$$f(x, y) \rightarrow \hat{f}(u, v)$$

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f(\tilde{x}, \tilde{y}) \rightarrow \iint\limits_{-\infty}^{\infty} f(\tilde{x}, \tilde{y}) \exp(-i2\pi(xu + yv) dx dy =$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}$$

$$f(\tilde{x}, \tilde{y}) \rightarrow \iint\limits_{-\infty}^{\infty} f(\tilde{x}, \tilde{y}) \exp(-i2\pi((\cos\theta\,\tilde{x} - \sin\theta\,\tilde{y})u + (\sin\theta\,\tilde{x} + \cos\theta\,\tilde{y})v) dx dy =$$

$$f(\tilde{x}, \tilde{y}) \rightarrow \iint\limits_{-\infty}^{\infty} f(\tilde{x}, \tilde{y}) \exp(-i2\pi(\tilde{x}(\cos\theta u + \sin\theta v) + \tilde{y}(\cos\theta\,v - \sin\theta\,u)) dx dy =$$

$$\begin{bmatrix} \dfrac{dx}{d\tilde{x}} & \dfrac{dx}{d\tilde{y}} \\ \dfrac{dy}{d\tilde{x}} & \dfrac{dy}{d\tilde{y}} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{vmatrix} \dfrac{dx}{d\tilde{x}} & \dfrac{dx}{d\tilde{y}} \\ \dfrac{dy}{d\tilde{x}} & \dfrac{dy}{d\tilde{y}} \end{vmatrix} = 1$$

$$dx dy = \left( \frac{dx}{d\tilde{x}}\frac{dy}{d\tilde{y}} - \frac{dy}{d\tilde{x}}\frac{dx}{d\tilde{y}} \right) d\tilde{x} d\tilde{y} = \begin{vmatrix} \dfrac{dx}{d\tilde{x}} & \dfrac{dx}{d\tilde{y}} \\ \dfrac{dy}{d\tilde{x}} & \dfrac{dy}{d\tilde{y}} \end{vmatrix} d\tilde{x} d\tilde{y} = d\tilde{x} d\tilde{y}$$

$$f(\tilde{x}, \tilde{y}) \rightarrow \iint\limits_{-\infty}^{\infty} f(\tilde{x}, \tilde{y}) \exp(-i2\pi(\tilde{x}(\cos\theta u + \sin\theta v) + \tilde{y}(\cos\theta\,v - \sin\theta\,u)) d\tilde{x} d\tilde{y} =$$

$$f(\tilde{x}, \tilde{y}) \rightarrow \iint\limits_{-\infty}^{\infty} f(\tilde{x}, \tilde{y}) \exp(-i2\pi(\tilde{x}\tilde{u} + \tilde{y}\tilde{v}) d\tilde{x} d\tilde{y} = \hat{f}(\tilde{u}, \tilde{v})$$

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$