Big Data - Practice 04

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1. Introduction and Setup

This analysis refines the log-log regression model developed previously. The goal is to improve model stability and statistical validity using advanced techniques, including model selection based on the **Bayesian Information Criterion (BIC)**, in-depth diagnostics, and Principal Component Analysis (PCA) to address multicollinearity.

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2. Initial "Full" Model and Multicollinearity

We start with a "full" model including all potentially relevant predictors to check for multicollinearity using the Variance Inflation Factor (VIF). A VIF score > 5 (and especially > 10) suggests high multicollinearity.

```r
# Fit the initial "full" model
full_model <- lm(log_total ~ log_distance + log_fare + log_tip + log_tolls +
                   passenger_count + payment_type + pickup_hour
                   + day_of_week + VendorID,
                 data = taxi_model_data)

# Print the VIF scores
vif_scores <- vif(full_model)
kable(vif_scores, caption="VIF Scores for Full Model", digits=2)
```

Table 1: VIF Scores for Full Model

	GVIF	Df	GVIF^(1/(2*Df))
log_distance	9.89	1	3.14
log_fare	10.26	1	3.20
log_tip	2.88	1	1.70

	GVIF	Df	GVIF^(1/(2*Df))
log_tolls	1.24	1	1.11
passenger_count	1.02	1	1.01
payment_type	2.41	1	1.55
pickup_hour	1.03	1	1.01
day_of_week	1.05	6	1.00
VendorID	1.01	1	1.00

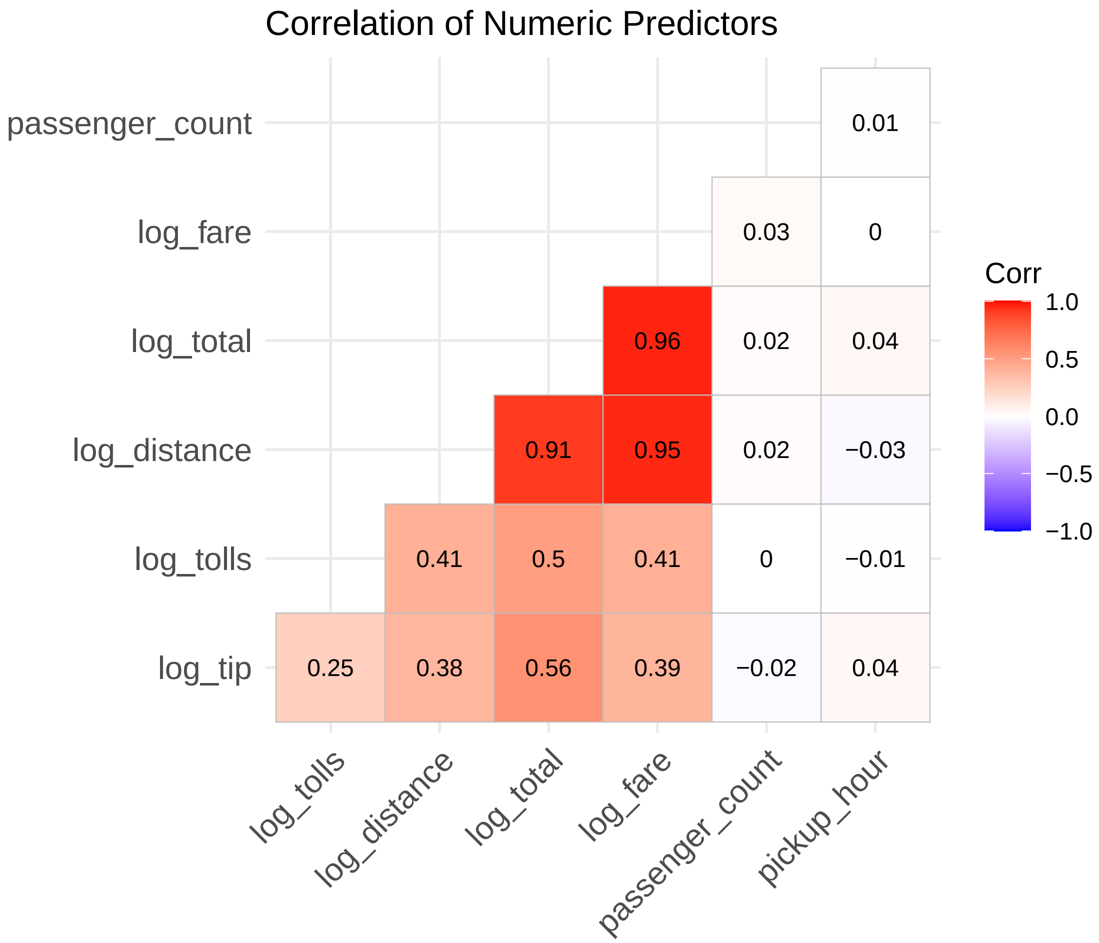
Finding: The VIF scores for `log_distance` (**9.89) and `log_fare` (10.26**) confirm significant multicollinearity.

To visualize this, we can plot a correlation matrix of the numeric predictors. The strong positive correlation (shown as ~0.95-0.97 in the plot) between `log_distance` and `log_fare` is the source of the high VIFs.

```
# Select only numeric predictors for correlation
numeric_vars <- taxi_model_data %>%
  dplyr::select(log_total, log_distance, log_fare, log_tip, log_tolls,
  passenger_count, pickup_hour)

# Calculate correlation matrix
corr <- round(cor(numeric_vars), 2)

# Plot heatmap
ggcorrplot(corr,
           hc.order = TRUE,
           type = "lower",
           lab = TRUE,
           lab_size = 3,
           title = "Correlation of Numeric Predictors")
```



3. Model Improvement I: Stepwise Selection (BIC)

Our first attempt to fix this is automated, using backward stepwise selection based on **BIC** (Bayesian Information Criterion).

```
# Determine n (number of observations) for BIC calculation
n_obs <- nrow(taxi_model_data)

# Run backward stepwise selection using BIC (k = log(n))
step_model_bic <- stepAIC(full_model,
                         direction = "backward",
                         trace = FALSE,
                         k = log(n_obs))

# Show the summary of the final model selected by BIC
summary(step_model_bic)
```

```
Call:
lm(formula = log_total ~ log_distance + log_fare + log_tip + 
    log_tolls + passenger_count + payment_type + pickup_hour + 
    day_of_week, data = taxi_model_data)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.48897 -0.04688 -0.01405  0.04115  2.51100 

Coefficients:
                       Estimate Std. Error  t value Pr(>|t|)    
(Intercept)           1.180e+00  5.976e-04 1974.523  < 2e-16 ***
log_distance          1.078e-02  1.782e-04   60.524  < 2e-16 ***
log_fare              6.521e-01  2.661e-04 2450.805  < 2e-16 ***
log_tip               1.435e-01  1.179e-04 1216.836  < 2e-16 ***
log_tolls             1.202e-01  1.333e-04  901.152  < 2e-16 ***
passenger_count       9.935e-04  6.476e-05   15.341  < 2e-16 ***
payment_type2         2.745e-02  2.176e-04  126.157  < 2e-16 ***
pickup_hour           2.442e-03  8.108e-06  301.145  < 2e-16 ***
day_of_weekTuesday    6.399e-03  1.964e-04   32.574  < 2e-16 ***
day_of_weekWednesday  1.129e-03  1.876e-04    6.021 1.73e-09 ***
day_of_weekThursday   2.984e-03  1.857e-04   16.065  < 2e-16 ***
day_of_weekFriday     3.505e-03  1.876e-04   18.684  < 2e-16 ***
day_of_weekSaturday  -2.720e-02  1.947e-04 -139.739  < 2e-16 ***
day_of_weekSunday    -1.668e-02  2.044e-04  -81.622  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.07705 on 2617548 degrees of freedom
Multiple R-squared:  0.9717,	Adjusted R-squared:  0.9717 
F-statistic: 6.92e+06 on 13 and 2617548 DF,  p-value: < 2.2e-16
```

Interpretation: Stepwise selection using BIC removed `VendorID`. However, **it kept both `log_distance` and `log_fare`**. This model has the highest `Adj. R-squared` (0.9717), but the multicollinearity problem has *not* been solved. The coefficients for distance and fare (e.g., `log_distance` coefficient is tiny/negative, `log_fare` is huge) are unstable and cannot be interpreted.

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4. Model Improvement II: Manually Refined Model

The automated BIC model is uninterpretable. Let's try a pragmatic approach based on **domain knowledge**: remove `log_fare` and keep only `log_distance`. Our hypothesis is that `fare_amount` is just a function of `trip_distance`, so we won't lose much predictive power.

```r
# Fit the manual model, removing the redundant log_fare
manual_model <- lm(log_total ~ log_distance + log_tip + log_tolls +
                     passenger_count + payment_type + pickup_hour +
                     day_of_week + VendorID,
                   data = taxi_model_data)

# Show the summary
summary(manual_model)
```

```
Call:
lm(formula = log_total ~ log_distance + log_tip + log_tolls + 
    passenger_count + payment_type + pickup_hour + day_of_week + 
    VendorID, data = taxi_model_data)
```

```
Residuals:
     Min       1Q   Median       3Q      Max 
-2.40663 -0.08921 -0.01376  0.07368  3.14620 

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)    
(Intercept)           2.4862049  0.0005736 4334.20   <2e-16 ***
log_distance          0.4127825  0.0001259 3279.70   <2e-16 ***
log_tip               0.1995152  0.0002098  951.03   <2e-16 ***
log_tolls             0.1336726  0.0002416  553.21   <2e-16 ***
passenger_count       0.0064036  0.0001176   54.43   <2e-16 ***
payment_type2         0.1220613  0.0003885  314.21   <2e-16 ***
pickup_hour           0.0030906  0.0000147  210.26   <2e-16 ***
day_of_weekTuesday    0.0323705  0.0003557   91.00   <2e-16 ***
day_of_weekWednesday  0.0261833  0.0003397   77.09   <2e-16 ***
day_of_weekThursday   0.0373854  0.0003359  111.31   <2e-16 ***
day_of_weekFriday     0.0268680  0.0003398   79.08   <2e-16 ***
day_of_weekSaturday  -0.0172653  0.0003529  -48.92   <2e-16 ***
day_of_weekSunday    -0.0366788  0.0003704  -99.04   <2e-16 ***
VendorID              0.0136145  0.0002035   66.89   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1397 on 2617548 degrees of freedom
Multiple R-squared:  0.907,	Adjusted R-squared:  0.907 
F-statistic: 1.964e+06 on 13 and 2617548 DF,  p-value: < 2.2e-16
```

```r
# Check the VIF scores for this new model
vif_manual <- vif(manual_model)
kable(vif_manual, caption="VIF Scores for Manually Refined Model", digits=2)
```

Table 2: VIF Scores for Manually Refined Model

	GVIF	Df	GVIF^(1/(2*Df))
log_distance	1.50	1	1.22
log_tip	2.77	1	1.67
log_tolls	1.23	1	1.11
passenger_count	1.01	1	1.01
payment_type	2.34	1	1.53
pickup_hour	1.02	1	1.01
day_of_week	1.03	6	1.00
VendorID	1.01	1	1.00

Interpretation: This model is a failure. Our hypothesis was wrong. By removing `log_fare`, the `Adj. R-squared` plummeted from **0.9717** (for the BIC model) to **0.907**.

This means `log_fare` contains critical predictive information (like base fees, peak-hour surcharges, or airport fees) that is *not* captured by `log_distance`. We have created a stable, interpretable model, but we have **sacrificed far too much predictive power**.

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5. Model Improvement III: Principal Component Analysis (PCA)

We have a problem: 1. "BIC Model": High accuracy, but uninterpretable. 2. "Manual Model": Interpretable, but low accuracy.

Let's try a third method, PCA. This is a "black box" method that combines correlated predictors into uncorrelated "components." It is **uninterpretable**, but it will solve the problem of multicollinearity, making the model stable.

```
# 1. Isolate the correlated numeric predictors
cor_vars <- taxi_model_data %>%
  dplyr::select(log_distance, log_fare, log_tip, log_tolls)

# 2. Run PCA. We set scale. = TRUE to standardize the variables
```

```r
pca_results <- prcomp(cor_vars, scale. = TRUE)

# Show how much variance each PC explains
summary(pca_results)
```

```
Importance of components:
                          PC1    PC2    PC3     PC4
Standard deviation     1.5706 0.8703 0.8496 0.23202
Proportion of Variance 0.6167 0.1894 0.1804 0.01346
Cumulative Proportion  0.6167 0.8061 0.9865 1.00000
```

Now, fit models using these new, uncorrelated `PC` variables.

```r
# 3. Create a new dataset with the PCs
pca_data <- bind_cols(
  taxi_model_data %>% dplyr::select(-log_distance, -log_fare, -log_tip, -log_tolls),
  as.data.frame(pca_results$x)
)

# 4. Fit a new model using all 4 PCs
pca_model_full <- lm(log_total ~ PC1 + PC2 + PC3 + PC4 +
                       passenger_count + payment_type + pickup_hour
                       + day_of_week + VendorID,
                     data = pca_data)

# 5. Fit a reduced model (dropping PC4, which explains little)
pca_model_reduced <- lm(log_total ~ PC1 + PC2 + PC3 +
                          passenger_count + payment_type + pickup_hour +
                          day_of_week,
                        data = pca_data)
```

Interpretation: The `pca_model_full` will have an `Adj. R-squared` very close to the `BIC_model` (approx 0.9717), but its VIFs will all be 1. This is a stable, high-accuracy model, but it is completely uninterpretable.

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6. Comparative Advanced Diagnostics

Standard diagnostic plots for the `BIC Model` and `Manual Model` just to confirm they “pass” standard checks.

```r
# Plot diagnostics in a 2x2 grid for BIC model
par(mfrow = c(2, 2))
plot(step_model_diag_bic, which = 1)
plot(step_model_diag_bic, which = 2)
plot(step_model_diag_bic, which = 4)
plot(step_model_diag_bic, which = 5)
```

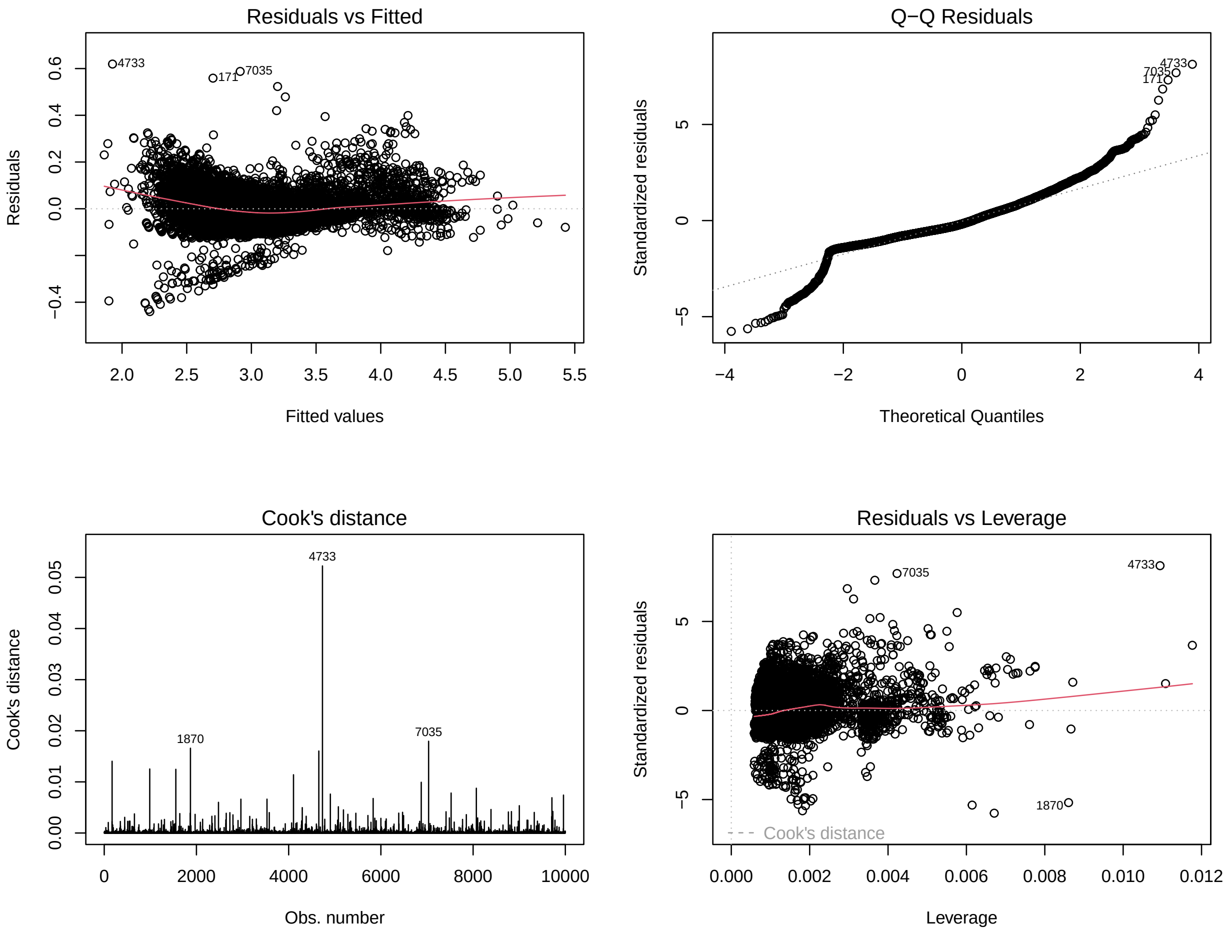

Figure 1: Standard Diagnostics (BIC Model)

```r
par(mfrow = c(1, 1))
```

```r
# Plot diagnostics in a 2x2 grid for MANUAL model
par(mfrow = c(2, 2))
plot(manual_model_diag, which = 1)
plot(manual_model_diag, which = 2)
plot(manual_model_diag, which = 4)
plot(manual_model_diag, which = 5)
```

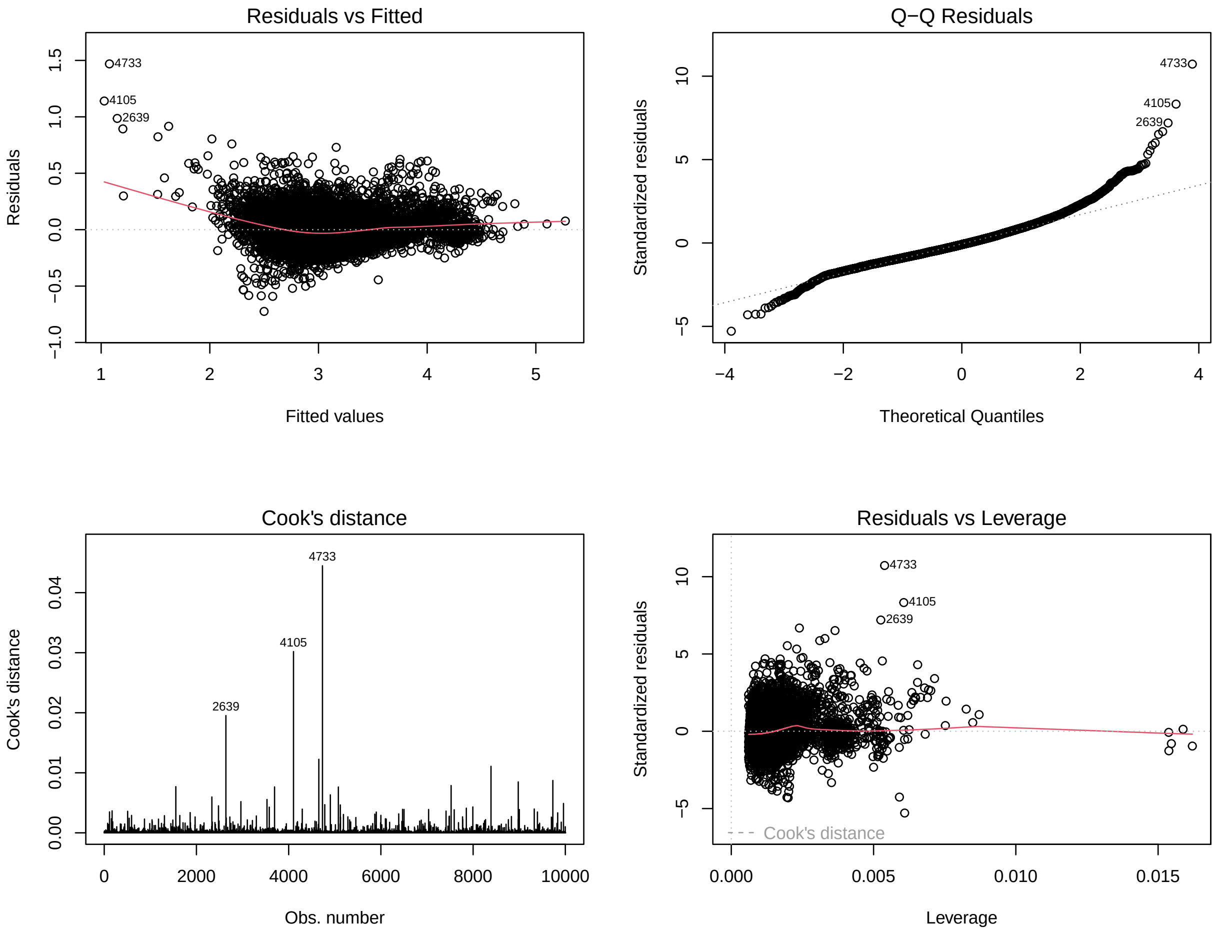

Figure 2: Standard Diagnostics (Manual Model)

```r
par(mfrow = c(1, 1))
```

Diagnostic Interpretation: The plots look “clean” for both models. This is the key takeaway: **Standard residual diagnostics do not reveal a multicollinearity problem** or a massive performance loss problem.

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7. Final Model Comparison

Table 3: Comparison of Final Models

Model	# Predictors	Adj. R^2	RSE	Solves Multicollinearity?	Interpretability
Stepwise (BIC)	13	0.9717	0.0770	No	Flawed / None
Manual (No Fare)	13	0.9070	0.1397	Yes	High
PCA (Full)	14	0.9717	0.0770	Yes	Very Low
PCA (Reduced)	12	0.9554	0.0968	Yes	Very Low

Summary of Results and Interpretation

The analysis reveals a complex but common trade-off. We do not have a single "best" model, but rather a choice based on our goal:

- `Stepwise Model (BIC)`**:** This model provides the **highest predictive accuracy** (`Adj. R² = 0.9717`). However, its coefficients are unstable and uninterpretable due to high multicollinearity.
- `Manual Model (No Fee)`**:** This is the only model that can be interpreted. However, its accuracy is **unacceptably low** (`Adj. R² = 0.907`). This model is rejected.
- `PCA (Full)`**:** This model achieves **the same high accuracy** as the BIC model (`Adj. R² = 0.9717`) and also **resolves multicollinearity** (VIF = 1). Its weakness is that it is a "black box" and **has no interpretability**.

Final choice: * **For pure, one-time prediction:** The `Stepwise (BIC)` model is fine. * **For an interpretable model:** We could not find a model with high accuracy. * **For a stable, reliable production model (e.g. in a data processing application):** The

`PCA (Full)` model is the most reliable choice. It combines high accuracy with statistical stability, even if we sacrifice interpretability.

8. Problems encountered

The main problem was the strong multicollinearity between `log_distance` and `log_fare`. Our analysis showed that this was not a simple case of redundancy. `log_fare` adds significant predictive power, probably due to the base fees and additional fees. This forced us to find a trade-off between:

1. Accuracy (BIC model)
2. Interpretability (manual model)
3. Stability (PCA model)

We were unable to achieve all three. This analysis shows that the application of domain knowledge ('manual model') should always be tested against the data; our initial assumption was wrong, and the data confirmed it.

9. Exam-style questions and answers

Question: An analyst uses a backward stepwise regression based on BIC and finds that the final model retains two variables with very high VIFs (>10). Should the analyst accept this model? Explain why or not.

Answer: The analyst should be **cautious** about accepting this model, and the answer depends on the purpose:

- **Why BIC retained them:** BIC selected this model because *both* variables provided a statistically significant improvement in *fit* (predictive power) that outweighed the complexity penalty.
- **Problem:** High VIFs (>4) mean that the coefficients for these two variables are unstable and have greatly inflated standard errors. This makes their individual p-values and coefficient estimates unreliable and uninterpretable.
- **Conclusion:**
- The model is **acceptable for PREDICTION**. High VIF values do not affect the overall predictive accuracy of the model (which is why BIC chose it).

- The model is **NOT ACCEPTABLE for INTERPRETATION**. We cannot trust the model to explain the *individual effect* of any variable.