Answers to the first part of the task in probability

Q 1 . a:

The equation for the probability of a case A given another case B is:

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)}$$

Both twin brother and an identical brother - $P(A \cap B) = \frac{1}{300} \cdot \frac{1}{2} = \frac{1}{600}$
Given that he is a twin brother and therefore there are two options:

1. he is an identical twin
2. he is a non - identical twin

$$P(B) = (\frac{1}{2} \cdot \frac{1}{300}) + (\frac{1}{2} \cdot \frac{1}{125}) = \frac{17}{3000}$$

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{600}}{\frac{17}{3000}} = 0.29$$

Q 1 . b:

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)}$$

Both chocolate and bowl number 1 - $P(A \cap B) = \frac{1}{2} \cdot \frac{30}{40} = \frac{30}{80} = \frac{3}{8}$
The probability that chocolate came out - $P(B) = (\frac{1}{2} \cdot \frac{1}{2}) + (\frac{1}{2} \cdot \frac{3}{4}) = \frac{5}{8}$

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{3}{8}}{\frac{5}{8}} = 0.6$$

Q 2:

in order to calculate the probability that the yellow candy came from the 1994 bag

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)}$$

both yellow and came from 1994 - $P(A \cap B) = \frac{1}{2} \cdot \frac{20}{100} = \frac{20}{200} = 0.1$
came out yellow - $P(B) = (\frac{1}{2} \cdot \frac{20}{100}) + (\frac{14}{100} \cdot \frac{1}{2}) = \frac{17}{100} = 0.17$

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{20}{200}}{\frac{17}{100}} = \frac{0.1}{0.17} = 0.588$$

Q 3.a:

The disease affects one in ten thousand people, so the probability that he is sick is: $P(s) = \left(\frac{1}{10000}\right)$, and according to this the probability of being healthy is: $P(h) = \left(1 - \frac{1}{10000}\right)$.

If a person is ill the test will be 100% correct and give a correct answer that he is ill, so the probability that he us sick and the test will show it is:

$P(p \setminus s) = (1)$.

If he is healthy the test will be wrong in 1% and show that he is ill, so the probability that the person is healthy but the test showed that he is ill is:

$P(p \setminus h) = \left(\frac{1}{100}\right)$.

s − sick

h − healthy

p − positive

$P\ (actual\ sick) = P(p \setminus s)P(s) + P(p \setminus h)P(h) = 1 \cdot \frac{1}{10000} + \frac{1}{100} \cdot (1 - \frac{1}{10000}) = 0.0100$

Q 3.b:

Now according to the new question that i came from Thailand and for those who came back from Thailand the probability that he is sick is now:

$P(s) = \left(\frac{1}{200}\right)$.

We will put this in the equation instead of $P(s) = \left(\frac{1}{10000}\right)$ and we will get,

$P\ (actual\ sick) = P(p \setminus s)P(s) + P(p \setminus h)P(h) = 1 \cdot \frac{1}{200} + \frac{1}{100} \cdot (1 - \frac{1}{200}) = 0.0149$

<u>Answers for random variables</u>

Q 1:

$E(x) = \sum_{i=0}^{n} x_1 \cdot P(x_1)$

$E(x) = (3 \cdot \frac{12}{36}) + (-3 \cdot \frac{24}{36}) = -1$

$\frac{12}{36}$ - this is the probability of getting a number divided by 3

$\frac{24}{36}$ - this is the probability of getting a number that is not divisible by 3

Q 2:

$$E(x) = \sum_{i=0}^{n} x_1 \cdot P(x_1)$$

a total of 25 options:
in 6 he wins.
in 15 he loses.
in 4 there is a tie breaker.

$$E(x) = (5 \cdot \frac{6}{25}) + (-6 \cdot \frac{15}{24}) = -2\frac{2}{5} = -2.4$$

$\frac{6}{25}$ - this is the probability that the sum is greater than 12
$\frac{15}{24}$ - this is the probability that the sum is lower than 12 (here it's 24 and not 25 because we have already taken out one marker before)

Q 3:

There are 200 employees and 40% of them are men.
So there are 80 men and 120 women.

$\mu = n \cdot p$

according to the data, $n = 8$ and $p = 0.4$

$\mu = 0.4 \cdot 8 = 3.2$

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4(1-0.4)}{8}} = 0.1732$$

the mean is: $\mu = 3.2$
the standard deviation is: $\sigma = 0.1732$

Q 4:

We will use the formula:

$P(a < X < b) = P(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma})$

when $Z = \frac{x-\mu}{\sigma}$

According to question:

$\mu = 26$ - mean

$\sigma = 2$ - standard deviation

$P(26 < X < 30) = P(\frac{26-26}{2} < Z < \frac{30-26}{2}) = P(0 < Z < 2)$

$P(Z < 2) = 0.9772$

$P(0 < Z) = 0.5000$

$P(Z < 2) - P(0 < Z) = 0.9772 - 0.5000 = 0.4772$

$P(26 < X < 30) = 0.4772$

Q 5:

This is a calculation of the area of the triangle below the graph in the range between 3 and 5.

$y = 0.4$

$x = 2$ (the length from 3-5 in the graph)

$P(x > 3) = \frac{0.4 \cdot 2}{2} = 0.4$

Q 6:

500 employees of which 300 with children and 200 without children.

$\frac{300}{500} \cdot \frac{299}{499} \cdot \frac{298}{498} \cdot \frac{200}{497} = 0.0865$ - here the first three have children

$\frac{200}{500} \cdot \frac{300}{499} \cdot \frac{299}{498} \cdot \frac{298}{497} = 0.0865$ - here the last three have children

$\frac{300}{500} \cdot \frac{200}{499} \cdot \frac{299}{498} \cdot \frac{298}{497} = 0.0865$ - here only number two has no children

$\frac{300}{500} \cdot \frac{299}{499} \cdot \frac{200}{498} \cdot \frac{298}{497} = 0.0865$ - here only number three has no children

$0.865 + 0.865 + 0.865 + 0.865 = 3.46$

Q 7:

$E(x) = \sum_{i=0}^{n} x_1 \cdot P(x_1)$

$E(x) = (-10 \cdot 0.1) + (-5 \cdot 0.35) + (0 \cdot 0.1) + (5 \cdot 0.35) + (10 \cdot 0.1)$

$E(x) = -1 + (-1.75) + 0 + 1.75 + 1$

$E(x) = 0$