CHAPTER 7

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7-1.

- (a) R1 + 2's complement of R2 = 2^n + R1 R2. If R1 \geq R2, the result is \geq 2^n . The 2^n gives C = 1. R1 + 2's complement of R2 = 2^n + R1 R2, if R1 < R2, the result is \leq 2^n giving C = 0.
- (b) If C = 1 then $R1 \ge R2$ and there is no borrow. If C = 0 then R1 < R2 and there is a borrow. Thus, the borrow is the complement of the C status bit.

7-2. *

1001 1001

1100 0011

1000 0001 AND

1101 1011 OR

0101 1010 XOR

7-3.

- (a) AND, 1010 1010 1010 1010 (b)
- (b) OR, 0000 0000 0000 1111
- (c) XOR, 1111 1111 0000 0000

7-4.*

sl 1001 0100

sr 0110 0101

7-5.*

 Q_i remains connected to MUX data input 0. Connect D_i to MUX data input 1 instead of Mux data input 3. Connect Q_{i-1} to MUX data input 2 instead of MUX data input 1. Finally, 0 is connected to MUX data input 3.

7-6.*

- a) 1000, 0100, 0010, 0001, 1000....
- b) # States = n

7-7.

- a) 000, 100, 110, 111, 011, 001, 000, ...
- b) # States = 2n

7-8.

a) 8

b) 3

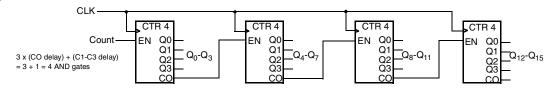
7-9.+

Examine an n-bit ripple counter and an n-bit synchronous counter. If either of these counters cycles through all of its states, there are $2(2^n) = 2^{n+1}$ transitions for the clock, and there are $2^{n+1}-2$ total transitions for all flipflop outputs. For the ripple counter, the clock transitions occur on the input of only one stage, the 0th stage. For the synchronous counter, the clock transitions occur on the inputs to all of the n stages. Combining the transition counts above, the ratio of the input + output transitions for the synchronous counter compared to the ripple counter is:

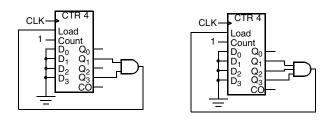
$$[n\ 2^{n+1}+2^{n+1}-2]/[2^{n+1}+2^{n+1}-2]\approx (n+1)\ 2^{n+1}/2\ (2^{n+1})=(n+1)/2$$

Thus, the power dissipated by the synchronous counter is at least as large as that dissipated by the ripple counter in all cases and grows more rapidly with the number of stages.

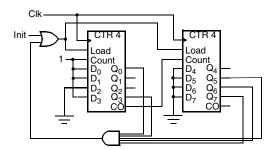
7-10.



7-11.

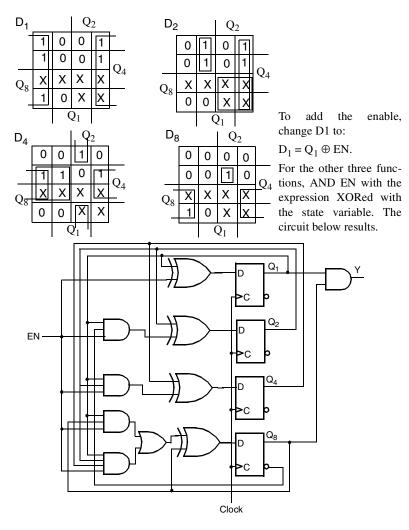


7-12.



7-13.*

The equations given on page 364-5 can be manipulated into SOP form as follows: $D_1 =$ $\begin{array}{l} \overline{Q}_1,\,D_2=Q_2\oplus Q_1\overline{Q}_8=Q_1\overline{Q}_2\overline{Q}_8+\overline{Q}_1Q_2+Q_2Q_8,\,D_4=Q_4\oplus Q_1Q_2=Q_1Q_2\overline{Q}_4+\overline{Q}_1Q_4\\ +\overline{Q}_2Q_4,\,D_8=Q_8\oplus (Q_1Q_8+Q_1Q_2Q_4)=\overline{Q}_8(Q_1Q_8+Q_1Q_2Q_4)+Q_8(\overline{Q}_1+\overline{Q}_8)(\overline{Q}_1+\overline{Q}_2\\ +\overline{Q}_4)=Q_1Q_2Q_4\overline{Q}_8+\overline{Q}_1\,Q_8. \end{array}$ These equations are mapped onto the K-maps for Table 7-9 below and meet the specifications given by the maps and the table.



7-14.*

Pre	Present state			Next state			
A	В	С	A	В	С		
	0	0		0	1		
	0	1		1	0		
	1	0		0	0		
0	0	0	0	0	1		
0	0	1	0	1	0		
0	1	0	0	1	1		
0	1	1	1	0	0		
1	0	0	1	0	1		
1	0	1	0	0	0		

a)
$$D_B = C$$

a)
$$D_B = C$$
 b) $D_A = BC + A\overline{C}$

$$D_C = \overline{B} \overline{C}$$

$$D_C = \overline{B} \, \overline{C} \qquad D_B = \overline{A} \, \overline{B} C + B \overline{C}$$

$$D_C = \overline{C}$$

7-15.

Pres	sent s	tate	Ne	ate	
Α	В	С	A	В	С
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	0	0	1
0	1	1	1	0	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	0	0	0

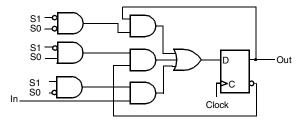
$$\mathrm{D}_{\mathrm{A}} = \mathrm{A} \overline{\mathrm{B}} + \mathrm{A} \overline{\mathrm{C}} + \overline{\mathrm{A}} \mathrm{B} \mathrm{C}$$

$$D_B = \overline{B}$$

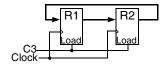
$$D_C = \overline{B}C + B\overline{C}$$

7-16.

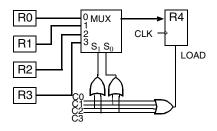
The basic cell of the register is as follows:



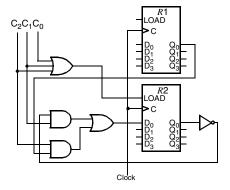
7-17.*



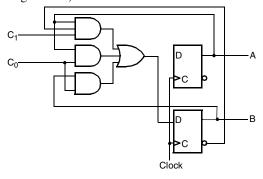
7-18.



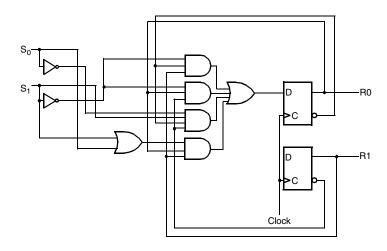
7-19.*



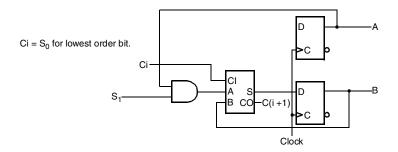
7-20. (Errata: Change "register A" to "register B")



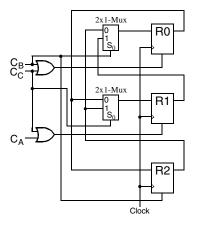
7-21.



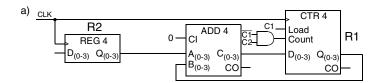
7-22.

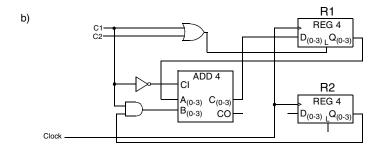


7-23.



7-24.*



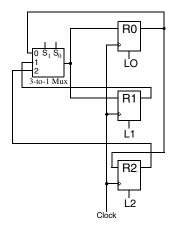


7-25.

The register transfer logic is as follows:

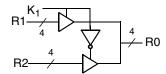
Operation		ect	Load		
	S 1	S 0	L0	L1	L2
CA: R1 <- R0	0	0	0	1	0
CB: R0 <- R1, R2 <- R0	0	1	1	0	1
CC: R1 <- R2, R0 <- R2	1	0	1	1	0



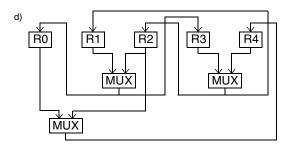


7-26.

Replace multiplexer with:

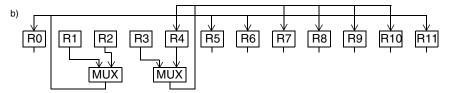


7-27.*

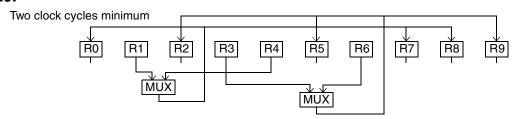


7-28.

a) Using two clock cycles, the minimum # of buses is 2 .



7-29.



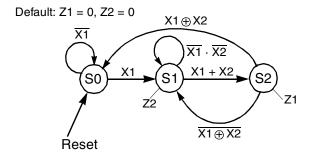
7-30.*

0101, 1010, 0101, 1010, 1101, 0110, 0011, 0001, 1000

7-31.*

Shifts:	0	1	2	3	4
A	0111	0011	0001	1000	1100
В	0101	0010	0001	0000	0000
C	0	1	1	1	0

7-32.*



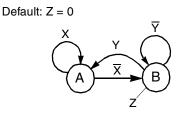
7-33.*

7-34.

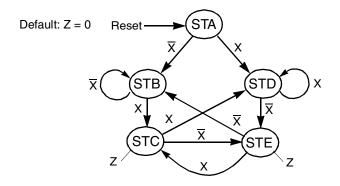
State	Input	Next State	Output
STA	$\overline{\mathbf{W}}$	STA	*
STA	W	STB	*
STB	$\overline{X} Y$	STA	*
STB	X	STC	*
STB	$\overline{X}\overline{Y}$	STC	Z
STC		STA	Z

*Default: Z = 0

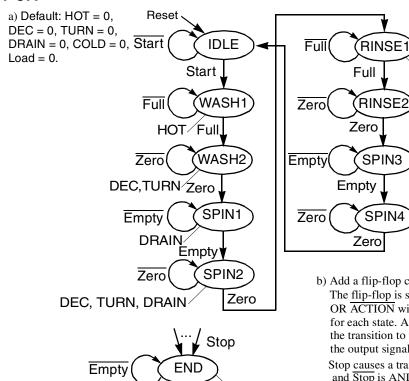
7-35.



7-36.*



7-37.+



Empty

To IDLE

b) Add a flip-flop called ACTION controlled by Pause and Start. The flip-flop is set by START and reset by Pause.

OR ACTION with each input condition on the "loop" for each state. AND ACTION with1) each input condition on t the transition to the following state from each state and 2) all the output signals.

DEC, TURN, DRAIN

DEC, TURN,

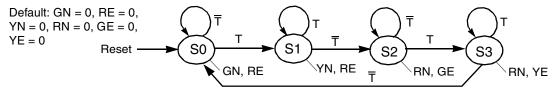
DRAIN

COLD

Stop <u>causes</u> a transition from each state to a new state END and $\overline{\text{Stop}}$ is ANDed with all of the input conditions on each of the states. The partial diagram for state END appears at the left.

DRAIN

7-38.



7-39.*

Present state		Input	Input Next state		ate	Output		
	A	В	C		A	В	С	
CTDA	1	0	0	$\overline{\mathbf{W}}$	1	0	0	
STA	1	0	0	W	0	1	0	
	0	1	0	$\overline{X}Y$	1	0	0	
STB	0	1	0	X	0	0	1	
	0	1	0	$\overline{X}\overline{Y}$	0	0	1	Z
STC	0	0	1		1	0	0	Z

$$D_{A} = A\overline{W} + B\overline{X}Y + C$$

$$D_{B} = AW$$

$$D_{C} = B(X + \overline{Y})$$

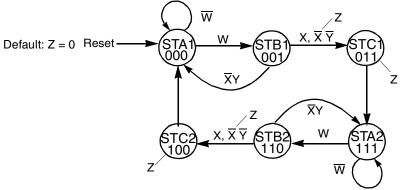
$$Z = B\overline{X}\overline{Y} + C$$

The implementation consists of the logic represented by the above equations and three D flip-flops with Reset connected to S on the first flip-flop and to R on the other two flip-flops.

7-40.

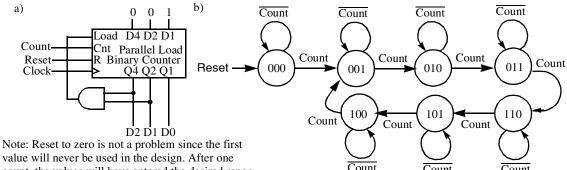
This state diagram has a closed loop of three transitions (STA to STB to STC to STA). In a Gray code, only one bit may change in going from one state to another. Any state machine diagram with a loop of an odd number of transitions is impossible to encode with a Gray code. For example, to go from STA to STB suppose bit B1 of the code changes. Then to go from STB to STC, some other bit, say B2 must change. Since two bits have changed, It is impossible to return to state STA. Thus, the answer to this problem is that this state diagram cannot be implemented with a Gray code.

But suppose that we use two equivaent states to represent each of the original states, STA1, STB1, STC1, STA2, STB2, and STC2. Is it possible to implement the new diagram generated such the it has exactly the same properties as the old diagram. Suppose that the codes are 000, 001, 011, 111, 110, 100, respectively, for the six states. The coded diagram is:



The behavior of this diagram is the same as that of the original and it has been successfully Gray coded by assigning two codes to each state. The implementation is a straightforward design problem with two unused states.

7-41.



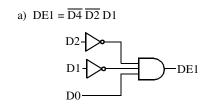
value will never be used in the design. After one count, the values will have entered the desired range

of 1 through 6.

Applying K-maps to the table entries:

State	Cnt = 0	Cnt = 1	$D_{Q4} = Q4 \overline{C} + Q4 \overline{Q2} + Q1 Q2 C$
000	000	001	$D_{Q2} = Q2 \overline{C} + Q1 Q2 C + \overline{Q4} Q2 \overline{Q1}$
001	001	010	$D_{O1} = Q1 \overline{C} + \overline{Q1} C$
010	010	011	
011	011	100	Cost comparison:
100	100	101	a) $3(14 + 2 + 8 + 6) + 1 + 2 = 93$
101	101	110	b) $3(14) + 4 + 6 + 11 + 10 = 73$
110	110	111	The gate input cost of b) is 78.5 % that of a).
111	ddd	ddd	

7-42.





greater than or equal to 100 in terms of powers of 2. The smallest value is $2^6 + 2^5 + 2^2$ and the largest value is $2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1$. This range of D can be described by saying that 2^6 and 2^5 must be present and any of 2^2 , 2^3 , or 2^4 must be present in D. The resulting equation is D6 D5 (D4 + D3 + D2). An alternative way of finding this is to contract the carry circuit for D + 2's comp of 1100100.

7-43.

Binary BCD (C ₀ = 0) BCD (C ₀ = 1) D3 = B3 $\overline{B2} \overline{B1}$ D2 = $\overline{B3} B2 + B2 B1$ D1 = $\overline{B3} B1 + B3 B2 \overline{B1}$	
$B_3B_2B_1B_0$ $C_4D_3D_2D_1D_0$ $C_4D_3D_2D_1D_0$ $D_0 = B_0$	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	+ B3 B2 B1 B0 - B3 B1 + B3 B0) 1 + B3 B2 B1 B0) 31 + B3 B2 B0 + B2 B1 B0 + B3 B2 B1 B0) B1 B0 + B3 B1 B0 + B3 B1 B0 + B3 B2 B1 B0 1 B0 B3 B0) + C0 B3 B2 B1 B0

7-44.

a) Transition constraint checking for Figure 7-28.

	Constraint 1:		Constraint 2:	
INIT: No possible conflicts since a single transition.			Condition implicitly = 1	OK
	BEGIN: $\overline{ROLL} \cdot ROLL = 0$	OK	BEGIN: $\overline{ROLL} + ROLL = 1$	OK
	ROL: ROLL $\cdot \overline{\text{ROLL}} = 0$	OK	ROL: ROLL + $\overline{\text{ROLL}}$ = 1	OK
	ONE: DIE1 · $\overline{DIE1} = 0$	OK	$DIE1 + \overline{DIE1} = 1$	OK
	ROH: ROLL \cdot ROLL \cdot HOLD = 0	OK	$ROH: ROLL + \overline{ROLL} \cdot HOLD + \overline{ROLL} \cdot \overline{HOLD} = 1$	OK
	$ROLL \cdot \overline{ROLL} \cdot \overline{HOLD} = 0$	OK		
	$\overline{\text{ROLL}} \cdot \text{HOLD} \cdot \overline{\text{ROLL}} \cdot \overline{\text{HOLD}} = 0$	OK		
	TEST: $WN \cdot \overline{WN} = 0$	OK	TEST: $WN + \overline{WN} = 1$	OK
	WIN: $NEW_GAME \cdot \overline{NEW_GAME} = 0$	OK	WIN: NEW_GAME + $\overline{NEW_GAME}$ = 1	OK

b) Implementation of state machine diagram Figure 7-28 using 1-hot code.

The order from LSB to MSB for the state variables is the same as the order of the states in the diagram from top to bottom. The state variables have the same respective names as the states, e.g., INIT, BEGIN, \dots

The flip-flop input equations:

```
D_{INIT} = INIT(t + 1) = WIN \cdot NEW\_GAME
```

 $D_{BFGIN} = BEGIN(t+1) = INIT + ONE \cdot DIE1 + TEST \cdot \overline{WN} + BEGIN \cdot \overline{ROLL}$

 $D_{ROL} = BEGIN \cdot ROLL + ROH \cdot ROLL + ROL \cdot ROLL$

 $D_{ONE} = ROL \cdot \overline{ROLL}$

 $D_{ROH} = ONE \cdot \overline{DIE1} + ROH \cdot \overline{ROLL} \cdot \overline{HOLD}$

 $D_{TEST} = ROH \cdot \overline{ROLL} \cdot HOLD$

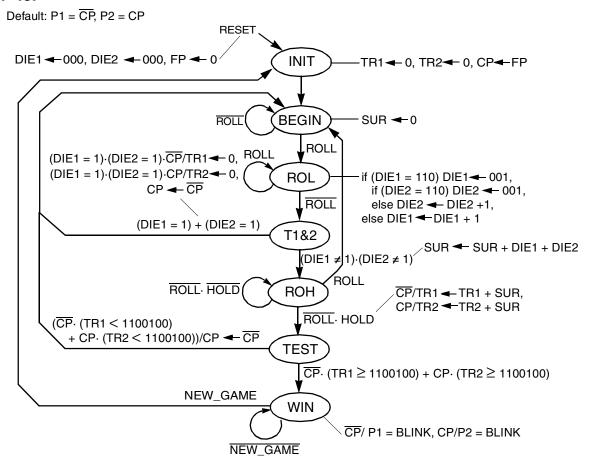
 $D_{WIN} = TEST \cdot WN + WIN \cdot \overline{NEW_GAME}$

The output equations:

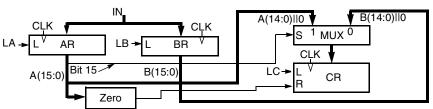
 $RST1 = INIT, RST2 = INIT, CPFI = INIT, LDCP = INIT + TEST \cdot \overline{WN} + ONE \cdot DIE1, RSSU = BEGIN, ENDI = ROL, LDSU = ONE, LDT1 = ROH \cdot \overline{CP} \cdot \overline{ROLL} \cdot HOLD, LDT2 = ROH \cdot CP \cdot \overline{ROLL} \cdot HOLD, BP1 = WIN \cdot \overline{CP}, BP2 = WIN \cdot CP$

The circuit consists of gates implementing the above equations with logic shared where possible, and seven D flip-flops. The flip-flop for INIT has Reset attached to S and the remaining flip-flops have Reset attached to R.

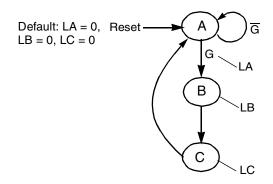
7-45.+

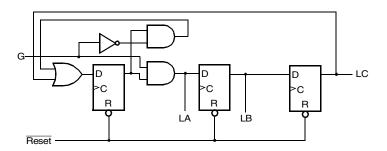


7-46.*



R is a synchronous reset that overides any simultaneous synchronous transfer.

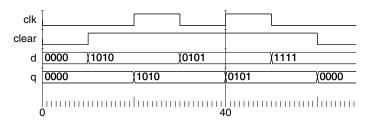




7-47.

```
C[0] = EN,
C[1] = C[0] & Q[0],
C[2] = C[1] & Q[1],
C[3] = C[2] & Q[2],
// 4-bit Binary Counter
// Positive Edge-Triggered D Flip-Flop with Reset:
module dff_v(CLK, RESET, D, Q); input CLK, RESET, D;
                                                                                                   CO = C[3] & Q[3];
  output Q;
                                                                                             \begin{array}{l} assign \\ D\_in[0] = C[0] \land Q[0], \\ D\_in[1] = C[1] \land Q[1], \\ D\_in[2] = C[2] \land Q[2], \\ D\_in[3] = C[3] \land Q[3]; \end{array}
  reg state;
  assign Q = state;
always @(posedge CLK or posedge RESET)
begin
if (RESET)
                                                                                             dff_v
  state <= 0;
else
                                                                                                   y
g1(Clock, Reset, D_in[0], Q[0]),
g2(Clock, Reset, D_in[1], Q[1]),
g3(Clock, Reset, D_in[2], Q[2]),
g4(Clock, Reset, D_in[3], Q[3]);
state <= D;
endmodule
                                                                                             endmodule
module\ Counter\_4bit\ (Clock,\ Reset,\ EN,\ Q,\ CO)\ ;
input Clock, Reset, EN;
output [3:0] Q;
output CO;
wire[3:0] Q;
wire [3:0] C, D_in;
// (continued in next column)
 Clock
 Reset
     ΕN
      Q
                                                                                                 1000 1001 1010 1011 1100 1101 1110 1111
          0000 (0001 (0010 (0011 (0100 (0101 (0110 (0111
    СО
           100 200 300
```

7-48.*

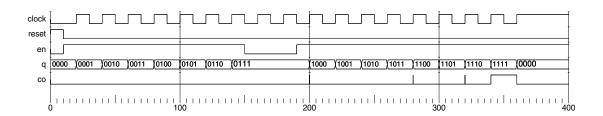


7-49.

```
library IEEE;
                                                                    architecture reg_4_bit_load_arch of reg_4_bit is
use IEEE.std_logic_1164.all;
                                                                    begin
                                                                    process (CLK)
entity reg_4_bit is
  port (
                                                                    begin
     LOAD, CLK: in STD_LOGIC;
D: in STD_LOGIC_VECTOR (3 downto 0);
Q: out STD_LOGIC_VECTOR (3 downto 0)
                                                                        if (CLK'event and CLK='1') then --CLK rising edge
if LOAD = '1' then
                                                                                 Q \leq D;
  );
                                                                             end if;
end reg_4_bit;
                                                                        end if;
                                                                    end process;
-- (continued in next column)
                                                                    endreg_4_bit_load_arch;
                   clk
                 load
                        0000
                                   (1010
                                                          (0101
                                                                                 (1111
                     d
                        UUUU
                                              (1010
                                                                                            (1111
                     q
```

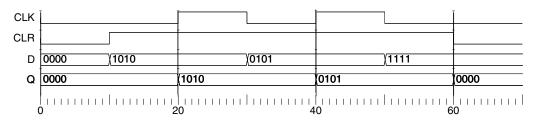
7-50.

```
library ieee;
use ieee.std_logic_1164.all;
                                                                                              architecture counter_4_bit_arch of counter_4_bit is
                                                                                              component dff
entity dff is port(CLK, RESET, D: in std_logic;
                                                                                                 port(CLK, RESET, D: in std_logic;
                                                                                                    Q: out std_logic
      Q : out std_logic);
end dff;
                                                                                              end component :
                                                                                              signal D_in, C, Q_out: std_logic_vector(3 downto 0);
architecture pet_pr of dff is
-- Implements positive edge-triggered bit state storage -- with asynchronous reset.
                                                                                             begin
                                                                                                   T(0) <= EN;
C(1) <= C(0) and Q_out(0);
C(2) <= C(1) and Q_out(1);
C(3) <= C(2) and Q_out(2);
  signal state: std_logic;
begin
Q <= state;
  process (CLK, RESET)
                                                                                                    CO \leftarrow C(3) and Q_{out}(3);
  begin
if (RESET = '1') then
                                                                                                    D_{in}(0) \le C(0) \text{ xor } Q_{out}(0);
     state <= '0';
                                                                                                    D_in(1) <= C(1) xor Q_out(1);
D_in(2) <= C(2) xor Q_out(2);
D_in(3) <= C(3) xor Q_out(3);
     if (CLK'event and ClK = '1') then
        state <= D;
      end if;
                                                                                                   port map (Clock, Reset, D_in(0), Q_out(0)); bit1: dff
   end if;
  end process;
                                                                                                   port map (Clock, Reset, D_in(1), Q_out(1)); bit2: dff
end;
                                                                                                    port map (Clock, Reset, D_in(2), Q_out(2)); bit3: dff
library IEEE;
use IEEE.std_logic_1164.all;
entity counter_4_bit is
                                                                                                          port map (Clock, Reset, D_in(3), Q_out(3));
  port (
Clock, Reset, EN: in STD_LOGIC;
Q: out STD_LOGIC_VECTOR (3 downto 0);
CO: out STD_LOGIC
                                                                                                    Q \leq Q_{out};
                                                                                             end counter_4_bit_arch;
end counter_4_bit;
```



7-51.*

```
\begin{array}{ll} module\ register\_4\_bit\ (D,\ CLK,\ CLR,\ Q)\ ; \\ input\ [3:0]\ D\ ; \\ input\ CLK,\ CLR\ ; \\ output\ [3:0]\ Q\ ; \\ reg\ [3:0]\ Q\ ; \\ always\ @(posedge\ CLK\ or\ negedge\ CLR) \\ begin \\ if\ (-CLR) \\ Q\ =\ 4"b0000; \\ else \\ Q\ =\ D; \\ end \\ endmodule \\ \end{array}
```



7-52.

```
module register_4_bit_load (D, CLK, LOAD, Q);
input [3:0] D;
input CLK, LOAD;
output [3:0] Q;
reg [3:0] Q;
always @(posedge CLK)
begin
  if (LOAD)
     Q = D;
end
endmodule
   CLK
  LOAD
     D
       0000
                (1010
                                   (0101
                                                      Q
                          1010
                                                                1111
                        20
```

7-53.*

```
library IEEE;
                                                                               if W = '1' then
use IEEE.std_logic_1164.all;
entity prob_7_53 is
port (clk, RESET, W, X, Y: in STD_LOGIC;
                                                                                   next_state <= STB;</pre>
                                                                               else
                                                                                   next_state <= STA;</pre>
        Z : out STD_LOGIC);
                                                                               end if;
end prob_7_53;
                                                                           when STB =>
                                                                               if X = '0' and Y = '1' then
architecture process_3 of prob_7_53 is
                                                                                   next_state <= STA;</pre>
type state_type is (STA, STB, STC);
signal state, next_state: state_type;
                                                                                   next_state <= STC;</pre>
begin
                                                                               end if;
                                                                           when STC =>
-- Process 1 - state register
                                                                                   next_state <= STA;</pre>
state_register: process (clk, RESET)
                                                                      end case;
                                                                  end process;
    if (RESET = '1') then
         state <= STA;
                                                                  -- Process 3 - output function output_func: process (X, Y, state)
    else if (CLK'event and CLK='1') then
         state <= next_state;
                                                                  begin
         end if;
                                                                      case state is
    end if:
                                                                        when STA =>
end process;
                                                                           Z \le '0':
                                                                           when STB =>
-- Process 2 - next state function
                                                                             if X = '0' and Y = '0' then
next_state_func: process (W, X, Y, state)
                                                                                 Z \le '1';
begin
                                                                               else
    case state is
                                                                                 Z \le '0';
        when STA =>
                                                                               end if;
-- Continued in next column
                                                                           when STC =>
                                                                               Z \le '1';
                                                                      end case;
                                                                   end process;
                                                                  end process_3;
```

7-54.*

```
// State Diagram in Figure 5-40 using Verilog module prob_7_54 (clk, RESET, W, X, Y, Z); input clk, RESET, W, X, Y;
                                                                                   next_state <= STA;
STB: if (X == 0 & Y == 1)
                                                                                           next_state <= STA;</pre>
output Z;
                                                                                       else
                                                                                            next_state <= STC;</pre>
reg[1:0] state, next_state;
                                                                                   STC:
parameter STA = 2'b00, STB = 2'b01, STC = 2'b10;
                                                                                           next_state <= STA;</pre>
reg Z;
                                                                                   endcase
                                                                              end
// State Register always@(posedge clk or posedge RESET)
                                                                              // Output Function
begin
                                                                              always@(X or Y or state)
if (RESET == 1)
                                                                              begin Z \le 0;
     state <= STÁ;
                                                                                  case (state)
STB: if (X == 0 \& Y == 0)
else
     state <= next_state;
end
                                                                                           Z \le 1;
                                                                                        else
// Next StateFunction
                                                                                            Z \le 0;
always@(W or X or Y or state)
                                                                                   STC:
                                                                                           Z \leq 1;
     case (state)
                                                                                   endcase
     STA: if (W == 1)
                                                                              end
             next_state <= STB;</pre>
                                                                              end module \\
         else
// (continued in the next column)
```