CENG 216 – Numerical Computation
Midterm Exam

2019–20 Spring Semester
May 12, 2020

- This exam contains 4 questions on 2 pages.
- The exam duration is **60 minutes**.
- Use your **own handriting**, no typesetting is allowed.
- Make sure to **write your name/student ID** on each page you submit.
- Make sure to **sign** each page.
- Good Luck!

Question	Q1	Q2	Q3	Q4	**Total**
Points	25	25	25	25	**100**
Grade					

Q1 *(25 points)* Floating-Point Representation

IEEE double-precision floating numbers are 64 bits each, 1 bit for the sign bit, 11 bits for the exponent, and 52 bits for the mantissa.

Take the last four digits of your Student Id, interpret it as a number X. For example, if your student ID is 123456789, then $X = 6789$. Set $Y = \frac{X}{100} + 0.003$, Y would be 67.893 in our example.

i. Write down the value of Y that you have obtained in decimal.

ii. Write down Y as a binary number.

iii. Write down the 52 bits of the significand/mantissa part of $Y' = float(Y)$ in binary, the possibly rounded value stored in your computer as an IEEE double precision number using the round to nearest even rule.

Q2 *(25 points)* Fixed-Point Iterations

We want to find the roots of the parabola $P(x) = (x - r_1)(x - r_2)$ using fixed point iterations. We will do so by converting $P(x) = 0$ into $f(x) = x$.

i. Write down the equation for $f(x)$.

ii. Write down the formulas for the first derivative of $f(x)$ evaluated at the roots r_1 and r_2.

iii. Write down the condition on the roots so that the fixed point iterations will converge towards r_1 but it will diverge from r_2.

Q3 *(25 points)* Linear Systems of Equations

Assume that your Student ID has the digits "$ABCDEFGHI$" (If your ID is 123456789, then $A = 1, B = 2, E = 5$, and $I = 9$). You are given the following linear system of equations

$$\begin{bmatrix} A & D & G \\ B & (E + 0.5) & H \\ C & F & I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \\ 18 \end{bmatrix}$$

Note the $(0.5 + E)$ **in the middle.**

i. Write down the linear system that you obtain as explained above.

ii. Find the solution to this system using LU decomposition, write down the two pivots p_0 and p_1 and the solution vector $\mathbf{x}$ as your final answer. In every step, you can write down the four digits after the decimal dot.

iii. For the approximate solution $\mathbf{x}_a = [1.0, 1.0, 1.0]^\top$, calculate and write down the relative forward, backward errors, and the error magnification factor.

Q4 *(25 points)* Interpolation

Assume that your Student ID has the digits "$ABCDEFGHI$" (If your ID is 123456789, then $A = 1, B = 2, E = 5$, and $I = 9$).

i. Write down the coordinates of the points $P_0 = (A, B)$, $P_1 = (H, I)$, and $P_2 = (C, D)$.

ii. Write down **the coefficients** a_0, a_1, and a_2 of the second degree polynomial interpolating the points P_0, P_1, and P_2 using a method of your choice.

iii. If we set the end points as P_0 and P_1, and set both control points as the mid-point in the line segment $[P_0, P_1]$, what will be the shape of the resulting Bézier Curve? Why?