Cauchy–Schwarz inequality

The Cauchy–Schwarz inequality states that for all vectors *x* and *y* of an inner product space it is true that $\langle x, y\rangle^2 \leq \langle x, x\rangle\langle y, y\rangle$ where $\langle x, y\rangle$ is an inner product.

Define: $z = x - \frac{\langle y, x\rangle}{\langle y, y\rangle} y$; $\langle z, y\rangle = \left\langle x - \frac{\langle y, x\rangle}{\langle y, y\rangle} y, y \right\rangle = \langle x, y\rangle - \frac{\langle y, x\rangle}{\langle y, y\rangle}\langle y, y\rangle = 0$.

Therefore $z \quad y$ are orthogonal vectors.

$$x = z + \frac{\langle y, x\rangle}{\langle y, y\rangle} y$$

$$\langle x, x\rangle = \left\langle z + \frac{\langle y, x\rangle}{\langle y, y\rangle} y, z + \frac{\langle y, x\rangle}{\langle y, y\rangle} y \right\rangle = \langle z, z\rangle + \left(\frac{\langle y, x\rangle}{\langle y, y\rangle}\right)^2 \langle y, y\rangle = \langle z, z\rangle + \frac{\langle y, x\rangle^2}{\langle y, y\rangle} \geq \frac{\langle y, x\rangle^2}{\langle y, y\rangle}$$

$$\langle x, x\rangle\langle y, y\rangle \geq \langle y, x\rangle^2$$

Define: $\langle x, y\rangle = \int\limits_{-\infty}^{\infty} x(t)\overline{y}(t)dt$

Assume $y(t) = x(t+\tau)$

$$\left(\int\limits_{-\infty}^{\infty} x(\tau)x(\tau)d\tau\right)^2 = \langle x, x\rangle^2 = \langle x, x\rangle\langle y, y\rangle \geq \langle y, x\rangle^2 = \left(\int\limits_{-\infty}^{\infty} x(t+\tau)x(\tau)d\tau\right)^2$$