

# Interaction

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## Interaction Overview

### Interaction R Part A

#### import data

```
IceCreamExcel <- read_excel('IceCreamExcel.xlsx')  
head(IceCreamExcel)
```

```
## # A tibble: 6 x 5  
##       id  cons income price  temp  
##   <dbl> <dbl>   <dbl> <dbl> <dbl>  
## 1    10 0.264     82 0.280    25  
## 2     39 0.256     79 0.277    24  
## 3      9 0.269     77 0.270    33  
## 4     38 0.269     76 0.265    32  
## 5     23 0.282     93 0.279    32  
## 6     52 0.284     94 0.277    32
```

```
str(IceCreamExcel)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':   58 obs. of  5 variables:  
## $ id      : num  10 39 9 38 23 52 11 40 8 37 ...  
## $ cons    : num  0.264 0.256 0.269 0.269 0.282 0.284 0.288 0.286 0.288  
##           0.289 ...  
## $ income: num  82 79 77 76 93 94 81 82 79 78 ...  
## $ price  : num  0.28 0.277 0.27 0.265 0.279 0.277 0.282 0.282 0.267 0.  
##           267 ...  
## $ temp   : num  25 24 33 32 32 32 28 28 47 48 ...
```

```
summary(IceCreamExcel)
```

```
##           id           cons           income           price  
## Min.      : 1.00    Min.      :0.2560    Min.      :76.00    Min.      :0.2540  
## 1st Qu.:15.25    1st Qu.:0.3103    1st Qu.:79.25    1st Qu.:0.2700  
## Median :29.50    Median :0.3465    Median :83.00    Median :0.2770  
## Mean      :29.50    Mean      :0.3534    Mean      :84.40    Mean      :0.2764  
## 3rd Qu.:43.75    3rd Qu.:0.3860    3rd Qu.:87.75    3rd Qu.:0.2820  
## Max.      :58.00    Max.      :0.4720    Max.      :96.00    Max.      :0.2920  
##           temp  
## Min.      :24.00  
## 1st Qu.:33.00  
## Median :47.50
```

```
## Mean :48.55
## 3rd Qu.:63.75
## Max. :73.00
```

### exclude variable x(id)

```
ICD <- subset(IceCreamExcel, select = -c(id))
str(ICD)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame': 58 obs. of 4 variables:
## $ cons : num 0.264 0.256 0.269 0.269 0.282 0.284 0.288 0.286 0.288
0.289 ...
## $ income: num 82 79 77 76 93 94 81 82 79 78 ...
## $ price : num 0.28 0.277 0.27 0.265 0.279 0.277 0.282 0.282 0.267 0.
267 ...
## $ temp : num 25 24 33 32 32 32 28 28 47 48 ...
```

### summary(ICD)

```
##      cons      income      price      temp
## Min.   :0.2560   Min.   :76.00   Min.   :0.2540   Min.   :24.00
## 1st Qu.:0.3103   1st Qu.:79.25   1st Qu.:0.2700   1st Qu.:33.00
## Median :0.3465   Median :83.00   Median :0.2770   Median :47.50
## Mean   :0.3534   Mean   :84.40   Mean   :0.2764   Mean   :48.55
## 3rd Qu.:0.3860   3rd Qu.:87.75   3rd Qu.:0.2820   3rd Qu.:63.75
## Max.   :0.4720   Max.   :96.00   Max.   :0.2920   Max.   :73.00
```

### run correlations

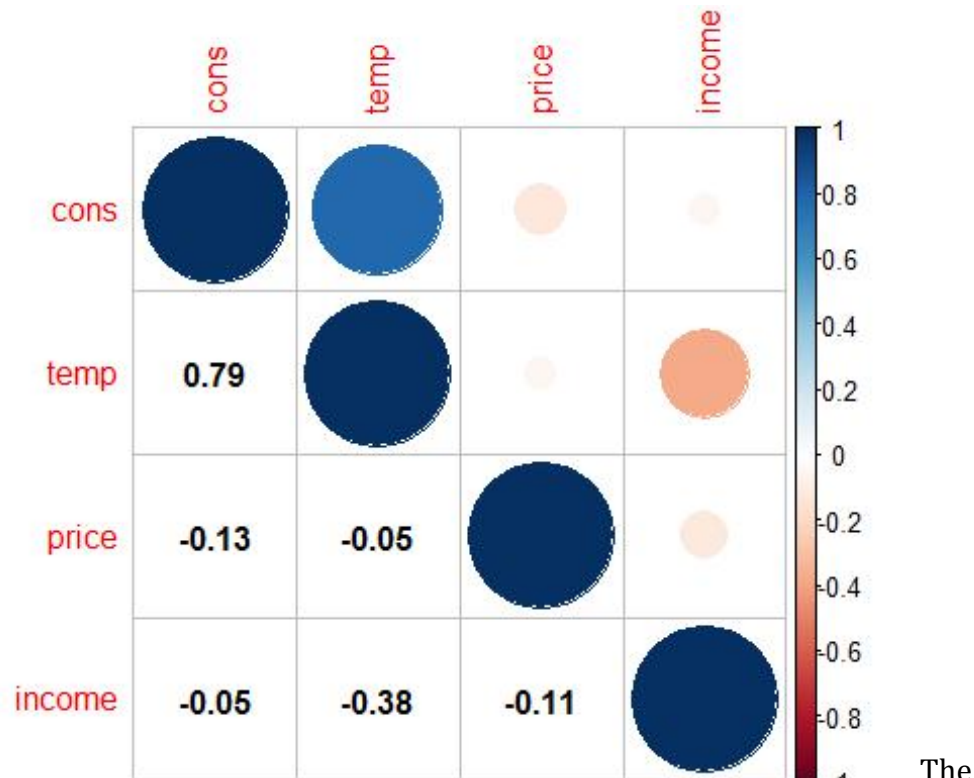
#### cor(ICD)

```
##      cons      income      price      temp
## cons  1.00000000 -0.04683888 -0.12709010  0.78761058
## income -0.04683888  1.00000000 -0.11173850 -0.37693836
## price  -0.12709010 -0.11173850  1.00000000 -0.04729122
## temp   0.78761058 -0.37693836 -0.04729122  1.00000000
```

The correlation matrix show us that,the correlation in variable *temp* and *cons* is about **0.788**,which mean these two variable have strong correlationship.And the other variables are almost independent.

### corr visualization

```
corr <- cor(ICD)
# corrplot::corrplot(corr = corr)
corrplot::corrplot(corr = corr, order = "AOE", tl.pos = "tp")
corrplot::corrplot(corr = corr,
  add = TRUE, type = "lower", method = "number", order = "AOE",
  col = "black", diag = FALSE, tl.pos = "n", cl.pos = "n")
```



The correlations plot only is another method for explanation of correlation matrix, what it tells us is the same.

### use Hmisc

```
library(Hmisc)
rcorr(as.matrix(ICD))

##      cons income price temp
## cons   1.00  -0.05 -0.13  0.79
## income -0.05   1.00 -0.11 -0.38
## price  -0.13  -0.11  1.00 -0.05
## temp   0.79  -0.38 -0.05  1.00
##
## n= 58
##
##
## P
##      cons  income price temp
## cons           0.7270 0.3418 0.0000
## income 0.7270           0.4037 0.0035
## price  0.3418 0.4037           0.7245
## temp   0.0000 0.0035 0.7245
```

The **rcorr** is not only provided correlation matrix, it also provides test in correlation across variable. And only *p-value* in test of *temp* and *cons* is smaller than 0.05, that is, *temp* is correlated with *cons*.

## Interaction R Part B

### create some new var, run correlations

```
ICD <- ICD %>% mutate(  
  Income_by_price = income * price,  
  Income_by_temp = income * temp ,  
  Price_by_temp = price * temp  
) %>% mutate(  
  income_mc = scale(income, center = TRUE),  
  price_mc = scale(price, center = TRUE),  
  temp_mc = scale(temp, center = TRUE)  
)
```

`summary(ICD)`

```
##      cons      income      price      temp  
## Min.   :0.2560   Min.   :76.00   Min.   :0.2540   Min.   :24.00  
## 1st Qu.:0.3103   1st Qu.:79.25   1st Qu.:0.2700   1st Qu.:33.00  
## Median :0.3465   Median :83.00   Median :0.2770   Median :47.50  
## Mean   :0.3534   Mean   :84.40   Mean   :0.2764   Mean   :48.55  
## 3rd Qu.:0.3860   3rd Qu.:87.75   3rd Qu.:0.2820   3rd Qu.:63.75  
## Max.   :0.4720   Max.   :96.00   Max.   :0.2920   Max.   :73.00  
## Income_by_price Income_by_temp Price_by_temp      income_mc.V1  
## Min.   :20.14   Min.   :1896   Min.   : 6.648   Min.   :-1.3880139  
## 1st Qu.:22.31   1st Qu.:2984   1st Qu.: 8.876   1st Qu.: -0.8507642  
## Median :23.27   Median :3834   Median :12.848   Median :-0.2308606  
## Mean   :23.32   Mean   :4062   Mean   :13.414   Mean   : 0.0000000  
## 3rd Qu.:24.63   3rd Qu.:5157   3rd Qu.:17.485   3rd Qu.: 0.5543505  
## Max.   :26.79   Max.   :5915   Max.   :20.586   Max.   : 1.9181383  
##      price_mc.V1      temp_mc.V1  
## Min.   :-2.7027504   Min.   :-1.5277786  
## 1st Qu.: -0.7734024   1st Qu.: -0.9677362  
## Median : 0.0706873   Median :-0.0654456  
## Mean   : 0.0000000   Mean   : 0.0000000  
## 3rd Qu.: 0.6736085   3rd Qu.: 0.9457422  
## Max.   : 1.8794510   Max.   : 1.5213414
```

`rcorr(as.matrix(ICD))`

```
##      cons income price temp Income_by_price Income_by_tem  
p  
## cons      1.00 -0.05 -0.13 0.79      -0.11      0.82  
## income -0.05  1.00 -0.11 -0.38      0.91      -0.19  
## price  -0.13 -0.11  1.00 -0.05      0.31      -0.08  
## temp    0.79 -0.38 -0.05  1.00     -0.39      0.98  
## Income_by_price -0.11 0.91 0.31 -0.39      1.00      -0.23  
## Income_by_temp 0.82 -0.19 -0.08 0.98     -0.23      1.00  
## Price_by_temp 0.78 -0.39 0.04 1.00     -0.36      0.97  
## income_mc -0.05  1.00 -0.11 -0.38      0.91      -0.19  
## price_mc -0.13 -0.11  1.00 -0.05      0.31      -0.08
```

```

## temp_mc      0.79 -0.38 -0.05 1.00      -0.39      0.98
##      Price_by_temp income_mc price_mc temp_mc
## cons      0.78      -0.05      -0.13      0.79
## income     -0.39      1.00      -0.11      -0.38
## price      0.04      -0.11      1.00      -0.05
## temp       1.00      -0.38      -0.05      1.00
## Income_by_price -0.36      0.91      0.31      -0.39
## Income_by_temp  0.97      -0.19      -0.08      0.98
## Price_by_temp  1.00      -0.39      0.04      1.00
## income_mc    -0.39      1.00      -0.11      -0.38
## price_mc     0.04      -0.11      1.00      -0.05
## temp_mc      1.00      -0.38      -0.05      1.00
##
## n= 58
##
##
## P
##      cons   income price   temp   Income_by_price Income_by_
temp
## cons      0.7270 0.3418 0.0000 0.4067      0.0000

## income     0.7270      0.4037 0.0035 0.0000      0.1529

## price      0.3418 0.4037      0.7245 0.0188      0.5261

## temp       0.0000 0.0035 0.7245      0.0025      0.0000

## Income_by_price 0.4067 0.0000 0.0188 0.0025      0.0865

## Income_by_temp  0.0000 0.1529 0.5261 0.0000 0.0865

## Price_by_temp  0.0000 0.0025 0.7590 0.0000 0.0049      0.0000

## income_mc    0.7270 0.0000 0.4037 0.0035 0.0000      0.1529

## price_mc     0.3418 0.4037 0.0000 0.7245 0.0188      0.5261

## temp_mc      0.0000 0.0035 0.7245 0.0000 0.0025      0.0000

##      Price_by_temp income_mc price_mc temp_mc
## cons      0.0000      0.7270      0.3418      0.0000
## income     0.0025      0.0000      0.4037      0.0035
## price      0.7590      0.4037      0.0000      0.7245
## temp       0.0000      0.0035      0.7245      0.0000
## Income_by_price 0.0049      0.0000      0.0188      0.0025
## Income_by_temp  0.0000      0.1529      0.5261      0.0000
## Price_by_temp  0.0025      0.0025      0.7590      0.0000
## income_mc    0.0025      0.0025      0.4037      0.0035

```

```
## price_mc      0.7590      0.4037      0.7245
## temp_mc       0.0000      0.0035      0.7245
```

### create two\_way interaction terms and view corr

```
ICD <- ICD %>% mutate(
  Income_by_price = income_mc * price_mc,
  Income_by_temp  = income_mc * temp_mc ,
  Price_by_temp   = price_mc * temp_mc
)
```

### summary(ICD)

```
##      cons      income      price      temp
## Min.   :0.2560   Min.   :76.00   Min.   :0.2540   Min.   :24.00
## 1st Qu.:0.3103   1st Qu.:79.25   1st Qu.:0.2700   1st Qu.:33.00
## Median :0.3465   Median :83.00   Median :0.2770   Median :47.50
## Mean   :0.3534   Mean   :84.40   Mean   :0.2764   Mean   :48.55
## 3rd Qu.:0.3860   3rd Qu.:87.75   3rd Qu.:0.2820   3rd Qu.:63.75
## Max.   :0.4720   Max.   :96.00   Max.   :0.2920   Max.   :73.00
## Income_by_price.V1 Income_by_temp.V1 Price_by_temp.V1
## Min.   :-5.184249 Min.   :-2.2416441 Min.   :-1.8871213
## 1st Qu.: -0.366100 1st Qu.: -1.0429021 1st Qu.: -0.6270359
## Median : -0.063060 Median : -0.2647573 Median : 0.0446646
## Mean   : -0.109812 Mean   : -0.3704394 Mean   : -0.0464759
## 3rd Qu.: 0.625419 3rd Qu.: 0.1130087 3rd Qu.: 0.5054885
## Max.   : 1.910356 Max.   : 1.4296031 Max.   : 1.4175626
## income_mc.V1      price_mc.V1      temp_mc.V1
## Min.   :-1.3880139 Min.   :-2.7027504 Min.   :-1.5277786
## 1st Qu.: -0.8507642 1st Qu.: -0.7734024 1st Qu.: -0.9677362
## Median : -0.2308606 Median : 0.0706873 Median : -0.0654456
## Mean   : 0.0000000 Mean   : 0.0000000 Mean   : 0.0000000
## 3rd Qu.: 0.5543505 3rd Qu.: 0.6736085 3rd Qu.: 0.9457422
## Max.   : 1.9181383 Max.   : 1.8794510 Max.   : 1.5213414
```

### rcorr(as.matrix(ICD))

```
##      cons income price temp Income_by_price Income_by_tem
p
## cons      1.00 -0.05 -0.13 0.79      -0.39      -0.21
## income    -0.05 1.00 -0.11 -0.38      -0.49      -0.26
## price     -0.13 -0.11 1.00 -0.05      0.26      -0.12
## temp       0.79 -0.38 -0.05 1.00      -0.09      -0.12
## Income_by_price -0.39 -0.49 0.26 -0.09      1.00      -0.09
## Income_by_temp  -0.21 -0.26 -0.12 -0.12      -0.09      1.00
## Price_by_temp   0.22 -0.13 -0.09 0.12      -0.31      0.07
## income_mc     -0.05 1.00 -0.11 -0.38      -0.49      -0.26
## price_mc      -0.13 -0.11 1.00 -0.05      0.26      -0.12
## temp_mc       0.79 -0.38 -0.05 1.00      -0.09      -0.12
## Price_by_temp income_mc price_mc temp_mc
## cons      0.22      -0.05      -0.13      0.79
```

```

## income          -0.13      1.00     -0.11    -0.38
## price           -0.09     -0.11      1.00    -0.05
## temp            0.12     -0.38     -0.05     1.00
## Income_by_price -0.31     -0.49      0.26    -0.09
## Income_by_temp  0.07     -0.26     -0.12    -0.12
## Price_by_temp   1.00     -0.13     -0.09     0.12
## income_mc       -0.13      1.00     -0.11    -0.38
## price_mc        -0.09     -0.11      1.00    -0.05
## temp_mc         0.12     -0.38     -0.05     1.00
##
## n= 58
##
##
## P
##           cons   income price  temp   Income_by_price Income_by_
temp
## cons                0.7270 0.3418 0.0000 0.0023           0.1100

## income            0.7270           0.4037 0.0035 0.0000           0.0530

## price             0.3418 0.4037           0.7245 0.0510           0.3822

## temp              0.0000 0.0035 0.7245           0.5115           0.3597

## Income_by_price  0.0023 0.0000 0.0510 0.5115           0.5042

## Income_by_temp   0.1100 0.0530 0.3822 0.3597 0.5042

## Price_by_temp     0.1024 0.3219 0.5146 0.3738 0.0172           0.6167

## income_mc         0.7270 0.0000 0.4037 0.0035 0.0000           0.0530

## price_mc          0.3418 0.4037 0.0000 0.7245 0.0510           0.3822

## temp_mc           0.0000 0.0035 0.7245 0.0000 0.5115           0.3597

##           Price_by_temp income_mc price_mc temp_mc
## cons          0.1024           0.7270   0.3418   0.0000
## income         0.3219           0.0000   0.4037   0.0035
## price          0.5146           0.4037   0.0000   0.7245
## temp           0.3738           0.0035   0.7245   0.0000
## Income_by_price 0.0172           0.0000   0.0510   0.5115
## Income_by_temp   0.6167           0.0530   0.3822   0.3597
## Price_by_temp    0.3219           0.5146   0.3738
## income_mc        0.3219           0.4037   0.0035
## price_mc         0.5146           0.4037           0.7245
## temp_mc          0.3738           0.0035   0.7245

```

This step add variable interaction term in raw data.

## Interaction R Part C

### testing linear model w/no interactions

```
model <- lm(cons ~ income + price + temp, data = ICD)
summary(model)

##
## Call:
## lm(formula = cons ~ income + price + temp, data = ICD)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.059146 -0.018587  0.007452  0.018436  0.071937
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0819503  0.1662346   0.493  0.62403
## income       0.0026093  0.0007533   3.464  0.00105 **
## price      -0.3575747  0.5095112  -0.702  0.48582
## temp        0.0030912  0.0002821  10.958 2.43e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03155 on 54 degrees of freedom
## Multiple R-squared:  0.696, Adjusted R-squared:  0.6791
## F-statistic: 41.21 on 3 and 54 DF,  p-value: 5.46e-14

library(car)

vif(model)

## income price temp
## 1.188929 1.022290 1.176717
```

According to the result of this model, and the function **Vif**, which compute each variable's **Variance Inflation Factors**; and vif value in each variable is smaller than 10, which mean that collinearity in model does not exist. And the *R.Square* in this model is about **0.68**.

### testing linear model w/no interactions without pequod, note use of mean center predictors

```
model1 <- lm(cons ~ income + price + temp + Income_by_temp +
             Income_by_price + Price_by_temp, data = ICD)
summary(model1)

##
## Call:
## lm(formula = cons ~ income + price + temp + Income_by_temp +
##      Income_by_price + Price_by_temp, data = ICD)
```



```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.055437 -0.014994 -0.001467  0.014653  0.075777
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.1940702  0.1684847   1.152   0.2548
## income        0.0007633  0.0010230   0.746   0.4590
## price        -0.1412090  0.4870840  -0.290   0.7731
## temp          0.0026738  0.0003001   8.910 5.66e-12 ***
## Income_by_temp -0.0079020  0.0050423  -1.567   0.1233
## Income_by_price -0.0127447  0.0048355  -2.636   0.0111 *
## Price_by_temp   0.0040494  0.0055626   0.728   0.4700
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02927 on 51 degrees of freedom
## Multiple R-squared:  0.7529, Adjusted R-squared:  0.7239
## F-statistic: 25.9 on 6 and 51 DF, p-value: 6.96e-14
```

```
vif(model1)
```

```
##      income      price      temp Income_by_temp
##      2.548701      1.085771      1.547520      1.358149
## Income_by_price Price_by_temp
##      2.206110      1.288576
```

This new model add interaction term. According to the summary of the new model, we can know that all predictors' **Variance Inflation Factors** in this model are still smaller than 10, which means that collinearity in new model does not exist too, but in **R-Square** is about **0.72**, that is to say this model performs better than origin model.

### testing linear model w/no interactions using pequod

```
library(pequod)
```

```
modelpe <- lmres(cons ~ income + price + temp, data = ICD)
summary(modelpe)
```

```
## Formula:
## cons ~ income + price + temp
## <environment: 0x00000000131e4ae8>
##
## Models
##      R      R^2  Adj. R^2    F    df1  df2  p.value
## Model 0.834 0.696   0.679 41.206  3.000   54 5.5e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residuals
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -0.0591 -0.0186  0.0075  0.0000  0.0184  0.0719
##
## Coefficients
##               Estimate   StdErr  t.value    beta p.value
## (Intercept)  0.08195   0.16623   0.49298      0.62403
## income       0.00261   0.00075   3.46405   0.2834 0.00105 **
## price       -0.35757   0.50951  -0.70180  -0.0532 0.48582
## temp        0.00309   0.00028  10.95810   0.8919 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Collinearity
##               VIF Tolerance
## income 1.1889    0.8411
## price  1.0223    0.9782
## temp   1.1767    0.8498

modelpe1 <- lmres(cons ~ income * price + income * temp + price * temp,
                  centered = c("income", "price", "temp"), data = ICD)
summary(modelpe1)

## Formula:
## cons ~ income + price + temp + income.XX.price + income.XX.temp +
##       price.XX.temp
## <environment: 0x000000000b225020>
##
## Models
##           R      R^2  Adj. R^2    F      df1  df2 p.value
## Model  0.868  0.753    0.724 25.904   6.000   51  7e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residuals
##   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -0.0554 -0.0150 -0.0015  0.0000  0.0147  0.0758
##
## Coefficients
##               Estimate   StdErr  t.value    beta p.value
## (Intercept)   0.34928   0.00442  78.94223      <2e-16 ***
## income        0.00076   0.00102   0.74609   0.0829 0.4590
## price       -0.14121   0.48708  -0.28991  -0.0210 0.7731
## temp        0.00267   0.00030   8.91026   0.7715 <2e-16 ***
## income.XX.price -0.25405   0.09639  -2.63563  -0.2725 0.0111 *
## income.XX.temp -0.00008   0.00005  -1.56715  -0.1271 0.1233
## price.XX.temp   0.03038   0.04174   0.72796   0.0575 0.4700
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

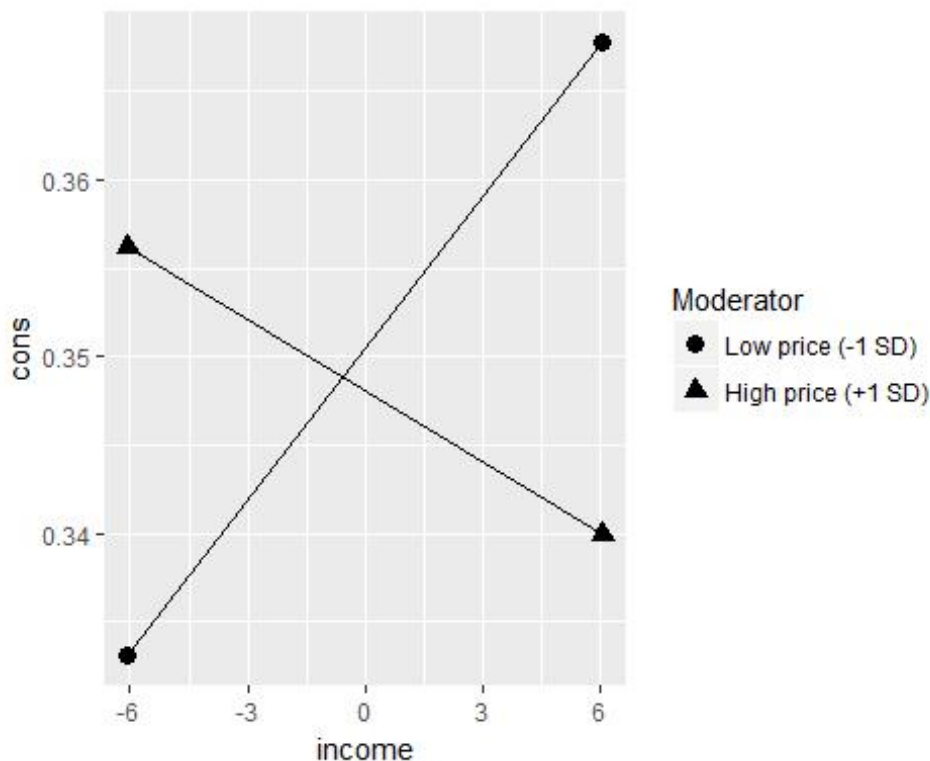
```
## Collinearity
##               VIF Tolerance
## income        2.5487    0.3924
## price         1.0858    0.9210
## temp          1.5475    0.6462
## income.XX.price 2.2061    0.4533
## income.XX.temp  1.3581    0.7363
## price.XX.temp   1.2886    0.7761
```

In model that lack of interactions, the coefficient of *income* and *temp* is significant, and these two variables both have a positive effect on response *cons*; But in the model that add interactions, the coefficient of *intercept*, *temp* and *interaction of income and price* are significant. The coefficient result change, but the latter model perform better based on *R.Square*. However, the coefficient of interaction is greater than other, and is negative, that is, the interaction has a more greater effect on response. And the comparison between the two model, interaction term should add in linear model.

## Interaction R Part D

### Simple slope test and plot for income by price interaction

```
S_slopes <- simpleSlope(modelpe1, pred = "income", mod1 = "price")
PlotIncome_by_proce <- PlotSlope(S_slopes)
PlotIncome_by_proce
```



```
summary(S_slopes)
```

```
##
## ** Estimated points of cons **
##
##               Low income (-1 SD) High income (+1 SD)
## Low price (-1 SD)           0.3331           0.3678
## High price (+1 SD)          0.3562           0.3400
##
##
##
## ** Simple Slopes analysis ( df= 51 ) **
##
##               simple slope standard error t-value p.value
## Low price (-1 SD)           0.0029           0.0008    3.79 0.0004 ***
## High price (+1 SD)          -0.0013           0.0017   -0.80 0.4257
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
##
## ** Bauer & Curran 95% CI **
##
##           lower CI upper CI
## price -0.0034  0.0348
```

The result show us that,the slope is significant when price is low,and is not significant when price is high.And the *cons* change whether is positive or not is effect signifivantly by price

## Interaction R Part E

### creating and testing:

testing three-way interactions without pequod, note use of maen center predictors

```
ICD <- ICD %>% mutate(threeway = income_mc * price_mc * temp_mc)
model2 <- lm(cons ~ income + price + temp + Income_by_temp +
              Income_by_price + Price_by_temp + threeway, data = ICD)
summary(model2)

##
## Call:
## lm(formula = cons ~ income + price + temp + Income_by_temp +
##     Income_by_price + Price_by_temp + threeway, data = ICD)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.053020 -0.010476 -0.000138  0.015241  0.069999
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)      0.4371217  0.1794095   2.436 0.018436 *
## income          -0.0003607  0.0010364  -0.348 0.729238
## price           -0.6627874  0.4916374  -1.348 0.183695
## temp            0.0025004  0.0002876   8.694 1.43e-11 ***
## Income_by_temp  -0.0141126  0.0052022  -2.713 0.009125 **
## Income_by_price -0.0175438  0.0048328  -3.630 0.000666 ***
## Price_by_temp    0.0014251  0.0052917   0.269 0.788793
## threeway        -0.0141151  0.0049541  -2.849 0.006349 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02742 on 50 degrees of freedom
## Multiple R-squared:  0.7874, Adjusted R-squared:  0.7577
## F-statistic: 26.46 on 7 and 50 DF,  p-value: 9.665e-15
```

```
vif(model2)
```

```
##          income          price          temp Income_by_temp
##      2.980614      1.260545      1.620030      1.647387
## Income_by_price Price_by_temp      threeway
##      2.511157      1.328835      1.431312
```

```
modelpe2 <- lmres(cons ~ income * price * temp,
                  centered = c("income", "price", "temp"), data = ICD)
summary(modelpe2)
```

```
## Formula:
## cons ~ income + price + temp + income.XX.price + income.XX.temp +
##      price.XX.temp + income.XX.price.XX.temp
## <environment: 0x0000000014b34db8>
##
## Models
##      R      R^2  Adj. R^2    F      df1 df2 p.value
## Model 0.887 0.787    0.758 26.461   7.000  50 9.7e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residuals
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -0.0530 -0.0105 -0.0001  0.0000  0.0152  0.0700
##
## Coefficients
##              Estimate StdErr t.value  beta p.value
## (Intercept)    0.34487  0.00442 77.96730    < 2e-16 ***
## income         -0.00036  0.00104 -0.34808 -0.0392 0.72924
## price          -0.66279  0.49164 -1.34812 -0.0987 0.18369
## temp           0.00250  0.00029  8.69365  0.7215 < 2e-16 ***
## income.XX.price -0.34971  0.09633 -3.63015 -0.3751 0.00067 **
##
## income.XX.temp  -0.00015  0.00005 -2.71283 -0.2270 0.00913 **
## price.XX.temp    0.01069  0.03971  0.26932  0.0202 0.78879
```

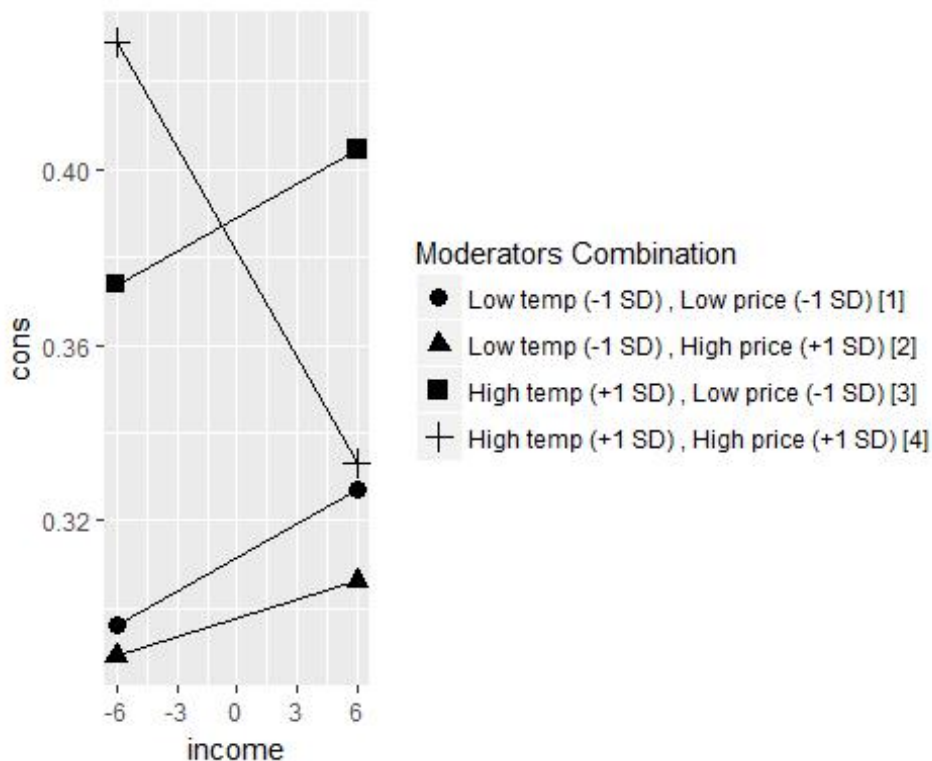
```
## income.XX.price.XX.temp -0.01751 0.00615 -2.84915 -0.2222 0.00635 **

## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Collinearity
##
## VIF Tolerance
## income 2.9806 0.3355
## price 1.2605 0.7933
## temp 1.6200 0.6173
## income.XX.price 2.5112 0.3982
## income.XX.temp 1.6474 0.6070
## price.XX.temp 1.3288 0.7525
## income.XX.price.XX.temp 1.4313 0.6987
```

According to the models show us that, the result is the model with threeway interaction perform better based on *R.Square*, The *R.Square* is about 0.75. And other result is similar with the model with twoway interaction. However, in this time, the coefficient of interaction of income\_price, income\_temp, income\_price\_temp are significant, and the effect of them are all negative on response.

### Simple slope test and plot for the three-way interaction

```
S_slopes_3way <- simpleSlope(modelpe2, pred = "income", mod1 = "temp", mod2 = "price")
Plot_threeway <- PlotSlope(S_slopes_3way)
Plot_threeway
```



```
summary(S_slopes_3way)
```

```
##
## ** Estimated points of cons **
##
##                               Low income (-1 SD)
## Low temp (-1 SD) , Low price (-1 SD) [1]          0.2963
## Low temp (-1 SD) , High price (+1 SD) [2]          0.2893
## High temp (+1 SD) , Low price (-1 SD) [3]          0.3738
## High temp (+1 SD) , High price (+1 SD) [4]          0.4289
##                               High income (+1 SD)
## Low temp (-1 SD) , Low price (-1 SD) [1]          0.3270
## Low temp (-1 SD) , High price (+1 SD) [2]          0.3063
## High temp (+1 SD) , Low price (-1 SD) [3]          0.4045
## High temp (+1 SD) , High price (+1 SD) [4]          0.3330
##
##
##
## ** Simple Slopes analysis ( df= 50 ) **
##
##                               simple slope standard error
## Low temp (-1 SD) , Low price (-1 SD) [1]          0.0025          0.0011
## Low temp (-1 SD) , High price (+1 SD) [2]          0.0014          0.0015
## High temp (+1 SD) , Low price (-1 SD) [3]          0.0025          0.0012
## High temp (+1 SD) , High price (+1 SD) [4]        -0.0079          0.0028
##                               t-value p.value
## Low temp (-1 SD) , Low price (-1 SD) [1]          2.37 0.0214 *
## Low temp (-1 SD) , High price (+1 SD) [2]          0.95 0.3449
## High temp (+1 SD) , Low price (-1 SD) [3]          2.05 0.0452 *
## High temp (+1 SD) , High price (+1 SD) [4]        -2.87 0.0061 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
##
## ** Slope Difference Test (( df= 50 ); Dawson & Richter, 2006) **
##
##                               t-value
## High temp (+1 SD) , High price (+1 SD) [4] vs. High temp (+1 SD) , L
ow price (-1 SD) [3] -3.9393
## High temp (+1 SD) , High price (+1 SD) [4] vs. Low temp (-1 SD) , Hi
gh price (+1 SD) [2] -3.2992
## High temp (+1 SD) , High price (+1 SD) [4] vs. Low temp (-1 SD) , Lo
w price (-1 SD) [1] -3.6964
## High temp (+1 SD) , Low price (-1 SD) [3] vs. Low temp (-1 SD) , Low
price (-1 SD) [1] 0.0003
## High temp (+1 SD) , Low price (-1 SD) [3] vs. Low temp (-1 SD) , Hig
h price (+1 SD) [2] 0.6544
## Low temp (-1 SD) , High price (+1 SD) [2] vs. Low temp (-1 SD) , Low
price (-1 SD) [1] -0.4266
```

```

##
      p.value
## High temp (+1 SD) , High price (+1 SD) [4] vs. High temp (+1 SD) , Low price (-1 SD) [3] 0.0003
## High temp (+1 SD) , High price (+1 SD) [4] vs. Low temp (-1 SD) , High price (+1 SD) [2] 0.0018
## High temp (+1 SD) , High price (+1 SD) [4] vs. Low temp (-1 SD) , Low price (-1 SD) [1] 0.0005
## High temp (+1 SD) , Low price (-1 SD) [3] vs. Low temp (-1 SD) , Low price (-1 SD) [1] 0.9998
## High temp (+1 SD) , Low price (-1 SD) [3] vs. Low temp (-1 SD) , High price (+1 SD) [2] 0.5159
## Low temp (-1 SD) , High price (+1 SD) [2] vs. Low temp (-1 SD) , Low price (-1 SD) [1] 0.6715
##
      Bonferroni.p
## High temp (+1 SD) , High price (+1 SD) [4] vs. High temp (+1 SD) , Low price (-1 SD) [3] 0.0015
## High temp (+1 SD) , High price (+1 SD) [4] vs. Low temp (-1 SD) , High price (+1 SD) [2] 0.0108
## High temp (+1 SD) , High price (+1 SD) [4] vs. Low temp (-1 SD) , Low price (-1 SD) [1] 0.0033
## High temp (+1 SD) , Low price (-1 SD) [3] vs. Low temp (-1 SD) , Low price (-1 SD) [1] 1.0000
## High temp (+1 SD) , Low price (-1 SD) [3] vs. Low temp (-1 SD) , High price (+1 SD) [2] 1.0000
## Low temp (-1 SD) , High price (+1 SD) [2] vs. Low temp (-1 SD) , Low price (-1 SD) [1] 1.0000
##

## High temp (+1 SD) , High price (+1 SD) [4] vs. High temp (+1 SD) , Low price (-1 SD) [3] **
## High temp (+1 SD) , High price (+1 SD) [4] vs. Low temp (-1 SD) , High price (+1 SD) [2] *
## High temp (+1 SD) , High price (+1 SD) [4] vs. Low temp (-1 SD) , Low price (-1 SD) [1] **
## High temp (+1 SD) , Low price (-1 SD) [3] vs. Low temp (-1 SD) , Low price (-1 SD) [1]
## High temp (+1 SD) , Low price (-1 SD) [3] vs. Low temp (-1 SD) , High price (+1 SD) [2]
## Low temp (-1 SD) , High price (+1 SD) [2] vs. Low temp (-1 SD) , Low price (-1 SD) [1]
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

According to the result of this test, the slope change is significant when temp, price change, which indicates that the *cons* is mainly affected by *income* and *temp*



In conclusion, *price* and *temp* are the main effective factor on response *cons*, and their interaction, *treeway* interaction still have effect on *cons*, but interactions' effect is less.