STAT211 Mandatory Homework 2

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1 Problem 3.1

2 Problem 3.2

Consider the MA(1) process given by

$$Y_t = Z_t + \theta Z_{t-1} \tag{1}$$

where Z_t id iid. N(0,4).

2.1 Part a

Show that

$$Z_t = \sum_{j=0}^{\infty} (-\theta)^j Y_{t-j}. \tag{2}$$

By rearranging equation (1) we get

$$Z_t + \theta Z_{t-1} - Y_t = 0. \tag{3}$$

Now we want to show that Z_t given by equation (2) is solution of (3). By inserting the expression of Z_t from equation (2) into equation (3), we get

$$\begin{aligned}
Z_t + \theta Z_{t-1} - Y_t &= \sum_{j=0}^{\infty} (-\theta)^j Y_{t-j} + \theta \left(\sum_{j=0}^{\infty} (-\theta)^j Y_{t-1-j} \right) - Y_t \\
&= Y_t + \sum_{j=1}^{\infty} (-\theta)^j Y_{t-j} + \theta \left(\sum_{j=0}^{\infty} (-\theta)^j Y_{t-1-j} \right) - Y_t \qquad (4) \\
&= \sum_{j=1}^{\infty} (-\theta)^j Y_{t-j} + \theta \sum_{j=0}^{\infty} (-\theta)^j Y_{t-1-j}
\end{aligned}$$

Now the first sum in the right expression of equation (4) can be written as

$$\begin{aligned}\sum_{j=1}^{\infty}(-\theta)^j Y_{t-j} &= -\theta Y_{t-1} + \theta^2 Y_{t-2} - \theta^3 Y_{t-3} + \theta^4 Y_{t-4} - \cdots \\ &= -\theta(Y_{t-1} - \theta^1 Y_{t-2} + \theta^2 Y_{t-3} - \theta^3 Y_{t-4} + \cdots) \\ &= -\theta \sum_{j=0}^{\infty}(-\theta)^j Y_{t-j-1}\end{aligned} \tag{5}$$

Therefore equation (4) can be rewritten as

$$Z_t + \theta Z_{t-1} - Y_t = -\theta \sum_{j=0}^{\infty}(-\theta)^j Y_{t-j-1} + \theta \sum_{j=0}^{\infty}(-\theta)^j Y_{t-1-j} = 0. \tag{6}$$

And we are done.

3 All R code Code

```
options( warn = -1 )

library(astsa)
data(varve)

# plot data
plot_data <- function(data){
  plot(varve,col="blue")
  title(main="Logarithm␣of␣glacial␣varve␣timeseries", col.main="blue")
  title(xlab="Time", col.lab="blue")
  title(ylab="Varve", col.lab="blue")
}

#histogram plot

histogram <- function(sample){
  hist(sample,main="Histogram␣of␣logarithm␣of␣varve␣data", col="blue", prob=TRUE,yl

  lines(density(sample),lwd=2,col="green")
}

# compute variance of a sample
get_variance <- function(sample){
  sample_variance <- var(sample)
  return(sample_variance)
```

```
}

# compute the logarithm of a sample
get_log <- function(sample){
  log_sample <- log(sample)
  plot(log_sample,col="blue",xlab="Time",ylab="Log␣varve", col.lab="blue")
  title(main="Logarithm␣of␣glacial␣varve␣timeseries", col.main="blue")
}

plot_difference <- function(sample){
  difference <- diff(sample,lag=1, differences=1)
  plot(difference,col="blue")
  title(main="Difference␣of␣log␣of␣varve␣data", col.main="blue")
  #title(xlab="Time", col.lab="blue")
  #title(ylab="Varve", col.lab="blue")

}



Y <- log(varve)
U <- diff(Y,lag=1, differences=1)
#emp_auto_corr_rho <- acf(U,type = "correlation")
#emp_auo_variance_gamma <- acf(U, type = "covariance")
#print(emp_auo_variance_gamma[1])
#print(emp_auo_variance_gamma[0])

x1 <- var(U)/(1-2*0.49+0.49*0.47)
x2 <- var(U)/(1-2*2+2*2)

print(x1)

print(x2)

print(var(U))
```