Time series analysis

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1 What is a time series

A time series is a set of observations x_t recorded at a specific time t. For discrete time series, the set T at which the observations are recorded is discrete. Example of time series: recording the arterial pressure every minute, recording the voltage of a computer every second.

2 Objective and methodology of time series analysis

The objective of time series analysis is to draw inferences from them. What does it mean to draw inferences? Before answering that question let define the steps in time series analysis.

1. Select a family of probability models
2. Estimate parameters. This will lead you to a specific model for the time series
3. Check goodness of fit to the data

Now drawing inferences from the model means using the model to enhance our understanding of the underlying process that generated the time series. Or we can simply say without lost of generality that drawing inferences is an umbrella expression for different aspects that will eventually lead us to enhance our understanding of the underlying process that generated to time series.

Drawing inferences as in

- use the model to provide a compact description of the data
- perform seasonal adjustment
- predict future values of a series (Predicting future sales for wine, predict future values of stock)

- use time series model to estimate the probability of emptiness of a reservoir, in reservoir simulation
- perform hypotheses testing

Now let get a bit technical

3 Definition of a time series model

Definition 1 *A time series models for the observed data $\{x_t\}$ is a specification of the join distribution (or possibly only the mean and covariance) of a sequence of random variables $\{X_t\}$ of which $\{x_t\}$ is postulated to be a realisation.*

Instead of specifying the joint probability we instead specify the first and the second order moment of the joint distributions: the expected value $E[X_t]$ and the expected products $E[X_{t+h}X_t]$.

In the particular case where all the joint distributions are multivariate normal, the second order properties of X_t completely determined the joint distributions and hence gives the complete probabilistic characterisation of X_t

4 Examples of time series models

4.1 The iid noise

The iid noise or Independent and identical distributed random variable with mean zero, is a sequence with no trend and no seasonality . Such random variable

$$X_1, \cdots, X_n \tag{1}$$

is said to be iid noise if and by definition

$$P[X_1 \leq x_1, \cdots X_n \leq x_n] = P[X_1 \leq x_1] \cdots P[X_n \leq x_n] = F(x_1) \cdots F(x_n) \tag{2}$$

where $F(.)$ is the cumulative distribution of each of the random variables $X_1, \cdots, X_n$

In this model, there is no dependence between observations. In particular, for all $h \geq 1$ and for all x, $x_1, \cdots, x_n$

$$P[X_{n+h} \leq x | X_1 = x_1, \cdots, X_n = x_n] = P[X_{n+h} \leq x] \tag{3}$$

This shows that $X_1, \cdots, X_n$ is of no value for predicting the behaviour of X_{n+h}. iid noise is not useful for forecasting, but it it plays an important role in building more complicated time series models

4.2 A binary process

References

[1] Petter J. Brocckwell, Richard A. Davis *Introduction to time series and forecasting.* Springer Texts in Statistics, second edition.