习题：2.6

2014年12月30日 20:47

练习 2.6. A. P_1.

2. 证明：当 $x\to 0$ 时 $\sin^2 x$ 是 x 的高阶无穷小。

pf: ∵ $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sin x}{x}\cdot\dfrac{\sin x}{x}=1$.

∴ $\sin^2 x$ 为 x^2 的2阶无穷小 ~~也~~.

3. 当 $x\to 1$ 时，下列无穷小

(1) $\frac{1}{2}(1-x^2)$. (2). $1-x^3$. (3) $(1-x)^2$. (4) $1-x^2$.

中，哪一个与无穷小 $1-x$ 等价.

(1). $\lim\limits_{x\to 1}\dfrac{\frac{1}{2}(1-x^2)}{1-x}=\lim\limits_{x\to 1}\frac{1}{2}(1+x)=1$. 等价.

(2). $\lim\limits_{x\to 1}\dfrac{1-x^3}{1-x}=\lim\limits_{x\to 1}(1+x+x^2)=3$ ×

(3). $\lim\limits_{x\to 1}\dfrac{(1-x)^2}{1-x}=0$. ×

(4) $\lim\limits_{x\to 1}\dfrac{1-x^2}{1-x}=\lim\limits_{x\to 1}1+x=2$ ×.

4. 求下列变量的等价无穷大.

(1). $2x^3+3x^2-5x+6$ ($x\to\infty$)

∵ $\lim\limits_{x\to\infty}\dfrac{2x^3+3x^2-5x+6}{2x^3}=1$. ∴ $2x^3$ 为等价无穷大

(2). $\sqrt{x+\sqrt{x+\sqrt{x}}}$ ($x\to+\infty$)

∵ $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}=1$. ∴ $\sqrt{x}$ 为等价无穷大.

B.

B.

1. 证明：当 $x \to 0$ 时，有

(1). $\tan x - \sin x \sim \frac{1}{2}x^3$.

Pf: $\tan x - \sin x = \frac{\sin x(1-\cos x)}{\cos x}$

练习2.6. B P2

$\because \lim\limits_{x\to 0} \frac{\tan x - \sin x}{\frac{1}{2}x^3} = \lim\limits_{x\to 0} \frac{2\sin x(1-\cos x)}{x^3 \cos x} = \lim\limits_{x\to 0} \frac{2x \cdot \frac{1}{2} \cdot x^2}{x^3} = 1.$

(2). $\arctan x \sim \frac{1}{4}\sin 4x$.

Pf: $\because \arctan x \sim x \quad (x \to 0)$.

$\therefore \lim\limits_{x\to 0} \frac{\arctan x}{\frac{1}{4}\sin 4x} = \lim\limits_{x\to 0} \frac{4x}{\sin 4x} = 1.$

2. 利用等价无穷小求下面极限.

(1). $\lim\limits_{x\to 0} \frac{\tan 5x}{2x} = \lim\limits_{x\to 0} \frac{\tan 5x}{5x} \cdot \frac{5}{2} = \frac{5}{2}$.

(2). $\lim\limits_{x\to 0} \frac{\sin(x^m)}{(\sin x)^n} = \lim\limits_{x\to 0} \frac{\sin(x^m)}{x^m} \frac{x^m}{(\sin x)^n}$

1°. $m=n$. $\lim\limits_{x\to 0} \frac{\sin x^m}{(\sin x)^n} = 1$.

2°. $m>n$. $\lim\limits_{x\to 0} \frac{\sin(x^m)}{x^m} \cdot \frac{x^n}{(\sin x)^n} \cdot x^{m-n} = 0$

3°. $m<n$. $\lim\limits_{x\to 0} \frac{\sin x^m}{x^m} \cdot \frac{x^m}{(\sin x)^m} \cdot \frac{1}{(\sin x)^{n-m}} = \infty$.

(3). $\lim\limits_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x} = \lim \frac{\frac{1}{2}x^3}{}$

$$(3).\ \lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}=\lim_{x\to 0}\frac{\frac{1}{2}x^3}{\sin^3 x}=\frac{1}{2}.$$

$$(4).\ \lim_{x\to 0}\frac{\sqrt{1+x^2}-1}{1-\cos x}=\lim_{x\to 0}\frac{\frac{1}{2}x^2}{\frac{1}{2}x^2}=1.$$

3. 证明下面各题

(1). $2x-x^2=O(x).\ (x\to 0)$

Pf: 只需证 $\lim\limits_{x\to 0}\frac{2x-x^2}{x}=2$

(2). $\sqrt{1+x}-1=o(1)\ (x\to 0)$

练习2.6. B P3

Pf: $\because \lim\limits_{x\to 0}(\sqrt{1+x}-1)=\lim\limits_{x\to 0}\frac{x}{2}=0.$

$\therefore \sqrt{1+x}-1=o(1).$

(3). $2x^3+2x^2=O(x^3)\quad (x\to\infty)$

Pf: $\because \lim\limits_{x\to\infty}\frac{2x^3+2x^2}{x^3}=\lim\limits_{x\to\infty}\frac{2+\frac{2}{x}}{1}=2$

$\therefore 2x^3+2x^2=O(x^3).$

(4). $(1+x)^n=1+nx+o(x)\quad (x\to 0)$

Pf: $\lim\limits_{x\to 0}\frac{(1+x)^n-1-nx}{x}=\lim\limits_{x\to 0}\frac{1+nx+C_n^2x^2+C_n^3x^3+\cdots+x^n-1-nx}{x}$

$=\lim\limits_{x\to 0}(C_n^2x+C_n^3x^2+\cdots+x^{n-1})=0.$

$\therefore (1+x)^n=1+nx+o(x).$

$(1+x)^n = 1+nx+o(x)$.

4. 设在某一极限过程中，α、β 均为无穷小．证明：若 $\alpha \sim \beta$．则 $\beta-\alpha=o(\alpha)$；反之，若 $\beta-\alpha=o(\alpha)$ 则 $\alpha\sim\beta$．

Pf: "$\Rightarrow$" $\because \alpha\sim\beta$．$\therefore \lim\frac{\alpha}{\beta}=1$.

$\therefore \lim\frac{\beta-\alpha}{\alpha}=\lim(\frac{\beta}{\alpha}-1)=\lim\frac{\beta}{\alpha}-1=0$.

$\therefore \beta-\alpha=o(\alpha)$.

"$\Leftarrow$" $\because \beta-\alpha=o(\alpha)$．$\therefore \lim\frac{\beta-\alpha}{\alpha}=0$

$\Rightarrow \lim(\frac{\beta}{\alpha}-1)=0$

$\therefore \lim\frac{\beta}{\alpha}=\lim\frac{\beta-\alpha+\alpha}{\alpha}=\lim(\frac{\beta-\alpha}{\alpha}+1)=0+1=1$.

$\therefore \alpha\sim\beta$．＃．

5. 证明：当 $x\to0$ 时，下列关系式成立．

(1) $o(x^n)+o(x^m)=o(x^n)$．$(0<n<m)$.

练习 2.6． B P4.

Pf: $\lim\limits_{x\to0}\frac{o(x^n)+o(x^m)}{x^n}=\lim\limits_{x\to0}\frac{o(x^n)}{x^n}+\lim\limits_{x\to0}\frac{o(x^m)}{x^m}\frac{x^m}{x^n}$

$=0+\lim\limits_{x\to0}\frac{o(x^m)}{x^m}\cdot x^{m-n}$

$=0+\lim\limits_{x\to0}\frac{o(x^m)}{x^m}\cdot\lim\limits_{x\to0}x^{m-n}=0$.

(2) $o(x^m) \cdot o(x^n)=o(x^{n+m})$

$$\lim_{x\to 0} \frac{\ldots}{x^m} \cdot \lim_{x\to 0} x^{m-n} = 0.$$

$\cancel{(2).\ o(x^m) = o(x^{n+m})}$

$\cancel{\text{Pf: } \lim_{x\to 0} \frac{o(x^m)}{x^{n+m}} = \lim_{x\to 0} \frac{o(x^m)}{x^m} \cdot \frac{x}{x^n} \quad (m>0,\ n>0)}$

(2). $o(x^m)\cdot o(x^n) = o(x^{n+m})$

Pf: $$\lim_{x\to 0} \frac{o(x^m)\cdot o(x^n)}{x^n \cdot x^m} = \lim_{x\to 0} \frac{o(x^m)}{x^m} \cdot \lim_{x\to 0} \frac{o(x^n)}{x^n} = 0.\ \#$$