Your Name: ______________________ Your Section's Professor: ______________________

You have $1\frac{1}{4}$ hours to complete the exam, i.e., from 7:00-8:15pm.. You can bring in a calculator and hand written notes on both sides of a single $8\frac{1}{2} \times 11$ inch sheet of paper. You should have no other materials on your desk. There are four questions spread out across six pages with space for working. You can use the back sheet of the exam for any additional scratch paper. Each question requires a brief but complete explanation or derivation. Only **neat** work in the space provided on the exam will be evaluated.

Q. 1 (20 points) A **two-sided** coin is sequentially tossed *three times*. The coin has faces labeled 1 and 2. The three successive coin tosses are fair and independent.

1. (5 pts) Describe/specify the *sample space* Ω for this experiment, and describe/specify the *event*

$$E = \{\text{the product of the faces of the } \textbf{first two} \text{ tosses is } 4\}.$$

$$\boxed{\Omega = \Big\{ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Big\}}$$

$$\boxed{E = \Big\{ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Big\}}$$

2. (5 pts) A discrete random variable X is defined on this sample space and corresponds to the **product of the three** tosses. Determine the probability mass function $p_X(\cdot)$.

$$\boxed{p_X(x) = \left\{ \qquad\qquad \right.}$$

3. (5 pts) Compute a numerical value for $E[X]$.

$E[X] =$

4. (5 pts) Let A be the event that at least one of the three coin tosses is a 1. Determine a numerical value for $P(X = 4|A)$.

$P(X = 4|A) =$

Q. 2 (15 points) A computer manufacturer uses chips from three sources, with $50\%, 10\%$ and 40% coming from sources S_1, S_2 and S_3 respectively. Chips from sources S_1, S_2 and S_3, are defective with probabilities $0.005, 0.001$, and 0.010 respectively.

1. (5 pts) Find the probability that a randomly selected chip used by the manufacturer is defective.

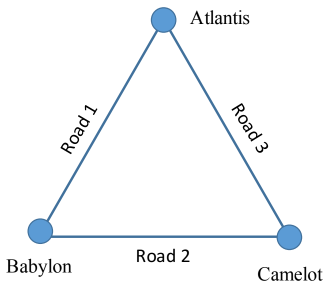
2. (10 pts) If a randomly selected chip is found to be defective, find the probability that it came from source S_1.

Q. 3 (15 points) The following two parts involve counting.

1. (5 pts) A two-sided coin (with the two sides labeled as Heads and Tails) is sequentially tossed 10 times. Count the number of possible ways that we can observe a sequence with exactly 4 Heads out of the 10 tosses.

2. (10 pts) There are 50 identical apples that need to be distributed among Alice, Bob, and Cathy. Determine the number of ways that this can be done such that each one of them gets at least 2 apples. As examples, possible distributions of the identical apples among Alice, Bob and Cathy include (A, B, C) = (46, 2, 2); (10, 10, 30); (9, 2, 39), etc., where A, B and C denote the number of apples allocated to Alice, Bob and Cathy respectively.

Q. 4 (20 points) There are three cities – Atlantis, Babylon and Camelot – located on the vertices of a triangle. Each pair of cities is connected by a road (the three edges of the triangle, see figure). Each of these roads could be blocked (independent of any other road), and the probability of a road being blocked is q, with $0 < q < 1$. In the questions below, a path between two cities is a connected sequence of roads leading from one city to the other. For example, there are two paths between Babylon and Camelot – one is the direct path (Road 2), and the other is the indirect path through Atlantis (Road 1 followed by Road 3).



1. (5 pts) Determine an expression (as a function of q) for the probability that none of the three roads are blocked.

2. (5 pts) Determine an expression (as a function of q) for the probability that there is a non-blocked path between Atlantis to Camelot.

3. (10 pts) Determine an expression (as a function of q) for the conditional probability that there is a non-blocked path between Atlantis to Camelot, given that exactly two of the roads are blocked (it is unknown which two).