ Geometric $\rightarrow P(X=x) = (1-p)^{x-1}$

$P(x) = (.7)^{x-1} \cdot (.3)$

$P(x \leq 5) = P(1) + P(2) + \dots + P(5) = 0.832$

Q4.

1) $X \sim$ Binomial $\rightarrow P(x=x) \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$

$\binom{6}{3} \cdot \left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^3 \rightarrow 0.132$

2) Let A = event that 1st rolls = 1

Let B = event that 2nd rolls = 1

$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} = \frac{\binom{5}{1}}{\binom{6}{2}} = 0.333$

3) $\binom{6}{3,3} = \frac{6!}{3!\,3!} = 20$

there is only one way to get 12:2:2 $= \frac{1}{20}$

4) $P(A)$ = probability that at least one 3 was rolled

$= 1 - \left(\frac{3}{4}\right)^6 = 0.822$

$P(X=k|A) = \frac{P(x=k \cap A)}{P(A)} \rightarrow \frac{\binom{6}{k}\left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{n-4}}{.822}$

$P(x=1|A) = 0.433$

$P(x=2|A) = 0.361$

$P(x=3|A) = 0.160$

$P(x=4|A) = 0.040$

$P(x=5|A) = 0.005$

$P(x=6|A) = 0.0003$

Q5.

1) $P_S(s) = \sum_r P_{RS}(r,s)$

$P_S(1) = P_{RS}(1,1) + P_{RS}(2,1) + P_{RS}(3,1) = \frac{12}{45}$

$P_S(2) = \frac{18}{45}$

$P_S(3) = \frac{15}{45}$

$P_{S|A}(s) = \frac{P(S=s, A)}{P(A)} \rightarrow P(B) = \frac{12}{45} + \frac{18}{45} = \frac{30}{45}$

$P_{S|A}(1) = \frac{\frac{11}{45} + \frac{6}{45} + \frac{2}{45}}{\frac{30}{45}} = 0.4$

$P_{S|A}(2) = \frac{\frac{6}{45} + \frac{9}{45} + \frac{3}{45}}{\frac{30}{45}} = 0.6$

2) $P_{RY} = (r,y) \rightarrow Y = R - S \rightarrow S = R - Y$

$Y_{min} = R_{min} - S_{max} = 1 - 3 = -2$

$Y_{max} = R_{max} - S_{min} = 3 - 1 = 2$

$P_{RY}(1,-2) \rightarrow P_{RS}(1, 1-2) \rightarrow P_{RS}(1,3) = 0.133$

$P_{RY}(1,-1) = 0.133$

$P_{RY}(1,0) = 0.039$

$P_{RY}(2,-1) = 0.200$

$P_{RY}(2,0) = 0.200$

$P_{RY}(2,1) = 0.133$

$P_{RY}(3,0) = 0$

$P_{RY}(3,1) = 0.067$

$P_{RY}(3,2) = 0.044$

3) $P_{X|A}(x) = \frac{P_X(x)}{P(A)}$

$P(A) = \frac{30}{45}$ from part 1)

$P_X(2) = \frac{4}{45}$

$P_X(3) = \frac{6}{45} + \frac{6}{45} = \frac{12}{45}$

$P_X(4) = \frac{6}{45} + \frac{9}{45} + \frac{2}{45} = \frac{17}{45}$

$P_X(5) = \frac{9}{45} + \frac{3}{45} = \frac{12}{45}$

$P_{X|A}(2) = 0.133$

$P_{X|A}(3) = 0.4$

$P_{X|A}(4) = 0.566$

$P_{X|A}(5) = 0.4$

Q6.

$P_X(x) = e^{-\lambda} \frac{\lambda^x}{x!}$, $\lambda = 2.2$

$P_X(0) = e^{-2.2} \frac{2.2^0}{0!} = 0.111$

$P_X(1) = 0.244$

$P_X(2) = 0.268$

$P_X(3) = 0.197$

$P_X(0) + P_X(1) + P_X(2) + P_X(3) = 0.82$

$\rightarrow$ 3 capacity