# Probabilistic Reasoning

Russell Chap 13, 14, 15

Luger Chap. 5, 9

# Managing Uncertainty

- Abduction
  - If P $\Rightarrow$ Q, Q then P → Not sound
- Most diagnostic rules are abduction
  - True implication (causal relation):
    - *Problem $\Rightarrow$ Symptom*

      Ex> Problem in battery $\Rightarrow$ Light gone
  - Expert system rule:
    - *Symptom $\Rightarrow$ Problem*

      Ex> Light gone $\Rightarrow$ Problem in battery

  → Not 100% true

  → Need measure of uncertainty

# Certainty Factor

- Certainty Theory
  - Confidence measure (expert heuristic) + combining rules

    **E(evidence) → H(hypothesis) : Confidence?**
  - Certainty Factor(CF) of H(hypothesis) given E(evidence)
    - CF(H | E) = MB(H | E) - MD(H | E)
    - MB: Measure of belief of H given E
    - MD: Measure of disbelief of H given E
  - Inference with CF
    - Rule: if P then Q (CF = 0.9)
    - Fact: P (CF = 0.9)

    → Q (CF = 0.9*0.9 = 0.81)
  - Combining CF
    - CF(P1 and P2) = min (CF(P1), CF(P2))
    - CF(P1 or P2) = max (CF(P1), CF(P2))

# Certainty Factor

- Example
  - Rule: (P1 and P2) or P3 $\rightarrow$ R1(0.7) and R2(0.3)
  - During production cycle, it is found that

    CF(P1) = 0.6

    CF(P2) = 0.4

    CF(P3) = 0.2
  - Then

    CF((P1 and P2) or P3) = max (min (0.6, 0.4), 0.2)

    = 0.4

    $\therefore$CF(R1) = 0.4 * 0.7 = 0.28

    CF(R2) = 0.4 * 0.3 = 0.12

# MYCIN

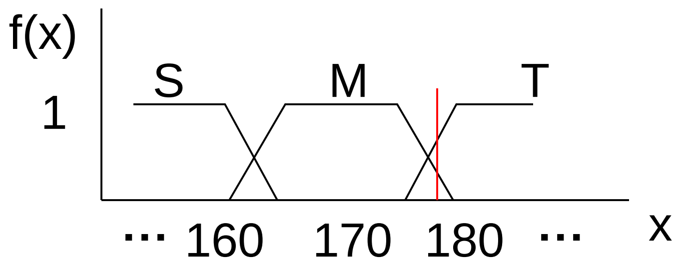
- Purpose
  - Expert system: determines the organism that causes meningitis and provides therapy
- Inference
  - Start from a hypothesis(organism), perform backward chaining
  - Use certainty factor
- Example rule
  - ```
    If   ((infection is primary-bacteremia) AND
          (portal-of-entry is gastrointestinal-tract))
    Then (conclude bacteroid 0.7)
    ```
    ```

# Fuzzy Logic

- Fuzzy set
  - Set membership is a function to [0, 1]

    Ex> S = set of small integer

    fs(1) = 1.0, fs(5) = 0.7, fs(100) = 0.001
  - Membership function
    - Sets: S, M, T




$f_T(178) = 0.7$, $f_M(178) = 0.3$

# Fuzzy Logic

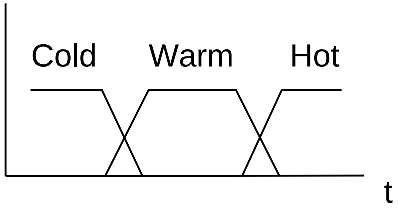
- Fuzzy logic
  - If $f_A(x) = a$, $f_B(x) = b$ then

    $f_{A \cap B}(x) = \min(a, b)$

    $f_{A \cup B}(x) = \max(a, b)$
- Fuzzy rules
  - If (speed = high) and (no. of cars = many) then (push brake)
  - If (temp = hot) then (fan speed = high)

# Reasoning with Fuzzy Logic

1. Define fuzzy sets
   - Temperature Hot, Warm, Cold, …




2. Build logical rules with the fuzzy sets
   - If (temp is Hot) then (fan_speed is High)

3. Perform inference
   1. Fuzzification : real value (30° C) → fuzzy set (Hot)
   2. Apply rules : use fuzzy logic
   3. Defuzzification : fuzzy set (High) → real_value (4500 RPM)

# Reasoning with Fuzzy Logic

- Example: Inverted Pendulum
  - Input: x1 = $\theta$, x2 = $d\theta/dt$
  - Output: u = movement M

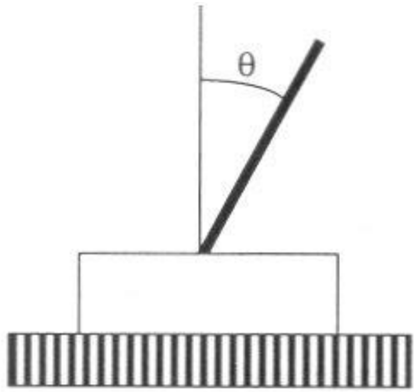



**Figure 7.15** The inverted pendulum and the angle $\theta$ and $d\theta/dt$ input values.

# Reasoning with Fuzzy Sets

- Fuzzy set
  - x1, x2: Z(zero), P(positive), N(negative)
  - u: Z, P, PB(positive big), N, NB(negative big)
- Rules
  1. If (x1 = P and x2 = Z) then u = P
  2. If (x1 = P and x2 = N) then u = Z
  3. If (x1 = Z and x2 = Z) then u = Z
  4. If (x1 = Z and x2 = N) then u = N
  5. . . .

# Reasoning with Fuzzy Sets

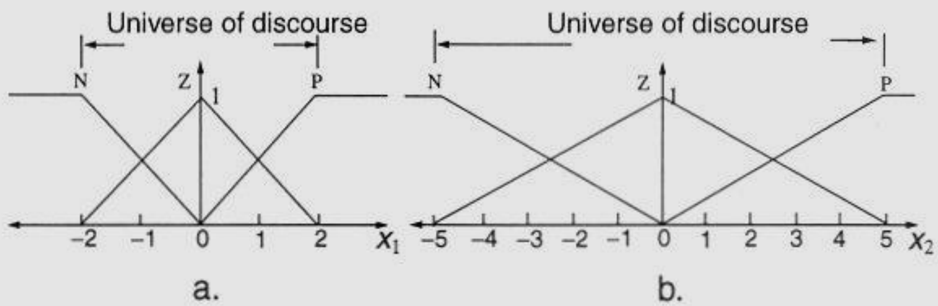



**Figure 7.16** The fuzzy regions for the input values θ a. and dθ/dt b.

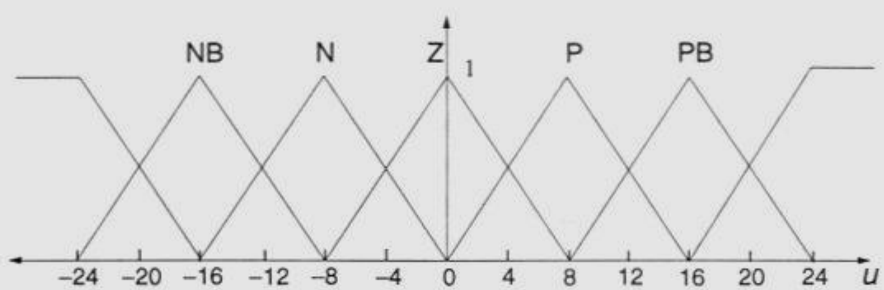



**Figure 7.17** The fuzzy regions of the output value *u*, indicating the movement of the pendulum base.

# Reasoning with Fuzzy Sets

- When x1 = θ = 1, x2 = dθ/dt = -4

  1. from fuzzy set membership

     x1: 0.5P, 0.5Z

     x2: 0.8N, 0.2Z

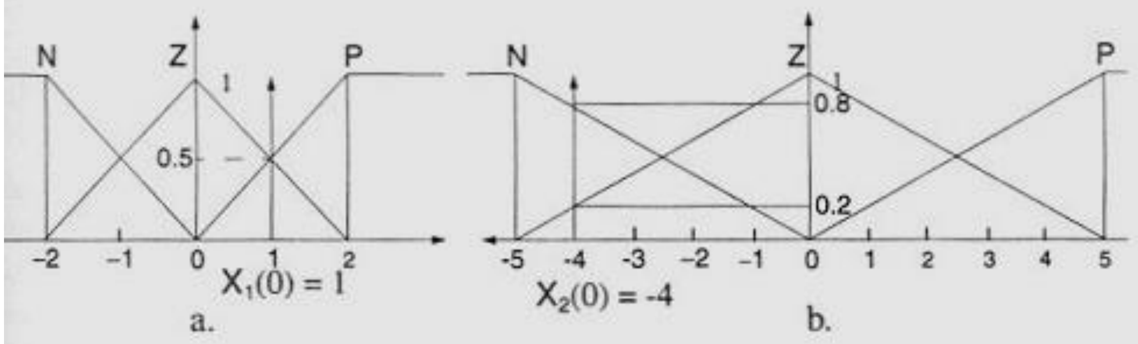
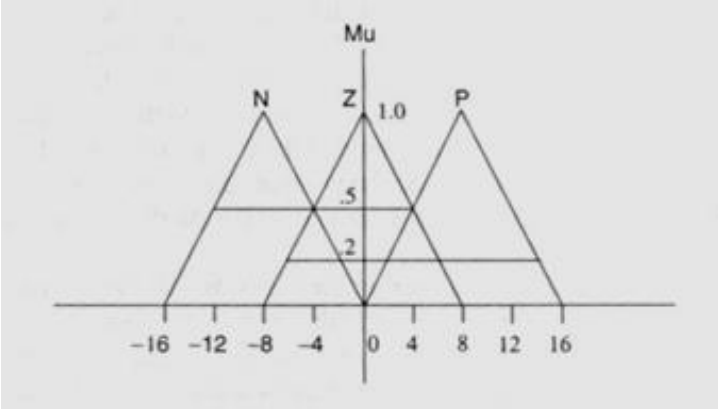



**Figure 7.18** The fuzzification of the input measures $x_1$=1, $x_2$ = -4.

# Reasoning with Fuzzy Sets

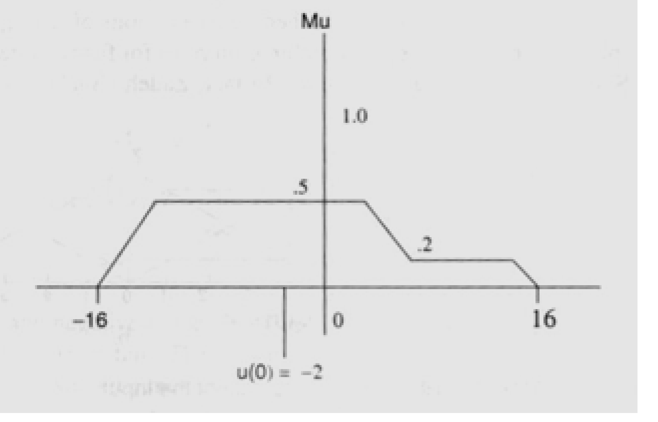
2. From fuzzy rules

u = 0.2P (1. If x1 = P(0.5) and x2 = Z(0.2) then u = P(min(0.5,0.2))
u = 0.5Z (2. If x1 = P(0.5) and x2 = N(0.8) then u = Z(min(0.5,0.8))
u = 0.2Z (3. If x1 = Z(0.5) and x2 = Z(0.2) then u = Z(min(0.5,0.2))
u = 0.5N (4. If x1 = Z(0.5) and x2 = N(0.8) then u = N(min(0.5,0.8))



# Reasoning with Fuzzy Sets

3. From fuzzy set membership, computing centroid of the union
   → u = -2




$$\begin{array}{ccc} X1 = 1, \; X2 = -4 & \xrightarrow{\textit{Fuzzification}} & P, Z, N \\ & & \Big\downarrow \textit{Fuzzy rules} \\ U = -2 & \xleftarrow{\textit{Defuzzification}} & P, Z, N \end{array}$$

# Probability

- For an experiment that produces outcomes
  - Sample space S: set of all outcomes
  - Event E: a subset of S
  - Probability P(E) = |E|/|S| (if outcomes are equally likely)
- Probability distribution
  - Function P: x → [0, 1] (x: random variable)

| S | x | P(x) | y(odd) | P(y) |
|---|---|---|---|---|
| 1 | 1 | 1/6 | 1 | 1/2 |
| 2 | 2 | 1/6 | 0 | 1/2 |
| … | … | … | … | |
| 6 | 6 | 1/6 | 0 | |

| S | x(# of H) | P(x) |
|---|---|---|
| HH | 2 | 1/4 |
| HT | 1 | 1/2 |
| TH | 1 | |
| TT | 0 | 1/4 |

# Probability

- Joint probability distribution
  - P(x,y) : represents joint probability distribution of x and y
  - Example
    - Gas{true,false}, Meter{empty, full}, Start{yes, no}
    - P(G,M,S):

| Gas | Meter | Start | P(G,M,S) |
|---|---|---|---|
| false | empty | no | 0.1386 |
| false | empty | yes | 0.0014 |
| false | full | no | 0.0594 |
| false | full | yes | 0.0006 |
| true | empty | no | 0.0240 |
| true | empty | yes | 0.0560 |
| true | full | no | 0.2160 |
| true | full | yes | 0.5040 |

# Probability

| Gas | Meter | Start | P(G,M,S) |
|---|---|---|---|
| false | empty | no | 0.1386 |
| false | empty | yes | 0.0014 |
| false | full | no | 0.0594 |
| false | full | yes | 0.0006 |
| true | empty | no | 0.0240 |
| true | empty | yes | 0.0560 |
| true | full | no | 0.2160 |
| true | full | yes | 0.5040 |

- P(Gas=false, Meter=empty, Start=no) = 0.1386
- P(Start=yes) = 0.5620

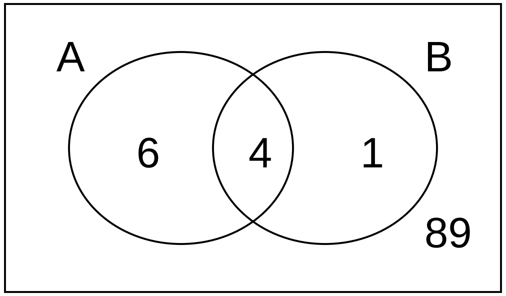
  ⇨ *Prior probability*

- P(Start=yes), given that Meter=empty ?

  ⇨ *Posterior probability (conditional probability)*

# Conditional Probability

- Conditional probability
  - $P(A|B) = P(A \wedge B) / P(B)$
  - Example
    - $P(A \wedge B) = 4 / 100 = 0.04$
    - $P(B) = 5 / 100 = 0.05$
    - $P(A \mid B) = 4 / 5 = 0.80$




- Independence
  - $P(A|B) = P(A)$
  - $P(A \wedge B) = P(A|B)\,P(B)$

    $= P(A)\,P(B)$ if A, B are independent

# Bayesian Reasoning

- Let H: Hypothesis (Problem)
  E: Evidence (Symptom)
- Then the probability of E → H is P(H | E)

$$P(H \cap E) = P(H \mid E) * P(E)$$
$$= P(E \mid H) * P(H)$$

$$\therefore \quad \boxed{P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(E)}}$$

# Bayesian Reasoning

- If we want to compare P(H1 | E), P(H2 | E), …

$$P(H1 \mid E) = \frac{P(E \mid H1) * P(H1)}{P(E)}$$

$$P(H2 \mid E) = \frac{P(E \mid H2) * P(H2)}{P(E)}$$

*same*

$$\therefore \quad \boxed{P(Hi \mid E) = \alpha\, P(E \mid Hi)\, P(Hi)}$$

# Bayesian Reasoning

- For multiple, conditionally independent evidences

  $$P(E1, E2, \ldots En \mid Hi) = P(E1 \mid Hi)P(E2 \mid Hi) \ldots P(En \mid Hi)$$

$$\therefore P(Hi \mid E1, E2, \ldots En)$$

$$= \alpha\, P(E1, E2, \ldots En \mid Hi)\, P(Hi)$$

$$= \alpha\, P(E1 \mid Hi)\, P(E2 \mid Hi) \ldots P(En \mid Hi)\, P(Hi)$$

$$= \alpha\, P(Hi) \underbrace{\prod_{i=1..n} P(Ei \mid Hi)}_{\text{These probabilities are obtained from sample data}}$$

*These probabilities are obtained from sample data*

# Bayesian Classifier Example

| No. | age | income | student | credit_rating | buys_computer |
|---|---|---|---|---|---|
| 1 | <=30 | high | no | fair | no |
| 2 | <=30 | high | no | excellent | no |
| 3 | 31…40 | high | no | fair | yes |
| 4 | >40 | medium | no | fair | yes |
| 5 | >40 | low | yes | fair | yes |
| 6 | >40 | low | yes | excellent | no |
| 7 | 31…40 | low | yes | excellent | yes |
| 8 | <=30 | medium | no | fair | no |
| 9 | <=30 | low | yes | fair | yes |
| 10 | >40 | medium | yes | fair | yes |
| 11 | <=30 | medium | yes | excellent | yes |
| 12 | 31…40 | medium | no | excellent | yes |
| 13 | 31…40 | high | yes | fair | yes |
| 14 | >40 | medium | no | excellent | no |

# Bayesian Classifier Example

E X: (age="<30", income="medium", student="yes", credit="fair") $\rightarrow$ yes / no ? H

- $P(Yes \mid X) = \alpha\, P(Yes)\, P(X|Yes)$
  $= \alpha\, P(Yes)\, P(<30|Yes)\, P(m|Yes)\, P(y|Yes)\, P(f|Yes)$
  $= \alpha \times 9/14 \times 2/9 \times 4/9 \times 6/9 \times 6/9 = 0.028\, \alpha$
- $P(No \mid X) = \alpha\, P(No)\, P(X|No)$
  $= \alpha\, P(No)\, P(<30|No)\, P(m|No)\, P(y|No)\, P(f|No)$
  $= \alpha \times 5/14 \times 3/5 \times 2/5 \times 1/5 \times 2/5 = 0.007\, \alpha$

$\Rightarrow$ X is classified to **yes**

# Bayesian Classifier Example

- Category C1 : # of sample documents – 60
  Keywords - information (2), network (12), algorithm (6), system (10)
- Category C2 : # of sample documents – 40
  Keywords - information (10), database (8), algorithm (4), system (8)
- New document **D**: has keywords - **information, system**

P(C1 | information, system)
= k * P(information | C1) * P(system | C1) * P(C1)
= k * 2/30 * 10/30 * 60/100 = 0.013 k
P(C2 | information, system)
= k * P(information | C2) * P(system | C2) * P(C2)
= k * 10/30 * 8/30 * 40/100 = 0.036 k

➡ D is classified to C2

# Bayesian Classifier Example

- Breast cancer classification
  - Compute P(recur | age=40-49, menop=ge40, … )
  - Based on P(recur), P(age=40-49 | recur), P(menop=ge40 | recur), …

Viewer

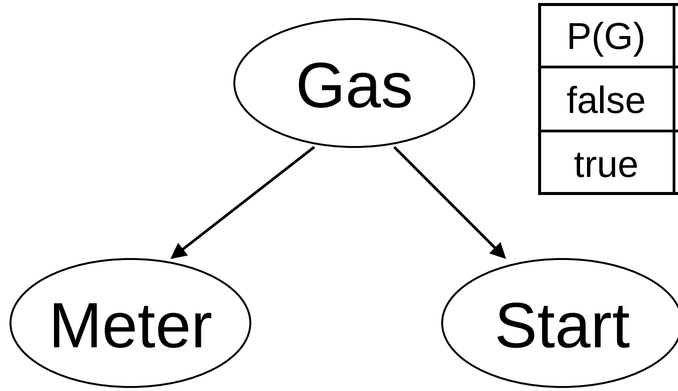
Relation: breast-cancer

| No. | age Nominal | menopause Nominal | tumor-size Nominal | inv-nodes Nominal | node-caps Nominal | deg-malig Nominal | breast Nominal | breast-quad Nominal | irradiat Nominal | Class Nominal |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 40-49 | premeno | 15-19 | 0-2 | yes | 3 | right | left_up | no | recurrence-events |
| 2 | 50-59 | ge40 | 15-19 | 0-2 | no | 1 | right | central | no | no-recurrence-events |
| 3 | 50-59 | ge40 | 35-39 | 0-2 | no | 2 | left | left_low | no | recurrence-events |
| 4 | 40-49 | premeno | 35-39 | 0-2 | yes | 3 | right | left_low | yes | no-recurrence-events |
| 5 | 40-49 | premeno | 30-34 | 3-5 | yes | 2 | left | right_up | no | recurrence-events |
| 6 | 50-59 | premeno | 25-29 | 3-5 | no | 2 | right | left_up | yes | no-recurrence-events |
| 7 | 50-59 | ge40 | 40-44 | 0-2 | no | 3 | left | left_up | no | no-recurrence-events |
| 8 | 40-49 | premeno | 10-14 | 0-2 | no | 2 | left | left_up | no | no-recurrence-events |
| 9 | 40-49 | premeno | 0-4 | 0-2 | no | 2 | right | right_low | no | no-recurrence-events |
| 10 | 40-49 | ge40 | 40-44 | 15-17 | yes | 2 | right | left_up | yes | no-recurrence-events |
| 11 | 50-59 | premeno | 25-29 | 0-2 | no | 2 | left | left_low | no | no-recurrence-events |
| 12 | 60-69 | ge40 | 15-19 | 0-2 | no | 2 | right | left_up | no | no-recurrence-events |
| 13 | 50-59 | ge40 | 30-34 | 0-2 | no | 1 | right | central | no | no-recurrence-events |
| 14 | 50-59 | ge40 | 25-29 | 0-2 | no | 2 | right | left_up | no | no-recurrence-events |
| 15 | 40-49 | premeno | 25-29 | 0-2 | no | 2 | left | left_low | yes | recurrence-events |
| 16 | 30-39 | premeno | 20-24 | 0-2 | no | 3 | left | central | no | no-recurrence-events |
| 17 | 50-59 | premeno | 10-14 | 3-5 | no | 1 | right | left_up | no | no-recurrence-events |
| 18 | 60-69 | ge40 | 15-19 | 0-2 | no | 2 | right | left_up | no | no-recurrence-events |
| 19 | 50-59 | premeno | 40-44 | 0-2 | no | 2 | left | left_up | no | no-recurrence-events |
| 20 | 50-59 | ge40 | 20-24 | 0-2 | no | 3 | left | left_up | no | no-recurrence-events |

Undo OK Cancel

# Bayesian Belief Network

- Nodes: Random variables
- Edges: Direct influence
- Each node x stores P(x | parents(x))



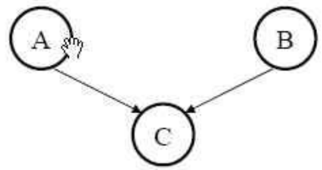

| P(G) | |
|---|---|
| false | 0.2 |
| true | 0.8 |

| P(M\|G) | G = f | G = t |
|---|---|---|
| empty | 0.7 | 0.1 |
| full | 0.3 | 0.9 |

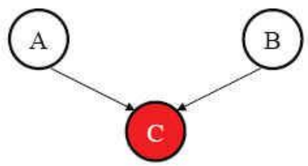
| P(S\|G) | G = f | G = t |
|---|---|---|
| no | 0.99 | 0.30 |
| yes | 0.01 | 0.70 |

$$P(G,M,S) = P(G)\,P(M \mid G)\,P(S \mid G)$$

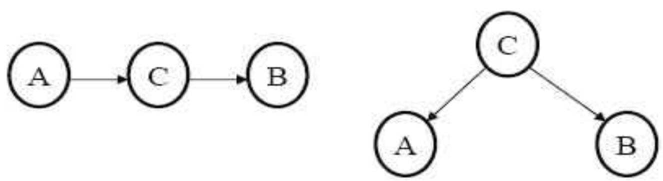
# Dependency



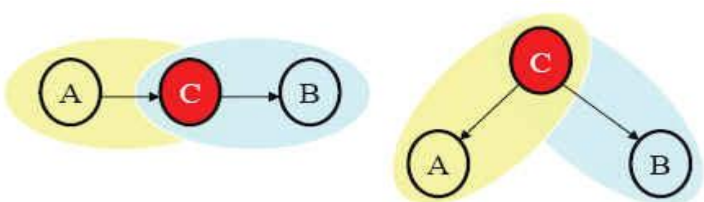

- A, B are independent
  - P(B|A) = P(B)




- A, B are *conditionally* dependent
  - P(B|A,C) ≠ P(B|C)



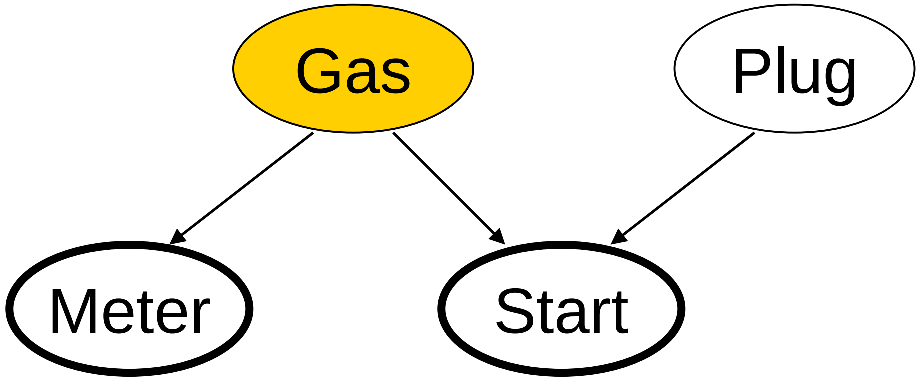

- A, B are dependent
  - P(B|A) ≠ P(B)




- A, B are *conditionally* independent
  - P(B|A,C) = P(B|C)

# Dependency

- Gas and Start are dependent
- Gas and Plug are independent
- Gas and Plug are conditionally dependent given Start
- Meter and Start are conditionally independent given Gas
  - $P(S \mid M, G) = P(S \mid G)$



# Chain Rule and Independence

- $P(A,B) = \frac{P(A,B)}{P(A)} P(A) = P(B|A)\,P(A)$
- $P(A,B,C) = \frac{P(A,B,C)}{P(A,B)} \frac{P(A,B)}{P(A)} P(A) = P(C|B,A)\,P(B|A)\,P(A)$
- $P(A,B,C,D) = P(D|C,B,A)\,P(C|B,A)\,P(B|A)\,P(A)$

- $P(G,M,S) = P(S \mid G,M)\,P(M \mid G)\,P(G)$ (Chain rule)

  $= P(S \mid G)\,P(M \mid G)\,P(G)$

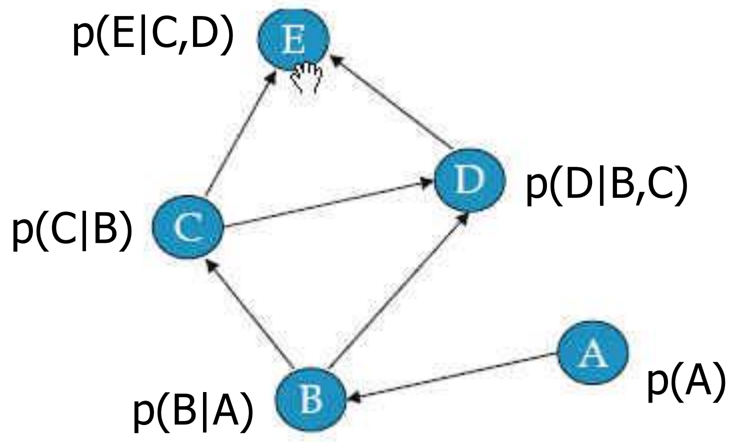
  (If S, M are conditionally independent given G)

  $= P(S)\,P(M)\,P(G)$

  (If S, M, G are all independent)

# Computing Joint Distribution from Bayesian Network

- In general,
  Joint probability ← *product of conditional probabilities*
- Ex>




$$P(A,B,C,D,E) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)P(E|A,B,C,D)$$
$$= P(A)P(B|A)P(C|B) \quad P(D|B,C) \quad P(E|C,D)$$

# Bayesian Reasoning with Full Probability Distribution

| Gas | Meter | Start | P(G,M,S) |
|---|---|---|---|
| false | empty | no | 0.1386 |
| false | empty | yes | 0.0014 |
| false | full | no | 0.0594 |
| false | full | yes | 0.0006 |
| true | empty | no | 0.0240 |
| true | empty | yes | 0.0560 |
| true | full | no | 0.2160 |
| true | full | yes | 0.5040 |

- P(Gas=false, Meter=empty, Start=no) = 0.1386
- P(Gas=false) = 0.2
- P(Start=yes | Meter=full) = P(S=yes, M=full) / P(M=full)
  = (0.5040+0.0006) / (0.5040+0.0006+0.2160+0.0594)
  = 0.6469

# Bayesian Reasoning with Full Probability Distribution

- P(Start=yes | Meter=full)

| Gas | Meter | Start | P(G,M,S) |
|---|---|---|---|
| false | full | no | 0.0594 |
| false | full | yes | 0.0006 |
| true | full | no | 0.2160 |
| true | full | yes | 0.5040 |

Select M=full

| Gas | Start | P(G,M,S) |
|---|---|---|
| ALL | no | 0.2754 |
| ALL | yes | 0.5046 |

Sum G

| Start | P(G,M,S) |
|---|---|
| no | 0.3531 |
| yes | 0.6469 |

Normalize

# Reasoning with Bayesian Belief Network

- Inference for P(H | E)
  - From the product of probability table,
  1. Remove all rows except E
  2. Compute product
  3. Sum over irrelevant variables
  4. Normalize
- Example
  - P(S=yes | M=full)

# Reasoning with Bayesian Belief Network

p(S,G,M)

- | P(G) | |
  |---|---|
  | false | 0.2 |
  | true | 0.8 |

  | P(M\|G) | G = f | G = t |
  |---|---|---|
  | ~~empty~~ | ~~0.7~~ | ~~0.1~~ |
  | full | 0.3 | 0.9 |

  | P(S\|G) | G = f | G = t |
  |---|---|---|
  | no | 0.99 | 0.30 |
  | yes | 0.01 | 0.70 |

Remove M=empty

p(S,G | M=f)

- | S | G = f | G = t |
  |---|---|---|
  | no | 0.0594 | 0.2160 |
  | yes | 0.0006 | 0.5040 |

Product

p(S | M=f)

- | S | |
  |---|---|
  | no | 0.2754 |
  | yes | 0.5046 |

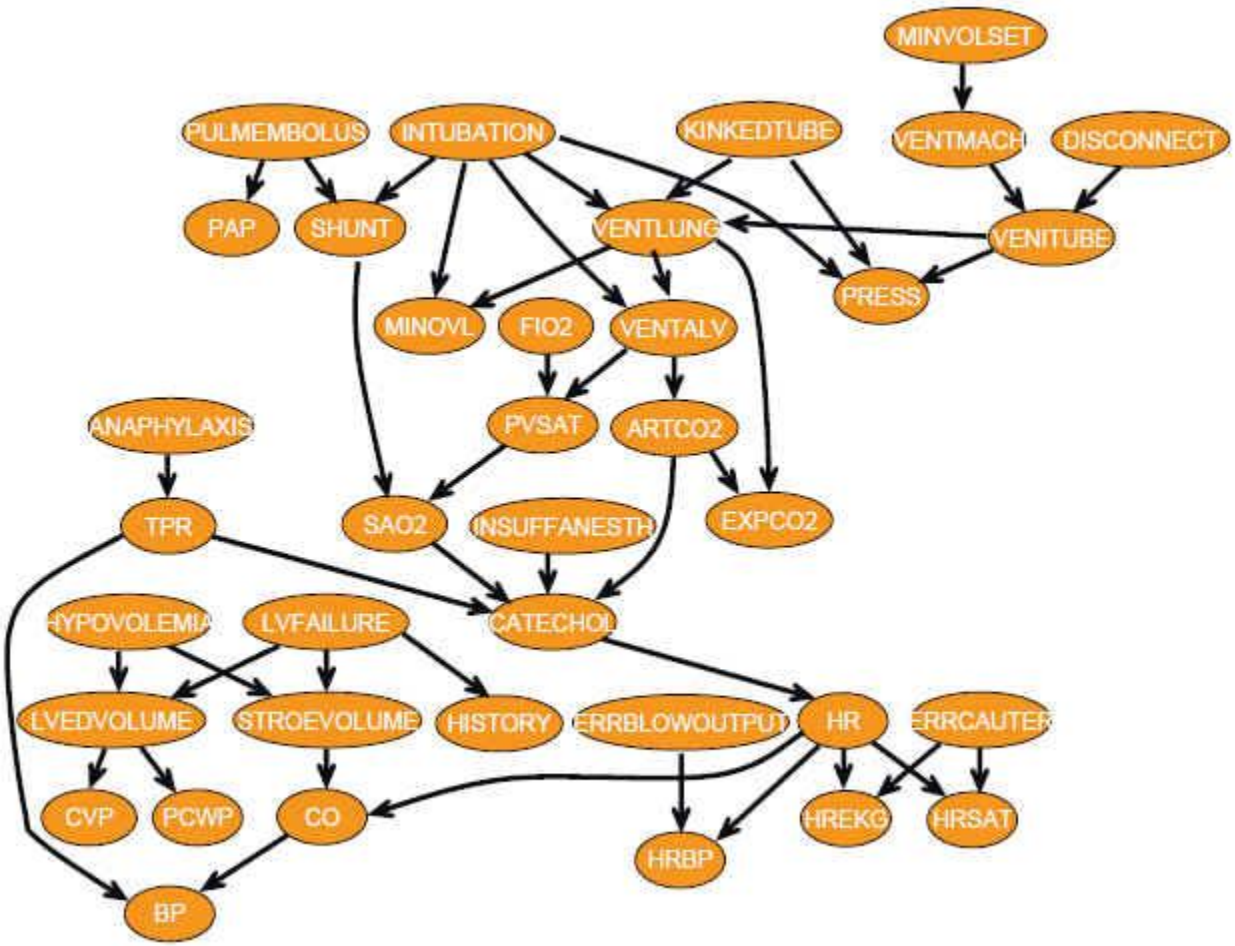
Sum over G

p(S | M=f)

- | S | |
  |---|---|
  | no | 0.3531 |
  | yes | **0.6469** |

Normalize

# Advantage of Reasoning with Bayesian Belief Network



# Advantage of Reasoning with Bayesian Belief Network

- Assume 20 boolean variables: 19 E → 1 H
- Compute P(H | E1, E2, …, E19)

$$= \frac{P(E1, E2, \ldots E19 \mid H)\, P(H)}{P(E1, E2, \ldots, E19)}$$

1) From full joint distribution

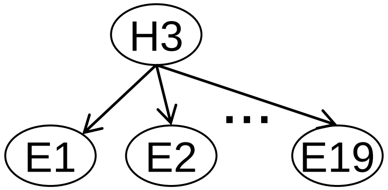
**P**(H, E1, E2, … E19)

→ We need to know $2^{20}$ = 1,048,576 prob.

⇨ Almost impossible

| H | E1 | E2 | … | P(H,E1,E2,…) |
|---|---|---|---|---|
| T | T | T | … | 0.xxxx |
| | | | … | … |
| F | F | F | … | 0.xxxx |

# Advantage of Reasoning with Bayesian Belief Network

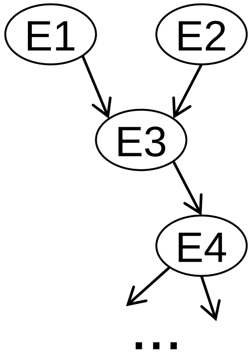



2) Assume complete independences (Naïve Bayesian)

**P**(E1 | H) **P**(E2 | H) … **P**(E19 | H) **P**(H)

→ 4*19 + 2 = 78 prob.

⇨ But the evidences are not actually independent




2) Consider actual dependences (Bayesian Network)

**P**(E1) **P**(E2) **P**(E3 | E1, E2) …

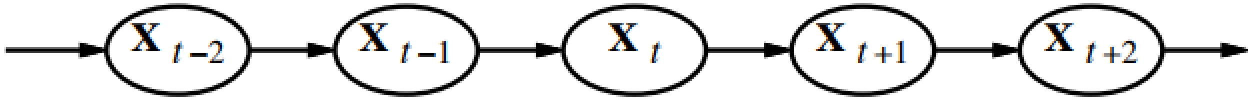
(assume less than 2 parents in Bayesian Network)

→ less than 8*20 = 160 prob.

⇨ Actual dependences are considered, yet need small # of probabilities.

# Markov Model

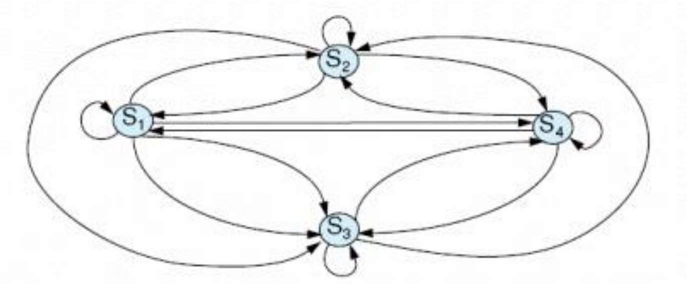
- Markov process (Markov chain)
  - Probability of a state at time t depends on its previous n states
    $P(X_t \mid X_{t-1}, X_{t-2}, X_{t-3}, \ldots, X_{t-n})$
- First-order Markov process
  - Probability of a state at time t depends on its previous 1 state
    $P(X_t \mid X_{t-1})$



# Markov Model

- Example
  - 4 states: S1(sunny), S2(cloudy), S3(Foggy), S4(Rainy)

| | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
|---|---|---|---|---|
| $s_1$ | 0.4 | 0.3 | 0.2 | 0.1 |
| $s_2$ | 0.2 | 0.3 | 0.2 | 0.3 |
| $s_3$ | 0.1 | 0.3 | 0.3 | 0.3 |
| $s_4$ | 0.2 | 0.3 | 0.3 | 0.2 |

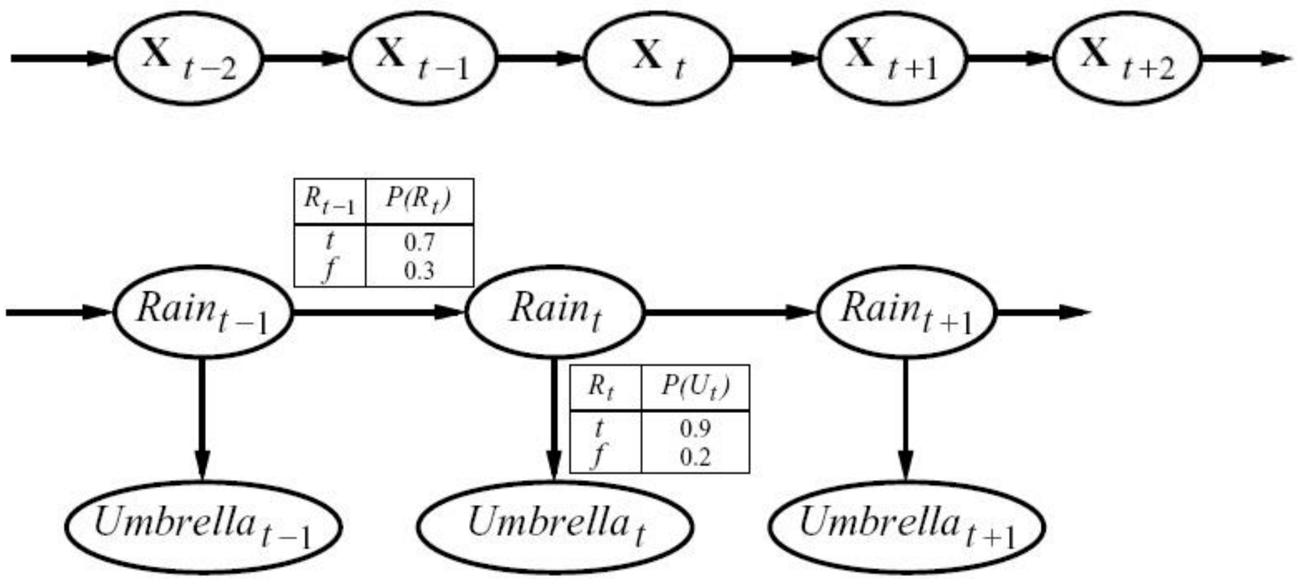



  - Today is sunny. Prob. of next 2 days are rainy?

$$P(S1, S4, S4) = P(S1)\, P(S4 \mid S1)\, P(S4 \mid S1,S4)$$
$$= P(S1)\, P(S4 \mid S1)\, P(S4 \mid S4)$$
$$= 1 * 0.1 * 0.2 = 0.02$$

# Hidden Markov Model (HMM)

- HMM
  - States are “hidden”
  - Probability of observation is given. P(Oj | Si)



# Hidden Markov Model (HMM)

$$P(s1 \dots sn \mid o1 \dots on)$$

$$= \frac{P(o1 \dots on \mid s1 \dots sn)\, P(s1 \dots sn)}{P(o1 \dots on)}$$

$$= \boldsymbol{\alpha}\, P(o1 \dots on \mid s1 \dots sn)\, P(s1 \dots sn)$$

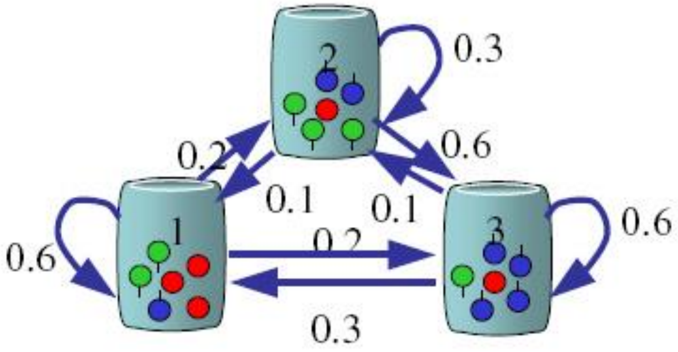
$$= \boldsymbol{\alpha}\, P(o1 \mid s1) P(o2 \mid s2) \dots P(on \mid sn)\, P(s1 \dots sn)$$

$$= \boldsymbol{\alpha}\, P(o1 \mid s1) P(o2 \mid s2) \dots P(on \mid sn)\, P(s1) P(s2 \mid s1) \dots P(sn \mid sn\text{-}1)$$

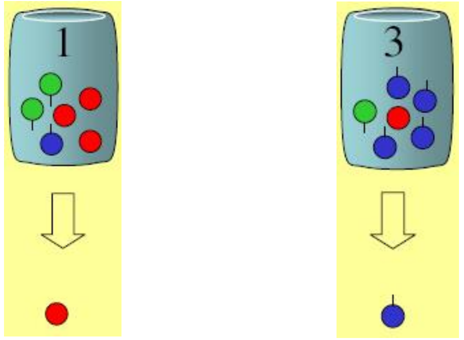
$$= \prod_{i=1..n} P(oi \mid si)\, P(si \mid si\text{-}1)$$

# Hidden Markov Model (HMM)

- Example
  - Markov process

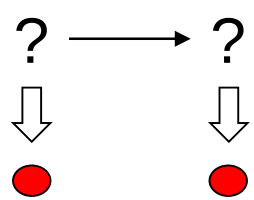


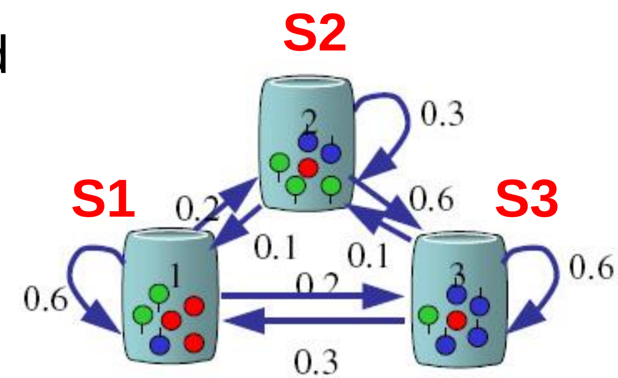

  - Output process



# Hidden Markov Model (HMM)

- Which cup is selected? → hidden
- Only output sequence is observed




- Most likely sequence

$= \text{argmax}_{X}\ P(X_0, X_1, \ldots, X_t \mid E_1, E_2, \ldots, E_t)$

$= \text{argmax}_{X1,X2}\ (1/3*(\mathbf{S1})*3/6 * 0.6(\mathbf{S1})*3/6\ ,$

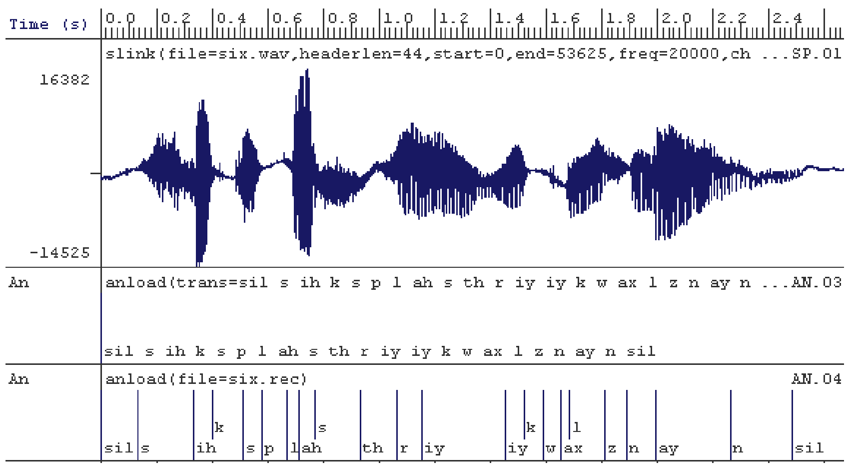
$1/3*(\mathbf{S1})*3/6 * 0.2(\mathbf{S2})*1/6\ ,$

$1/3*(\mathbf{S1})*3/6 * 0.2(\mathbf{S3})*1/6\ ,$

$1/3*(\mathbf{S2})*1/6 * 0.1(\mathbf{S1})*3/6\ , \ldots\ ) =$ **S1, S1**

# Speech Recognition

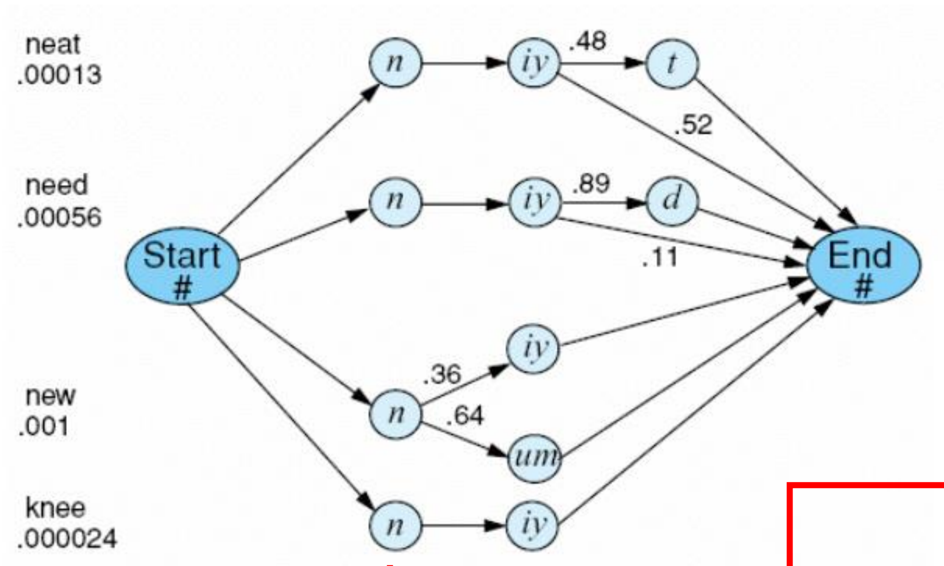
- The problem
  - Observed: sequence of acoustic signals
  - Determine: which phoneme? → which word?




➡ Compute P(phoneme | signal) by using HMM

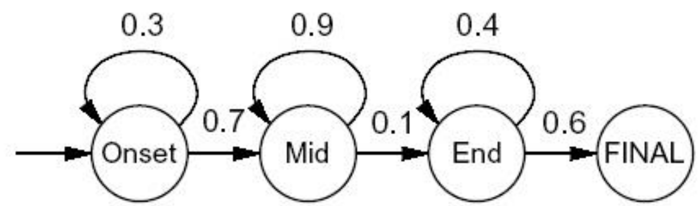
Find argmax P(*p1, p2, p3, .. | o1, o2, o3, ..*)
= argmax $\prod_{i=1..n}$ P(*oi | pi* ) P(*pi | pi-1*)

# Speech Recognition




state
(hidden)

**Phone HMM for [m]:**



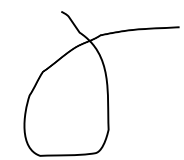

**Output probabilities for the phone HMM:**

| Onset: | Mid: | End: |
|---|---|---|
| C1: 0.5 | C3: 0.2 | C4: 0.1 |
| C2: 0.2 | C4: 0.7 | C6: 0.5 |
| C3: 0.3 | C5: 0.1 | C7: 0.4 |

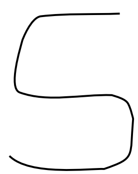
observation

# Handwriting Recognition

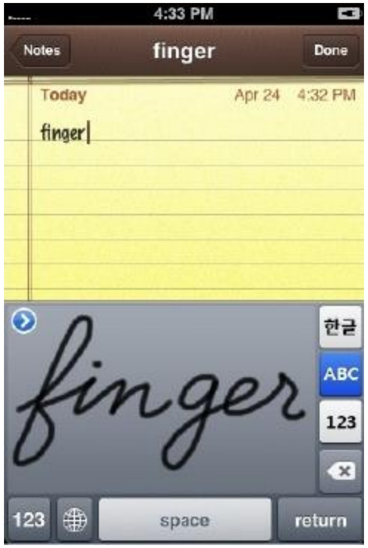
- Hand-written character recognition



*α* or *6* ?



*5* or *S* ?

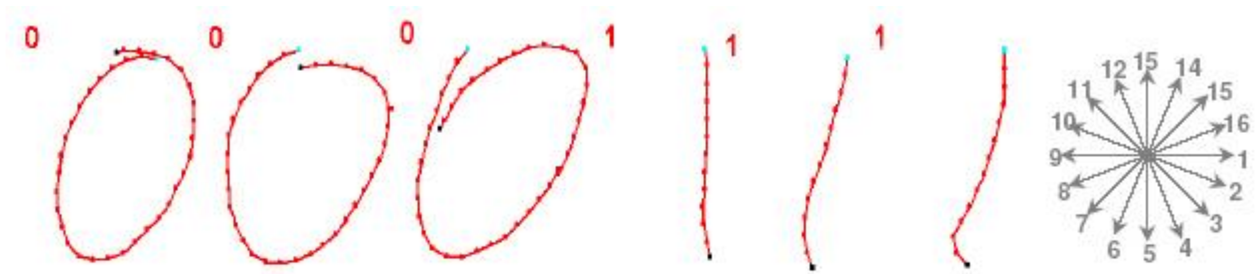


# Handwriting Recognition

- The problem
  - Observed: sequence of moving directions (d1.. dn )
  - Determine: which character? - states (s1.. sn)

Find argmax P(*s1 … sn* | *d1 … dn* )

$$= \text{argmax} \prod_{i=1..n} P(d_i \mid s_i)\, P(s_i \mid s_{i-1})$$

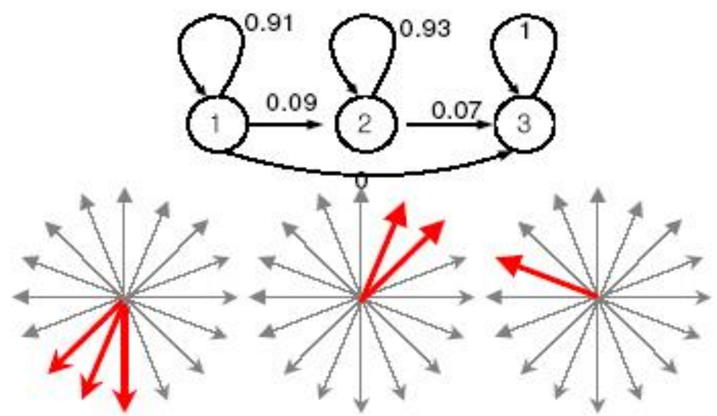



data0{1} = [9 8 8 7 7 7 6 6 6 5 5 5 4 4 3 2 1 16 15 15 15 15 14 14 14 13 13 13 12 12 11 10 9 9 8 ]

data1{2} = [5 6 6 6 6 6 6 6 6 5 5 4 ]

# Handwriting Recognition

- Example
  - Writing: [8, 8, 7, 7, 7, 6, 6, 5, 5, … ]
  - P(States of ‘zero’ | 8, 8, 7, 7, 7, 6, 6, 5, 5, … ) >> P(States of ‘one’ | 8, 8, 7, 7, 7, 6, 6, 5, 5, … )
  - Markov process for ‘0’:
  - Output process:




| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.33 | 0.21 | 0.12 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.09 | 0.07 | 0.07 | 0.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.28 | 0.11 |
| 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.06 | 0.16 | 0.23 | 0.13 | 0.10 | 0.10 | 0.16 | 0.00 | 0.00 | 0.00 |