# Design and Analysis of Algorithms
# Sorting

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# The problem of sorting

***Input:*** sequence $\langle a_1, a_2, \ldots, a_n \rangle$ of numbers.

***Output:*** permutation $\langle a'_1, a'_2, \ldots, a'_n \rangle$ such that $a'_1 \leq a'_2 \leq \cdots \leq a'_n$.

**Example:**

***Input:*** 8 2 4 9 3 6

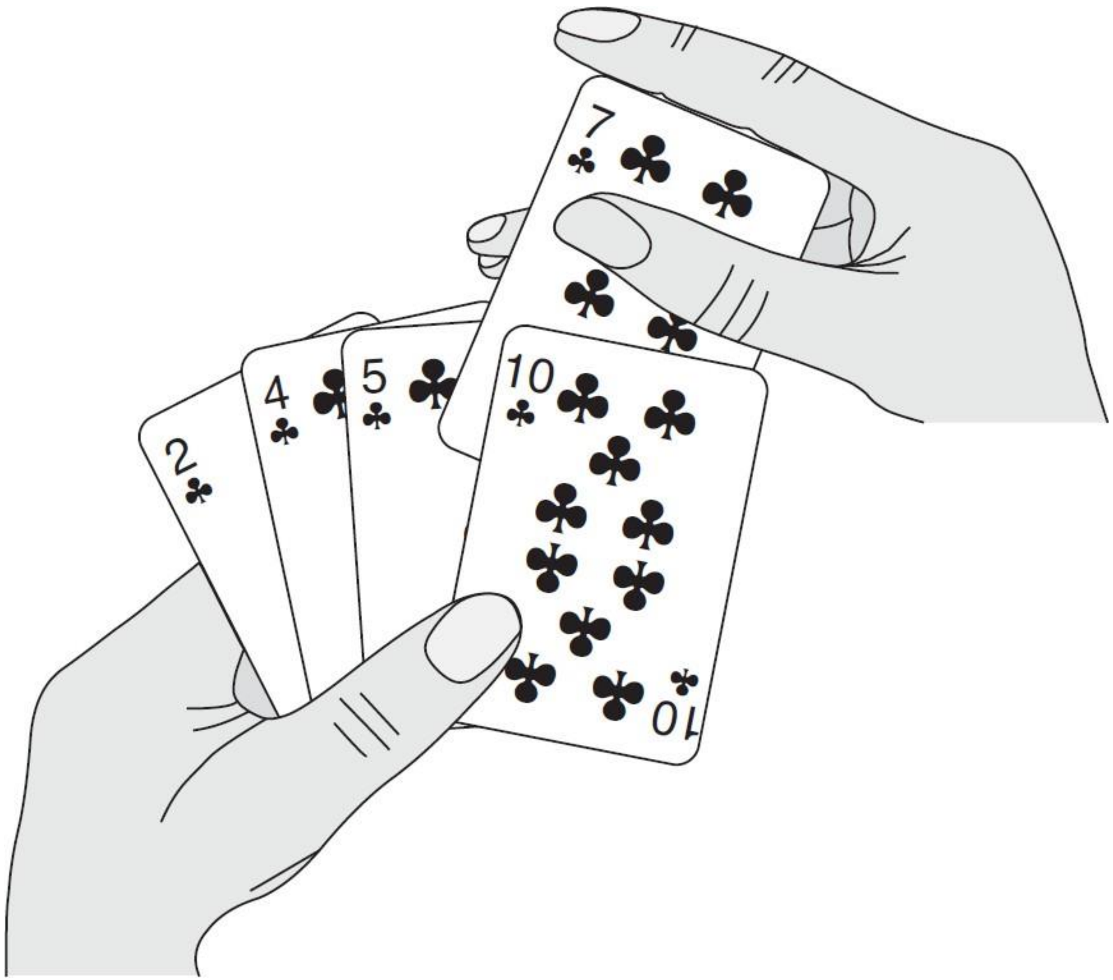
***Output:*** 2 3 4 6 8 9

# Overview

- ***Goals:***
  - Start using frameworks for describing and analyzing algorithms.
  - Examine two algorithms for sorting: insertion-sort and merge-sort.
  - See how to describe algorithms in pseudocode.
  - Begin using asymptotic notation to express running-time analysis.
  - Learn the technique of “divide and conquer” in the context of merge-sort.

# Insertion Sort

- Sorting a hand of cards using insertion sort.



# Insertion sort

"pseudocode"

INSERTION-SORT $(A, n)$ $\quad \triangleright A[1 \,..\, n]$

**for** $j \leftarrow 2$ **to** $n$

  **do** $key \leftarrow A[j]$

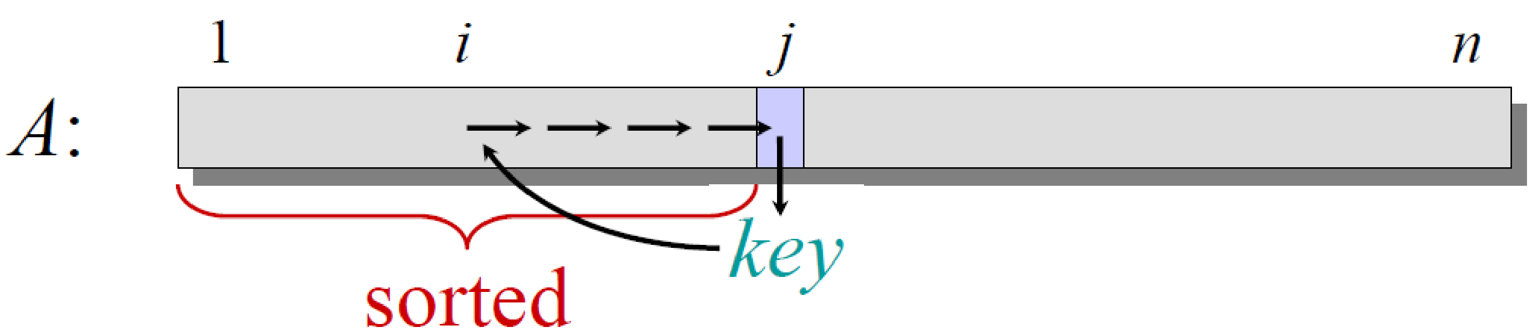
    $i \leftarrow j - 1$

    **while** $i > 0$ and $A[i] > key$

      **do** $A[i+1] \leftarrow A[i]$

        $i \leftarrow i - 1$

    $A[i+1] = key$



# Example of insertion sort

8 2 4 9 3 6

# Example of insertion sort

8 2 4 9 3 6

# Example of insertion sort

| | | | | | |
|---|---|---|---|---|---|
| 8 | 2 | 4 | 9 | 3 | 6 |
| 2 | 8 | 4 | 9 | 3 | 6 |

# Example of insertion sort

8 2 4 9 3 6

2 8 4 9 3 6

# Example of insertion sort

| | | | | | |
|---|---|---|---|---|---|
| 8 | 2 | 4 | 9 | 3 | 6 |
| 2 | 8 | 4 | 9 | 3 | 6 |
| 2 | 4 | 8 | 9 | 3 | 6 |

# Example of insertion sort

8 2 4 9 3 6

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2 4 8 9 3 6

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# Example of insertion sort

8 2 4 9 3 6

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# Example of insertion sort

| | | | | | |
|---|---|---|---|---|---|
| 8 | 2 | 4 | 9 | 3 | 6 |
| 2 | 8 | 4 | 9 | 3 | 6 |
| 2 | 4 | 8 | 9 | 3 | 6 |
| 2 | 4 | 8 | 9 | 3 | 6 |
| 2 | 3 | 4 | 8 | 9 | 6 |

# Example of insertion sort

8 2 4 9 3 6

2 8 4 9 3 6

2 4 8 9 3 6

2 4 8 9 3 6

2 3 4 8 9 6

# Example of insertion sort

8 2 4 9 3 6

2 8 4 9 3 6

2 4 8 9 3 6

2 4 8 9 3 6

2 3 4 8 9 6

2 3 4 6 8 9 *done*

# Insertion Sort (another example)

```
INSERTION-SORT (A, n) ▷ A[1 . . n]
for j ← 2 to n
      do key ← A[j ]
         i ← j -  1
         while i > 0 and A[i] > key
               do A[i + 1] ← A[i]
                  i ← i -  1
         A[i + 1] = key
```

Initial

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| -∞ | 8 | 5 | 7 | 9 | 6 | 4 |

j=2

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| -∞ | 8 | 5 | 7 | 9 | 6 | 4 |

j=3

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| -∞ | 5 | 8 | 7 | 9 | 6 | 4 |

j=4

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| -∞ | 5 | 7 | 8 | 9 | 6 | 4 |

j=5

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| -∞ | 5 | 7 | 8 | 9 | 6 | 4 |

j=6

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| -∞ | 5 | 6 | 7 | 8 | 9 | 4 |

# Running time

- **The running time depends on the input: an already sorted sequence is easier to sort.**

- **Parameterize the running time by the size of the input, since short sequences are easier to sort than long ones.**

- **Generally, we seek upper bounds on the running time, because everybody likes a guarantee.**

# $\Theta$-notation

***Math:***

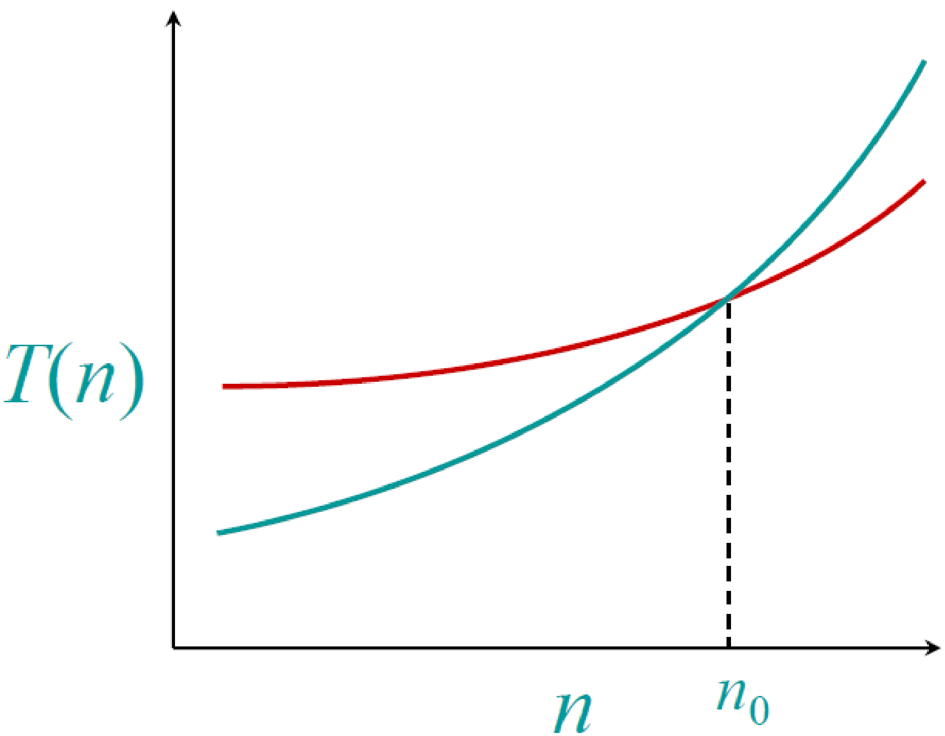
$\Theta(g(n)) = \{ f(n)$ : there exist positive constants $c_1$, $c_2$, and $n_0$ such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0 \}$

***Engineering:***

- Drop low-order terms; ignore leading constants.
- Example: $3n^3 + 90n^2 - 5n + 6046 = \Theta(n^3)$

# Asymptotic performance

When $n$ gets large enough, a $\Theta(n^2)$ algorithm *always* beats a $\Theta(n^3)$ algorithm.




- We shouldn't ignore asymptotically slower algorithms, however.
- Real-world design situations often call for a careful balancing of engineering objectives.
- Asymptotic analysis is a useful tool to help to structure our thinking.

# Analysis of INSERTION-SORT

| INSERTION-SORT (A, n) ▹ A[1 . . n] | cost | times |
| --- | --- | --- |
| for j ← 2 to n | $c_1$ | $n-1$ |
|     do key ← A[j ] | | |
|       i ← j - 1 | | |
|       while i > 0 and A[i] > key | | |
|           do A[i + 1] ← A[i] | | |
|             i ← i - 1 | | |
|       A[i + 1] = key | | |

# Analysis of INSERTION-SORT

| INSERTION-SORT (A, n) ▷ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
|     do key ← A[j ] | $c_2$ | $n-1$ |
|         i ← j - 1 | | |
|         while i > 0 and A[i] > key | | |
|             do A[i + 1] ← A[i] | | |
|                 i ← i - 1 | | |
|         A[i + 1] = key | | |

# Analysis of INSERTION-SORT

| INSERTION-SORT (A, n) ⊳ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
|     do key ← A[j ] | $c_2$ | $n-1$ |
|       i ← j - 1 | $c_3$ | $n-1$ |
|       while i > 0 and A[i] > key | | |
|           do A[i + 1] ← A[i] | | |
|             i ← i - 1 | | |
|       A[i + 1] = key | | |

# Analysis of INSERTION-SORT

| INSERTION-SORT (A, n) ⊳ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
|     do key ← A[j ] | $c_2$ | $n-1$ |
|        i ← j - 1 | $c_3$ | $n-1$ |
|        while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
|            do A[i + 1] ← A[i] | | |
|               i ← i - 1 | | |
|        A[i + 1] = key | | |

# Analysis of INSERTION-SORT

| INSERTION-SORT (A, n) ▷ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n - 1$ |
|     do key ← A[j ] | $c_2$ | $n - 1$ |
|         i ← j - 1 | $c_3$ | $n - 1$ |
|         while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
|             do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
|                 i ← i - 1 | | |
|         A[i + 1] = key | | |

# Analysis of INSERTION-SORT

| INSERTION-SORT (A, n) ▻ A[1 . . n] | cost | times |
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| for j ← 2 to n | $c_1$ | $n-1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j - 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
| do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | | |

# Analysis of INSERTION-SORT

| INSERTION-SORT (A, n) ▹ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j − 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
| do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| i ← i − 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | $c_7$ | $n-1$ |

# Analysis of INSERTION-SORT

| INSERTION-SORT (A, n) ▹ A[1 . . n] | cost | times |
| --- | --- | --- |
| for j ← 2 to n | $c_1$ | $n - 1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j - 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
| do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | $c_7$ | $n - 1$ |

Let $T(n)$ = running time of **INSERTION-SORT**.

$$T(n) = c_1 (n-1) + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

| INSERTION-SORT (A, n) ▷ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j - 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
| do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | $c_7$ | $n-1$ |

$$T(n) = c_1 (n-1) + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

***Best-case:*** The array is already sorted.

- Always find that **A[i ] ≤ key** upon the first time the while loop test is run (when i =j −1).
- All $t_j$ are 1.
- Running time is

$$T(n) = c_1 (n-1) + c_2 (n-1) + c_3 (n-1) + c_4 (n-1) + c_7 (n-1)$$
$$= (c_1 + c_2 + c_3 + c_4 + c_7) n - (c_1 + c_2 + c_3 + c_4 + c_7)$$

- Can express $T(n)$ as $an + b$ for constants $a$ and $b$ (that depend on the statement costs $c_i$ ) ⇒ $T(n)$ is a linear function of $n$. ⇒ $T(n) = \Theta(n)$

| INSERTION-SORT (A, n) ▷ A[1 . . n] | cost | times |
| --- | --- | --- |
| for j ← 2 to n | $c_1$ | $n-1$ |
|     do key ← A[j ] | $c_2$ | $n-1$ |
|         i ← j - 1 | $c_3$ | $n-1$ |
|         while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
|             do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
|                i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
|         A[i + 1] = key | $c_7$ | $n - 1$ |

$$T(n) = c_1(n-1) + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

***Worst-case:*** The array is in reverse sorted order.

- Always find that ***A[i ] > key*** in while loop test.

| INSERTION-SORT (A, n) ▹ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j - 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
| do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | $c_7$ | $n - 1$ |

$$T(n) = c_1(n-1) + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

***Worst-case:*** The array is in reverse sorted order.

- Have to compare ***key*** with all elements to the left of the $j$ th position ⇒ compare with $j - 1$ elements.

| INSERTION-SORT (A, n) ▷ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j - 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
| do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | $c_7$ | $n - 1$ |

$$T(n) = c_1(n\text{-}1) + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

***Worst-case:*** The array is in reverse sorted order.

- Since the while loop exits because i reaches 0, there's one additional test after the $j - 1$ tests $\Rightarrow t_j = j$.

| INSERTION-SORT (A, n) ▹ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j - 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
| do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | $c_7$ | $n - 1$ |

$$T(n) = c_1(n-1) + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

***Worst-case:*** The array is in reverse sorted order.

- $\sum_{j=2}^{n}(t_j - 1) = \sum_{j=2}^{n}(j - 1)$

| INSERTION-SORT (A, n) ▹ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j - 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
| do A[i + 1] ← A[i] | $c_5$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | $c_7$ | $n - 1$ |

$$T(n) = c_1(n-1) + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

***Worst-case:*** The array is in reverse sorted order. Running time:

- $T(n) = c_1(n-1) + c_2(n-1) + c_3(n-1) + c_4 \left(\frac{n(n+1)}{2} - 1\right) + c_5 \left(\frac{n(n-1)}{2} - 1\right) + c_6 \left(\frac{n(n-1)}{2} - 1\right) + c_7(n-1)$

$$= \overbrace{\boxed{\frac{c_4}{2} + \frac{c_5}{2} + \frac{c_6}{2}}}^{a} n^2 + \overbrace{\boxed{\left(c_1 + c_2 + c_3 - \left(\frac{c_4}{2} + \frac{c_5}{2} + \frac{c_6}{2}\right) + c_7\right)}}^{b} n - \overbrace{\boxed{(c_1 + c_3 + c_4 + c_7)}}^{c}$$

| INSERTION-SORT (A, n) ⊳ A[1 . . n] | cost | times |
|---|---|---|
| for j ← 2 to n | $c_1$ | $n-1$ |
| do key ← A[j ] | $c_2$ | $n-1$ |
| i ← j - 1 | $c_3$ | $n-1$ |
| while i > 0 and A[i] > key | $c_4$ | $\sum_{j=2}^{n} t_j$ |
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| i ← i - 1 | $c_6$ | $\sum_{j=2}^{n} (t_j - 1)$ |
| A[i + 1] = key | $c_7$ | $n - 1$ |

$$T(n) = c_1(n-1) + c_2(n-1) + c_3(n-1) + c_4 \sum_{j=2}^{n} t_j + c_5 \sum_{j=2}^{n} (t_j - 1) + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7(n-1)$$

## ***Worst-case:*** The array is in reverse sorted order.

- *Can express* $T(n)$ *as* $an^2 + bn + c$ *for constants* $a$, b, c
- $T(n)$ *is a quadratic function of* $n$. $\Rightarrow$ $T(n) = \Theta(n^2)$

# Order of Growth

**We will only consider order of growth of running time:**

- **We can ignore the lower-order terms, since they are relatively insignificant for very large n.**
- **We can also ignore leading term's constant coefficients, since they are not as important for the rate of growth in computational efficiency for very large n.**
- **For the insertion-sort algorithm, we just said that best case was linear to n and worst/average case quadratic to n.**

# Designing Algorithms

- **We discussed insertion sort**

**-- Can we design better than $n^2$ sorting algorithms?**

**-- We will do so using one of the most powerful algorithm design techniques.**

# Divide - and - Conquer

- **To solve problem P:**

  **-- Divide P into smaller problems $P_1$, $P_2$, …, $P_k$.**

  **-- Conquer by solving the (smaller) subproblems recursively.**

  **-- Combine the solutions to $P_1$, $P_2$, …, $P_k$ into the solution for P.**

# Merge-Sort Algorithm

- **Using divide-and-conquer, we can obtain a merge-sort algorithm**

**-- Divide: Divide the n elements into two subsequences of n/2 elements each.**

**-- Conquer: Sort the two subsequences recursively.**

**-- Combine: Merge the two sorted subsequences to produce the sorted answer.**

# Merge-Sort (A, p, r)

- **INPUT**: **a sequence of n numbers stored in array A**
- **OUTPUT**: **an ordered sequence of n numbers**

```
MERGE-SORT (A, p, r)
1     if p < r
2           then q ← ⌊(p + r) / 2⌋
3                 MERGE-SORT(A, p, q)
4                 MERGE-SORT(A, q + 1, r)
5                 MERGE(A, p, q, r)
```

# Merge (A, p, q, r)

```
MERGE (A, p, q, r)
n1 ← q – p + 1
n2 ← r – q
create arrays L[1..n1 + 1] and R[1..n2 + 1]
for i ← 1 to n1
        do L[i] ← A[p + i - 1]
for j ← 1 to n2
        do R[j] ← A[q + j]
L[n1 + 1] ← ∞
L[n2 + 1] ← ∞
i ← 1
j ← 1
for k ← p to r
        do if L[i] ≤ R[j]
              then A[k] ← L[i]
                   i ← i + 1
              else A[k] ← R[j]
                   j ← j + 1
```

R

| | |
|---|---|
| 20 | 12 |
| 13 | 11 |
| 7 | 9 |
| 2 | 1 |

# Merging two sorted arrays

| | |
|---|---|
| 20 | 12 |
| 13 | 11 |
| 7 | 9 |
| 2 | 1 |

# Merging two sorted arrays

| | |
|---|---|
| 20 | 12 |
| 13 | 11 |
| 7 | 9 |
| 2 | 1 |

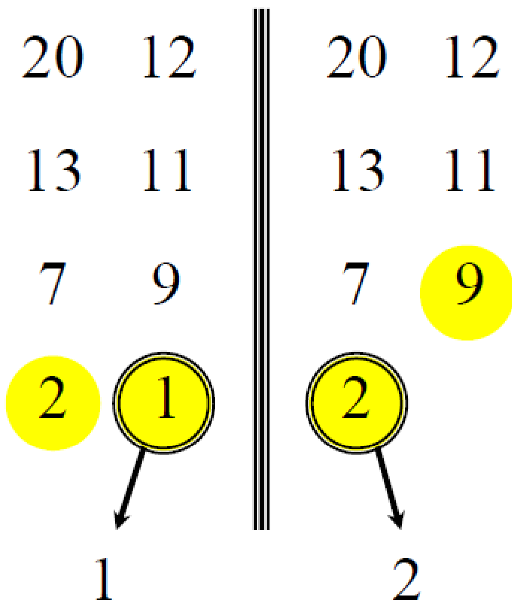
1

# Merging two sorted arrays

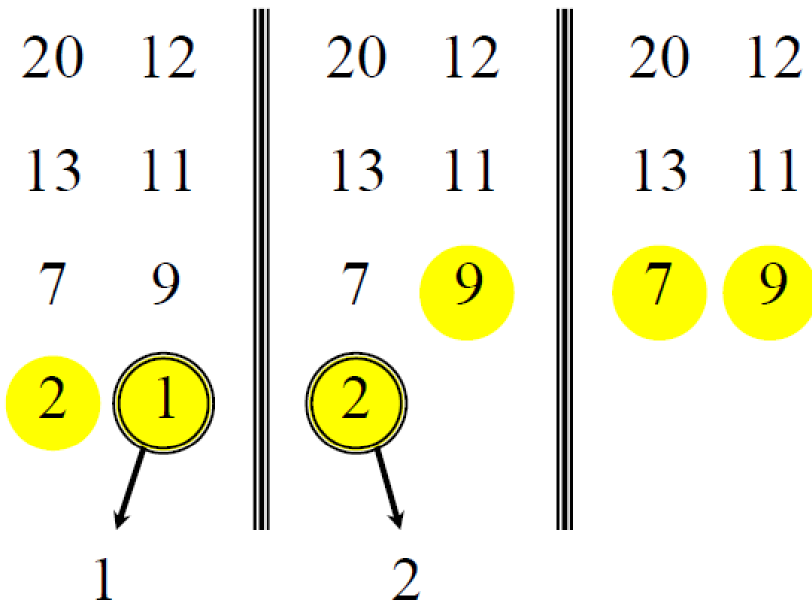
| | | | |
|---|---|---|---|
| 20 | 12 | 20 | 12 |
| 13 | 11 | 13 | 11 |
| 7 | 9 | 7 | 9 |
| 2 | 1 | 2 | |

1

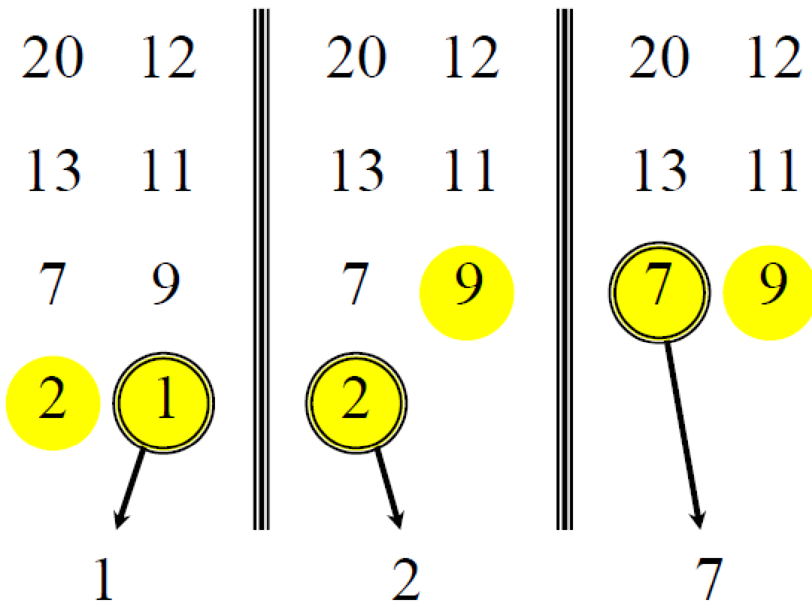
# Merging two sorted arrays



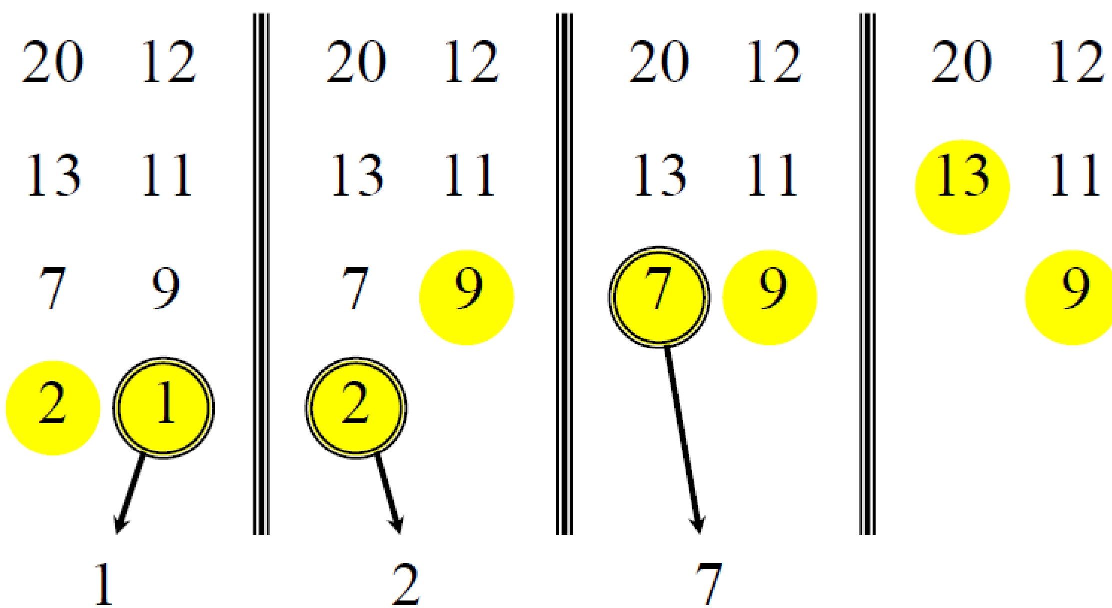
# Merging two sorted arrays



# Merging two sorted arrays



# Merging two sorted arrays



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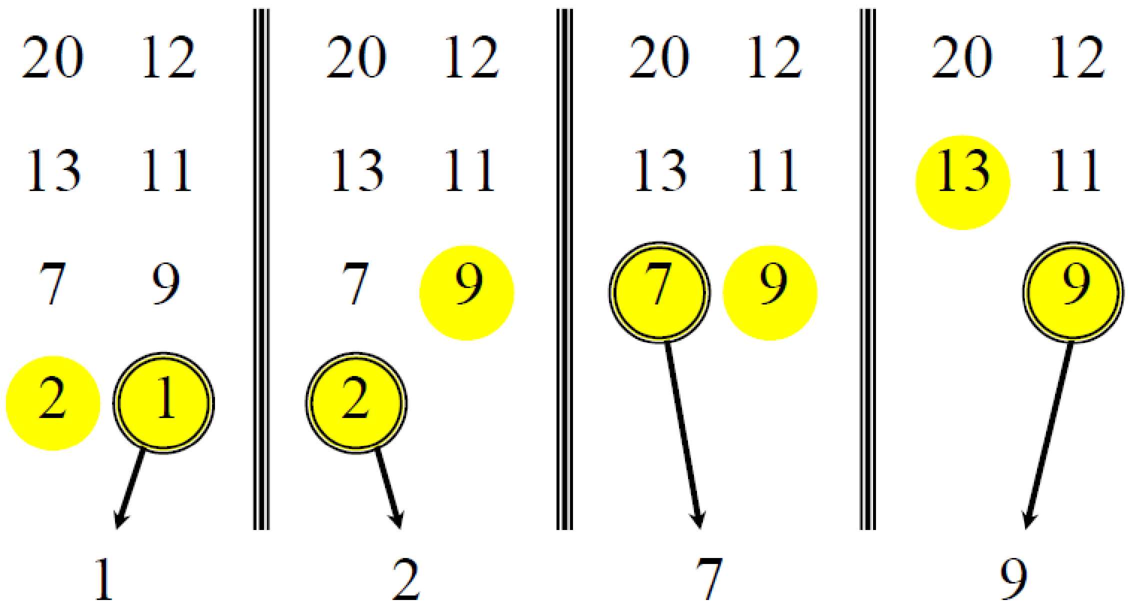

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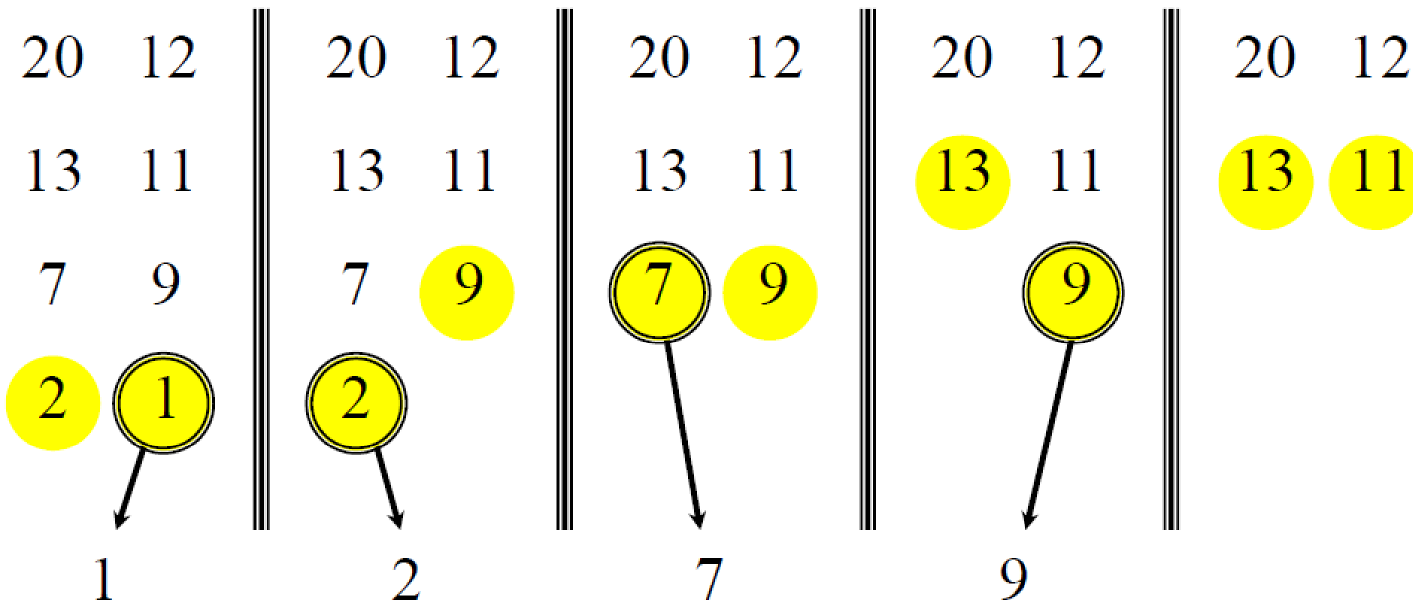

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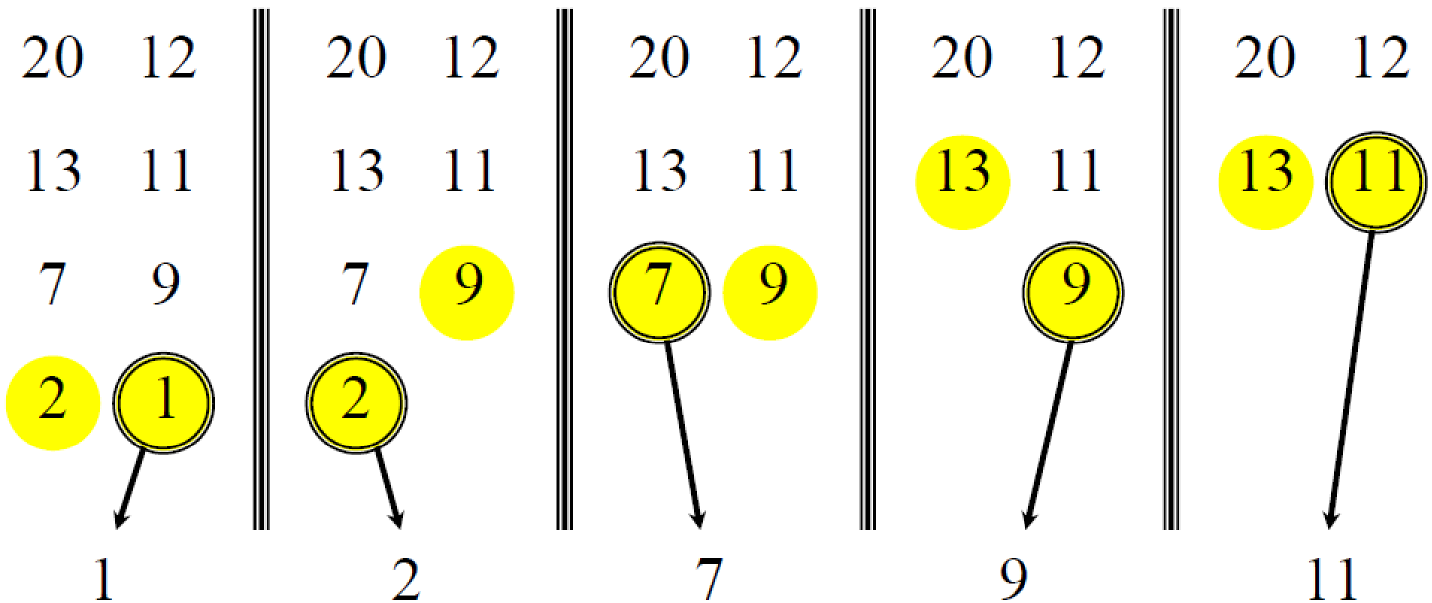

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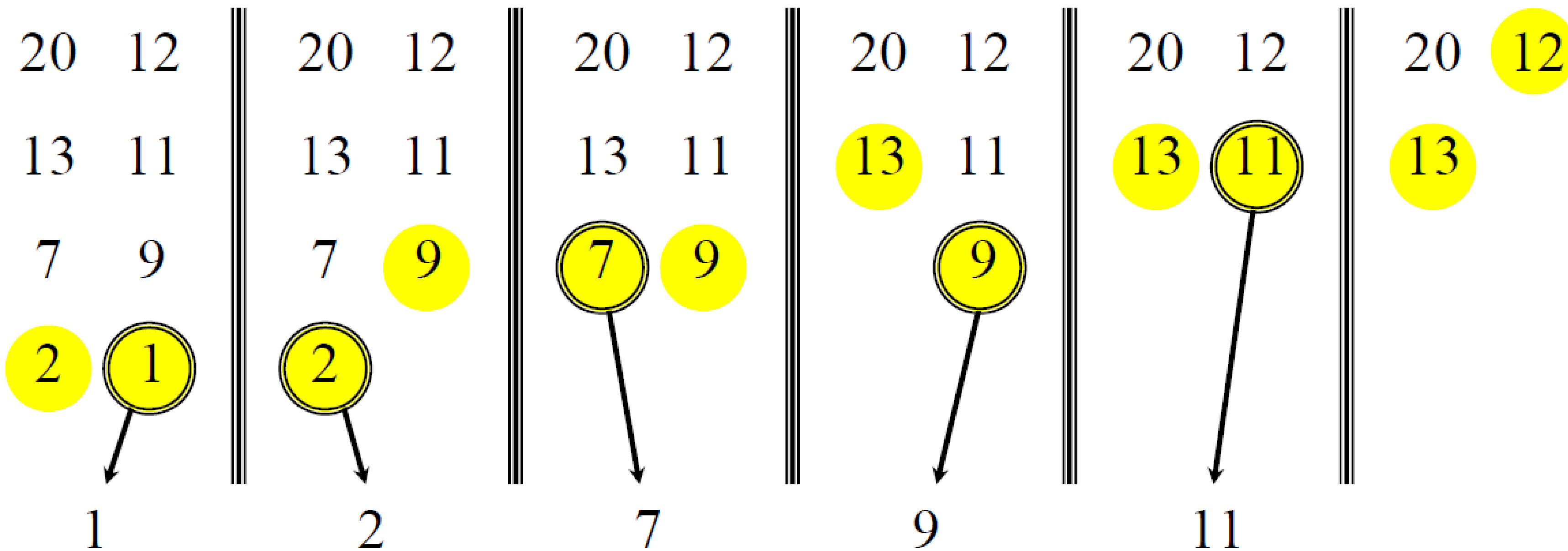

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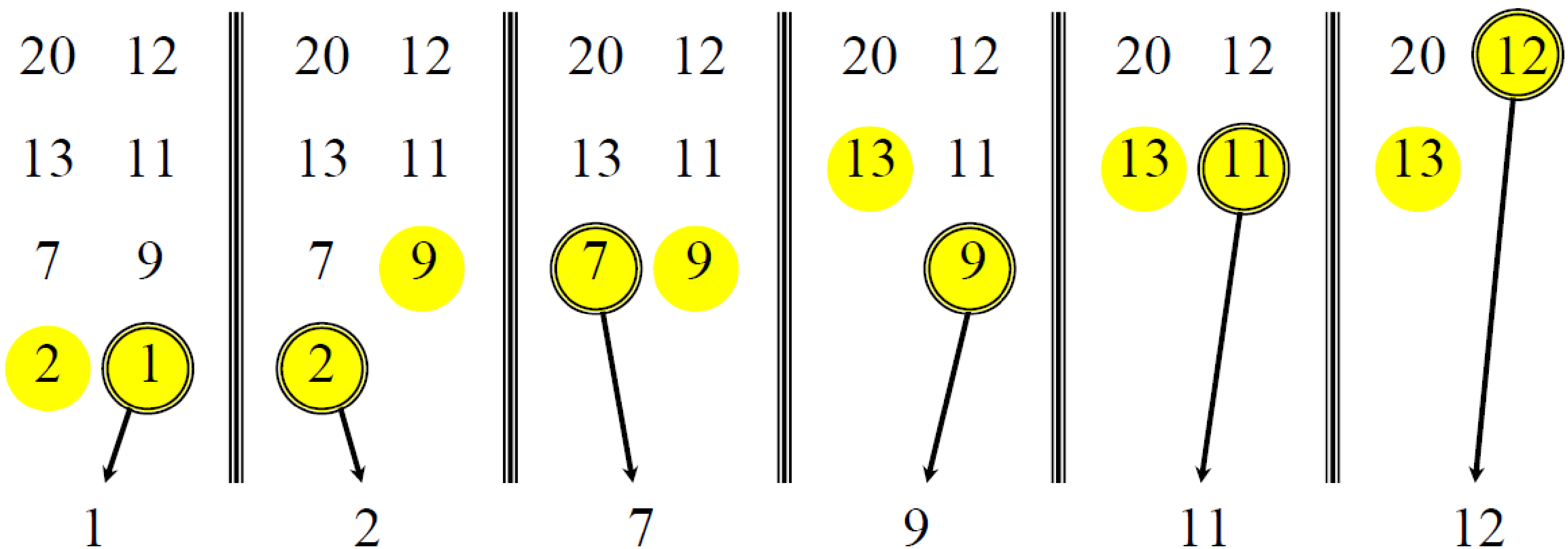

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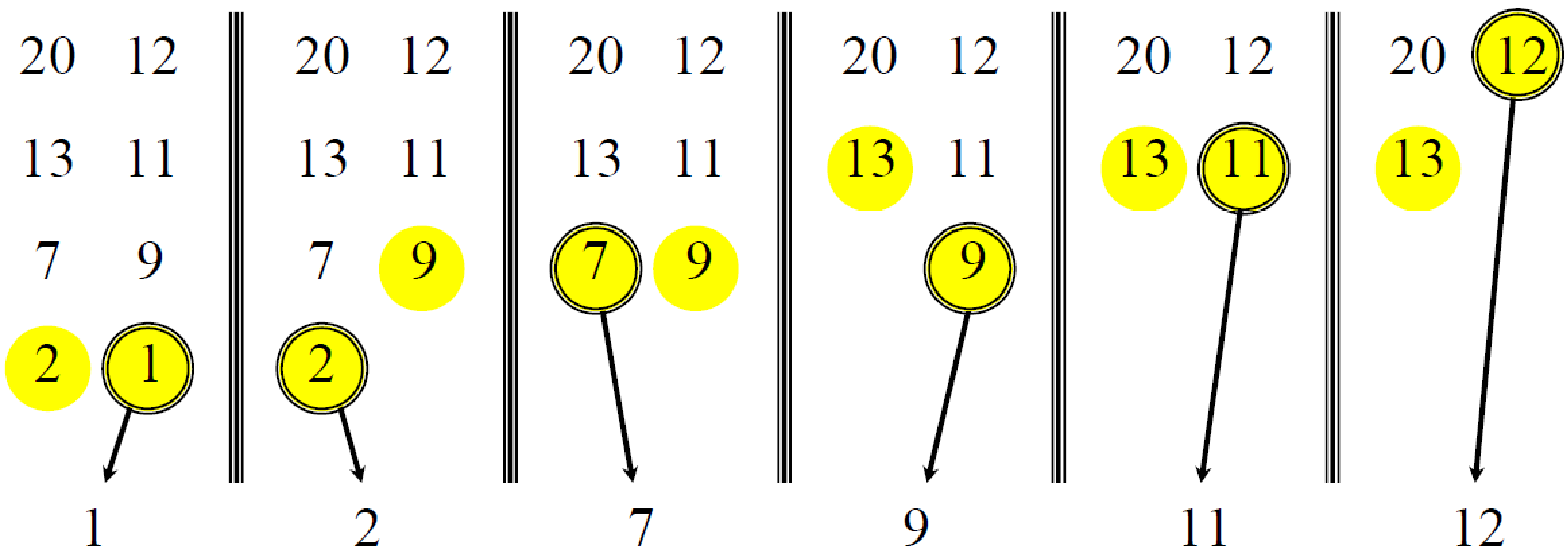
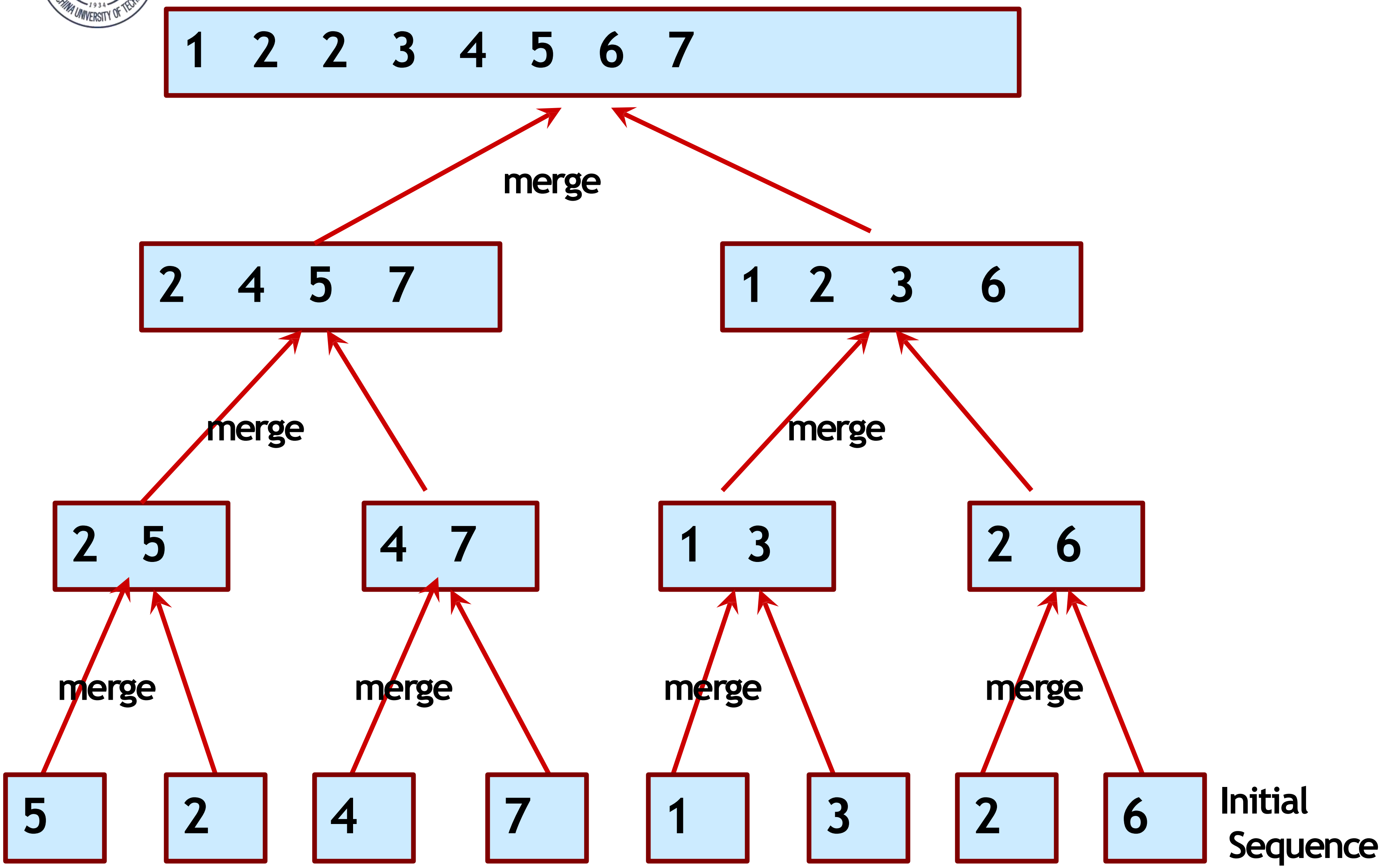



Time = $\Theta(n)$ to merge a total of $n$ elements (linear time).

# Action of Merge Sort



# Analyzing Merge-Sort

- **How long does merge-sort take?**

  **-- Bottleneck = merging (and copying).**

  **>> merging two files of size *n/2* requires *n* comparisons**

  **-- *T(n)* = comparisons to merge sort *n* elements.**

  **>>to make analysis cleaner, assume n is a power of 2**

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ \underbrace{2T(n/2)}_{\text{Sorting both halves}} + \underbrace{\Theta(n)}_{\text{merging}} & \text{otherwise} \end{cases}$$

•**Claim. $T(n) = n \log_2 n$**

**-- Note: same number of comparisons for ANY file.**

**>> even already sorted**

# Recursion tree

Solve $T(n) = 2T(n/2) + cn$, where $c > 0$ is constant.

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$$T(n)$$

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$$
\begin{array}{ccc}
 & cn & \\
\swarrow & & \searrow \\
T(n/2) & & T(n/2)
\end{array}
$$

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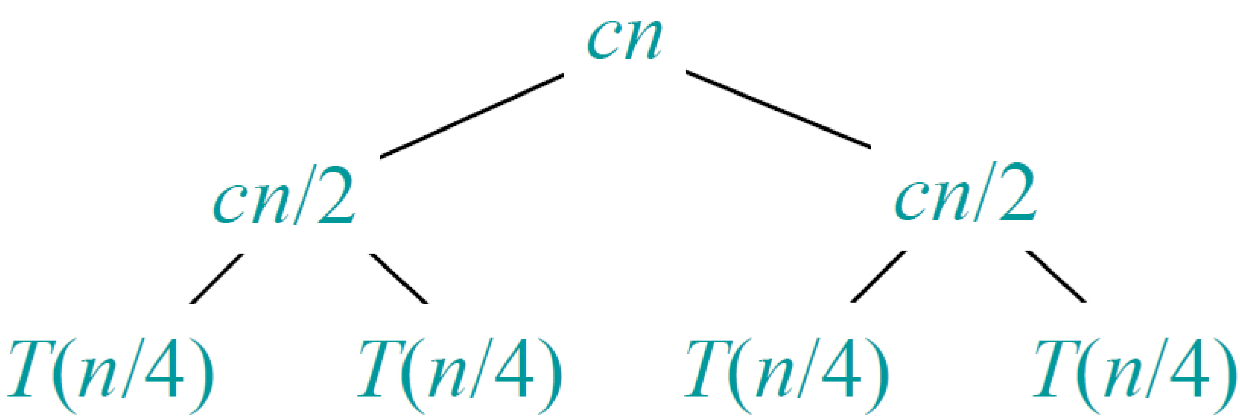

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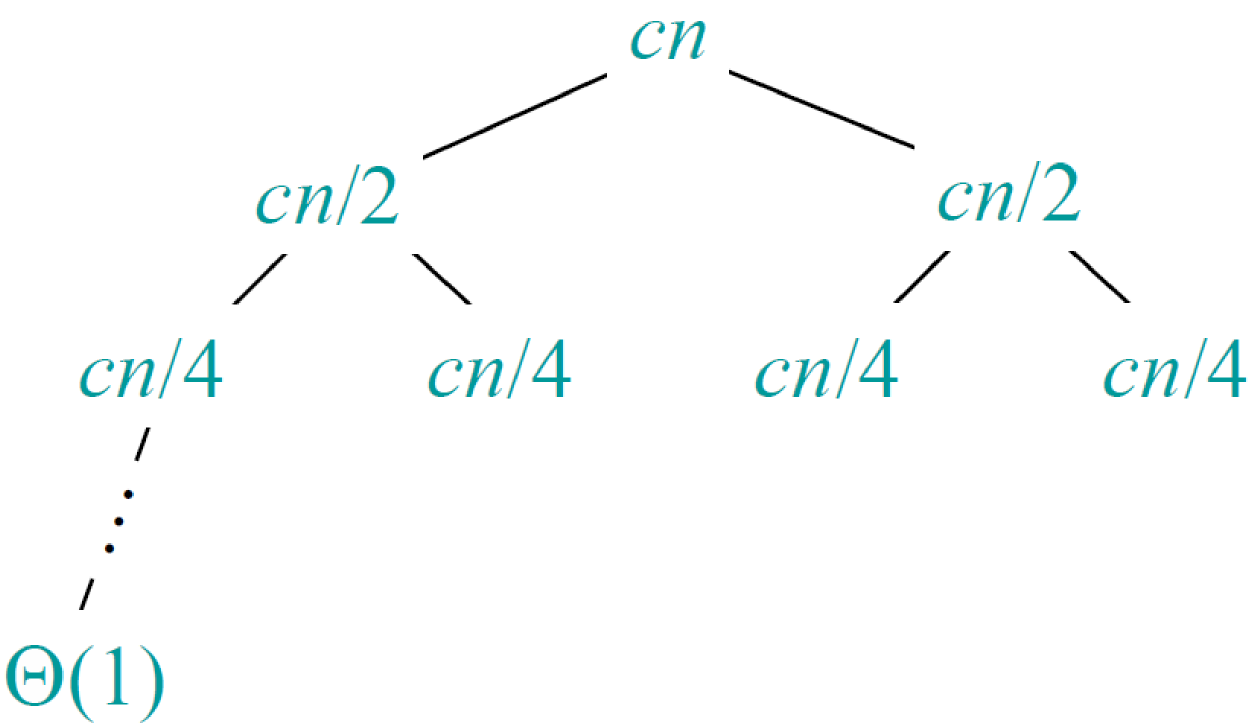

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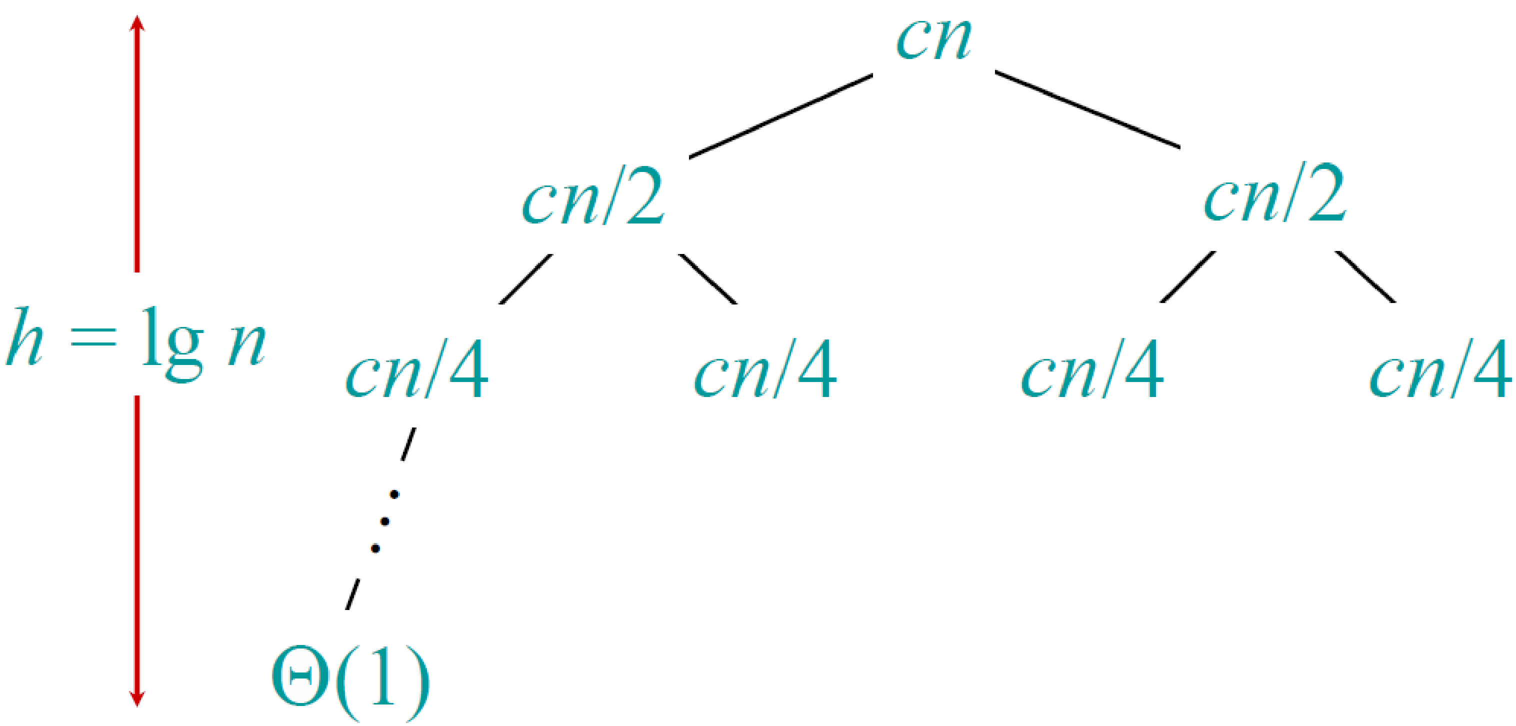

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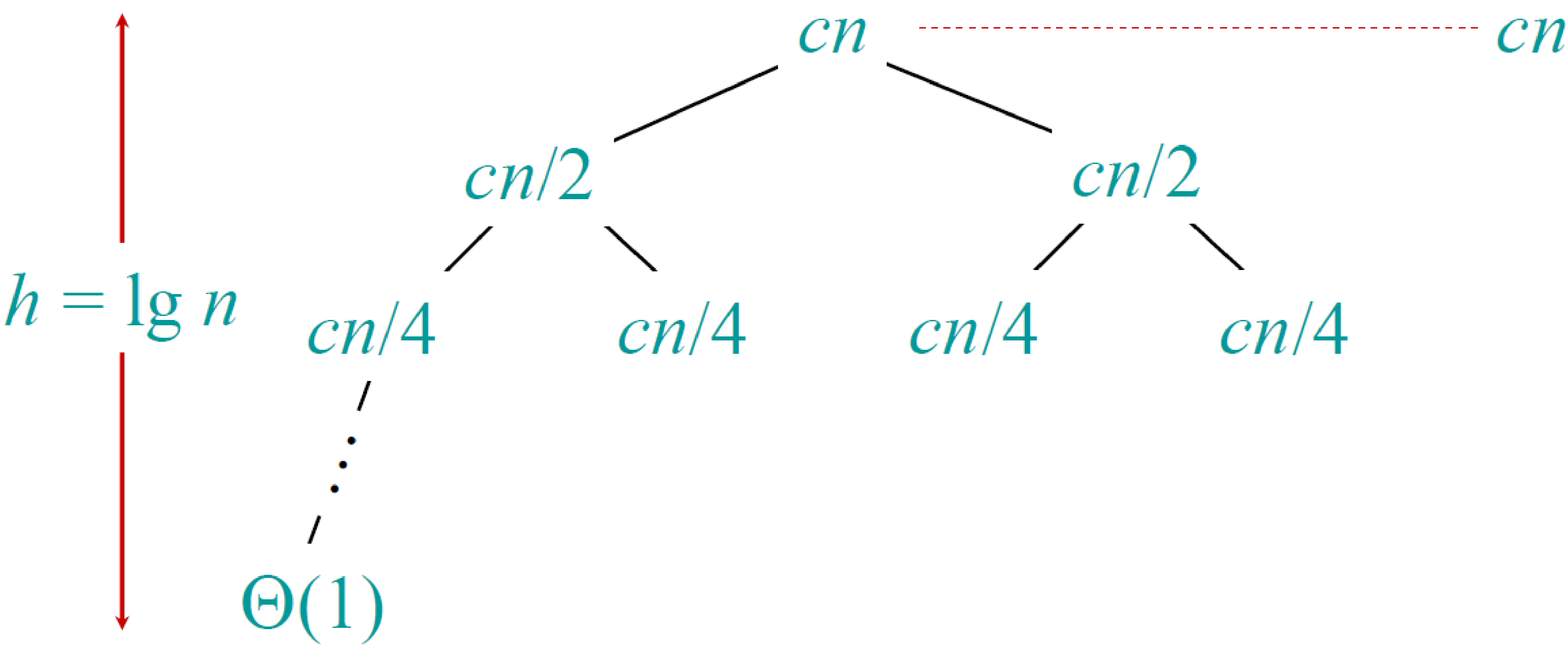

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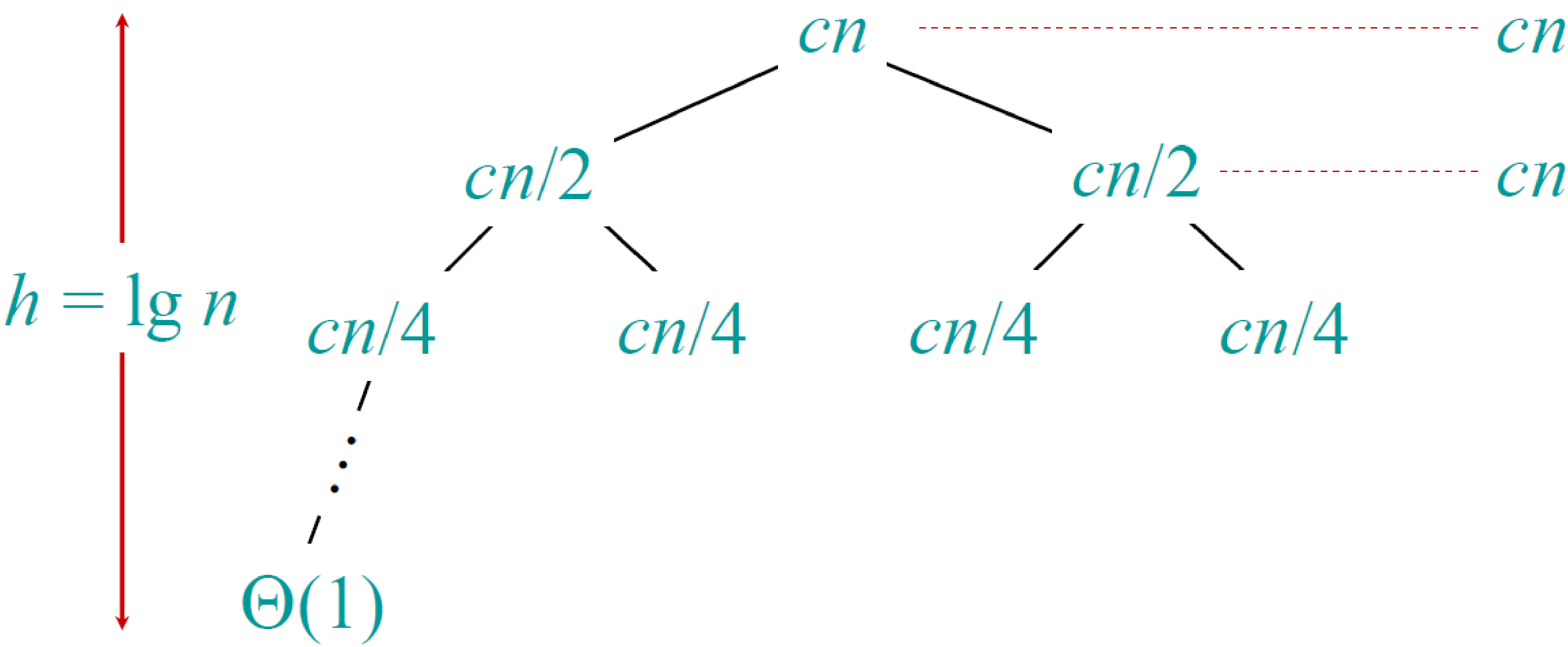

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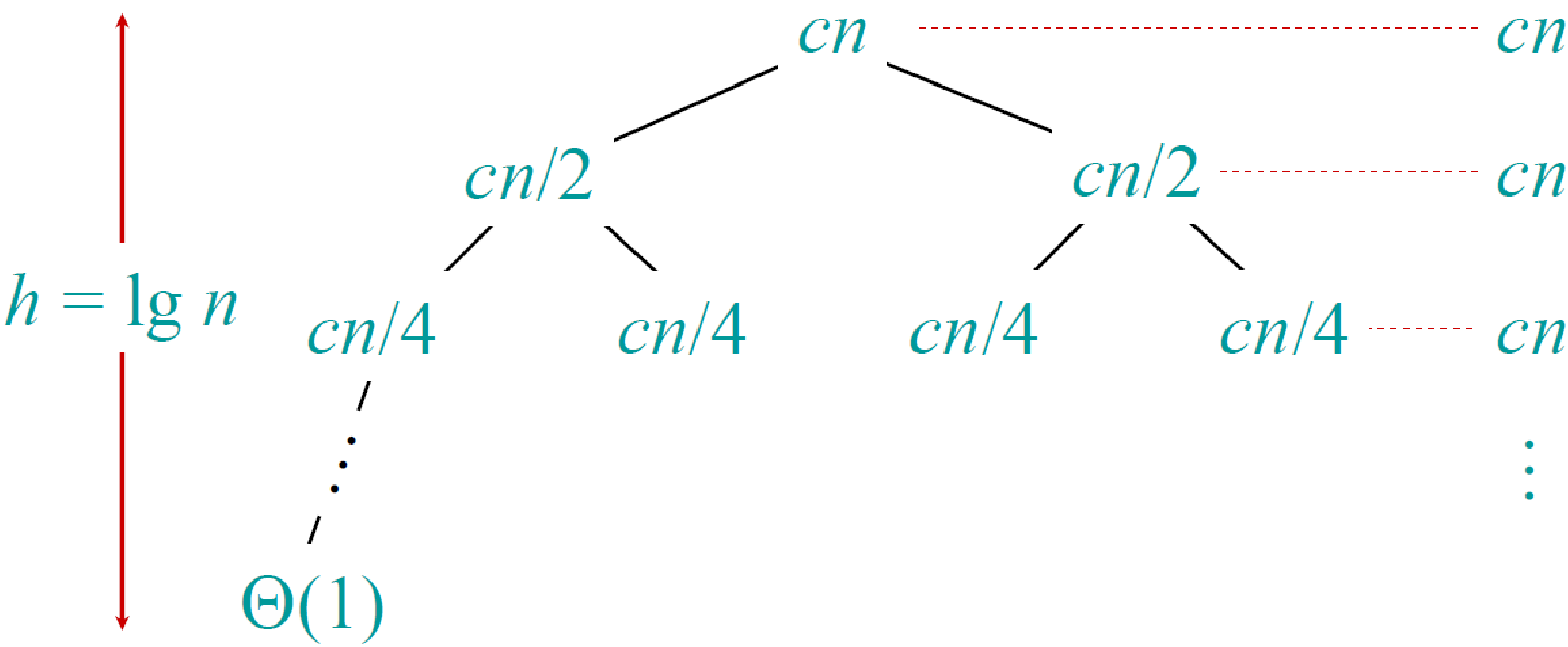

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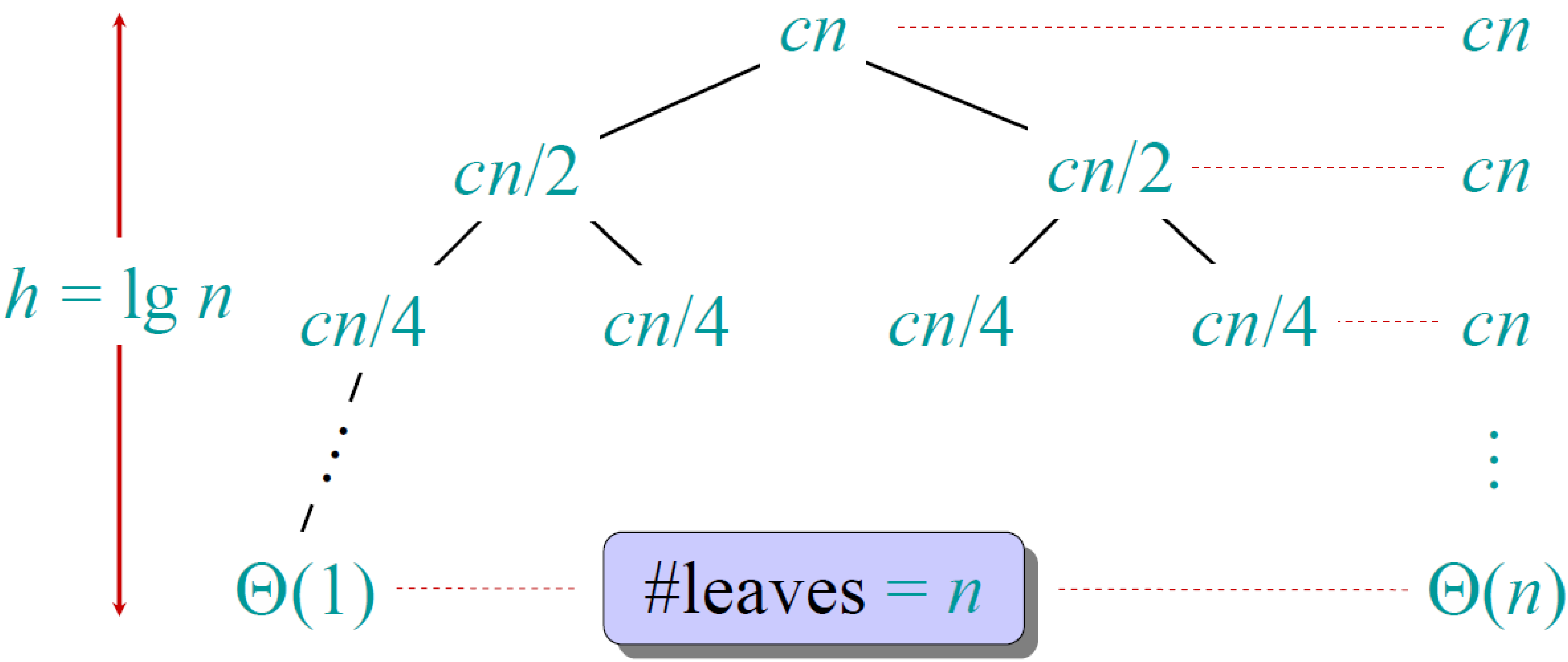

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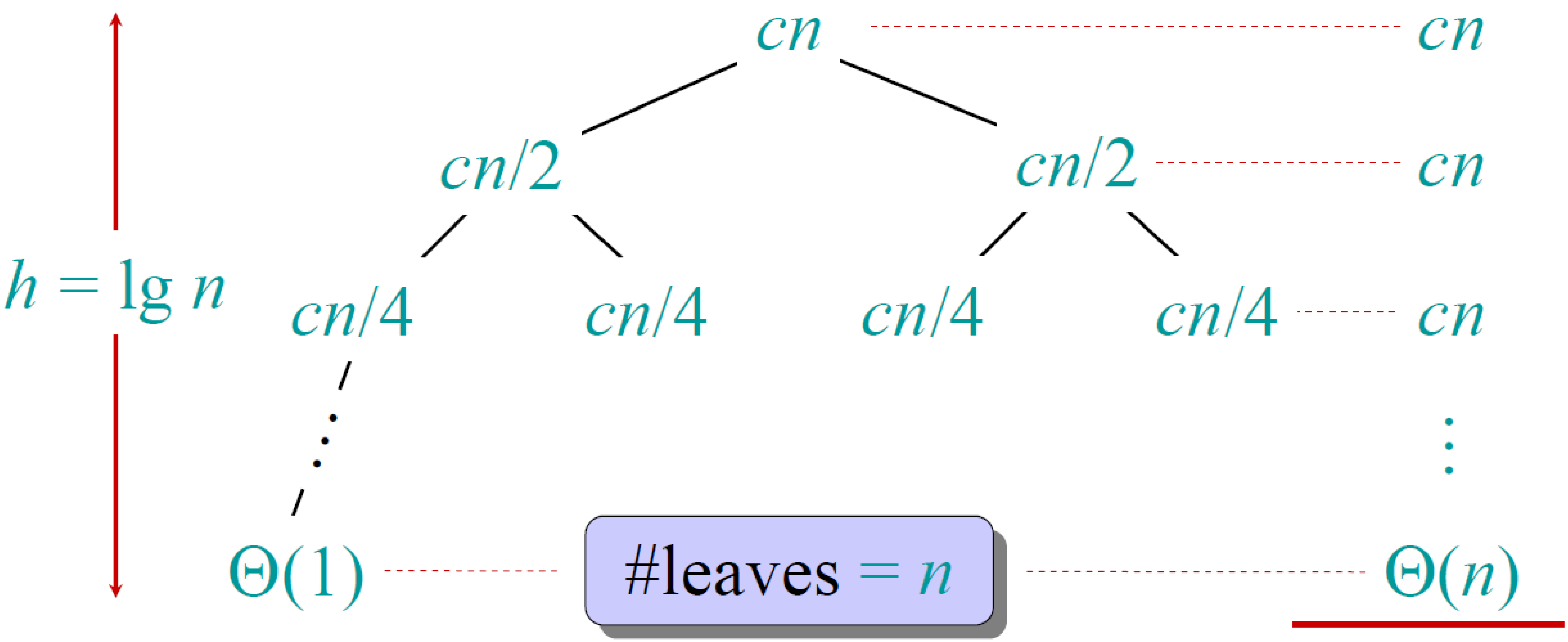


Total $= \Theta(n \lg n)$

# Conclusions

- $\Theta$ (nlgn) grows more slowly than $\Theta(n^2)$
- Therefore, merge sort asymptotically beats insertion sort in the worst case.
- In practice, merge sort beats insertion sort for ***n > 30*** .