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Guest Editorial

The Problem with Algebraic Models of Marriage and Kinship Structure

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Introduction

Algebraic models of marriage and kinship systems have been developed for nearly 50 years, but they continue to be a matter of controversy. Most anthropologists have found algebraic models abstract and unnecessary, while mathematical scientists have tended to consider the value of these models self-evident. I argue here that the anthropologists have been largely correct, and that the crux of the matter comes down to the question: What constitutes a mathematical model?

Abstract algebraic modeling of marriage and kinship systems began in 1949 as an addendum to Claude Levi-Strauss's seminal *The Elementary Structures of Kinship* [1969]. The addendum, by the great algebraist André Weil, was "at Levi-Strauss's request" [1969, 221]. Variations of the same algebraic model have appeared subsequently in articles and books into the 1990s (for example, Ascher [1991]). Perhaps the most influential use of the model was in the finite mathematics textbook by Kemeny et al. [1966], and its most ambitious application was in the text by Harrison C. White [1963]. I even wrote a paper myself on the subject [Cargal 1978].

To some extent I am embarrassed by that paper, as I now believe that my work on algebraic marriage and kinship systems, like the other works, has little value. Many anthropologists have attacked such models; Korn and Needham [1970] offer perhaps the best attack. However, though they make some stinging points, Korn and Needham are caught up too much in mathematical notation,

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as opposed to mathematical substance. The problem with the models does not lie in the mathematics itself but in what these models do *not* do.

The rest of this essay is concerned with:

- What is a (good) mathematical model?
- How good are algebraic models of marriage and kinship systems?

What Is a Mathematical Model?

Essays on the nature of a mathematical model abound in texts on operations research, simulation, probability, and applied mathematics—but are strangely absent from physics and engineering books. This is not because physicists and engineers are less philosophical than their counterparts in other mathematical sciences, but because physicists routinely use models in introductory courses; there is no need for a transition in later courses to modeling.

Mathematical models are generally defined as mathematical representations of the subject at hand. They are abstractions that represent ideal assumptions, and they are supposed to capture the salient features of the subject and to leave out the irrelevant features. If the model is a good model, predictions made from the model should be true of the subject that is modeled. *The question of whether the model is a good model is the core philosophical question.* Related questions are:

- Which features of the subject are relevant for the model?
- To what extent can predictions based on the model be used?
- How accurate are the predictions?

Models and Predictions

Prediction is core to the subject of models and core to this critique of algebraic models of marriage and kinship systems. I suggest that a model is only as good as the predictions that it accurately makes. What is the purpose of a model if not to make predictions? The usual answer is that the model can help us understand the subject. But if the model does not yield predictions, what is the value of this understanding? I will give two historical examples of models in physics and astronomy and will then re-examine the algebraic models of marriage and kinship.

Two Models by Kepler

Although mathematical models of nature seem to have been important to the Greeks, mathematical models in the modern sense took off around 1600 with Galileo, Kepler, and others. Kepler constructed two models of interest to this paper.

- **Planetary motion:** If we view Kepler's laws of planetary motion as a mathematical model, we can see that it is very much a predictive model. The model not only constructs the path of the planets but also determines the speed of their revolution, and use of the model enabled more accurate forecasting of planetary positions. It may be relevant that this model was based on painstaking analysis of data that were themselves of unprecedented accuracy.

- **Planets as platonic solids:** In his *Harmonice Mundi*, Kepler devised a less well-known model of the planetary system (see Kappraff [1991, 265]). He showed that the orbits of the six known planets could be inscribed about the five platonic solids. The five solids are nested inside one another, and the six planets nest within the solids. Kepler published this model and was proud of it for his entire life. The model was predictive in only one sense: It implies that there are no new planets to be discovered. For its time, it is not a bad model; it is less mystical than prior Greek theories of the universe. For our time, the model is primitive. We do not reject the model because its one prediction failed; the prediction could well have turned out to be true. We reject the model because by our standards it is contrived, and because it is not predictive enough.

I contend that the algebraic models of marriage and kinship systems are more in the style of Kepler's platonic solids model than in the style of his laws of planetary motion.

The Quantum Physics Model

Quantum physics is difficult to understand and completely unintuitive, but it is has been perhaps the most successful model of modern physics. The quantum theory of physics has been totally dominant in its domain (atomic mechanics) because of two features:

- It is mathematically consistent.

- It has been the source of thousands of predictions, which in all testable cases have been correct. The most famous such prediction might be Bell's theorem (for an elementary account, see Peat [1990]).

Hence, a mathematical model that is best known for its statement of what cannot be predicted (i.e., the Heisenberg uncertainty principle) has been as successful as any model in science history, because it is the exemplary predictive model.

Following the example of quantum physics, we should evaluate algebraic models of marriage and kinship systems according to the criterion of how well they predict.

Algebraic Models of Marriage and Kinship

The Core of the Algebraic Models

Algebraic models of marriage and kinship are based on clans and their relationships. In some cases—specifically, the Kariera, the Aranda, perhaps the Tarau, and perhaps the Murngin—the structure of the kinship system is an algebraic group. This single observation, which is apparently due to André Weil, is the heart of 40-plus years of writing on mathematical aspects of marriage and kinship systems.

However, it is not in any way a remarkable observation. Clan systems are either hierarchical or they are not. If, as in the above cases, the clan relationships are not hierarchical, they are likely to be symmetric. Group theory could be described as the mathematics of symmetry (see Armstrong [1988] and Stewart [1992, Chapter 9: The Duellist and the Monster, 115–129]). It would be a remarkable discovery indeed if there were symmetric clan structures that could not be described in the language of mathematical groups!

If we use groups to describe symmetrical clan structures, there should be a payoff or return to anthropologists: There needs to be some reason to bring group theory into anthropology. This may seem obvious, but it is not. Anyone who has worked in certain areas of industry has seen mathematical models (and simulation models in particular) that seem to have no purpose. On the subject of simulation modeling, E.C. Russell says: “The goal of a simulation project should never be ‘To model the . . . .’ Modeling itself is not a goal; it is a means of achieving a goal” [1983, 16]. That models without a purpose exist is a consequence, I believe, of the divergence of mathematics and physics that has been going on for 200 years but which has greatly accelerated in the last 30 years.

The purpose of a mathematical model is to predict. What do algebraic models of marriage and kinship systems predict? The answer is, *nothing*.

Let us ask a simpler question: What information do algebraic models of marriage and kinship systems provide? In his genesis of algebraic models, André Weil [1969, 221] says “I propose to show how a certain type of marriage and kinship laws can be interpreted algebraically, and how algebra and the study of groups . . . can facilitate its study and classification.” Given the forty-plus years since Weil’s essay, it is reasonable to ask whether this has been accomplished: Has anyone aided the study and classification of marriage and kinship systems through the use of group theory (or any use of abstract algebra whatsoever)?

Another approach is this: How are the traditional anthropological methods for classifying marriage and kinship structures deficient? How does group theory make up for such purported deficiency?

Prediction and Falsifiability

A key element in the twentieth-century view of scientific theories is falsifiability, a key doctrine in the work of the eminent philosopher of science Karl R. Popper (see, for example, Popper [1959]). A scientific theory must be falsifiable, that is, there must potentially be observations or experimental results that would force abandonment of the theory.

Falsifiability seems nearly equivalent to predictability. A scientific theory must make predictions so that the theory itself can be tested. This seems like a reasonable criterion for scientific theories and for models, but it certainly is not a criterion met by algebraic models of marriage and kinship systems. What predictions do the algebraic models make?

If algebraic models of marriage and kinship systems are not predictive models, perhaps they are so-called explanatory models. This begs several questions:

- Can a model that makes no predictions be explanatory?
- What exactly does an explanatory model explain?
- How do we judge the merits of an explanatory model?

Explanatory models don't predict but perhaps they give us insights. Let us take an example from my own paper [Cargal 1978]. The Kariera have four clans, and their clan system can be represented by the graph in **Figure 1**.

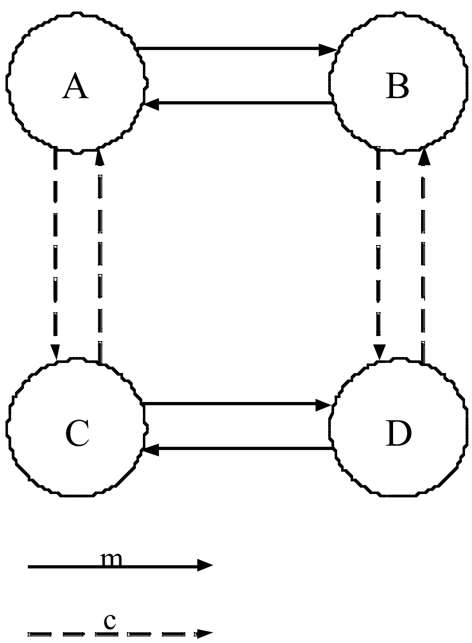

Figure 1. Diagram of the Kariera kinship system. Solid arcs represent a clan of marriage and kinship and the dotted lines represent the clan of a child.

This representation, the traditional algebraic view of the Kariera, is a valid way to introduce algebra, since the graph depicted is also the Cayley graph of the Klein four-group $Z_2 \times Z_2$. In my paper, I suggest looking at the subclans of each sex: "[T]here is sex differentiation among the clans referred to. Natives think in terms of men of Clan A or women of Clan A ..." [1978, 161]. I give a graph of the eight subclans and then relabel that graph to realize a Cayley graph of a group. **Figure 2** shows the group of subclans of the Kariera.

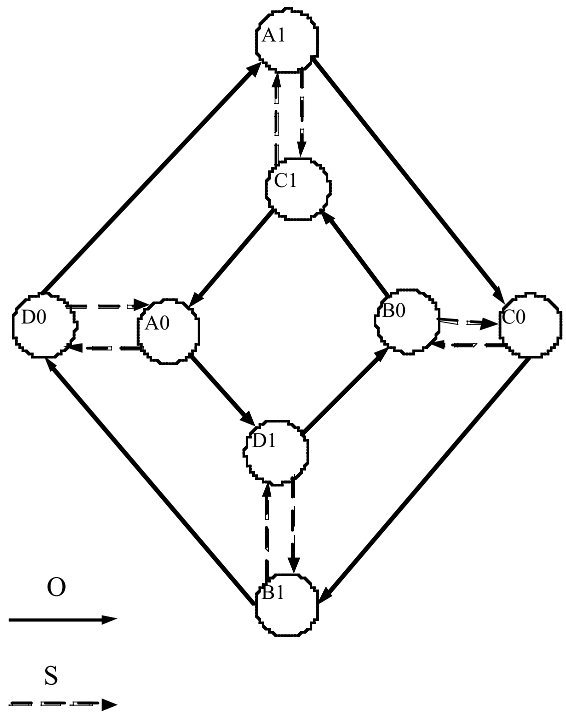

Figure 2. A1 denotes men of Clan A and C0 denotes women of Clan C. The relations are S and O. An S means subclan of children of same sex and an O means subclan of children of opposite sex.

The group of **Figure 2** happens not to be homomorphic to the group of **Figure 1**. This is a formal mathematical statement to the effect that the two groups are fundamentally different. The one group, of order eight, is not merely a refinement of the other group, of order four. To my knowledge, the group of order eight does not appear anywhere else in the literature; but it is as valid as the standard group (to whatever extent that group can be said to be valid).

We have two algebraic representations of marriage and kinship relations in the Kariera people, that is, two fundamentally distinct models. If these models have anthropological content, they should give conflicting information, or at least the second model should provide new information. My paper gives a great deal of analysis of this and other groups. It is rather enthusiastic analysis and to some extent I find myself a little impressed as I look at it now. But with the distance of more than 20 years since doing the research, I can ask, as anthropologists asked: What significance does this have to anthropology?

Cultures of Mathematics

Different branches of the mathematical sciences have different understandings of what a model is. Mathematical logic, for example, is concerned a great deal with models that are quite different from the models that interest physicists (see Bell and Slomson [1971]). Models in statistics are different from either of those (see Hinkelmann and Kempthorne [1994]). In much of mathematical culture, a model is a mathematical system that is formally specified and internally consistent; there is absolutely no requirement that it be useful. Operations researchers and statisticians are interested in models that they can use to solve problems, but the importance of making predictions is not emphasized (it certainly wasn't in my own training in those areas).

To the few anthropologists who can understand the mathematics, the (positive) worth of algebraic models of marriage and kinship is self-evident. The remaining anthropologists are skeptical and tend to ask, "What good is this model? What will it do for me?" Mathematicians tend to respond condescendingly, knowing that the anthropologists are mystified by the mathematics, "It will help you understand the structure of the marriage and kinship system." Unfortunately, I do not remember any anthropologist being prescient enough to ask, "What will this model predict?" In the sciences, one manifestation of different subcultures is the understanding of what a model is. One consequence of this is that in the case of what constitutes a mathematical model, there may be an unlikely alliance between anthropologists and physicists against mathematicians.

Mathematics Is Not a Science

Though it is said to be the language of science, mathematics is generally considered outside of science; this is because scientific facts must be verified empirically but mathematical truths must only be true to their axioms.

Is Cultural Anthropology a Science?

The models referred to in this paper concern cultural anthropology, a field that may not be hospitable to a traditional concept of science (see Fox [1992]). So anthropologists may not *care* for or about mathematical models of marriage and kinship. Nonetheless, mathematicians who apply their models outside of mathematics should use the paradigm of the scientific method and of predictions, testing, and falsifiability.

Lastly, when mathematicians apply mathematics to previously non-mathematical areas, people who fail to understand the mathematics may nonetheless be quite right in their criticism. It is important that the needs of the applied area be recognized: that in applying mathematics, mathematicians should look outside their own needs and their own culture.

Addendum

What happened in the last 20 years that I should now have an attitude far more sympathetic to anthropologists? Why did I come to reject an application of mathematics that I had published?

Until recent times almost all mathematics majors studied physics. They are still encouraged to do so, since physics provides much motivation for calculus, not to mention most examples in differential equations and vector calculus. However, it is becoming common for mathematics majors to "get around" the physics requirement (as I did), and today a smaller proportion of mathematicians have any training in physics than in the past.

When I did my work in anthropology, my background was a master's degree in mathematics with emphasis on logic and abstract algebra. When I published the paper, I was working in aerospace and had made it my business to be comfortable in statistics and computer science. Subsequently I did a Ph.D. in operations research, a mathematical discipline with strong ties to statistics and computer science—but not to physics.

In 1988, when I was again working in military aerospace as an applied mathematician, a problem came up in classical Newtonian physics. I needed merely to use computer models already constructed for the problems that I faced. However, I felt that I should understand the underlying physics, so I began the study of physics—not realizing that it is addictive! *It was exposure to physics that changed my appreciation of the meaning of mathematical models.*

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Acknowledgments

An earlier version of this paper was presented at a session devoted to Humanistic Mathematics at the Joint Mathematics Meetings, January 1996, in Orlando, Florida.

About the Author

James M. Cargal has a B.S. in mathematics from San Diego State University (1973) and an M.S. in mathematics from Purdue (1975); his Ph.D., in operations research, is from the Industrial Engineering Dept. of Texas A & M. He is currently Professor of Mathematics and Chair of Mathematical Sciences at Troy State University Montgomery, where he has been since 1990. He has worked off and on in the military aerospace industry and still consults in that area on statistical questions. His original work on algebraic systems of marriage was done in the summer of 1976 (at a long defunct pizza joint in Florida), and about 10% of it was published in his 1978 paper (the other 90% was destroyed in the 1980s). He has been Reviews Editor of this *Journal* since 1986.

Editor's Note

A future issue of the *Journal* will contain an article responding to this editorial.

Modeling Forum

Results of the 2001 Interdisciplinary Contest in Modeling

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Introduction

A total of 83 teams of undergraduates, from 58 institutions in 5 countries, spent the second weekend in February working on an applied mathematics problem in the 3rd Interdisciplinary Contest in Modeling (ICM).

The 2001 ICM began at 12:01 A.M. on Friday, Feb. 9 and officially ended at 11:59 P.M. on Monday, Feb. 12. During that time, teams of up to three undergraduates were to research and submit an optimal solution for an open-ended modeling problem. The 2001 ICM marked the inaugural year for the new online administration contest, and it was a great success. Students were able to register, obtain contest materials, download the problems at the appropriate time, and enter data through COMAP'S ICM website.

After a weekend of hard work, solution papers were sent to COMAP on Monday. Three of the top papers appear in this issue of *The UMAP Journal*.

Results and winning papers from the first two contests were published in special issues of *The UMAP Journal* in 1999 and 2000.

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COMAP's Mathematical Contest in Modeling and Interdisciplinary Contest in Modeling are unique among modeling competitions in that they are the only international contests in which students work in teams to find a solution. Centering its educational philosophy on mathematical modeling, COMAP uses mathematical tools to explore real-world problems. It serves the educational community as well as the world of work by preparing students to become better informed—and prepared—citizens.

This year's problem was about limiting or preventing the spread of zebra mussels in the Great Lakes and inland waterways of the United States and Canada.

Problem: The Zebra Mussel Problem

Our Waterways—An Uncertain Future

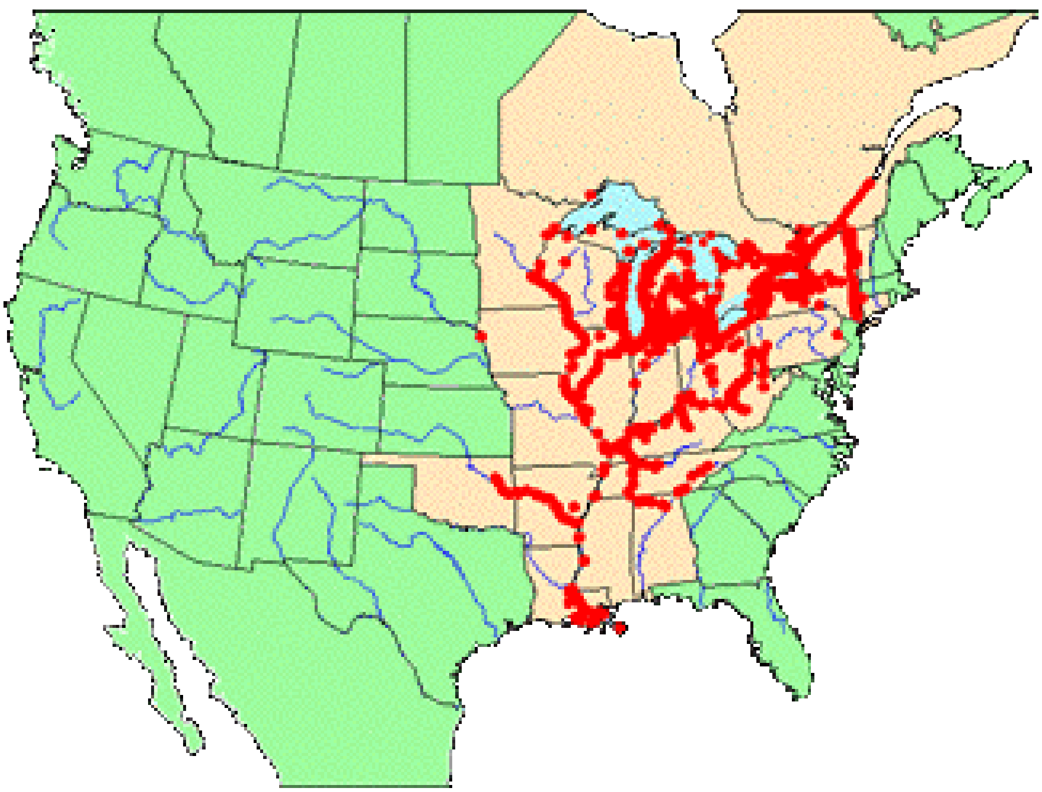

Figure 1. Map showing U.S. states and Canadian provinces with zebra mussels in inland and adjacent waters in July 2000.

Zebra mussels (*Dreissena polymorpha*) are small, fingernail-sized, freshwater mollusks unintentionally introduced to North America via ballast water from a transoceanic vessel. Since their introduction in the mid-1980s, they have spread through all of the Great Lakes and to an increasing number of inland waterways in the United States and Canada. Zebra mussels colonize on various surfaces, such as docks, boat hulls, commercial fishing nets, water intake pipes and valves, native mollusks, and other zebra mussels. Their only known predators—some diving ducks, freshwater drum, carp, and sturgeon—are not numerous enough to have a significant effect on them. Zebra mussels have significantly impacted the Great Lakes ecosystem and economy. Many communities are trying to control or eliminate these aquatic pests. (Source: Great Lakes Sea Grant Network `http://www.sgnis.org/` .)

Researchers are attempting to identify the environmental variables related to the zebra mussel infestation in North American waterways. The relevant factors that may limit or prevent the spread of the zebra mussel are uncertain. You will have access to some reference data to include listings of several chemicals and substances in the water system that may affect the spread of the zebra mussel throughout waterways. Additionally, you can assume individual zebra mussels grow at a rate of 15 mm per year with a life span between 4–6 years. The typical mussel can filter 1 liter of water each day.

Requirement A

Discuss environmental factors that could influence the spread of zebra mussels.

Requirement B

Utilizing the chemical data provided at
`http://www.comap.com/undergraduate/contests/icm/imagesdata/LakeAChem1.xls`
and the mussel population data provided at
`http://www.comap.com/undergraduate/contests/icm/imagesdata/LakeAPopulation1.xls` ,
model the population growth of zebra mussels in Lake A. Be sure to review the information below about the collection of the zebra mussel data.

Requirement C

Utilizing additional data on Lake A from another scientist provided at
`http://www.comap.com/undergraduate/contests/icm/imagesdata/LakeAChem2.xls`
and additional mussel population data provided at
`http://www.comap.com/undergraduate/contests/icm/imagesdata/LakeAPopulation2.xls` ,
corroborate the reasonableness of your model from Requirement B. As a result of this additional data, adjust your earlier model. Analyze the performance of your model. Discuss the sensitivity of your model.

Requirement D

Utilizing the chemical data from two lakes (Lake B and Lake C) in the United States provided at
`http://www.comap.com/undergraduate/contests/icm/imagesdata/LakeB.xls`
and
`http://www.comap.com/undergraduate/contests/icm/imagesdata/LakeC.xls`,
determine if these lakes are vulnerable to the spread of zebra mussels. Discuss your prediction.

Requirement E

The community in the vicinity of Lake B (in Requirement D) is considering specific policies for the de-icing of roadways near the lake during the winter season. Provide guidance to the local government officials regarding a policy on “de-icing agents.” In your guidance, include predictions on the long-term impact of de-icing on the zebra mussel population.

Requirement F

It has been recommended by a local community in the United States to introduce round goby fish. Zebra mussels are not often eaten by native fish species, so those fish represent a dead-end ecologically. However, round gobies greater than 100 mm feed almost exclusively on zebra mussels. Ironically, because of habitat destruction, the goby is endangered in its native habitat of the Black and Caspian Seas in Russia. In addition to your technical report, include a carefully crafted report (3-page maximum) written explicitly for the local community leaders that responds to their recommendation to introduce the round goby. Also, suggest ways to help reduce the growth of the mussel within and among waterways.

Collection of the Zebra Mussel Data

The developmental state of the zebra mussel is categorized by three stages: veligers (larvae), settling juveniles, and adults. Veligers (microscopic zebra mussel larvae) are free-swimming, suspended in the water for one to three weeks, after which they begin searching for a hard surface to attach to and begin their adult life. Looking for zebra mussel veligers is difficult because they are not easily visible by the naked eye. Settled juvenile zebra mussels can be felt on smooth surfaces like boats and motors. An advanced zebra mussel infestation can cover a surface, even forming thick mats sometimes reaching very high densities. The density of juveniles was determined along the lake using three 15×15 cm settling plates. The top plate remained in the water for the entire sampling season (S—seasonal) to estimate seasonal accumulation. The middle and bottom plates are collected after specific periods (A—alternating) of time denoted by "Lake Days" in the data files.

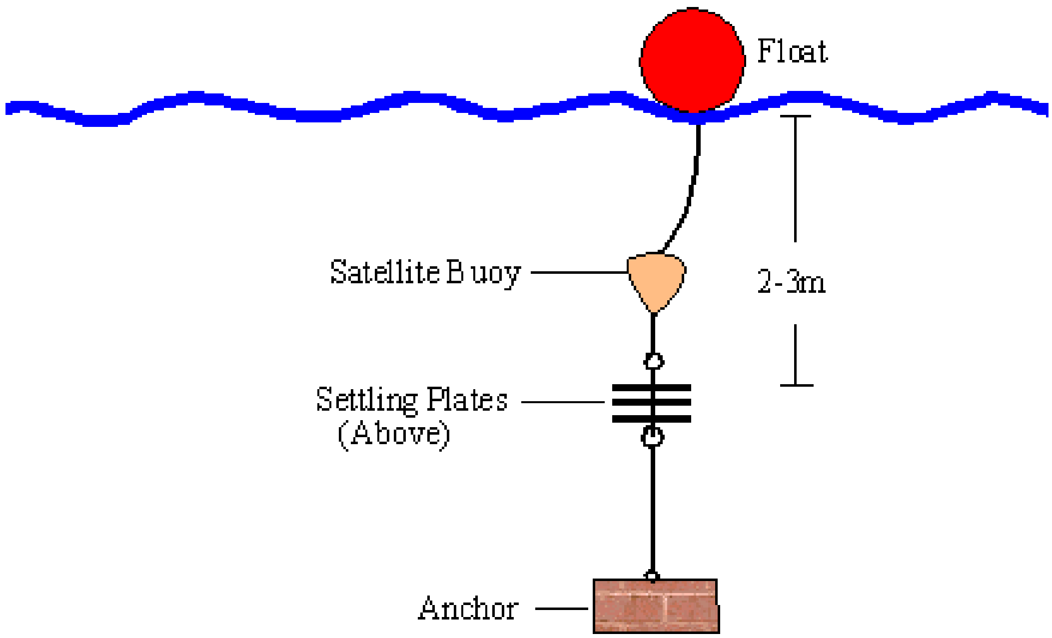

Figure 2. Diagram of collection apparatus.

The settling plates are placed under the microscope, all juveniles on the undersides of the plate are counted, and densities are reported as juveniles / m^2.

The Results

The solution papers were coded at COMAP headquarters so that names and affiliations of the authors would be unknown to the judges. Each paper was then read preliminarily by two "triage" judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary and overall organization are the basis for judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree on a score, a third judge evaluated the paper.

Final judging took place at the College of St. Rose, Albany, NY. The judges classified the papers as follows:

	Outstanding	Meritorious	Honorable Mention	Successful Participation	Total
Zebra Mussel	3	14	28	38	83

The three papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with commentaries. We list those

Figure 3. A settling plate after collection.

teams and the Meritorious teams (and advisors) below; the list of all participating schools, advisors, and results is in the **Appendix**.

Outstanding Teams

Institution and Advisor	Team Members
"A Multiple Regression Model to Predict Zebra Mussel Population"	
Harvey Mudd College	Michael Schubmehl
Claremont, CA	Marcy LaViolette
Michael E. Moody	Deborah Chun
"Identifying Potential Zebra Mussel Colonization"	
Humboldt State University	David E. Stier
Arcata, CA	Mark Alan Leisenring
Eileen M. Cashman and Jeffrey B. Haag	Matthew Glen Kennedy
"Waging War Against the Zebra Mussel"	
Lewis and Clark College	Nasreen A. Ilias
Portland, OR	Marie C. Spong
Robert W. Owens	James F. Tucker

Meritorious Teams (14 teams)

Beijing University of Posts & Telecommunications, Beijing, China
(He Zuguo and Luo Shoushan) (two teams)
Dickinson College, Carlisle, PA (Brian S. Pedersen)
East China University of Science & Technology, Shanghai, China
(Xiwen Lu and Yan Qin)
Harvey Mudd College, Claremont, CA (Michael E. Moody)
North Carolina School of Science and Mathematics, Durham, NC
(Dot Doyle and Dan Teague)
Northwestern Polytechnical University, Xian, China (Yong Xiao Hua and Yi Lu Quan)
South China University of Technology, Guangzhou, China
(He Chunxiong and Tao Zhisui)
Southeast University, Nanjing, China (Sun Zhi-zhong)
University College Dublin, Dublin, Ireland (Ted Cox)
University of Science and Technology of China, Hefei, China
(Tao Dacheng and Ma Jianxin)
Villa Julie College, Stevenson, MD (Eileen C. McGraw)
Zhejiang University, Hangzhou, China (Yang Qifang and He Yong) (two teams)

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Directors and the Head Judge.

Judging

Director
David C. “Chris” Arney, Dean of the School of Mathematics and Sciences,
The College of Saint Rose, Albany, NY

Associate Directors
Michael Kelley, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY
Gary W. Krahn, Dept. of Mathematical Sciences, U.S. Military Academy,
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Wayne Jerzak, Dept. of Mathematical Sciences, Rensselaer Polytechnic
Institute, Troy, NY
Sandra Nierzwicki-Bauer, Dept. of Biology and Darrin Fresh Water Institute,
Rensselaer Polytechnic Institute, Troy, NY

Triage Judges
Darryl Ahner, Mike Corson, Alex Heidenberg, Jerry Kobylski, Gary Krahn, Joe Myers, Mike Phillips, Kathi Snook, Gideon Weinstein, and Brian Winkel, all of the U.S. Military Academy, West Point, NY

Source of the Problem

The Zebra Mussel Problem was contributed by Sandra Nierzwicki-Bauer, Dept. of Biology and Darrin Fresh Water Institute, Rensselaer Polytechnic Institute.

Acknowledgments

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- the ICM judges and ICM Board members for their valuable and unflagging efforts;
- the staff of the Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY, for hosting the triage judging; and
- the staff of the School of Mathematics and Sciences, The College of Saint Rose, Albany, NY, for hosting the final judging.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the student papers here is the result of undergraduates working on a problem over a weekend; allowing substantial revision by the authors could give a false impression of accomplishment. So these papers are essentially au naturel. Light editing has taken place: minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. Please peruse these student efforts in that context.

To the potential ICM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Appendix: Successful Participants

KEY:
P = Successful Participation
H = Honorable Mention
M = Meritorious
O = Outstanding (published in this special issue)

INSTITUTION	CITY	ADVISOR	I
CALIFORNIA			
Harvey Mudd College	Claremont	Michael E. Moody	O,M
Humboldt State University	Arcata	Eileen M. Cashman and	
		Jeffery B. Haag	O
COLORADO			
Colorado Northwestern Comm. College	Rangely	Richard S. Knaub	P,P
University of Southern Colorado	Pueblo	Bruce N. Lundberg	P
FLORIDA			
Florida A&M University	Tallahassee	Bruno Guerrieri	P
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Monmouth College	Monmouth	Christopher Gerard Fasano	H
IOWA			
Luther College	Decorah	Reginald D. Laursen	H
Simpson College	Indianola	Steve Emerman and	
		Jeff Parmelee	P
KENTUCKY			
Asbury College	Wilmore	David L. Coulliette	H,H
MARYLAND			
Villa Julie College	Stevenson	Eileen C. McGraw	M,P
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Gordon College	Wenham	Dorothy F. Boorse and	
		Mike Veatch	H
MIT	Cambridge	Michael P. Brenner and	
		L. Mahadevan	H
MINNESOTA			
Macalester College	St. Paul	Peter W. Vaughan and	P
		A. Romero	P
MONTANA			
Carroll College	Helena	Kyl Strode	H
NEVADA			
Sierra Nevada College	Incline Village	Steven D. Ellsworth	P

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NEW YORK			
Roberts Wesleyan College	Rochester	Carlos A. Pereira	H
SUNY Geneseo	Geneseo	Christopher C. Leary and Gregg Hartvigsen	P
U.S. Military Academy	West Point	Scott Nestler and Suzanne DeLong	H
		Michael Jaye and Michael Huber	H
NORTH CAROLINA			
Elon College	Elon College	Crista L. Coles and and J. Todd Lee	H,P
N.C. School of Science & Mathematics	Durham	Dot Doyle and Dan Teague	M
OHIO			
Youngstown State University	Youngstown	Thomas Smotzer	H
		Scott Martin	H
OKLAHOMA			
University of Central Oklahoma	Edmond	Jesse W. Byrne and Charlotte K. Simmons	P
OREGON			
Clatsop Community College	Astoria	Michael C. Vorwerk	H
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Lewis and Clark College	Portland	Robert W. Owens	O
Rogue Community College	Grants Pass	John T. Salinas	P
PENNSYLVANIA			
Clarion University	Clarion	Andy M. Turner and Sharon L. Challener	H
Dickinson College	Carlisle	Brian S. Pedersen	M
SOUTH DAKOTA			
Mount Marty College	Yankton	Bonita L. Gacnik	P,P
TENNESSEE			
Austin Peay State University	Clarksville	Nell K. Rayburn and James Bateman	P
VIRGINIA			
Chesterfield County Math & Science H.S.	Midlothian	Diane C. Leighty	P

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Beijing University of Aero. & Astronautics	Beijing	Lin Guiping and Peng Linping	P
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Dalian University of Technology	Dalian	Zhao Lizhong	H
East China University of Science & Technology	Shanghai	Lu Xiwen and Qin Yan	M
		Lu Yuanhong and Su Chunjie	P
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University of Science and Technology of China	Hefei	Tao Dacheng and Ma Jianxin	M
		Sun Guangzhong and Song Zhiwei	H

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University College Dublin	Dublin	Ted Cox	M,H

Editor's Note

For team advisors from China, we have endeavored to list family name first, with the help of Susanna Chang '03.

A Multiple Regression Model to Predict Zebra Mussel Population Growth

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Summary

Zebra mussels (*Dreissena polymorpha*) are an invasive mollusk accidentally introduced to the United States by transatlantic ships during the mid-1980s. Because the mussels have few natural predators and adapt quickly to new environments, they have spread quickly from the Great Lakes into many connected waterways. Although the mussel is hardy, sometimes little or no growth is observed in lakes to which it has been introduced; extensive research indicates that the chemical concentrations in these bodies of water may be unsuitable for the mussels.

To quantify the relationship between chemical contents and mussel population growth, we first use the logistic equation,

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right),$$

to model *Dreissena* population as a function of time. After modeling growth rates under a variety of conditions, we used multiple regression to determine which chemicals affect this growth rate. An extensive literature search supported our findings that population growth is linearly dependent on two primary factors: calcium concentration and pH. After further refining our model using the second set of data from Lake A, we obtained the regression equation

$$\text{maximum growth rate} = 2338\,[\mathrm{Ca}^{2+}] + 39202\,\mathrm{pH} - 334089,$$

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where the maximum growth rate is in juveniles settling per day, and $[Ca^{2+}]$ is in mg/L. Using this model, we predict that lakes B and C cannot support *Dreissena* population. Because the levels of calcium in Lake B are close to those required to support a *Dreissena* population, however, we advise the community near Lake B to use de-icing agents that do not contain calcium.

Environmental Factors Affecting *Dreissena*

A large body of research links environmental factors such as temperature, pH, calcium ion concentration, and alkalinity to the success or failure of zebra mussel populations. The two factors repeatedly most closely associated with survival are calcium concentration and pH. In a survey of 278 lakes, for example, Ramcharan et al. [1992] found no populated lakes with pH below 7.3 or Ca content below 28.3 mg/L. Recent studies have lowered the minimum Ca concentration to 15 mg/L for adults and 12 mg/L for larvae [McMahon 1996]. The upper bound for pH is somewhere near 9.4 [McMahon 1996]. The optimum conditions for growth are a pH of 8.4 and 34 mg/L of Ca [McMahon 1996].

Other requirements for survival include alkalinity, which must be kept above 50 mg/L [Balog et al. 1995], and dissolved oxygen, which must be above 0.82 ppm (approximately 10% of saturation) [Johnson and McMahon 1996]. *Dreissena* also cannot survive in magnesium-deficient water; they require a minimum concentration of 0.03 mM for a low-density population [Dietz and Byrne 1994]. Sulfate (SO_4) is also required in small amounts for survival [Dietz and Byrne 1999].

Zebra mussels can survive in an amazingly wide range of temperatures, but Van der Velde et al. [1996] determined that exposure to 34°C is lethal within 114 minutes and that any temperature above 25°C inhibits movement and feeding. Some individuals can tolerate short-term sub-freezing air temperatures [Paukstis et al. 1996].

Although not used by the mussels themselves, phosphorus and nitrogen are essential for freshwater phytoplankton survival, and phytoplankton are the main source of food for *Dreissena*. Densities of mussel populations are negatively related to both phosphates and nitrates; but iron, chlorine, and sodium have no relationship to the existence or density of populations [Ramcharan et al. 1992]. Chlorophyll content measures the density of phytoplankton and thus decreases drastically after the establishment of a zebra mussel colony [Miller and Haynes 1997].

Surprisingly, food availability is not an important factor once a zebra mussel is established. In one study, *Dreissena* were able to survive without food for 524 days with only a 60% mortality rate [Chase and McMahon 1995]. Once a population has acclimatized, limited reproduction can occur in brackish water below 7.0 ppt salinity [Fong et al. 1995], with little mortality even up to 10 ppt [Kennedy et al. 1996]. Potassium can be tolerated only in low concentrations up to 0.3–0.5 mM. Ammonia (NH_3) is lethal in doses as low as 2 mg/L [Baker et al. 1994]. An extensive literature search revealed no correlation between NH_4 and zebra mussel populations.

Constructing the Model

We need to quantify *Dreissena* population growth, then examine how this growth is affected by the environment. We use the logistic equation, a standard modeling device in ecology [Gotelli 1998]. We choose a continuous approach because of the huge number of individuals involved, and the logistic equation in particular because its simplicity allows us to make as few assumptions as possible.

Standard techniques for examining the influence of variables like calcium ion concentrations, pH, and temperature on *Dreissena* populations include multiple regression and discriminant analysis [Ramcharan et al. 1992]. We want to predict actual population growth rates and not just state whether or not a population could exist in certain conditions, so we use multiple regression to relate population growth to chemical concentrations.

Assumptions

- **Population growth rate is proportional to total population.**

  We assume that the growth rate of an area's population is proportional to the rate at which juveniles settle on plates there. This rate is, in turn, proportional to the total number of larvae present in the water, which is proportional to the total population. Thus, the population growth rate is proportional to the population level.

- **Carrying capacity is constant.**

  Larvae can be thought of as a resource necessary for juveniles to exist. Each breeding season, only a certain number of larvae are produced, so the population can increase only to a certain point. Thus, there is effectively a carrying capacity at work. We assume that this carrying capacity does not depend explicitly on time once the breeding season begins.

- **Migration, genetic structure, and age structure do not affect the population.**

  Although *Dreissena* populations spread quickly from one region to another, individuals can move only at a slow crawl. Thus, migration of existing population into or out of a region is negligible. Also, there is no evidence for the existence of individuals whose ages or genes dramatically affect their influence on the population, so we neglect age and genetic variation.

- **Predation is negligible.**

We assume that *Dreissena* are so numerous that any species that prey on them—and there are few—do not have a substantial impact.

- **Sites within a lake can be treated as distinct lakes.**

  Although all of the data came from a single lake, we model each site as a separate lake. That is, we assume that the introduction of mussels from another part of the lake is equivalent to their introduction into a fresh lake, and we model the population at the new site independently.

Population Growth Model: The Logistic Equation

We model a *Dreissena* population with the logistic equation

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right),$$

where r is the intrinsic growth rate of the population and K is the carrying capacity. For simplicity, we let $a = r$ and $b = r/K$, so that

$$\frac{dy}{dt} = ay - by^2.$$

With the initial condition $y(0) = y_0$, the equation has closed-form solutions

$$y(t) = \frac{ae^{at}y_0}{a - by_0 + be^{at}y_0},$$

shown in **Figure 1**. Because the data from Lake A measure the population growth rate, what we really want to fit to the data is the derivative of this function,

$$y'(t) = \frac{a^2e^{at}y_0(a - by_0)}{(a + b(-1 + e^{at})y_0)^2},$$

whose graph is shown in **Figure 2**. We can convert the parameters a, b, and y_0 into the position, height, and full width at half maximum (FWHM) of this peak, making it easy to fit to data.

Because the first data set did not include information about changes in chemical concentration over time, we average the population growth rates over all years after the introduction of *Dreissena* and fit the model curve to this "average year" at each site. The position and width of the peak are fairly constant from site to site, as we expect, since the breeding season usually peaks around mid- to late August and lasts for about three months. The peak heights, however, are radically different at different sites, ranging from about 38,000 juveniles per day at site 2 (**Figure 3**) to just 1 juvenile per day at site 10. This variation can be explained only by the environmental conditions there, so we determine how these growth rates varied with chemical concentrations.

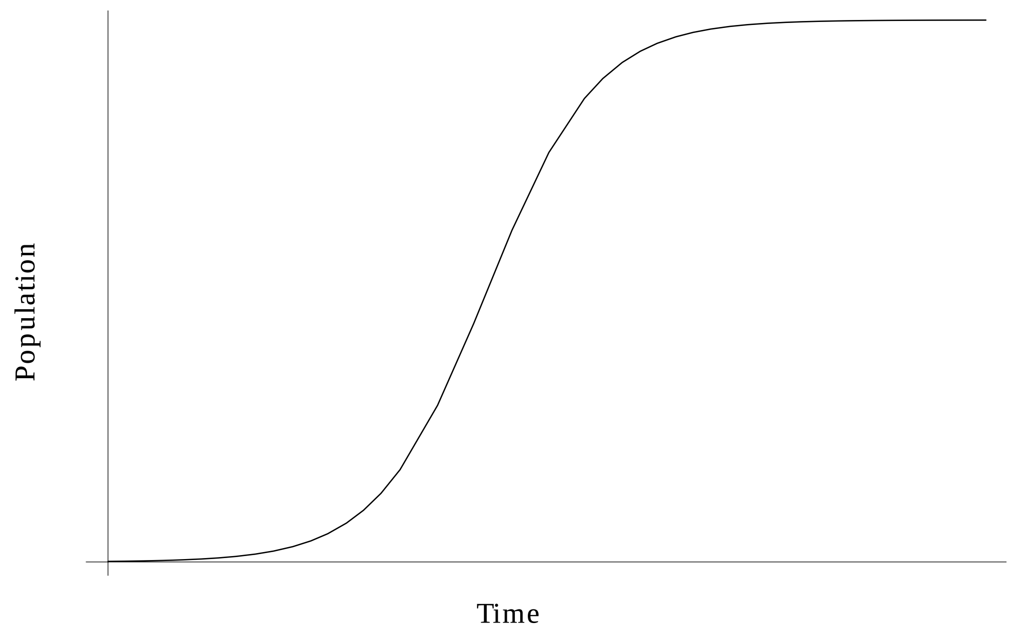

Figure 1. Solution to a generic logistic equation, $y' = ay - by^2$, with population plotted as a function of time.

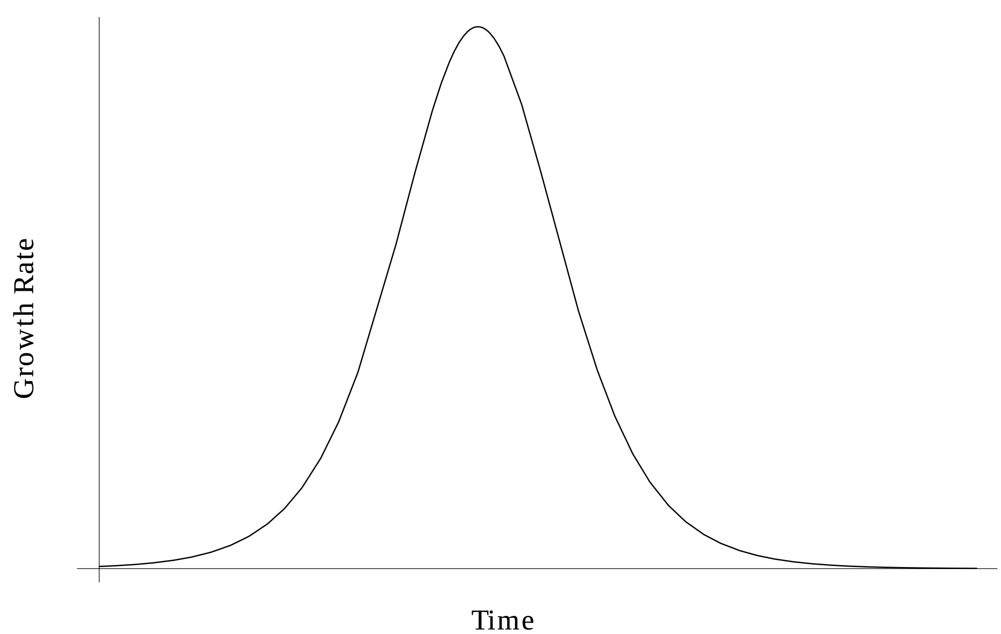

Figure 2. The derivative of the solution to a generic the logistic equation, showing the time rate of change of population. The peak corresponds to *Dreissena* breeding season in our model.

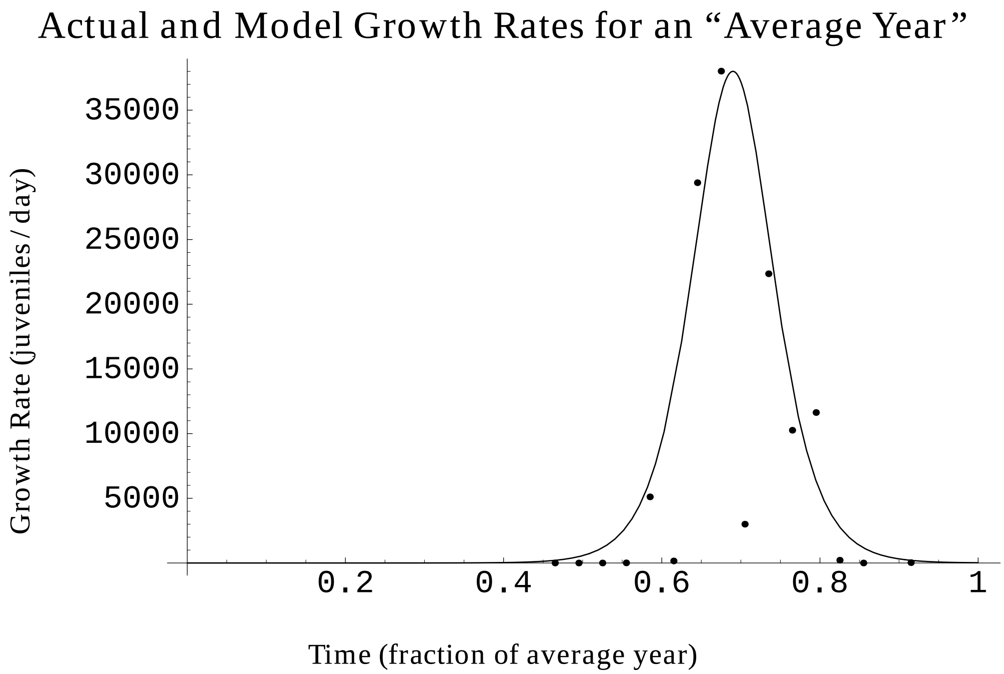

Figure 3. The derivative of the population growth model, along with data for an average year at Lake A (at site 2, the most populous site). The peak height of 38,000 is the quantity that best characterizes the population's success, so it is used in the regression analysis.

Influence of the Environment: Multiple Regression Analysis

To determine the effect of environmental conditions on growth rates, we must correlate the peak growth rates in the logistic model with the chemical concentrations at each site. To this end, we perform a multiple regression with peak growth rate as the dependent variable and some or all of the chemical concentrations as independent variables.

There are only 10 data points, far fewer than needed to separate the effects of all 11 variables. Fortunately, the literature provides guidance in selecting which variables to use. The dominant factors influencing the success of a *Dreissena* population are the concentration of calcium and the pH. Although alkalinity seems to be somewhat important, it is included in only the first data set; moreover, it also appears to be closely correlated with calcium concentration, so we exclude it. Another marginally important factor, dissolved oxygen, was not measured in the first data set. According to the literature, other chemical factors are negligible as long as they are present in trace amounts. Thus, we perform the regression on just two variables: calcium concentration and pH.

The equation we obtain is

$$\text{maximum rate} = 1687\,[\mathrm{Ca}^{2+}] + 55703\,\mathrm{pH} - 454995, \qquad \textbf{(1)}$$

where the maximum growth rate is in juveniles settling per day and $[\mathrm{Ca}^{2+}]$

is in mg/L. Thus, by measuring the concentration of Ca^{2+} and the pH of the water, we can predict the population growth rate.

Tests and Refinements

The population growth model fits the data surprisingly well, considering its simplicity. Although in some cases the model could be strengthened by allowing two peaks of different heights, doing so would introduce at least one more degree of freedom and thus make it difficult to perform a meaningful regression with just 10 sites. Because we are interested in the overall success or failure of the population, we accept some inaccuracy in the population model in order to set up a better regression.

As a first check on the model, we use it to predict the growth rates at sites 1–10 in Lake A and compared the predictions to the actual rates (**Table 1**).

Table 1.

Actual growth rates in Lake A (first data set) vs. predicted growth rates, in thousands per day.

Site	Actual	Model
1	12	18
2	38	28
3	15	6
4	1	10
5	30	20
6	0.002	−100
7	0.003	0.2
8	0.2	9
9	3	14
10	0.001	3

Although far from perfect, the agreement gave us confidence that the model can give at least a qualitative idea of how well a *Dreissena* population will do in a given calcium concentration and pH.

For a second test of the model, we use it to predict the minimum pH and calcium concentration tolerable to *Dreissena*. At a pH of 7.7, which is typical of the data available for Lake A, the regression equation predicts that the lowest tolerable concentration of Ca^{2+} would be 15.4 mg/L—very close to the accepted value of 15 mg/L [McMahon 1996]. At a calcium concentration of 25 mg/L, also typical of freshwater lakes, the model predicts a minimum pH of 7.4; this is only slightly higher than the literature value of about 7.3.

Having established some confidence in our model, we test it against the second data set for Lake A. Because this data set does not include pH, we assume that the values reported in the first data set are accurate and use them in concert with the new calcium concentrations to predict growth rates (**Table 2**).

Although this agreement is coincidentally somewhat better than that with the first data set, we perform a new regression on both data sets at once to see

Table 2.
Actual growth rates in Lake A (second data set) vs. predicted growth rates, in thousands per day.

Site	Actual	Model
1	16	16.5
2	50	27
3	45	6
4	10	9.5
5	30	20
6	15	−10
7	0.02	−0.1
8	0.5	8
9	8	150
10	0.03	5

if we can improve the model. This gives us the new regression equation

$$\text{maximum rate} = 2338\,[\mathrm{Ca}^{2+}] + 39202\,\mathrm{pH} - 334089. \tag{2}$$

Using this new equation, we predict the peak growth rates at all ten sites, based on data from both sets. We found the results given in **Table 3**.

Table 3.
Actual growth rates in Lake A (from both data set) vs. predicted growth rates from combined regression, in thousands per day.

Site	Set 1	Model	Set 2	Model
1	12	30	16	28
2	38	32	50	30
3	15	11	45	10
4	0.001	12	10	12
5	30	20	30	19
6	0.002	−5	0.015	−5
7	0.0033	6	0.020	5
8	0.150	11	0.450	10
9	3	14	8	15
10	0.001	2	0.030	5

The revised model illustrates the sensitivity of the coefficients to changes in the data. Although the additional data incorporated are from the same physical locations as the first data set, they have a significant impact on the regression equation. This modification improves some predictions and worsens others.

Strengths and Weaknesses

Like any model, the one presented above has its strengths and weaknesses. Some of the major points are presented below.

Strengths

- **Applies widely accepted techniques.**

  The logistic equation is often used to model population growth under the conditions set forth in our assumptions [Gotelli 1998]. Multiple regression analysis has been used effectively in predicting *Dreissena* populations previously [Ramcharan et al. 1992].

- **Produces predictions in agreement with the data and other models.**

  Although agreement with the data provided is far from perfect, our model produces peak growth rates that are largely consistent with observed growth rates. The model also correctly predicts minimum [Ca^{2+}] and pH levels for *Dreissena* survival. Additionally, it is consistent with other models in the literature. Ramcharan et al. [1992], for instance, give a probability-of-survival model

  $$A = 0.045\,[\mathrm{Ca}^{2+}] + 1.246\,\mathrm{pH} - 11.696$$

  that is very nearly a constant multiple of our **(1)**.

- **Correctly predicts results at Lakes B and C.**

  Equation **(2)** predicts population growth rates of −8,000 juveniles / day for Lake B and −145,000 juveniles / day for Lake C. That is, the lakes are incapable of supporting mussel populations. This is consistent with the fact that both lakes are well below the minimum calcium and pH requirements.

Weaknesses

- **Extremely sensitive to changes in experimental data.**

  Based on the results described above, this seems to be a fairly substantial problem with the model. Given the extraordinarily small amount of data available, though, it is hardly remarkable that a change in any given peak value changes the model significantly. If more data were available, we would expect much better averaging-out of error and a regression equation with much better predictive power.

- **Neglects the effects of all factors but [Ca^{2+}] and pH.**

  Again, while this would initially appear to limit the predictive power of the model, the literature supports our selection of these two factors as the dominant ones influencing population growth [Ramcharan et al. 1992].

Results and Interpretation

To apply the model to the data for Lakes B and C, we assume that the values given for the concentrations are representative of the entire lake. With only one data point for each lake, we must extrapolate. Thus, the model's predictions might not hold in areas where the concentration or pH differs significantly from this value.

The model clearly indicates that there is no chance of zebra mussel infestation in Lake C, consistent with the fact that the pH in the lake is far too low to support a mussel population. The literature indicates zero growth at a pH below about 7.3; the highest measurement of pH in Lake C is 6.0, which is clearly far too acidic. In addition, the calcium concentration must be greater than 12 mg/L for larvae survival; Lake C is far below this cutoff, with a mere 1.85 mg/L at maximum.

The chemical data for Lake B are less clear cut. The pH is in the required range but the calcium concentration is too low for adult survival. Our model and the literature both indicate that it would take a significant shift in the lake's calcium content for it to support zebra mussels.

Although taken over the course of several years, the data for Lakes B and C are not spread out spatially. It is possible that some region in either lake has much higher pH and calcium concentrations. For example, Lake George in the Adirondacks was initially thought to be immune to zebra mussels because of the water chemistry, but they were later discovered in a small region near a culvert with elevated calcium concentrations. Scientists are now concerned about *Dreissena*'s potential to spread to other parts of Lake George, as the mussels have an amazingly ability to adapt once they have settled [Revkin 2000].

Other models strongly agree with our conclusions about Lakes B and C. Hincks and Mackie's model [1997] also found that zebra mussel populations depend only on pH and calcium concentration. Their formula,

$$p = \frac{e^L}{1 + e^L},$$

where

$$L = 134.7 - 3.659\,[\mathrm{Ca}^{2+}] - 15.868\,\mathrm{pH} + 0.43\,[\mathrm{Ca}^{2+}]\mathrm{pH},$$

predicts 100% mortality in Lake C and 99% in Lake B; a population might be able to make some headway if it could establish itself in Lake B.

Ramcharan et al. [1992] modeled the probability of a population becoming established, finding through discriminant analysis that only pH and calcium levels are significant factors. The discriminant function is

$$A = 1.246\,\mathrm{pH} + 0.045[\mathrm{Ca}^{2+}] - 11.696,$$

where A must be greater than -0.638 for a population to exist. This equation, which is nearly a constant multiple of our **(1)**, suggests that no populations would establish themselves in either lake.

Recommendations on De-Icing

Since the 1940s, America has used de-icing agents, primarily road salt (NaCl), to break the bond between road and ice. Ions in the water decrease the freezing temperature and melt the ice, promoting safer driving during icy weather by increasing the wheel / road traction. This proposal will show why Ice Ban or potassium acetate is a better candidate for de-icing near a lake, and discuss anti-icing as an alternative method of combating ice.

Using standard road salt near freshwater lakes is not a good idea. While doing so *might* have a negative effect on the zebra mussel population, it would certainly have a greater negative effect on other aquatic life. Since zebra mussels are able to adapt to environmental changes more quickly than other freshwater species, any change in the chemical content of the lake will probably result in an even greater abundance of mussels [Kennedy et al. 1996]. A good de-icing method should remove the road hazard without profoundly impacting any ecosystem.

Another consideration specific to zebra mussels is that de-icing chemicals used should not *promote* their growth. Common agents including calcium, such as calcium chloride and calcium magnesium acetate, should therefore not be used. This is especially important near lakes with low levels of calcium, such as Lake B, where the introduction of a source of calcium might lead to successful colonization by zebra mussels.

According to the comprehensive report *Liquid Road Deicing Environment Impact* [Cheng and Guthrie 1998], the common de-icing chemicals remaining include common road salt, magnesium chloride, potassium acetate, and Ice Ban. This authoritative report lays out the effects of each agent on vegetation, soils, water quality, aquatic life, and people. Sodium chloride (NaCl), the most common, tested worst of all. It damages vegetation, kills some freshwater fish, pollutes groundwater, and causes air-quality concerns. The primary concern with magnesium chloride is that the chloride readily separates from the compound and can pollute groundwater or freshwater lakes. The chlorine often tastes unpleasant to people, and it can kill freshwater fish.

Potassium acetate and Ice Ban, however, are less ecologically intrusive. Potassium acetate may affect plant growth slightly and is a mild skin and eye irritant. In very high concentrations, it has been shown to kill rats, but the report does not predict any animal deaths at normal concentrations. Acetate is biodegradable in soil and the remaining potassium has no negative affect on the surrounding environment. Ice Ban, on the other hand, is completely biodegradable. Since it is completely organic, it has no effect on the vegetation, aquatic life, or air quality [Cheng and Guthrie 1998]. We therefore suggest both potassium acetate and Ice Ban as possible de-icing agents, favoring the later as the most ecologically benign and less expensive.

Potassium acetate is available as a liquid de-icing agent under the names Enviro-MLT TM (from Midwest Industrial Supply), Cryotech CF7 or E36LRD (from Cryotech), or Safeway KA Liquid (from Clariant) [Cheng and Guthrie

1998]. Ice Ban is available from Ice Ban America. For your examination, we have included cost and other information in **Table 4**.

Table 4.
Data on de-icing materials and the anti-icing solution RWIS.

Company	Amount required per 1,000 sq ft	Cost / gal	Phone Number
Enviro-MLT	0.x	4.67	1.800.321.0699
Cryotech CF7	0.5	3.30	1.800.346.7237
E36LRD	0.5	2.80	1.800.364.7237
Safeway KA Liquid	0.4	4.00	1.419.479.8650
Ice Ban	0.76	0.75	1.888.488.4273
RWIS	variable	$3000 / unit	1.800.363.6224

De-icing, however, is no longer considered to be the best solution to the problem of icy roads. The Strategic Highway Research Program (SHRP), a five-year program started to investigate anti-icing roads, has led an anti-icing initiative in 9 states and shown it to be effective there. Currently, more than 20 states are experimenting with the strategy, including states like Michigan that have zebra mussel populations, making the solution especially relevant [Federal Highway Administration 1997].

Anti-icing is a preventative measure that stops the ice from bonding to the road. Simply put, the roads are salted before the snow hits. The precipitation remains slushy and wet instead of frozen and slick. The slush is easily plowed off the road, and the "salt" does not need to be replaced as often, since it is not plowed off with the ice. This leads to many other benefits, but first we discuss cost.

Instead of reacting to snow, the state's Dept. of Transportation anticipates it from weather forecasts and road conditions. Many states have installed Road Weather Information Systems (RWIS), which report real-time information on pavement conditions. Installation is the major cost of the conversion. In states where there are at least 5 snowstorms a year, the initial cost is quickly offset [Chollar 1996].

There are several ways anti-icing has saved money. It requires less "salt" to keep the roads safe. Applying the chemical to the road before or early into a winter storm ensures that when the snow is plowed off the road, the anti-icing agent remains on the surface, while de-icing agents need to be reapplied after each plowing. Using less salt is also beneficial since de-icing agents are notoriously corrosive. This means that vehicles applying the "salt" are less corroded each year and don't need to be replaced as often. The study predicted that overall the states could save almost $108 million a year [Federal Highway Administration 1997]. Less anti-icing agent on the road means less agent on the passing cars and less corrosion of the cars. Less "salt" also helps protect the environment by reducing pollution from foreign chemicals [Chollar 1996].

For this method, we recommend using de-icing agents (which also work as anti-icing agents) and the RWIS. In areas with only 100 hours of storms per winter, the savings in de-icing chemicals is $659.50 per mile sanded or salted; with an RWIS unit every mile, the savings pay for the cost of installation in five years [Chollar 1996].

Conclusions

We treated each data site from Lake A as an independent "lake," providing 10 data points. Using these data, previous work showing that zebra mussel population is linearly dependent on pH and calcium concentration, and multiple regression, we constructed a model for zebra mussel population. We refined the model using the second set of data taken from the same lake. The resulting model is good but would be better with data from more lakes.

We have presented a recommendation on how to de-ice roads near Lake B. Lake B lacks the high levels of calcium required to support zebra mussel populations, so we suggest that the local community not use de-icing salts containing calcium but an anti-icing strategy using environmentally non-intrusive chemicals, such as Ice Ban.

Report on Controlling Zebra Mussel Populations

To the Lakeside Community:

You are planning to control the zebra mussel population by introducing a nonindigenous species, the round goby fish. We strongly encourage you not do so. This report explains how round gobies can adversely impact the surrounding ecological system, without producing a substantial impact on the zebra mussel population.

What round gobies lack in size, they more than make up in aggression. Their fierce nature allows them to dominate prime spawning grounds, forcing native species to move elsewhere or die off. In addition to eating zebra mussels, the gobies will attack other fish species' eggs and young, thus diminishing the population of previously well-established species. With their well-developed sensory system, round gobies can feed in complete darkness—an obvious advantage over competitors. Round gobies can quickly come to dominate a new area. In the rocky parts of Calumet Harbor, for example, the population of gobies already exceeds 20 individuals per square meter—that's 20 fish in a space the size of a bathtub! Their presence has caused a significant drop in the biodiversity of the ecosystem.

Round gobies are zebra mussels' natural predator in their native habitat, the Black Sea. Adult gobies can eat an average of 47 mussels per day. However, this number isn't particularly impressive when there can be up to 1 million mussels per square meter. Experts agree that gobies are a hopelessly inadequate method for controlling a zebra mussel population.

Another reason not to introduce the gobies is the safety of the people in your city. The potential danger arises because zebra mussels are filter feeders, often processing a liter of water per day. The mussels accumulate pollutants from the water, particularly PCBs (polychlorinated biphenyl), in large amounts in their tissues. Usually these toxins are not passed on, falling to the lake floor when the mussels die. If round gobies eat the mussels, however, the toxins are passed to the gobies. Because many game fish eat gobies, the toxins are passed on again. This time, though, the toxins are passed to fish that might be sold and consumed; the threat to human health is obvious.

We would like to review some possible alternatives to solve your initial zebra mussel problem.

The most commonly considered method for removing zebra mussels is chemical treatment. This method does not take into account the fact that zebra mussels are amazingly tolerant of large ranges of all chemicals—much more so than other species. Raising the toxicity level high enough to ensure the complete removal of the zebra mussels would cause other species to die.

Some methods of removal are effective only in a laboratory and seem ridiculously unsuitable for use in an external body of water. Experiments have shown, for example, that electric current and ultrasonic cavitation can kill zebra mussels. Even if such a method could be controlled and implemented, it would undoubtedly harm other species. Inventions such as the vacuum pump and the blasting hose are not practical except in factories. It has been shown that the neurotransmitter serotonin forces the mussels to spawn before the proper season, thus killing the young. The suggestion of the use of serotonin has some promise; however, this chemical may affect other species, including humans. More research is required in this area.

We regret that we must close this presentation on a gloomy note. Currently, the only available method of controlling this species is to stop it from spreading to other lakes. Since zebra mussels usually spread on the hulls of boats, we encourage you to inform your residents about the following procedures to prevent the spread of zebra mussels as suggested by the Sea Grant Extension:

- Inspect your boat's hull carefully. If surfaces feel grainy, tiny zebra mussels may be attached. Scrape off any "hitchhiking" mussels.
- Drain all water from the boat, including the bilge water where they often reside.
- Dry your boat in the sun for two to five days, or use a pressurized steam cleaner to ensure the hull is sterile.
- Throw leftover live bait away or give it to someone to use at the same water body.

We also encourage you to post signs at boat launches in your area to promote these simple guidelines for boaters.

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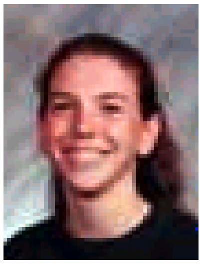

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Identifying Potential Zebra Mussel Colonization

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Summary

Both environmental and anthropogenic factors influence the spread of zebra mussels to new areas. Variations in water quality can affect both proliferation and mortality, which greatly influence colonization rate. High levels of calcium and alkalinity in fresh waters tend to increase juvenile zebra mussel population. *Dreissena* also requires specific ranges of pH, temperature, and potassium concentration for propagation. Consumption by predators and spread by humans also influence colonization and population dynamics.

We develop a lumped-parameter stochastic model using data from a lake with known water quality, using optimal water quality parameter ranges for zebra mussel survival. The model predicts the susceptibility to colonization of a lake with known water quality.

We find a significant probability for seasonal colonization in Lake B but negligible probability for Lake C.

The use of de-icing agents in the vicinity of Lake B may increase the probability of colonization, due to elevated calcium concentrations in the lake.

Literature Review

History

The zebra mussel originated in the Caspian and Black Sea regions. By the early 19th century, a well-developed population was established throughout

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the major drainages of Europe in connection with extensive canal building [USGS 2001]. Researchers surmise that the zebra mussel first arrived in North America in the mid-1980s in a ballast tank of a commercial vessel; the first recorded population appeared in Lake St. Clair, Canada [Herbert et al. 1989]. By 1990, the zebra mussel habitat encompassed the Great Lakes and soon after entered the Mississippi River drainage via the Illinois River. Today, zebra mussels exist in at least 21 states [USGS 2001].

Factors Influencing Propagation

Physical Mechanism of Propagation

Anthropogenic activities are considered the most influential factor in spreading zebra mussels [Mackie and Schloessler 1996]. Zebra mussels attach themselves to firm surfaces including boat hulls, nets, buoys, and floating debris [Balcom and Rohmer 1994; Ram and McMahon 1996]. A zebra mussel dislodged in transport can start a new population.

Natural dispersion mechanisms include birds, water currents, insects, and other animals [Mackie and Schloesser 1996; Hincks and Mackie 1997]. When carried by currents, microscopic zebra mussel larvae, called *veligers*, can quickly disperse themselves [Mackie and Schloesser 1996]. The mussels can travel large distances in the two- to three-week free-swimming veliger stage [Rice 1995].

The species has demonstrated resilience to long-overland trips. Zebra mussels survive longest under cool, moist conditions, similar to the environment in a boat hull [Payne 1992].

Habitat

Zebra mussel habitat includes freshwater lakes and reservoirs, as well as cooling ponds, quarries, and irrigation ponds of golf courses. However, the species can survive where salinity does not exceed 8 to 12 parts per thousand (ppt) [Mackie and Schloesser 1996].

Zebra mussels prefer hard substrates [Heath 1993] but can survive on soft sediment [Stoeckel et al. 1997]. Current velocities up to 2 m/s provide optimal settlement conditions, while speeds ranging from 0.5 m/s to 1.5 m/s best support growth [Rice 1995].

Water Quality

pH Zebra mussels have colonized areas with pH values ranging from 7.0 to 9.0. A pH of 7.5 promotes optimum growth [Rice 1995].

Potassium The optimal range of potassium in the environment is 0.5–1.5 mg/L, with survival at 2–3 mg/L [Dietz et al. 1996].

Calcium and Alkalinity Calcium and alkalinity are the strongest influences on zebra mussel growth and reproduction [Heath 1993]. Zebra mussels require a Ca^{+2} concentration of 12 mg / l and $CaCO_3$ concentration of 50 mg / l [Heath 1993]. Ramcharan et al. [1992] found that European lakes with pH below 7.3 and Ca^{+2} concentration below 28.3 mg / l lacked zebra mussels, but in North America there are numerous examples of invasion at far lower calcium concentrations.

Dissolved Oxygen Heath [1993] indicates a minimum oxygen threshold of 25% oxygen saturation, or 2 mg / l at 25°C. Dense overgrowths of zebra mussels may deplete dissolved oxygen enough to cause large die-offs of *Dreissena* and other aquatic species [Ramcharan et al. 1992].

Nutrients and Phytoplankton A water body's chlorophyll-*a* concentration is a factor in growth variability of the zebra mussel [Mackie and Schloesser 1996]. Zebra mussels compete with herbivorous zooplankton and fish for phytoplankton [Ramcharan et al. 1992]. Zebra mussels collect their food through ciliary filter feeding processes [McMahon 1996]; that filtering increases water clarity, and light penetration fosters growth in the lake's benthic population [MacIsaac 1996], which can increase the nuisance aquatic weed biomass.

Salinity Research suggests optimal salinity for adults is 1 ppt at high temperatures (18–20°C) and 2–4 ppt in lower temperatures (3–12°C) [Kilgour et al. 1994; Mackie and Schloesser 1996]. Rice [1995] suggests 1 ppt as optimal for growth and short-term tolerance of 12 ppt; but zebra mussels have high adaptive ability to nonideal conditions in salinity and other water quality parameters.

Temperature For reproduction, the zebra mussel requires prolonged periods above 12°C and maximum temperatures ranging from 18 to 23°C [Heath 1993; McMahon 1996]. It can't survive in temperatures greater than 32°C; the lower temperature survival threshold is 0°C [Heath 1993].

Predators Crustacean zooplankton and larval fish consume the larval stages of the mussel [Mackie and Schloesser 1996]. Adult *Dreissena* provide food for crayfish, fish, and waterfowl [Mackie and Schloessler 1996]. Fish observed consuming zebra mussels include yellow perch, white perch, walleye, white bass, lake whitefish, lake sturgeon, and the round goby [MacIsaac 1996; French 1993]. Potential consumers include the freshwater drum, redear sunfish, pumpkinseed, copper and river redhorse, and common carp. Round gobies consume 50–100 zebra mussels per day, depending on the size of the mollusk [Ghedotti et al. 1995]. Diving waterfowl consume significant amounts of zebra mussels in proper conditions. Hamilton et al. [1994] found the ducks devoured 57% of the autumn mussel biomass in Lake Erie; but due to icing over of the lake and consequent lack of winter predation, continued juvenile growth diminished the effects of the consumption.

Modeling Zebra Mussels

Zebra mussel populations demonstrate high sensitivity to small changes in water quality parameters. In some lakes, the long-term population size remains fairly constant, while populations in other lakes fluctuate greatly from year to year.

Modeling History

Some of the more common types of models developed include multivariate, bioenergetic, and probabilistic:

- Multivariate models have been used to determine the environmental factors that most influence the ability of *Dreissena* to establish viable populations [Ramcharan et al. 1992].
- Bioenergetic models focused on modeling individual zebra mussel growth as a function of certain environmental factors [Schneider 1992].
- Probabilistic models used discrete probabilities associated with environmental variables known to contribute to the successful colonization of freshwater bodies to evaluate the susceptibility of certain lakes to zebra mussel colonization [Miller and Ignacio 1994].

Model Development

Model Choice and Approach

We develop an analytical model that is transient, lumped-parameter, and stochastic.

We obtained from the literature ranges of water quality the parameters that are necessary for survival. Using a time step of one year, we determine the probability of survival based on those and determine the population. We use the data on Lake A to calibrate and verify the model's ability to predict colonization.

Data Considerations

The data files provided contain water quality and population data for Lake A. Shared by most files were calcium concentration (mg/L), chlorophyll concentration (μg/L), potassium concentration (mg/L), temperature (°C), and pH, all of which the literature shows are important factors.

We use the average juvenile population for a given year for comparison with the model results, regardless of the amount of data available for that year. Therefore, for each time step, we need an annual average and standard deviation for each parameter and each population. We assume that the average value is the average for the year.

Review of Literature

Calcium, alkalinity, phytoplankton, potassium, water temperature, and pH are important for survival. Because of the dependence between alkalinity and calcium concentration, we use only calcium. We use chlorophyll-*a* in place of phytoplankton to represent available food. We summarize in **Table 1** the ranges of water quality parameters required for survival.

Table 1.
Optimal water quality conditions for survival of each age class.

Age Group	Constituents									
	Ca (mg/L)		Chl-a (μg/L)		K (mg/L)		pH		Temp	
	LL	UL	LL	UL	LL	UL	LL	UL	LL	UL
Birth	20	50+	0	15	0.05	1.2	7.7	8.5	12	21
1	20	50+	0	15	0.05	1.2	7.7	8.5	12	21
2	15	50+	3	20	0.05	1.3	7.3	8.7	5	28
3	10	50+	8	30	0.05	1.5	5.2	9.3	0	31
4	10	50+	8	30	0.05	1.5	5.2	9.3	0	31

Methodology

The model uses assumptions about probabilities of survival at specific age classes.

Age Classes

We divide zebra mussels into four distinct age classes: class 1 (0–1 years), class 2 (1–2 years), class 3 (2–3 years), and class 4 (3–4 years). At the end of each time step (= one year), the population of each age class moves into the next age class, except that class 4 dies. Values for each water quality parameter are specified at each time step.

Survival Probabilities

The ranges of values for each parameter are divided into smaller ranges and assigned survival probabilities. A normal distribution is used to create a probability distribution for each parameter. For each age class, we take the mean of the optimal range found in the literature. Newborns and age class 1 use the same ranges and probabilities; classes 3 and 4 also use their own same ranges and probabilities; age class 2 has its own ranges and probabilities. A normal distribution is fit to the average; we assume that the limits of the optimal ranges in the literature represent one standard deviation from the mean.

Constraints and Assumptions

For each age group, the probabilities of survival at each time step for each of the water quality parameters are assumed to be mutually independent. Thus, the probability of survival of each age class is the product of the probabilities of its survival at each water quality value.

Additional constraints are also included:

- Age classes 2, 3, and 4 are able to reproduce in water above 12°C.
- The survival of eggs and larvae to age class 1 depends on their probability of migration out of the system and the probability of survival at the current water quality conditions. The probability of migration is calculated as a function of calcium concentration [Hincks and Mackie 1997].
- Since the number of eggs per adult female varies in the literature (4000–100,000), we use its value as a parameter for calibration.
- An initial number of juveniles (age class 1), specified by the user, is introduced at the first time step, and no additional veligers or juveniles enter the system from outside sources.
- The model allows the user to decide which parameters to consider in the probability calculations depending on the availability of data.

The model was programmed in Fortran 90 with a Lahey compiler under a Suse Linux operating system.

Calibration

The model was calibrated using the data in the files `LakeAChem1.xls` and `LakeAPopulation1.xls`. The water quality data are provided as the median, maximum, minimum, and 25th and 75th percentiles of data for 1992 to 1999. We assume that the median equals the mean and that the average difference between the mean and the 25th and 75th percentiles is the standard deviation.

We use a random number generator to create two sets of random numbers between 0 and 1, for n years. The value of each water quality parameter for each of the years is given by

$$X_i = \bar{X} + P_{\text{var } i} \times P_{\text{ran1 } i} \times \sigma_X,$$

where

- X_i is the value of the parameter at time step i,
- $\bar{X}$ is the parameter mean,
- σ_X is the parameter standard deviation,
- $P_{\text{ran1 } i}$ is the random number at time step i, and

- $P_{\text{var } i} = \begin{cases} -1, & \text{if } P_{\text{ran1 } i} < 0.5 \\ +1, & \text{if } P_{\text{ran1 } i} \geq 0.5. \end{cases}$

Using this method, we created a file of n years of generated data for each parameter for each of 10 sites at Lake A. We calibrated the model for its ability to predict susceptibility of a location to colonization by varying the initial population of juveniles and adjusting the number of eggs per adult female.

At these sites, trends in the model results replicate trends in the populations. At a site susceptible to colonization, a higher initial population of juveniles yields faster establishment and propagation; at a site not susceptible to infestation, the population does not establish any structure and dies off. However, increasing the number of eggs per female produces colonization at some sites that were not possible at lower levels of egg production; at these sites, water quality is near a juvenile survival threshold. [EDITOR'S NOTE: Space does not permit reproducing the authors' graphs illustrating these conclusions.]

The model is qualitatively accurate. It predicts zebra mussel colonization where and under circumstances when colonization actually occurs, and predicts no colonization when observed juveniles are low or nonexistent. The ability of a population to proliferate is apparent in the development of a population age class structure over time; if an age structure is not established, the location does not experience successful colonization.

Verification

The model predicts whether or not colonization will occur, but the speed and magnitude of the colonization are not accurately approximated. Also, since the water quality levels were artificially generated from descriptive statistics, the performance of the model with actual data is unknown. With data on the annual accumulation of zebra mussels and the distribution of water quality constituents, as provided in the files `LakeAChem2.xls` and `LakeAPopulation2.xls`, the model can be tested, adjusted, and verified.

Figures 1 and **2** compare 5 of the 10 sites for the two data sets at Lake A; similar trends appear at each site. Running the model with the second set of data indicates that populations proliferate where they have been observed in high numbers. Though the model predictions for juveniles are an order of magnitude greater than the observed values, the model correctly predicts whether populations survive; we attribute the difference to incomplete calibration.

Model Sensitivities

The dominant model sensitivities in predicting the magnitude of proliferation are to the number of water quality constituents incorporated and to the initial juvenile population. When more probabilities are considered in the calculation, overall probability is lowered. Since the model was calibrated using all parameters, using fewer parameters results in a more conservative estimate,

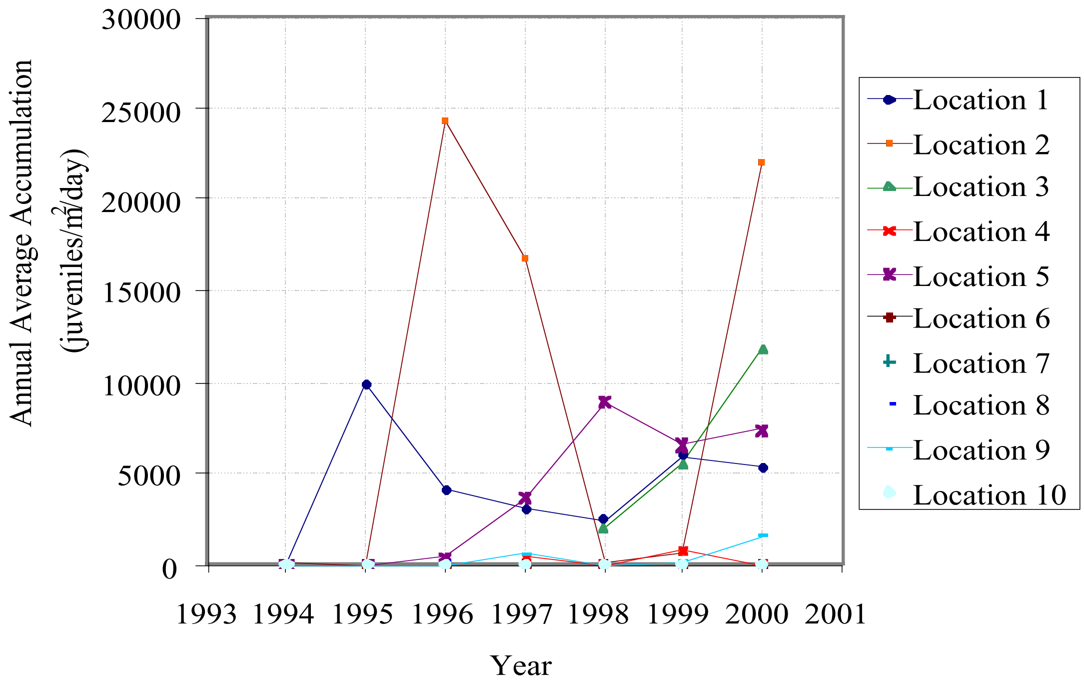

Figure 1. Annual average accumulation rates using the 1st population data for Lake A.

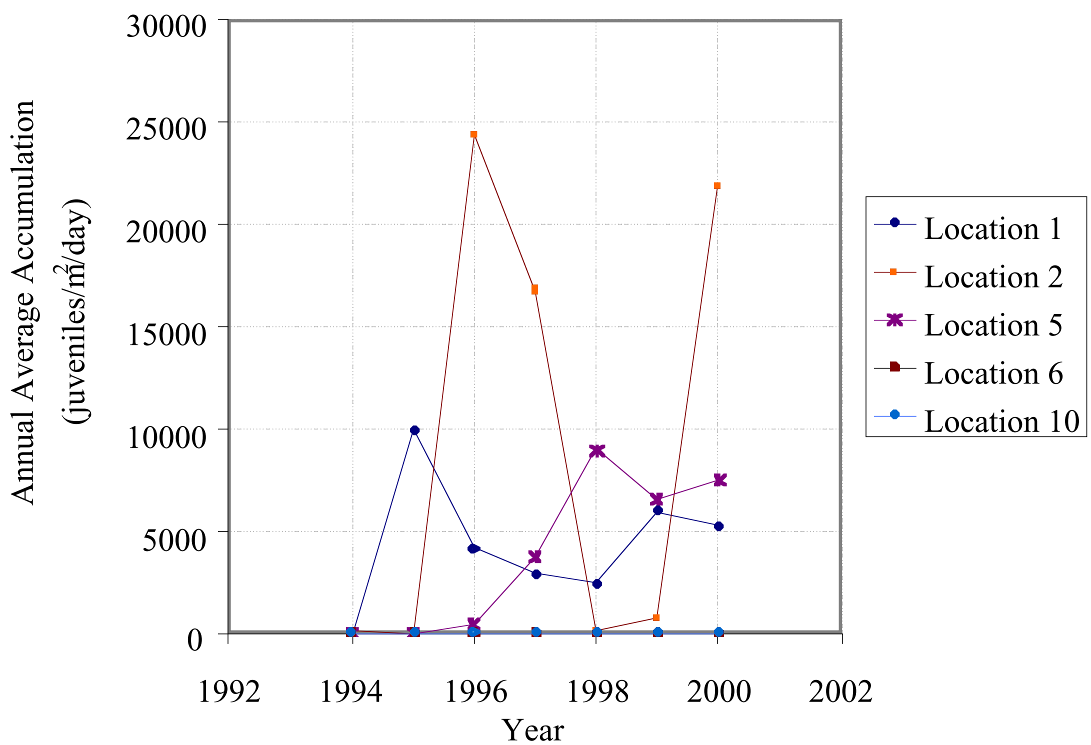

Figure 2. Annual average accumulation using the 2nd population data set for Lake A.

that is, the model over-predicts. The dominant factor in the rate of proliferation is the number of eggs or veligers that are allowed to survive.

Model Limitations

The model becomes more conservative as the number of variables considered decreases. It predicts either the occurrence of a large outbreak or that a population never establishes.

The model assumes that the survival probabilities for each parameter range are independent, but in actuality some parameters have strong dependencies, such as between pH and calcium concentration [Hincks and Mackie 1997].

Application

Lake B

Lake B is at the threshold for zebra mussel survival for the only variables on which we have data: pH, calcium concentration, and chlorophyll concentration. With so few water quality indicators, we expect a conservative estimate (i.e., an overestimate of survivability and colonization potential). We ran the model with an initial juvenile population of 1,000; only 10 survive to age class 2. A population introduced to Lake B will not proliferate.

Lake C

Lake C has a very low average pH and a low annual average calcium concentration; it is not suitable for colonization. The probability of survival predicted by the model is zero.

Impacts of De-icing Near Lake B

Many de-icing agents used to remove snow and ice from roads during the winter contain calcium salts, specifically calcium chloride ($CaCl_2$).

Repeated application of calcium chloride to roads may accumulate calcium in Lake B. A small increase in its available calcium level of 11.5 mg/L could allow colonization. The model indicates that a calcium concentration of 21.5 mg/L would allow zebra mussel colonization, but continuing low values for pH and and chlorophyll concentration force the colony to die out eventually.

Other de-icing agents, such as sodium chloride (NaCl), increase sodium concentrations in freshwater bodies, which can inhibit propagation of zebra mussels; however, zebra mussels can adapt to higher levels of salinity.

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Assessment of Introduction of the Round Goby

Ironically, the round goby and the zebra mussel both entered North American fresh waters by ballast-water discharge into the Great Lakes region at approximately the same time. They favor similar environments: slow water velocity and higher turbidity.

The diet of the round goby consists of small mollusks, especially the zebra mussel. The round goby has molar teeth well suited to crushing mollusk shells.

Biological control agents such as the round goby can have ecological advantages over chemical control. Natural enemies tend to be more specific to a certain pest, while chemical control measures often affect multiple species and the targeted pests can develop a tolerance to the chemical.

However, although the round goby can consume appreciable numbers of zebra mussels, the round goby violates the requirement of being specific to the target pest. They consume also the fry and eggs of habitat-sharing fish, including smallmouth bass, walleye, and perch, and their aggressive nature allows them to restrict native fish from utilizing optimal spawning locations.

After the zebra mussels reach a certain size, they are too large for the round goby. Spawning of larger mollusks then prevents the population from dying out.

During its filter-feeding process, the zebra mussel accumulates and stores pollutants, including PCBs. As the round goby consume the mussels, contaminants bioaccumulate in the fish. The accumulation pattern potentially continues as sport fish eat the round goby and as humans in turn consume the sport fish.

Thus, both environmental ethical and practical considerations require that additional alternatives be explored.

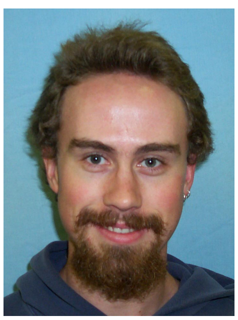
Research continues on types of biological control techniques other than round goby fish. Over the past 10 years, some microorganisms have shown promise of inducing very high zebra mussel mortality.

Until an ideal alternative exists, communities must take other measures to limit the spread of the zebra mussel. Since studies attribute the spread to movement of watercraft between bodies of water, an aggressive education campaign could inform recreational boaters and fishermen how to avoid contributing to proliferation of zebra mussels. If climate conditions necessitate de-icing of highways, a community should consider materials that don't promote zebra mussel growth.

About the Authors

David Stier attended high school in South Burlington, VT, before migrating to California in 1993. His current work activities include assessment of highway culverts in Northern California for anadromous [migrating upriver from the sea to breed in fresh water, as salmon do] fish-passage issues. He is also completing his senior year in Environmental Resources Engineering at Humboldt State University. David is an avid world traveler but unsure of his plans after graduation.

After graduating with honors with a B.S. in Environmental Resources Engineering, Marc Leisenring joined GeoSyntec Consultants in August, 2001. He has experience in both one- and two-dimensional hydrodynamic modeling, and now (with the ICM) also in stochastic modeling. As a Staff Engineering Specialist at GeoSyntec, his primary responsibilities have included technical analysis, report preparation and review, and preliminary design; future responsibilities may include model development and implementation, final design, and development of stormwater management plans.



Matthew Kennedy attended Santa Rosa Junior College in Santa Rosa, CA, before transferring to Humboldt State University in 1998. During the summers of 1998 and 1999, he worked with the Hydrology Research Group at the Pacific Northwest National Laboratory in Richland, WA. There he assisted in the development of hydrodynamic and water quality computer models of the Columbia River system. He graduated with honors in 2001 with a B.S. in Environmental Resources Engineering. Matthew is currently a research assistant at the University of Massachusetts, Amherst, where he is working on an M.S. in Environmental Engineering.

Waging War Against the Zebra Mussel

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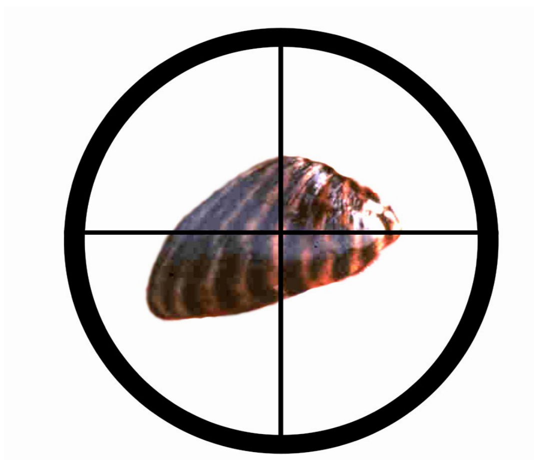

Summary

We design a mathematical model that accounts for pH, calcium concentration, and food availability, the most important factors in zebra mussel reproduction and in growth and survival of juvenile mussels. Our model can predict whether a given site is likely to be a suitable environment for a zebra mussel population as well as its potential density. Our model corresponds well with the population data provided and with the threshold values of pH (7.4) and calcium (12 mg/L) for zebra mussel viability.

We recommend to the community of Lake B that they limit their use of deicing agents containing calcium, because our model predicts that an increase in

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the calcium concentration in the lake will significantly enhance its suitability as zebra mussel habitat.

We find that using the goby fish to reduce zebra mussels is not a feasible option if the community is concerned with ecological impact, due to the invasive nature of the goby.

Environmental Factors in the Spread of Zebra Mussels

We first discuss the characteristics of a suitable breeding habitat and then address how the population is unintentionally introduced to new areas.

Population growth depends on successful reproduction and survival to adulthood. Veligers, zebra mussel larvae, are more sensitive to stress in their surrounding environment and therefore have more stringent survival requirements. Hence, we examine environmental conditions that can cause stress for the zebra mussel, especially in the larval and juvenile stages.

Ion Concentrations and pH

Calcium is required for the viability of zebra mussel populations because it is a major component in their shells. Alkalinity, which is directly linked to calcium concentrations, is an important variable in determining habitat suitability for zebra mussels. Calcium concentrations of 12 mg/L and alkalinity corresponding to 50 mg $CaCO_3$/L are required for adult zebra mussel populations [Heath 1993]. A calcium concentration of 12 mg/L is also the minimum required for embryo survival, though higher concentrations enhance egg fertilization and embryo survivorship [Sprung 1987].

Phosphorous and nitrogen are significant factors to zebra mussel population growth because they are critical nutrients for the freshwater phytoplankton that comprise the primary food source of the zebra mussel. Thus, they are an indirect measure of food availability [Baker et al. 1993].

The pH of the water is another critical factor. Adults require a pH of about 7.2; in lower pH environments, they experience a net loss of calcium, sodium, and potassium ions, and in very acidic waters adult zebra mussels eventually die because of ion imbalance [Heath, 1993]. Adults can survive in pH 7 environments, but eggs survive only between pH 7.4 to 9.4 [Baker et al. 1993].

Temperature

Adult mussels can survive temperatures from 0°C to 32°C, but growth occurs only above 10°C [Morton 1969] and breeding is triggered only in temperatures of at least 12°C [Heath 1993]. Higher temperatures increase overall egg production [Borcherding 1995] but also increase metabolism and demand for dissolved oxygen. Zebra mussels require 25% oxygen saturation (2 mg/L) at 25°C [Heath 1993]. Based on these values and the data provided for Lake A, we find that neither temperature nor dissolved oxygen is a limiting factor of zebra mussel proliferation there.

Saltatory Spread

Saltatory spread is the movement of a species in large leaps rather than by gradual transitions. It is believed that zebra mussels were introduced to the Great Lakes system in 1986 from larvae discharged in ballast water from a commercial ship [Griffiths et al. 1991]. As of 1996, zebra mussels had spread to 18 states in the United States (as far south as Louisiana) and two provinces in Canada, almost entirely within commercially navigated waters [Johnson and Padilla 1996]—strong evidence that commercial shipping was the primary vector of initial zebra mussel spread in the United States and Canada.

Most of the United States contains environments suitable for zebra mussel infestation [Strayer 1991], so the identification and elimination of saltatory spread to inland water systems is key to preventing infestation of the western United States. Transient recreational boating seems to be the most likely candidate for inland spread of the species. Based on this and other studies, it appears that recreational boating represents a substantial threat to the containment of the zebra mussel infestation in America.

Advective and Diffusive Spread

Zebra mussels live the first few weeks of their lives as planktonic larvae that are easily diffused or carried by moving water. This allows for the widespread dissemination of offspring by diffusion, currents, and wind-driven advection within a lake or watershed [Johnson and Carlton 1996], which largely explain the species rapid spread [Martel 1993]. However, veligers have been shown to have high mortality in turbulent waters, and mussel density in streams flowing out of infested lakes has been shown to decrease exponentially with the distance downstream [Horvath and Lamberti 1999]. Post-metamorphic zebra mussels have the ability to secrete long monofilament-like mucous threads that increase hydrodynamic drag and allow for faster advective spread [Martel 1993]. These juveniles can survive turbulence much better than veligers, which implies that they are the primary vector of downstream advective spread.

Zebra Mussel Population Model for Lake A

Using our model, we attempt to answer two important questions:

1. Given chemical information for a given site, is the site suitable for zebra mussels?

2. If a site is determined to be a suitable habitat, will it support a low- or a high-density zebra mussel population?

Rather than focusing on developing a complicated model that would predict the exact size of the population, we devised a simple, comprehensive model that answers these questions. The inspiration for our model was derived from Ramcharan [1992].

Assumptions

- The density of juveniles collected on the settling plates is proportional to the size of the adult population; this assumption allows us to use the provided data to predict the severity of the zebra mussel infestation.
- The chemical composition and concentrations (such as calcium levels) do not significantly vary with changes in the size of the zebra mussel population.

Examining the first data set from Lake A, we find that pH and calcium concentration are the two most important factors in determining whether a zebra mussel population is viable in a given site. This is reasonable, considering that the zebra mussels are very sensitive to pH and they need calcium to build their shells when developing from veligers to juveniles and onto adults.

We do not include temperature, because although it is important to the life cycle of the zebra mussel, as long as the temperature is high enough to signal spawning, reproduction will occur. All 10 sites in Lake A had suitable temperatures for spawning.

We developed a model equation (Model 1) utilizing the values provided for pH and calcium concentration for the 1992 to 1999 period that give a simple measure to predict the viability (V) of a zebra mussel invasion at a particular site. The coefficients of the two variables (pH and [Ca]) are used to weight the relative importance of the two factors. The range of values for pH for the ten sites is smaller than the range of values for calcium concentration, thus the coefficients function to equalize the importance of these two factors. The exact values of the coefficients were determined by successively modifying and refining the values until an equation was found that accurately reflected whether the lake site was a suitable habitat or not based on the population data. We chose the threshold value of 10.4 for viability because there appears to be a break there between the sites where zebra mussels survived and the sites where they were absent, and because 10.4 is close to the value from the equation with 7.4 for pH and 12 mg/L for calcium concentration.

$$V = 1.0\,\mathrm{pH} + 0.2\,[\mathrm{Ca}]$$

If $V > 10.4$, the site is a suitable habitat for zebra mussels.

Applying Model 1 to sites 1–10 in Lake A produces **Table 1**.

Table 1.

Calculated viability values for sites 1–10 in Lake A using model 1.

Site	pH	[Ca] mg/L	V
1	7.68	26.8	13.04
2	8.00	22.3	12.46
3	7.74	17.6	11.26
4	7.84	16.5	11.14
5	8.02	16.9	11.40
6	7.59	13.4	10.27
7	7.66	16.9	11.04
8	7.82	16.6	11.14
9	7.95	15.7	11.09
10	7.86	12.0	10.26

The model predicts that sites 6 and 10 should not be suitable habitats, while the other eight sites should be. **Figure 1**, which plots date vs. juveniles/day for each of the sites, shows that the data agree well with our model. Sites 6 and 10 have virtually no zebra mussel population growth, and sites 1, 2, 3, 4, 5, and 9 all show evidence of infestation. Although it is predicted that sites 7 and 8 should be susceptible to invasion, enlargement of **Figure 1** shows that these two sites are not supporting large populations; correspondingly, V for sites 7 and 8 is relatively low. Also, the source of the zebra mussel invasion was site 1, hence the more southerly sites have had longer to form stable populations than the northern sites 7 and 8. With threshold pH of 7.4 and threshold calcium level of 12 mg/L, the model—which predicts that sites 6 and 10, whose values border on the threshold, are not likely to be habitable—is consistent with the literature.

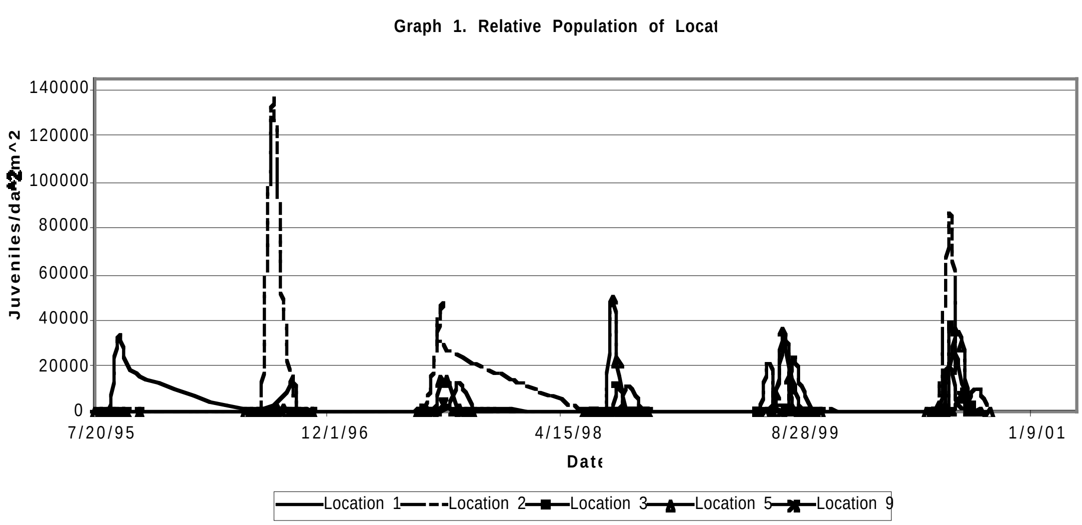

Figure 1. Relative populations at sites 1–10.

To improve upon Model 1, we account for trends observed in the second data set from Lake A in constructing a more descriptive model to answer question (2). By including parameters for total phosphorus and total nitrogen, we account for the role of food availability on density. Following Ramcharan [1992], we employ the natural logarithms of total phosphorus and total nitrogen. Once again, by successively altering the coefficients, we determine an equation for the density of populations in the lake sites. We define high density as more than 400,000 juveniles / m^2 on the settling plates collected at the peak of the reproductive season.

$$D = 1.0\,\mathrm{pH} + 0.2\,[\mathrm{Ca}] + 0.1\ln\,[\mathrm{TP}] + 0.4\ln\,[\mathrm{TN}].$$

$$\text{If } \begin{cases} D < 9.9, & \text{there will be no zebra mussels;} \\ 10 < D < 10.4, & \text{the site will support a low-density population;} \\ D > 10.5, & \text{the site will support a high-density population.} \end{cases}$$

By averaging the total phosphorus (TP) and total nitrogen (TN) values for each site in the second set of chemical data for Lake A, we calculated [TP] and [TN]. Using those values in Model 2, we calculated the density (D) for each site, as shown in **Table 2**.

Table 2.
Density values in sites 1–10 in Lake A.

site	ln[TP] mg / L	ln[TN] mg / L	D	low / high
1	−2.99	−0.598	12.5	high
2	−3.51	−0.892	11.8	high
3	−4.30	−0.796	10.5	high
4	−4.47	−0.814	10.3	low
5	−4.40	−0.879	10.6	high
6	−4.56	−0.852	9.5	absence
7	−4.12	−0.971	10.2	low
8	−4.39	−0.862	10.3	low
9	−4.16	−0.965	10.3	low
10	−3.01	−0.405	9.8	absence

Model 2 predicts that sites 1, 2, 3, and 5 should be able to support high density populations. The second set of population data used in **Figure 2** is consistent with the first set of population data. **Figure 2** shows that all four of the high-density sites have an average of more than 400,000 juveniles / m^2, which agrees with the prediction made by our model. In the enlargement of **Figure 2**, sites 4, 7, 8, and 9 have an average of less than 400,000 juveniles / m^2, while sites 6 and 10 have virtually no juvenile zebra mussels.

The most significant weakness of our model is that it does not predict population versus time. Our model simply classifies an area's risk of invasion by examining the levels of critical chemicals to which the zebra mussels are sensitive.

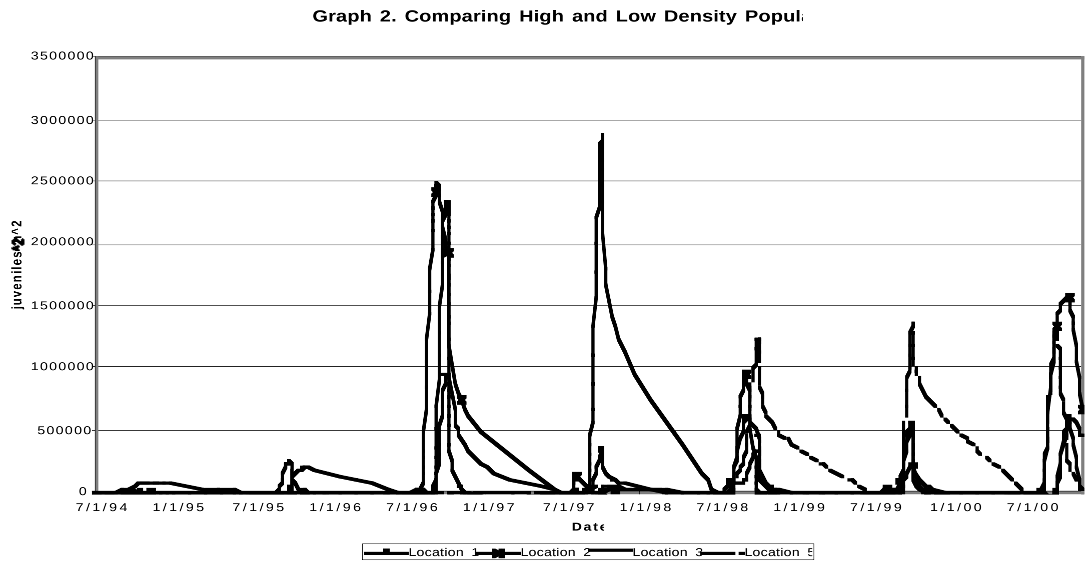

Figure 2. Comparison of high- and low-density populations.

Another weakness of our model is that it relies on chemical and population data from only one lake. By slightly varying the values of the coefficients and observing whether the altered model more accurately predicts the density of the zebra mussels in the newly incorporated lakes, a better model can be achieved. Information from other lakes could also be used to refine the value chosen for the division between low and high densities. Other factors, such as total ion concentration, could also be included in the model if the factor were shown in a variety of lakes to correspond to population densities.

We are not able to predict, using our model, how fast a population of zebra mussels will spread from one site to another within a lake. However, by qualitatively examining the data from Lake A, it appears to take only a few years for the population to spread from one area to another as long as the new site is suitable for zebra mussels. For example, in site 5 in 1994 and 1995, there were no zebra mussels collected, but from 1996 to 1998, the population rapidly increased to a high density. Since zebra mussels can very quickly reach high density populations in a supportive environment, it seems that knowing whether a given site is a suitable habitat is a more useful piece of information than the rate at which the population grows.

Using Model for Lake A to Predict for Lake B and Lake C

Using the equations from our models, we can average pH, calcium concentration, total phosphorus concentration, and total nitrogen concentration for

Lake B and Lake C and determine the level of risk of successful zebra mussel invasion in these two lakes. We averaged the values together for all of the years. We also assume that these two lakes are fairly uniform in chemical composition.

Table 3.

Viability and density values for Lake B and Lake C.

	pH	[Ca] mg/L	[TP] mg/L	[TN] mg/L	V	D
Lake B	7.63	11.5	6.02×10^{-3}	0.182	9.93	8.74
Lake C	4.74	1.15			4.97	

According to our Model 1, Lake B should not be at risk for a zebra mussel invasion because it is not a suitable habitat ($V < 10.4$); this prediction makes sense because the average calcium concentration is 11.5 mg/L, which is below the 12 mg/L threshold. Lake C is in no danger to an invasion, since $D = 4.97$, which corresponds to the fact that both the pH and the calcium concentration are far below the threshold values.

De-icing Policy for Community of Lake B

De-icing compounds increase the solute concentration in the melted ice, lowering its freezing temperature and preventing the ice from reforming. Thus, de-icing compounds are water soluble and can easily enter the water supply. The most commonly used de-icers are calcium chloride, calcium magnesium acetate, sodium chloride, and potassium acetate salts. Calcium magnesium acetate is popular because it has fewer negative environmental impacts, whereas calcium chloride is widely used because it lowers the freezing point of water more than sodium chloride.

Although these calcium containing compounds may be excellent choices for de-icing agents, our model indicates that using these compounds increases the risk of zebra mussel invasion. According to Model 2, if calcium levels increase in Lake B by 50% ($D = 9.9$), a low density population of zebra mussels can exist. Doubling the calcium levels ($D = 11.0$) will support a high density population. De-icing agent can therefore have a significant impact on the zebra mussel population. *We recommend that this community use sodium chloride or potassium acetate salts*, or decrease the amount of calcium salts used by mixing them with the other noncalcium salts or sand. We also suggest pre-wetting the salts before they are applied to the roads, to reduce the amount entering the water system. Lastly, the community should develop a strategy for anti-icing, applying de-icing agents before ice forms, thus decreasing the amount of de-icing agent used in each storm. These efforts should help prevent Lake B from becoming habitable by zebra mussels.

Methods for Reducing Zebra Mussel Populations

It is estimated that $3 billion will be spent in the next decade combating the zebra mussel infestation [Magee et al. 1996]. Besides damaging infrastructure (pipes, tubing, gratings), the zebra mussel is able to out-compete native species for space and food and can destroy commercial and recreational fish stocks. Since the zebra mussel body fat stores toxic chemicals, the introduction of these mussels into the food-chain could lead to human consumption of these harmful chemicals. There are three available options for dealing with zebra mussel infestation:

(1) Introduce a natural predator (the round goby).

(2 & 3) Eradicate and / or control the zebra population by utilizing preventative and reactive control strategies.

Introducing a natural predator, such as the round goby, may be more problematic than the zebra mussel infestation. Although the round goby shows selectivity in consuming zebra mussels over native clams, the goby will non-selectively consume a variety of bait, fishes, and invertebrates [Ghedotti et al. 1995]. In addition, the goby is extremely territorial and can aggressively occupy prime breeding areas and successfully compete for food against native species. Fortunately, there are more environmentally sound methods of controlling zebra mussel infestations.

Preventive and Reactive Strategies

Preventive control methods include implementing restrictive legislation and periodic monitoring of waterways to minimize introduction of zebra mussels and to improve early detection, thereby facilitating the development of appropriate strategies to eradicate or control the mussel population. Reactive strategies are a more aggressive mode of action in response to a potential or ongoing invasion and should be dependent on the level of infestation.

Preventive Control Strategies: Legislation and Monitoring

Legislation is a useful way to coordinate research with monitoring facilities, commercial industries, and the public. The United States Nonindigenous Aquatic Nuisance Prevention Control Act of 1990 (P.L. 101–646) [Florida Caribbean Science Center 2001] recommends that recreational vessels exchange ballast water before entering new waters, since this is the primary mode of saltatory non-native species introduction [Boleman et al. 1997]. In addition, the U.S. Code [Legal Information Institute 2001] suggests implementing alternative ballast water management, including modifying the ballast tank and intake system to prevent the unintentional introduction of new species. The improved sighting, reporting, and education under this plan will help the public and commercial sectors prevent the spread of zebra mussels.

Reactive Control Strategies

Acute Zebra Mussel Infestation In cases of acute or localized infestations, applying the least expensive method of preventing infrastructure damage is to employ a *foul release coating* in concert with mechanical cleanings and mechanical filtration. Coating pipes and surfaces in contact with the water with antifouling polymers, such as silicones and fluorochemicals, creates a slippery surface that makes it difficult for zebra mussels to attach [Magee et al. 1996]. These reagents are effective for 2–5 years [Boelman et al. 1997].

An alternative and equally successful method of infrastructure protection is the application of *zinc thermal spray* (ZTS) on metal surfaces. In addition to preventing corrosion, ZTS is the most durable and long-lasting zebra mussel repellent. The slow dissolution of heavy metal ions from ZTS is toxic to zebra mussels. In addition, the US Army Corps of Engineers Zebra Mussel Control Handbook suggests that low release of heavy metals and a large dilution factor produce minimal secondary effects on nontarget species. However, before implementing this strategy, it is critical that the environmental effects studied and the implementation meet federal standards.

Mechanical cleaning is a labor-intensive method of removing zebra mussel from infrastructure. The drawback to simply brushing and scraping zebra mussels off surfaces is that the scrubbings need to be repeated regularly. The removed zebra mussels also have to be transported and disposed of in landfills.

The final strategy for dealing with acute zebra mussel infestation is installing *mechanical filtration systems*. Water screen filters and strainers can be placed on water intakes. A mesh size of 25–40 mm is able to stop the inflow of veligers and translocation of larger zebra mussels. However, this system requires continuous maintenance.

Global Zebra Mussel Infestation Severe and large-area infestation and population expansion need to be treated with aggressive methods, since it is more beneficial to address the widespread infestation problem rather than fight specific site-related mussel-density problems. Since these methods require widespread application, the expense associated with implementation is higher than the strategies for dealing with acute infestation. There is also a potential for harming native organisms and commercial industries. However, after intense scrutiny, the following methods are the most effective ways to control and potentially eradicate severe zebra mussel infestations.

Thermal treatment. The discharge of heated water is a cost-effective and efficient method for controlling and eradicating the macrofouling zebra mussel. Since zebra mussels are able to acclimate to temperature changes, extreme temperature changes are required to kill the mussels. These extreme temperature changes will also kill a number of native species residing in the lake. There are two thermal treatment strategies that can be employed: acute thermal treatment and chronic thermal treatment [Boelman 1997]. Acute thermal treatment involves rapidly increasing the water temperature to lethal levels followed by a rapid return to original temperature levels. This method is most appropriate for treating infestation in waterways where a higher temperature cannot be maintained for an extensive period of time. Greatly increasing the water temperature for a period of 3–9 hours can yield 100% mortality.

Chronic treatment involves continuously maintaining a higher water temperature and is a cost-effective strategy for industries that generate and discharge heated water. This method prevents new zebra mussel infestations. This strategy is lethal to most if not all organisms that use the water. The water temperature, in this method, must be raised to greater than or equal to 34°C and must be maintained for 6–24 hours to kill the entire zebra mussel population.

Chemical treatments. Chemical treatments are an alternative to thermal treatment but are more environmentally invasive. Both oxidizing and nonoxidizing chemical treatments are available. Oxidizing treatments are most toxic to zebra mussels when applied rapidly due to the mussel's sensitivity to oxidizing compounds, whereas nonoxidizing chemicals can be administered over a longer period of time with equal effectiveness.

Of the oxidation treatments available, *chlorination* is the most widely used method for eradicating zebra mussels. There are large environmental consequences to this method, and terrestrial organisms and birds may also be killed.

Potassium permanganate is another commonly used oxidizing chemical. To obtain 100% zebra mussel mortality, a higher concentration of and a longer exposure to potassium permanganate is required than for chlorinated compounds. The advantage to using potassium compounds is that they are nontoxic to higher organisms like fish but are highly toxic to zebra mussels. Also, potassium permanganate by-products do not form carcinogenic compounds as is the case when using chlorinated reagents.

Nonoxidizing molluscicides, such as Mexel 432, are the best available chemical treatments, albeit more expensive than oxidation treatments. The greatest advantage to this strategy is that molluscicides have fewer direct consequences on native organisms and fewer long-term environmental impacts since many of these molluscicides rapidly biodegrade into harmless substances. These reagents induce their effect in three ways:

- On clean surfaces, the film prevents settlement.
- On infested surfaces, the molluscicides attack the zebra mussel byssal threads, causing the mussels to detach.

- The molluscicides form a film on zebra mussels that remain in the system, causing lesions on the gill and ultimately killing the organism. For this reason molluscicides are also lethal to other mussels. Application of these chemicals needs to be repeated on a daily basis to sustain the film until all zebra mussels are killed.

Future species-specific treatments. Although target-specific chemicals are not currently available, research is developing methods for targeting invasive species and interfering specifically with their reproduction cycle through biochemical compounds like serotonin. These targeted treatments would be highly advantageous in terminating zebra mussel propagation without affecting other aquatic organisms or damaging the environment.

Response to Community Leaders

For such small critters, zebra mussels can range from being a mild nuisance to a large environmental and economic cost. The introduction of these species into our lakes and rivers has created situations where communities are forced to control or eradicate zebra mussel populations. The most important question is how to do this in the most environmentally and economically sound manner. In order to develop a solution for this irritating infestation problem, we must first assess how extensive the problem is. We must identify

- how the zebra mussels were or are being introduced to the lake,
- if the lake provides a supportive environment for zebra mussels, and
- if there other aquatic organisms or terrestrial organisms (including humans!) that depend on the lake or use it as a food source.

Isolating the source of zebra mussel introduction to the lake is important so that the community can prevent reintroduction of the mussel or other non-native species that are a threat to indigenous aquatic organisms. This preventive measure will contribute to making the reactive strategies for controlling the zebra mussel invasion more successful and therefore more cost effective.

There are two types of reactive control strategies that can be implemented:

- introduction of a natural predator to the lake system or
- the use of mechanical or chemical methods to control or eradicate the zebra mussel population.

Introducing a natural zebra mussel predator, such as the round goby fish, to the lake system can be a cost-effective and simple solution to the infestation problem. However, if the lake sustains other aquatic organisms or is used by commercial industries (such as fishing), the costs associated with introducing the goby may be much higher. The goby is an aggressive territorial fish that prefers zebra mussels but will nonselectively consume bait, fish, and invertebrates. As a consequence, the goby can destroy fishing stocks and out-compete native species for food.

Another alternative is the use of mechanical or chemical strategies to control the zebra mussel population. For mild to moderate infestation, the following strategies are effective:

- Mechanical cleaning of pipes and surfaces exposed to water, followed by coating these surfaces with foul release coating. This coating contains environmentally sound antifouling polymers such as silicones and fluorochemicals, which create a slippery surface making it difficult for zebra mussels to attach.
- Installing simple mechanical filtration systems requires periodic maintenance but effectively prevents zebra mussels from clogging intake pipes.

Severe infestation requires more aggressive and environmentally abrasive strategies to control the zebra mussel population. Both of the following strategies are more expensive than the two methods discussed above and have more extensive environmental impacts.

- Thermal treatment is the discharge of heated water into the lake system. The water temperature can be raised rapidly (acute treatment) or slowly for an extended period of time (chronic treatment). In either case, 100-percent of the zebra mussels can be killed. However, this method kills most other aquatic organisms as well.
- An equally effective method is treating the lake with chemicals. There are two viable options in this approach. The first is using chlorinated compounds, which in a short duration will kill the entire zebra mussel population, as well as many other aquatic organisms and even birds. The drawback of this approach is the production of carcinogenic by-products that may remain in the environment for an extended period of time. A better alternative to chlorinated compounds is potassium permanganate. This chemical must be applied at larger concentrations for a longer period of time to kill mussels (including native species) without harming other organisms.

With any environmental problem, a balance has to be reached between the needs of the community and the effects on the environment. The community will have to weigh carefully the problems caused by the zebra mussels with both the economic and environmental costs associated with each method of removal.

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Judge's Commentary: The Outstanding Zebra Mussel Papers

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Introduction

The papers were assessed on

- their ability to transform the data into useful information;
- the application of an appropriate modeling process; and
- the integration of environmental science to render appropriate recommendations.

The judges appreciated the effort and valued the results of the papers. It was a very difficult problem that required a blend of science, mathematics, and conviction to solve a complex interdisciplinary problem during the four-day contest. It was clear that a solution was not going to jump out of the 40 pages of data; rather, it had to be pulled out skillfully.

The Problem

Zebra mussels were introduced to North America in 1980. They are an ecological "dead end," since native fish do not eat them. Researchers are currently attempting to identify environmental factors that may influence the population of zebra mussels within our waterways. Zebra mussels are now spread throughout the eastern waterways of the United States, causing tremendous problems for the ecosystem and the regional economies.

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The data in the problem statement are real: Prof. Nierzwicki-Bauer of Rensselaer Polytechnic Institute, a leading researcher of zebra mussels, provided data from several lakes in New York. Several population models appear in the literature; however, the collection of environmental factors that influence the rate of population growth of the zebra mussel is still unknown. This is a genuine interdisciplinary problem that confronts North America today.

The Data

The data appear to have created an "uncomfortable" feeling in the hearts and minds of the modelers. It was difficult for many to digest all of the data and either incorporate all of it into a model or else justify eliminating portions of the data. Often, teams did not address how they managed "missing" data or why they accepted or refuted data that appeared to be erroneous. In most cases, teams had done a significant amount of work in an attempt to understand the data. Most teams categorized the population data by month in order to synthesize the data into a more useful form. Similarly, they attempted to align the chemical data by averaging several time periods into a single data point. Many teams had difficulty describing their analysis and the interpretation of their results. The successful teams discussed how they transformed data and how they confronted missing or confusing data. **Tables 1** and **2** show portions of the data, the zebra population of one lake from 1994–2000 and the chemical information on the same lake for 1999. Confusing—yes, but real.

The entire set of data included the following categories: stratum, total phosphorus, dissolved phosphorus, calcium, magnesium, total nitrogen, temperature, chlorophyll, alkalinity chloride, iron, potassium, sodium, pH, secchi disk transparency, and population levels. It was essential to explain how the data would be organized for analysis. The judges expected teams to describe why they selected certain data to remain in their analysis and why other chemicals were eliminated. It was clear that contestants had to make several decisions to transform the data into a useful form. This problem, like last year's problem, was not clear-cut. Once again, we found that as the contestants formulated and refined their assumptions, they confronted the complexities typically associated with an open-ended problem. Last year they had reasonably clean data, while this year they had some "dirty" data.

The characteristic of a strong paper was the ability to uncover the uncertainties of the population growth of zebra mussels due to chemical concentrations using science and mathematical models. In some cases, the incomplete data and large unexplainable fluctuations in the population obscured the affect of specific chemicals. The data alone cannot reveal the complete interaction among the chemicals affecting population growth. For that reason, successful teams had to take an interdisciplinary problem-solving approach.

Table 1.
Zebra mussel population of one lake.

Date	Population
7 / 1 / 94	100
8 / 1 / 94	70
9 / 1 / 94	50
10 / 1 / 94	248
11 / 1 / 94	1,045
7 / 12 / 95	222
8 / 1 / 95	50
9 / 1 / 95	70
10 / 1 / 95	40,000
11 / 1 / 95	200,385
7 / 1 / 96	39
8 / 1 / 96	4,843
9 / 1 / 96	30,033
10 / 1 / 96	949,433
11 / 1 / 96	49,333
7 / 1 / 97	0
8 / 1 / 97	20,456
9 / 1 / 97	44,678
10 / 1 / 97	345,555
11 / 1 / 97	98,789
7 / 1 / 98	605
8 / 1 / 98	84,132
9 / 1 / 98	599,432
10 / 1 / 98	454,932
11 / 1 / 98	49,332
7 / 1 / 99	93
8 / 1 / 99	45
9 / 1 / 99	83,962
10 / 1 / 99	539,229
11 / 1 / 99	30,012
7 / 1 / 00	0
8 / 1 / 00	50
9 / 1 / 00	9,483
10 / 1 / 00	592,339
11 / 1 / 00	467,876

Table 2.

Chemical profiles of the same lake.

Date	Ca mg/L	Mg mg/L	TN mg/L	Temp °C	Chl-a
4/15/99	23.20	5.07	0.44	8.50	4.72
5/17/99			0.32	15.50	7.27
5/18/99	27.50	6.71	0.45		
6/1/99			0.49	18.20	10.18
6/9/99			0.42		
6/14/99			0.47	21.00	11.64
7/1/99			0.52	20.80	9.45
7/19/99	26.80	6.72	0.86	21.00	10.18
7/20/99	27.20	6.61	0.56		
7/29/99			0.44	21.80	13.58
8/4/99			0.52		
8/11/99			0.51		6.30
8/23/99			0.44	21.00	5.09
8/25/99			0.41		
9/7/99			0.38	22.00	12.12
9/13/99			0.38		
9/24/99	24.80	5.67	0.89	16.00	1.09
10/7/99			0.73	13.50	3.64

The Science

If science is defined to be the knowledge and study of "what is," then most of the teams got half of the science—the knowledge part. Almost every team was able to find an enormous amount of information from the open literature by using the Internet. The stronger teams not only gathered information, but they also explained the impact of specific environmental conditions on the life-cycle process of the zebra mussel. If chemicals such as nitrate and magnesium were eliminated without explaining why, the grader immediately suspected that the student did not know why. Likewise, if variables such as chlorophyll, pH level, and calcium were kept in the model, the outstanding teams explained why, from both a modeling and a scientific perspective. An explanation of the model using both science and mathematics was a characteristic of an outstanding paper.

An understanding of the ecological fabric of the waterways was important in the design of an outstanding solution to this problem. Environmental science was the thread that related the data to the model and the model to a "realistic" solution.

The Model

It was important that the modeling process be well formulated and that the rationale of the selected model be clearly explained. The definition of variables, identification of simplifying assumptions, and a discussion of the ramifications of these assumptions were important ingredients in the paper. Finally, it was important that the model developed was used to answer the question regarding the expected growth of the mussels in lakes B and C. An interdisciplinary discussion of the ramification of the de-icing policy required in Part E was also directly tied to the model. Surprisingly, many teams did not take advantage of their model to address follow-on questions.

The explanations of the modeling process varied tremendously. Some papers contained models that were well designed with results that were analyzed and interpreted. The teams also recognized their models to be both predictive and descriptive. Unfortunately, other papers had wonderful models that utilized a commercial package or constructed models, but they never explained how the model functioned. It appeared that providing the details of the model's underpinnings impacted the entire paper. Groups who had good explanations of their models also related these models nicely to the environmental science of the zebra mussel rate of growth.

The analysis of the data tied in nicely to how the students performed their modeling. Some students saw the problem as fitting a growth differential equation, and others as fitting a multivariable regression. The approach did not affect the assessment of the paper. Furthermore, whether a team used a discrete dynamical system, curve fitting, or simulation, or adapted the logistic model, a correlation analysis was very important. The stronger papers tended to perform this analysis graphically. Those groups providing useful graphs and explanations of these graphs faired quite well.

The Analysis

The problem was an interdisciplinary endeavor. Teams that did great mathematics but revealed little knowledge on environmental science could not capture the relationships required to solve this problem. Since the data were not clean, it was impossible to use only the data to uncover the essential relations affecting population growth. Similarly, teams that had a tremendous knowledge of the science but little mathematics were not able to create an appropriate predictive model. A thorough explanation of the implication of each variable on the growth of the mussel population was essential. Good teams shared a modeling process that was well thought out and justified the rationale of the selected model. In Part C (adjustment of the model) a clear explanation of the process involved in modifying the model was important. Finally, in Part D it was important that the analysis of the model was used to answer the question regarding the expected growth of the mussels in Lakes B and C.

An interdisciplinary discussion of the ramification of the de-icing policy was directly tied to the model. Those students who answered all the requirements had a significantly greater chance of going forward than those groups who either did not answer the requirements or who only addressed one or more requirements superficially.

Presentation

Some papers revealed tremendous analysis but lacked clarity in the presentation. The strong papers presented the problem, discussed the data and explained their analysis, and finally revealed the development of their mathematical methods/models. The big difference in papers was whether they informed the reader of what they did and, more important, how they did it. A clear presentation allowed the judge to comprehend their logic and reasoning. One judge noted that he wished he was a mind reader because there was clearly lots of outstanding work; however, only the result was revealed. The strong papers revealed their analysis, not just the results.

Very broadly, we saw two types of weak presentations. The first consisted of reports that had a significant narrative, but no support in the form of mathematical modeling or analysis. In these reports, the groups appeared to rely on qualitative observations and the information from the literature (web sites) to reach conclusions. The other type of poor-quality report was those that had a significant amount of mathematics in the form of tables and graphs, but no modeling or analysis to pull it together. These papers appeared to dump their computer runs into the report but did not really know what to do with them.

This year we noticed that the stronger teams clearly documented information they gathered from outside sources. When constructed models aligned very closely with models found in the open literature, it became difficult for judges to determine what was original work.

Conclusion

The effort and creativity of almost every team was inspiring. It appears, however, that most teams can reason better than they can communicate. Often, wonderful ideas were not revealed to the reader. The necessity to work with large data sets appeared much more difficult than anticipated. The top papers, however, did an amazing effort of blending and revealing the science, research, and mathematics. The best teams revealed the power of interdisciplinary problem solving.

About the Author

Gary Krahn received his Ph.D. in Applied Mathematics at the Naval Postgraduate School. He is currently the Head of the Dept. of Mathematical Sciences at the U.S. Military Academy at West Point. His current interests are in the study of generalized de Bruijn sequences for communication and coding applications. He enjoys his role as a judge and Associate Director of the ICM.

Author's Commentary: The Outstanding Zebra Mussel Papers

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Introduction

One cannot underestimate the potential impact of exotic aquatic species. In particular, the zebra mussel, a small, fingernail-sized freshwater mollusk that was unintentionally introduced to North America via ballast water from a transoceanic vessel, has caused havoc to say the least! Zebra mussels have significantly impacted electrical power generation stations, drinking water treatment plants, industrial facilities, navigation lock and dam structures, and recreational water bodies. In fact, zebra mussels cause an estimated $5 billion in economic damage annually, with this amount continuing to escalate. The zebra mussel problem is a national one, which impacts over half of the fifty states. In light of the ecologically devastating and costly consequences of zebra mussels, it is imperative that there is increased education, research, and science-based policy.

As revealed in this year's contest, the use of real data sets means working with numerous variables and sometimes incomplete information. Additionally, the facts that need to be considered when trying to address issues surrounding the success or failure of zebra mussels to spread and survive are complex. Many important and complex environmental problems lie at the interface of disciplines and therefore require interdisciplinary approaches to be addressed. Interdisciplinary training is more than learning and acquiring the ability to talk different languages across disciplinary boundaries. It is an approach that promotes teamwork, innovation, creativity, and "out-of-the-box" thinking for solving "real-world" issues and problems. The interdisciplinary problem contest plays a vital role in this experiential training bringing together

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teams of students that are focused for four days on “solving” a complex problem. The breadth of approaches that were used by the teams this year was truly impressive.

Basis for Contest Question: Queen of the American Lakes, Lake George, NY

Until recently, it was thought that zebra mussels had not invaded Lake George, New York, the home of the Darrin Fresh Water Institute (DFWI). Since 1995, the DFWI had carried out a zebra mussel monitoring program in Lake George where zebra mussel larvae had been observed in only two of the years. In 1997, larval zebra mussel numbers at 1 of 11 locations were comparable to those observed in the Hudson River, an area of high zebra mussel colonization. Despite the presence of larvae, no adult zebra mussels or settled juveniles had been observed. In December of 1999, the situation changed when two divers from the Bateaux Below Inc., a nonprofit organization dedicated to underwater archaeology, found adult zebra mussels at the southern end of Lake George.

In response to the discovery of these mussels, the DFWI has been working intensively at the site to determine why adult zebra mussels were able to survive and reproduce, ways in which they could have been introduced to this location, and an appropriate action to eradicate them from this location.

The discovery of zebra mussels in Lake George was particularly surprising given the low calcium content and low pH of the lake; laboratory tank experiments had previously shown that zebra mussel larvae would not survive under these conditions. However, water chemistry analyses conducted at the site where the mussels were found revealed calcium and pH levels higher than that characteristic of the majority of Lake George. Further investigation revealed that water entering the lake from a nearby culvert was introducing stormwater runoff and groundwater into the lake with calcium levels four times higher than that characteristic of the rest of the lake. In addition, the site contains numerous concrete and rock aggregates that are likely sources of additional calcium. Finally, there is potential contribution of calcium from a concrete boardwalk that was built approximately a year before the discovery of zebra mussels at this location.

Introduction of zebra mussels may have occurred when boats contaminated from other lakes entered Lake George at the boat launch adjacent to the site. Introduction could also have occurred during the construction of the nearby boardwalk via contaminated equipment. The exact mechanism(s) by which they were introduced may never be known.

After discovering zebra mussels in Lake George, the DFWI and Bateaux Below SCUBA divers carried out an extensive survey of the location to determine the size of the affected area. The mussels were confined to a 15,000 square-foot area. After consultation with state and local agencies, it was agreed that hand-harvesting of the relatively low-density mussels was the best solution. Diving at the site to remove all visible zebra mussels began and has been ongoing since April 2, 2000. This approach has been extremely labor intensive and, while hopefully effective, would not be feasible if multiple sites were found throughout Lake George.

Currently, a number of activities are being continued at Lake George, including monitoring and removal of any remaining zebra mussels at this site. Removal of any remaining zebra mussels is critical to reduce the likelihood of successful reproduction. In addition, mussels that are not removed may adapt to the lower calcium and pH conditions and spread into surrounding areas. Water samples are continuing to be checked for microscopic larvae and chemical parameters. This information will be used to evaluate success of removal efforts, determine whether to extend the monitoring area beyond the present site and better understand the local water chemistry.

As can be seen from the above "story," the questions asked in this year's contest—examining environmental factors that could influence the spread of zebra mussels and the potential impacts of human activities and policy issues—are real ones. I read with great interest the solutions provided by this year's teams. In fact, I plan to reread a number of them as we continue to work on these research questions.

Proactive vs. Reactive

There are many ways in which we can be proactive against the potential threat and spread of zebra mussels. Perhaps of primary importance is education of individuals, through which it is hoped that the spread of zebra mussels can be reduced. The primary mode by which zebra mussels are transported to new bodies of water or to new locations within single water bodies is by human activities: mussels attached to boat bottoms, or veligers hitching a ride in bait buckets or scuba gear, for example. Therefore education can be viewed as a preventive measure for the spread of zebra mussels.

A second critical activity is monitoring for the first appearance of zebra mussel larvae (veligers), young juvenile mussels and adult zebra mussels. Of course, the earlier the detection, the better the opportunity to minimize a widespread colonization. Thus, monitoring programs are paramount in being proactive about zebra mussel infestations.

Third, and to the point of the contest question, there is a need for development of mathematical models that can be made robust using the numerous data sets that already exist for water bodies that either have or lack zebra mussels. These models may then be used to predict possible new infestations within water bodies potentially in jeopardy of zebra mussel introductions. At the time of the contest only three such models had been published in the scientific literature. To have interdisciplinary student teams and worldwide focus on this important issue was a fantastic opportunity.

Another aspect of this year's question related to policy. Too often policy development is the result of being reactive. The most beneficial outcomes are likely to occur if we are proactive and policy decisions are put into place before, rather than after there is a serious and sometimes uncorrectable problem. In order to facilitate this scientists must accept the responsibility of effectively conveying scientific findings and results in "layman's" terms. It is only then that policy can be an informed decision influenced by the scientific fact finders.

Data Sets for Competition

Just as the students in the contest worked in teams, the collection of data for this year's problem was also an example of collaboration and teamwork. The sharing of scientific information is critical when working on complex problems, where the saying that "the whole is greater than the individual parts" is truly the case. Data were kindly provided for the contest by

- Cathi Eliopoulos of the Vermont Department of Environmental Conservation, for Lake A (Lake Champlain);
- Larry Eichler of the Darrin Fresh Water Institute, Rensselaer Polytechnic Institute, for Lake B (Lake George, NY); and
- Scott Kishbaugh of the New York Dept. of Environmental Conservation, for Lake C.

Zebra mussels were discovered in Lake Champlain in 1993 and have since continued to expand in their distribution throughout the lake. In 1999, adult zebra mussels were found for the first time at the southern end of Lake George. This remains the only location in that lake where they have been observed to date, although the search for additional colonies continues. No zebra mussels have been found in Lake C, and this is likely to remain the case unless there are significant increases in calcium concentrations within the lake.

Acknowledgment

I would like to thank Chris Arney and Gary Krahn for their invaluable contributions in brainstorming and the development of this year's problem.

About the Author

Sandra A. Nierzwicki-Bauer received a B.A. and a Ph.D. in Microbiology at the University of New Hampshire. After a two-year postdoc at the University of Chicago, she joined Rensselaer Polytechnic Institute in 1985 as Assistant Professor of Biology. She has served in a number of positions at RPI, including Chair of the Biology Department and most recently Interim Dean of the School of Science, and now Professor of Biology and Director of the Darrin Fresh Water Institute.

"Although my formal training was as a microbiologist, it did not take long for me to recognize the power of interdisciplinary research and education, as well as the national importance that the zebra mussel problem was taking on." In 1995, when zebra mussels began encroaching closer and closer to the beloved Adirondacks and Lake George, she began a new program that focused on research, education and outreach activities related to the pesky mollusk. Six years later, this exciting work continues. "Participating as a judge for this year's contest reminds me of one of the joys of working on interdisciplinary problems: having the best of both worlds . . . being a student and a teacher."

Reviews

Solow, Daniel. 1998. *The Keys to Linear Algebra.* Ashland, OH: BookMasters. ix + 543 pp, $49.95. ISBN 0–9644519–2–1.

The UMAP Journal has published many reviews of linear algebra ("l.a.") texts here, including the most important l.a. text of the last forty years, Gilbert Strang's *Linear Algebra and Its Applications* [1988; Cargal 1989], as well as the editions of his sophomore-level text *Introduction to Linear Algebra* [1993, 1998; Cargal 1994, 2000]. (It is time that we review some of the texts for a *second* undergraduate course in l.a.; any volunteers?)

Solow's *The Keys to Linear Algebra* is a very good book, better than 90% of the competing texts for a first course and possibly as good as any for self-study (l.a. may be required in all mathematics majors, but in some mathematical fields people often pick l.a. up on their own).

One of the virtues of the book is that it is unique; Solow was not following a template, to produce a "cookie-cutter" imitation of another popular book, as is sometimes mandated by publishers. If his book resembles any other book, it is indeed the first edition of Strang's *Introduction to Linear Algebra.* In particular, the first chapter of each book explores the mathematics of lines and planes in three-dimensional space. I know from classroom experience that many students have trouble with this topic; I assume that this is the reason Strang took it out of the second edition. Solow goes less far with this topic but spends more time on its development, pursuing and surpassing in this topic what are the main strengths of Strang's books, motivation and development.

Solow clearly wrote this book out of love of the subject and of teaching. He spends a great deal of time explaining the logic and reasoning of the proofs. There are copious examples, and he is very good at explaining applications.

Strang has self-published many of his books including *Introduction to Linear Algebra* because he wants to, and his reputation has sufficed to sell the books. There can be substantial financial advantages (as well as risks) in self-publishing. Out of choice or otherwise, Solow's book too is self-published (Bookmasters is just the distributor), like some of his previous books. Given that Solow's text is as good as I say it is (and it is!), could it have been turned down by major and not-so-major publishers? I have no idea whether in fact it was—but it certainly could have been. I have been closely watching publishers for 25 years. I do not completely understand the publishing business, but I understand it well enough that I could give 15 reasons why publishers would turn down an excellent text such as this. But that is an essay for another day.

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Lesmoir-Gordon, Nigel, Will Rood, and Ralph Edney. *Introducing Fractal Geometry*. Cambridge, UK: Totem Books. 176 pp, $11.95. ISBN 1–84046–123–3.

Sardar, Ziauddin, and Iwona Abrams. 1998. *Introducing Chaos*. Cambridge, UK: Totem Books. 176 pp, $11.95. ISBN 1–84046–078–4.

I was late (39) getting into physics. I knew that Newtonian mechanics was the place to start, which was convenient since I was facing a classic problem in that area (I was then working in aerospace). From Newtonian mechanics, special relativity was a good second place to go. Special relativity is elementary mathematically and can be viewed as an extension of Newtonian mathematics. However, reading various books, one thing escaped me. I could not see the logical flow. Just where and how did special relativity start? Yes, I could tell one set of laws from the other; but I was going up the wall about foundations.

My dilemma was solved by a cartoon book, *Einstein for Beginners* [Schwartz and McGuinness 1979]. It explains that Einstein based everything on two postulates and that the main point of his first paper on special relativity is that these postulates lead to a consistent theory. Though this is a cartoon book, it compares favorably to the book I read in high school. Both ignored Einstein's work on Brownian motion, quantum mechanics, and general relativity. But the cartoon book does special relativity better and provides a far more complex picture of its subject. Even what I do not like in the cartoon book is in its favor: It is irritating in its leftist politics (a characteristic of the series), but the book in high school did not have the nerve to be provocative.

Einstein for Beginners is in the Pantheon series *XYZ for Beginners*. That and the Icon Books (Totem Books in the U.S.) series, *Introducing XYZ*, which includes the two books reviewed here, are the principal academic cartoon book series. Both series are strongest on philosophy and politics.

The second author of *Introducing Fractal Geometry*, Will Rood, has significant mathematical training. Probably most mathematicians would quibble with some points (was Weierstrass ever a student under Cauchy?) The book certainly treats Benoît Mandelbrot in herculean terms. But the book is competent. This may seem weak praise; but given its format, it is the easiest introduction to fractal geometry that there is. Fractal geometry is one of the most graphic of all geometries; if there is a mathematical topic that lends itself to a cartoon book, fractal geometry is it.

Introducing Chaos by Sardar and Abrams is another story. Here, the art, by Abrams, seems more suited to a book on philosophy. Sardar's background is along the lines of journalism, philosophy, and Islamic studies; he is the author of another Icon / Totem book, *Introducing Mathematics*. Presumably his value to the series is his sympathy to a multicultural viewpoint, a topic to which I will return. (His book *Thomas Kuhn and the Science Wars* [2000], comprised of 74 small pages, seems to be a somewhat postmodern essay on the science wars.)

The very first heading of *Introducing Chaos* (p. 3) is "Ying, Yang, and Chaos." On p. 5, we are informed that "Chaos theory is a new and exciting field of scientific inquiry." On p. 17, "Non-linear equations, on the other hand, cannot be solved." P. 19: "Most forces in real life are nonlinear. So why have we not discovered this before? . . . Galileo (1564–1642), an Italian physicist, disregarded small nonlinearities [viz., friction] in order to get neat results. Since the advent of 'modern' Western science, we have been living in a world which acts as if the platypus was the only animal in existence."

Poincaré's results on the three-body problem are now considered an important landmark in the history of chaos theory, but his achievement is not mentioned in this volume. Instead, we are told on p. 23 that "[We] have 'solved' the three-body problem by demonstrating that the orbits are inherently unpredictable. Such a solution would have been considered nothing less than sacrilege a few years ago." The middle of the book may be better, but at the end we return to this false East=West dichotomy. We learn on p. 160: "Chaos theory and complexity are tools of understanding. But these new sciences contain understanding that has been indigenous to non-Western societies." If this understanding has ever been translated into mathematical form, we are not told about it, so it is impossible not to be extremely skeptical. Given the book's hype, its questionable mathematics, science, and history, and lastly its clichéd multiculturalism, I cannot recommend it. In fact, I find it somewhat offensive.

I speak of multiculturalism as it appears in science and mathematics education, where it precedes the current era in which multiculturalism in a broader sense has become a component of academia. In the 1970s, the idea of a close link between quantum physics and Eastern philosophy was very trendy [Shermer 2001]. In the 1980s and 1990s, this phenomenon recurred with fuzzy logic (which goes back at least to the early 1970s). In the best-seller *Fuzzy Thinking: The New Science of Fuzzy Logic* [1994], Bart Kosko stresses the point that whereas scientists in the West think in binary terms, Eastern philosophy is more subtle and is more compatible with fuzzy logic and its fuzzy shadings of truth.

My problem with all this is that the stereotype of Western scientists being binary—and linear, and all that—is utter nonsense. The history of Western science shatters this stereotype. Similarly, Eastern philosophy and religion are just that: philosophy and religion, and they do not bear comparison with Western science but with Western philosophy and religion. The Eastern philosophy that is presented in books like *Introducing Chaos* and Kosko's *Fuzzy Thinking* . . . is superficial and in my view glib and patronizing. The corresponding technical discussion invariably seems superficial as well; for example, Kosko claims that fuzzy logic and fuzzy algorithms are a great new development in technology. We get a bit of philosophy; we learn that Kosko is an accomplished fellow, and at one point we learn that he had a particularly brilliant idea while sitting in a hot tub. As to fuzzy algorithms, we get remarkably little detail.

Last, a little disclosure. I am conservative by most standards and live in the state of Alabama, so one might suppose that I am hostile to Eastern philosophy and religion. On the contrary, I have an affinity for it going back to 1958, when I was a boy of seven in Bangkok. At that age, I was too young for the ideas, but one should never underestimate the "feel" of a culture and the importance of that feel. (In fairness, I should mention that living in a society is not by itself sufficient for exposure to the culture, and some kids are resistant to other cultures by the ripe age of six.)

When we look at multiculturalism in the recent sense, I am struck by the fact that students who are exposed to it do not seem to know more about other cultures than earlier generations did. As a student and as a professor, I have found that what earns the respect of foreign students is knowledge of their countries. But to get a knowledge of international politics, one must look beyond the extremely shallow coverage of *Newsweek*, *Time*, the network television news, and all but a handful of America's daily newspapers.

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Shermer, Michael. 2001. Starbucks in the Forbidden City: Eastern and Western science are put to political uses in both cultures. *Scientific American* (July 2001): 34–35.

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Annual Index for Vol. 22, 2001

Author Index

Reviews Index

(Names of authors are in plain type; names of reviewers are in bold.)

Acknowledgments

I would like to express my great appreciation for the help of the associate editors, whose names appear on the masthead. Not only do they do the bulk of work of evaluating manuscripts, but they also solicit new works and encourage and guide potential authors.

I am also indebted to the additional individuals listed below who have reviewed manuscripts during the past year. Their careful evaluation and judgment have enhanced the quality of the articles and Modules that have appeared in the *Journal* and in *Tools for Teaching*. (Reviewers of some papers considered for Vol. 22 of the *Journal* were acknowledged already in Vol. 21, No. 4. Reviewers of the ILAP Modules in *UMAP/ILAP Modules 2000–01: Tools for Teaching* are acknowledged on the frontispiece of the corresponding ILAP Module.)

The *Journal* offers many opportunities for participation in COMAP's work. If you would like to

- referee manuscripts—please contact me;
- review books, software, or films—please contact the Reviews Editor;
- write a self-contained expository essay about an area of mathematics, however grand or small—an era, a concept, a theorem, an idea, a term—please contact the On Jargon Editor;
- contribute an Interdisciplinary Lively Applications Project (ILAP) Module—please contact the ILAP Editor;
- contribute an article, UMAP Module, or Minimodule—please contact me;
- encourage and stimulate colleagues to prepare and submit suitable material—please contact me about being nominated to join the Editorial Board.

Contact information for the associate editors in charge of Reviews, of On Jargon, and of ILAPs, as well as my own information, are on the masthead of every issue.

Finally, the associate editors and I would like to thank the *Journal*'s authors, without whom none of this would be possible, and its readers, whose benefit and enjoyment are the culmination of our enterprise.

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Errata

Vol. 15, No. 4

p. 312, l. 3: The multiplication sign should be a division sign.

Vol. 16, No. 4

p. 353, Table 1: The table claims that there is no win in the game Tchouka Ruma for 8 holes and 4 stones per hole, but Jeroen Donkers of the University of Maastricht (Netherlands) communicates the solution: 5 8 2 5 1 8 3 6 8 7 2 8 2 8 4 8 7 8 5 8 6 8, which can be tried at his Website at `http://fanth.cs.unimaas.nl/games/ruma` .

UMAP/ILAP Modules 2000–01: Tools for Teaching

p. iii, l. 9: Joe Myers $\longrightarrow$ Joseph D. Myers

p. iii, ll. 13–14: Mark Smillie, Charlotte Jones, David Westlake, and Mary Pietrukowic were early reviewers of this ILAP but should not be listed as authors.

p. 31, left sidebar: Berry $\longrightarrow$ Beery

Vol. 22, No. 1

back cover, page numbers: 29 $\longrightarrow$ 27, 65 $\longrightarrow$ 63

p. 40, l. 22: a relation $R \longrightarrow$ a relation r

p. 76, l. 11: **Figure 7** $\longrightarrow$ **Figure 9**

p. 79, l. 7: **(91)** $\longrightarrow$ **(9a)**

Vol. 22, No. 2

p. 164, Figure 5: The beam waist should be denoted by ℓ instead of by a sans serif L (which resembles a vertical bar).