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A temperature-dependent mathematical model of malaria transmission with stage-structured mosquito population dynamics

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Abstract: In this paper, we formulate a temperature-dependent model for malaria transmission dynamics which includes immature stages of mosquitoes. The model is constructed by using ordinary differential equations with some parameters which are periodic functions. Two thresholds dynamics associated to the model have been derived: the vector reproduction ratio \mathcal{R}_{ν} and the basic reproduction ratio \mathcal{R}_{0} . Through a rigorous analysis via theories and methods of dynamical systems, we prove that the global behavior of the model depends strongly on these two parameters. More precisely, we show that if \mathcal{R}_{ν} is greater than one and \mathcal{R}_{0} is less than one then, the disease-free periodic equilibrium is globally attractive. If \mathcal{R}_{ν} is greater than one and \mathcal{R}_{0} is greater than one, the disease remains persistent and the system admits at least one positive periodic solution. Finally, using the reported monthly mean temperature for Burkina Faso, numerical simulations are carried out to illustrate our mathematical results.

Keywords: Malaria, basic reproduction ratio, vector reproduction ratio, persistence, global stability, periodic solution.

MSC: 34D20, 34D23, 34D45, 37C75

1 Introduction

Malaria is a potentially deadly disease caused by protozoan parasites known as *Plasmodium*, that infect and replicate within human blood cells. It is spread between humans via the bite of the infectious female adult *Anopheles* mosquito and is one of the greatest infectious diseases to beset mankind. There are five (previously four) *Plasmodium* species that commonly infect humans, namely *P. falciparum*, *P. vivax*, *P. ovale*, *P. malariae*, and very recently, *P. knowlesi*. In 2019, the World Health Organization's report revealed that an estimated 200 million people and more than 400, 000 deaths due to malaria occured worldwide in 2018. Over 90% of all malarial mortality in sub-Saharan Africa is attributable to *P. falciparum*, where children under five, pregnant women and non-immune adults are most at risk. However, Asia, Latin America and to a lesser extent the Middle East of Europe are also infected [10, 18, 48].

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The spread of infectious diseases has always been of big concerns and poses a threat to public health, as well as the economic and social development of human society. Thus, Epidemic Dynamics formulates mathematical models, based on the occurrence and progressions of diseases and the surroundings, to characterize the infectious agents, to describe the transmission processes, to analyze the origins of the diseases and the factors involved in the transmissions, and to predict the prevalence of the diseases and their patterns [5, 13, 15, 31]. With deep understanding and the knowledge that we obtain from the Epidemic Dynamics studies, and good estimates of important factors and parameters, prevention and control strategies can be made [30, 33, 34]. Epidemic Dynamics can employ a well-developed modern dynamic theory to better characterize the inherent patterns and to investigate long-term behavior of disease transmission. Further, qualitative and quantitative studies, and sensitivity analysis on model parameters can help us make more realistic simulations and reliable transmission prediction which may not be feasible by experiments or field studies [14, 16, 24, 42].

Malaria is one of the first human diseases to be subject to mathematical inquiry. In fact, in the early 1900s, Sir Ronal Ross proposed a mathematical model to understand parasite transmission mechanism of malaria including the human host and the mosquito [29]. Following Ross's footsteps, George Macdonald improved Ross's model in the early 1950s and showed that there exists a threshold density, the basic reproduction number. He performed a sensitivity analysis of the basic reproduction number on the parameters and proved the potential use of this threshold for malaria intervention [25]. Other reviews on mathematical modeling in malaria include Chitnis, Cushing and Hyman, Ngwa and Shu, Chiyaka et al. and the references therein [8, 9, 28]. Moreover, malaria is one of the diseases that exhibits seasonality. Indeed, the transmission and the distribution of the disease are heavily influenced by environmental and climatic factors such as temperature, rainfall, humidity and wind [2, 3, 37]. A number of recent studies showed that climate change has significant impact on the dynamics of malaria transmission [20, 21, 47, 50]. For example, the average lifespan of mosquitoes decreases for higher temperatures. Increases in temperature generally cause the malaria vector to feed more frequently. Another important aspect about the mathematical modeling of malaria transmission is the stage structure of the vector [18]. Indeed, mosquitoes undergo complete metamorphosis going through four distinct stages of development during a lifetime: egg, larva, pupa, and adult. While it is appropriate to assume that only adult mosquitoes are involved in malaria transmission, the dynamics of the juvenile stages (pupae and larvae) have significant effects on the disease transmission dynamics. Thus, since the immature control is a hotly recommended strategy in fighting against malaria, then, it is important to incorporate the immature stage in the malaria models [1, 23, 26, 32].

In a recent paper, Wang et al. [47] have formulated a mathematical model of malaria with the use of Insecticide-Treated bed nets in periodic environments and age-structure for the vector population. They showed that the global behaviors of their model is determined by two thresholds dynamics: the vector reproduction ratio, \mathcal{R}_{V} and the basic reproduction ratio, \mathcal{R}_{0} . However, in their model, the total size of human hosts is assumed to be constant, the oviposition rate of juvenile mosquito is linear and the exposed classes of human hosts and mosquitoes are not incorporated. Moreover, in recent works, Traoré et al. [40, 41] have studied a seasonal malaria transmission model by taking into account the life-cycle of mosquitoes. It emerged from their study that incorporating the juvenile stage in the malaria model has great impact on the virus transmission. They proved that the carrying capacity of juvenile mosquitoes is an important parameter for controlling malaria transmission. Despite its virtues, the model of Traoré et al. did not explicitly consider the effect of temperature in the life-cycle of immature mosquitoes. This simplification is due to the difficulty to establish the global stability of the periodic solutions for vector population growth model as a three-dimensional system, especially in the use of Floquet theory in periodic linear systems and the Lyapunov method in stability theory.

In this paper, motivated by the works of Traoré et al. [40, 41] and Wang et al. [47], we formulate a mathematical model of malaria transmission in periodic environments with age structure of vector population. The model takes into account the juvenile mosquitoes which are subject to temperature variations, the contact rate between mosquitoes and humans is periodic and both the intrinsic incubation period in mosquitoes and the the intrinsic incubation period in human hosts are incorporated. Thus, the formulated model is a nonautonomous system of ordinary differential equations. It is constructed thanks to the coupling of two

subsystems. The first subsystem describes the dynamics of mosquito populations and the second subsystem describes the transmission of malaria parasites due to the interaction between mosquitoes and humans. So, this model is an extension of models studied in [41, 47]. The aim of this article is to study the dynamic properties of the model through the use of Floquet's theory of the periodic system [38], the theory of uniform persistence, the theory of chain transitive set and the limit system. Moreover, one of the major contributions of this paper is the analysis of the global attractiveness of the vector population model under the condition: $\Re_V > 1$ via the use of theory of subhomogeneous dynamical systems which is a part of theory of monotone dynamics. To our knowledge, this point has never been explicitly addressed in the literature. [39, 51].

The paper is organized as follows. In Section 2, we formulate the model. In Section 3, we introduce some basic results and then analyze the global behavior of the models. In Section 4, we perform some numerical analysis in order to support our main results. In Section 5, a brief discussion concludes the work and some possible future works are presented.

2 Mathematical model formulation

2.1 Life cycle of Anopheles

Malaria is transmitted to humans by female mosquitoes of the genus *Anopheles*. Female mosquitoes take blood meals to carry out egg production, and such blood meals are the link between the human and the mosquito. There exists approximately 430 *Anopheles* species, but only 30 – 40 can transmit malaria in the nature. In the life-cycle of mosquitoes, there are mainly two major stages: the mature stage (adult mosquitoes) and the immature stage (eggs, larvae and pupae). The immature stage is largely aquatic and the mature stage is aerial [4, 22, 23].

Adult females lay 50 - 200 eggs per oviposition. Eggs are laid singly directly on the surface of water. Some days after, depending on the temperature, they hatch to larva. *Anopheles* mosquitoes develop through four larval sizes or instars before pupating. Larvae are very small in the first instar and increase in size until reaching 5 to 6 mm by the completion of the fourth instar. They feed on organic matter and algae. Thus, after the larvae have completed moulting, depending on the temperature, they become pupae. A pupa is a resting, non-feeding development stage. This is the time the pupa changes into an adult. When development is complete, depending on the temperature the pupae skin splits and the adult mosquito emerges [1]. For convenience, we group the immature mosquitoes into a single compartment [41, 47]. Let J and N_V be the state variables representing the immature and adult mosquitoes, respectively.

(H1): We suppose that all vector population measures refer to densities of female mosquitoes.

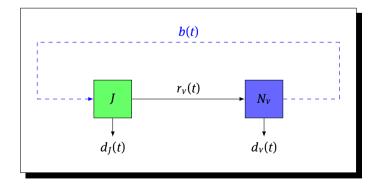


Figure 1: Transfer diagram: the solid arrows represent the transition from one class to another and the dashed arrow represents the eggs laying of female mosquitoes.

From Figure 1, the number of mosquitoes which survive from recruitment into one class, to the next is given by the following sequence of ordinary differential equations:

$$\begin{cases}
\dot{J}(t) = b(t) \left(1 - \frac{J(t)}{K(t)} \right) N_{\nu}(t) - (d_{J}(t) + r_{\nu}(t)) J(t), \\
\dot{N}_{\nu}(t) = r_{\nu}(t) J(t) - d_{\nu}(t) N_{\nu}(t),
\end{cases} \tag{1}$$

where

- K(t) is the temperature-dependent carrying capacity of juvenile mosquitoes $(K(t) \ge J(t))$. It depends on the available nutrients and breeding space,
- b(t) is the temperature-dependent eggs laying rate per female *Anophele* mosquito,
- $r_{\nu}(t)$ is the temperature-dependent maturation rate of juvenile mosquitoes to adult mosquitoes,
- $d_I(t)$ is the temperature-dependent death rate of juvenile mosquitoes,
- $d_{v}(t)$ is the temperature-dependent death rate of adult mosquitoes,
- $b(t)\left(1-\frac{J(t)}{K(t)}\right)$ represents the temperature-dependent logistic growth rate of juvenile mosquitoes.

2.2 Interaction between humans and mosquitoes

In presence of the disease, the human population is divided into four epidemiological categories representing the state variables: the susceptible class S_h , the exposed class E_h , the infectious class I_h and the immune (asymptomatic, but slightly infectious humans) class R_h . Indeed, humans in the class R_h have some immunity to the disease and do not get clinically ill, but they still harbour low levels of parasite in their blood stream and can pass the infection to the susceptible mosquitoes. This phenomenon of incomplete immunity permitting transmission is known to exist for malaria and complicates disease control strategies as the reservoir of infection [7, 17, 28]. Similarly, we divide the adult mosquito population into groups of susceptible, exposed, and infectious, denoted by S_v , E_v and I_v , respectively. Due to the short lifespan of mosquitoes, we assume that infectious mosquitoes die before they are recovered [35].

When males and females finished to mate, females will take a blood meal for their eggs development. Thus, if a susceptible mosquito bites an infectious human, the mosquito becomes infected and it moves into the class E_v . Some time after, it becomes infectious and enters into the class I_v . Hence, this infectious mosquito can infect a susceptible human through another bite. Some time after infection, the human becomes infectious, and if he is continually exposed to mosquito bites, he becomes immune. However, he can lose this immunity and become susceptible if the exposure to infection is stopped. Humans leave the total population through natural death rate and malaria death rate and mosquitoes leave the population through natural death [28].

At any time $t \ge 0$, the total size of humans and adults mosquitoes is given respectively by

$$N_h(t) = S_h(t) + E_h(t) + I_h(t) + R_h(t),$$
 (2)

$$N_{\nu}(t) = S_{\nu}(t) + E_{\nu}(t) + I_{\nu}(t). \tag{3}$$

The parameters of the model are given in Table 1.

It is assumed throughout this paper that, [41]

(H2): the mosquitoes bite only humans,

(H3): there is no direct transmission (blood transfusion or from mother to baby) of malaria,

(H4): all the new recruits are susceptible,

(H5): there is no immigration of infectious humans.

Using the standard incidence, we define respectively the infection incidence from mosquitoes to humans, $k_h(t)$ and from humans to mosquitoes, $k_v(t)$:

$$k_h(t) = c_{vh}\beta(t)\frac{I_v(t)}{N_h(t)}$$

Table 1: Parameters of the model.

Parameters	Biological descriptions
Λ	constant recruitment for humans
d_h	human natural death rate
α	transfer rate of humans from exposed class to infectious class
d_p	disease-induced death rate for humans
r_h	transfer rate of humans from infectious class to immune class
γ	per capita rate of loss of immunity for humans
$\nu_{ u}$	transfer rate of mosquitoes from exposed class to infectious class
c_{vh}	probability of transmission of infection from I_h to S_h
c_{hv}	probability of transmission of infection from I_h to S_v
\bar{c}_{hv}	probability of transmission of infection from R_h to S_v
$\beta(t)$	temperature-dependent biting rate of mosquito

and

$$k_{\nu}(t) = c_{h\nu}\beta(t)\frac{I_h(t)}{N_h(t)} + \bar{c}_{h\nu}\beta(t)\frac{R_h(t)}{N_h(t)}. \label{eq:knu}$$

where $\beta(t)$ is the average number of bites per mosquito per unit time t.

According to the above assumptions, our model of malaria transmission is represented by Figure 2.

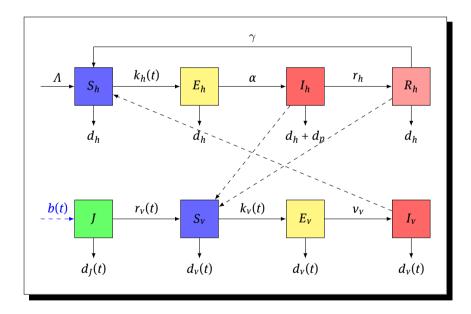


Figure 2: The black dashed arrows indicate the direction of the infection and the solid arrows represent the transition from one class to another.

2.3 Mathematical model

Following the above assumptions, the dynamics of the malaria transmission with the stage structure of mosquitoes are described by the following sequence of ordinary differential equations:

$$\begin{aligned}
\dot{J}(t) &= b(t) \left(1 - \frac{J(t)}{K(t)} \right) N_{V}(t) - (d_{J}(t) + r_{V}(t)) J(t), \\
\dot{S}_{V}(t) &= r_{V}(t) J(t) - (d_{V}(t) + k_{V}(t)) S_{V}(t), \\
\dot{E}_{V}(t) &= k_{V}(t) S_{V}(t) - (v_{V} + d_{V}(t)) E_{V}(t), \\
\dot{I}_{V}(t) &= v_{V} E_{V}(t) - d_{V}(t) I_{V}(t), \\
\dot{S}_{h}(t) &= \Lambda + \gamma R_{h}(t) - (d_{h} + k_{h}(t)) S_{h}(t), \\
\dot{E}_{h}(t) &= k_{h}(t) S_{h}(t) - (d_{h} + \alpha) E_{h}(t), \\
\dot{I}_{h}(t) &= \alpha E_{h}(t) - (d_{h} + d_{p} + r_{h}) I_{h}(t), \\
\dot{R}_{h}(t) &= r_{h} I_{h}(t) - (d_{h} + \gamma) R_{h}(t).
\end{aligned} \tag{4}$$

Furthermore, using the relation

$$S_{\nu}(t) = N_{\nu}(t) - E_{\nu}(t) - I_{\nu}(t), \forall t \geq 0.$$

system (4) reads as follows:

$$\begin{cases}
\dot{J}(t) = b(t) \left(1 - \frac{J(t)}{K(t)} \right) N_{V}(t) - (d_{J}(t) + r_{V}(t)) J(t), \\
\dot{N}_{V}(t) = r_{V}(t) J(t) - d_{V}(t) N_{V}(t). \\
\dot{E}_{V}(t) = k_{V}(t) N_{V}(t) - k_{V}(t) I_{V}(t) - (v_{V} + d_{V} + k_{V}(t)) E_{V}(t), \\
\dot{I}_{V}(t) = v_{V} E_{V}(t) - d_{V}(t) I_{V}(t), \\
\dot{S}_{h}(t) = \Lambda + \gamma R_{h}(t) - (d_{h} + k_{h}(t)) S_{h}(t), \\
\dot{E}_{h}(t) = k_{h}(t) S_{h}(t) - (d_{h} + \alpha) E_{h}(t), \\
\dot{I}_{h}(t) = \alpha E_{h}(t) - (d_{h} + d_{P} + r_{h}) I_{h}(t), \\
\dot{R}_{h}(t) = r_{h} I_{h}(t) - (d_{h} + \gamma) R_{h}(t),
\end{cases} (5)$$

with initial conditions:

$$S_h(0) > 0$$
, $E_h(0) > 0$, $I_h(0) > 0$, $R_h(0) > 0$, $I_v(0) > 0$, $I_v(0) > 0$, $I_v(0) > 0$, $I_v(0) > 0$.

Note that at any time $t \ge 0$, we have:

$$\dot{N}_h(t) = \Lambda - d_h N_h(t) - d_p I_h(t), \tag{6}$$

$$\dot{N}_{V}(t) = r_{V}(t)J(t) - d_{V}(t)N_{V}(t).$$
 (7)

3 Mathematical analysis

In this section, we focus on the global behavior of the model (5). Before starting our study, we enumerate some hypotheses and recall some fundamental results which will be used in our further analysis.

(H6): Constant parameters of the model are positive.

(H7): $d_1(t)$, $r_2(t)$, $d_2(t)$, b(t), b(t), b(t) are positive, continuous and periodic functions with the same period ω .

3.1 Preliminaries

Let w be a continuous positive ω -periodic function. We define

$$\hat{w} = \sup_{t \in [0, \omega]} w(t) \text{ and } \bar{w} = \inf_{t \in [0, \omega]} w(t).$$

Let us consider the following linear ordinary differential system:

$$\dot{z}(t) = N(t)z(t),\tag{8}$$

where N(t) is a continuous, cooperative, irreducible and ω -periodic $n \times n$ matrix function. Let $\Phi_N(t)$ be the fundamental solution matrix of (8) and $\rho(\Phi_N(\omega))$ be the spectral radius of the matrix $\Phi_N(\omega)$. By the Perron-Frobenius theorem (see [36], Theorem A.3), $\rho(\Phi_N(\omega))$ is the principal eigenvalue of $\Phi_N(\omega)$ which is associated to the eigenvector $v^* \gg 0$. The following result is useful for our subsequent comparison arguments.

Lemma 3.1. [49] Let $r = \frac{1}{\omega} \ln \rho(\Phi_N(\omega))$, then there exists a positive ω -periodic function v(t) such that $v(t)e^{rt}$ is a solution of (8).

Definition 3.1. Let X be a metric space with metric d and $g: X \to X$ a continuous map. For any $x \in X$, we denote

$$g^{n}(x) = g(g^{n-1}(x))$$
 for any integer $n > 1$ and $g^{1}(x) = g(x)$.

g is said to be compact in \mathfrak{X} if for any bounded set $H \in \mathfrak{X}$, the set

$$g(H) = \{g(x) : x \in H\}$$

is precompact in X.

g is said to be point dissipative if there is a bounded set $\mathcal{B}_0 \in \mathcal{X}$ such that \mathcal{B}_0 attracts each point in \mathcal{X} . The positive semiorbit through x^0 is defined by

$$\gamma^+(x^0) = \{x_n = g^n(x^0), n = 1, 2, ...\}$$

and the negative semiorbit through x^0 is defined as a sequence $\gamma^-(x^0) = \{x_k\}$ satisfying $g(x_{k-1}) = x_k$ for integers $k \le 0$.

The omega limit set of $\gamma^+(x^0)$ is defined by

$$\omega'(x^0) = \{c \in \mathcal{X} : \text{ there is a sequence } n_k \to \infty \text{ such that } \lim_{k \to \infty} x_{n_k} = c\}$$

and the alpha limit set of $\gamma^-(x^0)$ is defined by

$$\alpha'(x^0) = \{c \in \mathcal{X} : \text{ there is a sequence } n_k \to -\infty \text{ such that } \lim_{k \to \infty} x_{n_k} = c\}.$$

A nonempty set $C \subset X$ is said to be invariant if $g(C) \subseteq C$. A nonempty invariant set M of X is called to be isolated in X if it is the maximal invariant set in the neighborhood of itself.

For a nonempty set M of X, the set

$$W^s(M) := \{x \in \mathcal{X} : \lim_{n \to \infty} d(g^n(x), M) = 0\}$$

is called the stable set of M.

Let A_1 and A_2 be two isolated invariant sets. The set A_1 is said to be chained to set A_2 , written $A_1 \to A_2$ if there exists a full orbit through some $x \notin A_1 \cup A_2$ such that $\omega'(x) \subset A_2$ and $\alpha'(x) \subset A_1$.

A finite sequence $\{M_1, ..., M_k\}$ of isolated invariant sets is called a chain if $M_1 \to M_2 \to M_3 \to ... \to M_k$, and if $M_k = M_1$ then the chain is called a cycle. Let \mathfrak{X}_0 be a nonempty open set of \mathfrak{X} . We denote

$$\partial \mathcal{X}_0 = \mathcal{X} \setminus \mathcal{X}_0$$
, and $M_{\partial} := \{x \in \partial \mathcal{X}_0 : g^n(x) \in \partial \mathcal{X}_0, \forall n \ge 0\}.$

Lemma 3.2. [45, 51] Let $g: X \to X$ be a continuous map. Assume that the following conditions hold:

- (i) g is compact and point dissipative, and $g(\partial X_0) \subset \partial X_0$,
- (ii) there exists a finite sequence $\mathcal{M} = \{M_1, ..., M_k\}$ of compact and isolated invariant sets such that
 - (a) $M_i \cap M_j = \emptyset$ for any i, j = 1, 2, ..., k and $i \neq j$;
 - (b) $\cup_{x\in M_{\partial}}\omega'(x)\subset \cup_{i=1}^k M_i$;
 - (c) no subset of M forms a cycle in ∂X_0 ;
 - (d) $W^s(M_i) \cap X_0 = \emptyset$ for each $1 \le i \le k$.

Then, g is uniformly persistent with respect to $(\mathfrak{X}_0, \partial \mathfrak{X}_0)$, that is, there exists a positive constant η such that $\lim \inf_{n \to \infty} d(g^n(x), \partial \mathfrak{X}_0) \ge \eta$.

3.2 Analysis of the mosquito population model

Let us consider the following system:

$$\begin{cases}
\dot{J}(t) = b(t) \left(1 - \frac{J(t)}{K(t)} \right) N_{V}(t) - (d_{J}(t) + r_{V}(t)) J(t), \\
\dot{N}_{V}(t) = r_{V}(t) J(t) - d_{V}(t) N_{V}(t).
\end{cases} \tag{9}$$

System (9) can be written as follows:

$$\begin{cases}
\dot{Z}_1(t) = f_1(t, Z_1(t)), \\
Z_1(0) > 0,
\end{cases}$$
(10)

where

$$Z_1(t) = (J(t), N_{\nu}(t))^T.$$

3.2.1 Positivity and boundedness of solutions

Lemma 3.3. For any positive initial condition $\varphi_1 = (J(0), N_v(0))$, the system (9) admits a unique positive solution $u_1(t, \varphi_1)$ for all $t \ge 0$. Moreover, the closed set

$$\Delta_1 = \left\{ (J, N_v) \in \mathbb{R}^2_+ : J \leq \hat{K}, \ N_v \leq \frac{\hat{r}_v}{\bar{d}_v} \hat{K} \right\}$$

is positively invariant and attracting under the flow described by system (9).

Proof. For any positive initial condition φ_1 , the function $f_1(t, \varphi_1)$ is continuous in (t, φ_1) and Lipschitzian in φ_1 . So, thanks to Theorem 2.2.1 and 2.2.3 of Hale and Verduyn Lunel [11], the system (9) has a unique solution $u_1(t, \varphi_1)$ on its maximal interval $[0, t_{max})$ of existence.

In addition, for all $t \ge 0$, let

$$e(t) = \min \{J(t), N_{\nu}(t)\}.$$

Suppose that there exists $t_1 > 0$ such that $e(t_1) \notin \mathbb{R}_+^*$ and e(t) > 0 for all $t \in [0, t_1)$.

If $e(t) = N_v(t)$ then, from the second equation of system (9), we have

$$\dot{N}_{\nu}(t) > -d_{\nu}(t)N_{\nu}(t).$$

It then follows that

$$N_{\nu}(t_1) > N_{\nu}(0) \exp \left[- \int_{0}^{t_1} d_{\nu}(t) dt \right] > 0,$$

which leads to a contradiction.

If e(t) = J(t) then, from the first equation of system (9), we have

$$\dot{J}(t) > -(r_{\nu}(t) + d_{I}(t))J(t).$$

It then follows that

$$J(t_1) > E(0) \exp \left[- \int_0^{t_1} (r_v(t) + d_J(t)) dt \right] > 0,$$

which leads to a contradiction.

Hence, the solution $u_1(t, \varphi_1)$ is positive. Moreover, let

$$Q_1 = \{(J, N_v) \in \mathbb{R}^2 : J \leq \hat{K}\} \text{ and } Q_2 = \{(J, N_v) \in \mathbb{R}^2 : N_v \leq \frac{\hat{r}_v}{\bar{d}_v} \hat{K}\}.$$

Let us show that Q_1 and Q_2 are positively invariant.

Let

$$G_1(J, N_V) = J, G_2(J, N_V) = N_V.$$

It is clear that G_1 and G_2 are differentiable and we have:

$$\nabla G_1(J, N_V) = (1, 0)$$
 and $\nabla G_2(J, N_V) = (0, 1)$.

Moreover, let us define

$$\Gamma_1 = \{(J, N_v) \in \mathbb{R}^2_+ : J = \hat{K}\} \text{ and } \Gamma_2 = \{(J, N_v) \in \mathbb{R}^2_+ : N_v = \frac{\hat{r}_v}{\bar{d}_v} \hat{K}\}.$$

Thus, if:

- $x \in \Gamma_1$, then $\langle f_1(t,x), \nabla G_1(x) \rangle \leq -(r_v(t) + d_J(t)) < 0$.
- $x \in \Gamma_2$, then $\langle f_1(t,x), \nabla G_2(x) \rangle = r_v(t)P(t) d_v(t)\frac{\hat{r}_v}{\bar{d}_v}\bar{K} \le \hat{r}_v(J(t) \hat{K}) \le 0$.

So, thanks to Theorem 3.1 in [39], we conclude that the compact set Δ_1 is positively invariant and then the solution $u_1(t, \varphi_1)$ is bounded. It then follows that $t_{max} = +\infty$.

3.2.2 Threshold dynamics: the vector reproduction ration, \Re_{ν}

We introduce the basic reproduction ratio for model (9) according to the theory developed by Wang and Zhao [46] which is a generalization of Van den Driesshe and James Watmough method, [43]. Thus, linearizing system (9) at the unique mosquito-free equilibrium $H^+ = (0, 0)$, we obtain the following system:

$$\begin{cases}
\dot{J}(t) = b(t)N_{V}(t) - (r_{V}(t) + d_{J}(t))J(t), \\
\dot{N}_{V}(t) = r_{V}(t)J(t) - d_{V}(t)N_{V}(t),
\end{cases}$$
(11)

which can be written as follows:

$$\dot{U}_1(t) = \left(F_1(t) - V_1(t)\right)U_1(t)$$

with $U_1(t) = (J(t), N_v(t))^T$ and the matrix $F_1(t)$, $V_1(t)$ defined as follows:

$$V_1(t)=\left(\begin{array}{cc} r_v(t)+d_J(t) & 0\\ -r_v(t) & d_v(t) \end{array}\right),\ F_1(t)=\left(\begin{array}{cc} 0 & b(t)\\ 0 & 0 \end{array}\right).$$

For all $t \ge s$, let Y(t, s) be the evolution operator of the linear periodic system

$$\dot{y} = -V_1(t)y.$$

For each $s \in \mathbb{R}$, the 2 × 2 matrix Y(t, s) satisfies the equation

$$\dot{Y}(t,s) = -V_1(t)Y(t,s), \forall t \ge s, \ Y(s,s) = I,$$
 (12)

where *I* is the 2×2 identity matrix.

Let C_{ω} be the ordered Banach space of all ω -periodic functions from \mathbb{R} to \mathbb{R}^2 which is equipped with the maximum norm $\|.\|$ and the positive cone

$$C_{\omega}^+ := \{ \phi \in C_{\omega} : \phi(t) \ge 0, \forall t \in \mathbb{R} \}.$$

Suppose $\phi(s) \in C_{\omega}$ is the initial distribution of juvenile and adults mosquitoes. Then $F_1(s)\phi(s) \in C_{\omega}$ is the distribution of new juvenile mosquitoes in the breeding habits produced by the adult ones which were introduced at time s. Hence, for any $t \ge s$, $Y(t,s)F_1(s)\phi(s)$ is the distribution of those mosquitoes which were newly born into the juvenile mosquito compartment at time s and remain alive. Thus,

$$\psi(t) = \int_{0}^{t} Y(t,s)F_1(s)\phi(s)ds = \int_{0}^{\infty} Y(t,t-a)F_1(t-a)\phi(t-a)da, \ a \in [0,\infty)$$

is the distribution of new eggs at time t, hatched by all female adult mosquitoes $\phi(s)$ introduced at the previous time s.

Let $\mathcal{L}: C_{\omega} \longrightarrow C_{\omega}$ be the linear operator defined by

$$(\mathcal{L}\phi)(t) = \int_{0}^{\infty} Y(t, t-a)F_1(t-a)\phi(t-a)da, \ \forall t \in \mathbb{R}, \ \phi \in C_{\omega}.$$

Then, the vector reproduction ratio is $\mathcal{R}_{\nu} := \rho(\mathcal{L})$, the spectral radius of \mathcal{L} .

Lemma 3.4. Let $\tilde{\mathbb{R}}_{V}$ be the vector reproduction ratio associated to the temperature-dependent egg laying rate, $\tilde{b}(t)$ and κ be a positive real. Then we have

$$\tilde{b}(t) = \kappa b(t) \Longrightarrow \tilde{\mathcal{R}}_{V} = \kappa \mathcal{R}_{V}$$

Proof. Let us consider the following linear ω -periodic system:

$$\dot{w}_{1}(t) = \left[\frac{1}{\xi}F_{1}(t) - V_{1}(t)\right]w_{1}(t), \ \forall t \in \mathbb{R}_{+}, \ \xi \in (0, \infty). \tag{13}$$

Let $W_1(t, s, \xi)$, $t \ge s$, $s \in \mathbb{R}$, be the evolution operator of system (13) on \mathbb{R}^2 . It is obvious that

$$W_1(t, 0, 1) = \Phi_{F_1 - V_1}(t), \forall t \ge 0.$$

If $\tilde{b}(t) = \kappa b(t)$ then, the linear system (13) becomes:

$$\dot{w}_1(t) = \left\lfloor \frac{\kappa}{\xi} F_1(t) - V_1(t) \right\rfloor w_1(t), \ \forall t \in \mathbb{R}_+, \, \xi \in (0, \infty).$$

Let $\tilde{F}_1(t) = \kappa F_1(t)$, $\tilde{V}_1(t) = V_1(t)$ and $\tilde{W}_1(\omega, 0, \xi)$ the monodromy matrix of the following system:

$$\dot{\tilde{w}}_1(t) = \left[\frac{1}{\xi}\tilde{F}_1(t) - \tilde{V}_1(t)\right]\tilde{w}_1(t), \ \forall t \in \mathbb{R}_+, \xi \in (0, \infty).$$

It is evident that

$$\tilde{W}_1(\omega,0,\xi) = W_1\left(\omega,0,\frac{\xi}{\kappa}\right).$$

Thus, thanks to Theorem 2.1 in [46], we have

$$\rho(\tilde{W}_1(\omega,0,\tilde{\mathcal{R}}_{v})) = 1 \Longleftrightarrow \rho\left(W_1\left(\omega,0,\frac{\tilde{\mathcal{R}}_{v}}{\kappa}\right)\right) = 1.$$

Hence, $\tilde{\mathbb{R}}_{\nu} = \kappa \mathbb{R}_{\nu}$.

3.2.3 Global stability of mosquito-free equilibrium

Lemma 3.5. [46]

- i) $\Re_{V} = 1$ if and only if $\rho(\Phi_{F_1-V_1}(\omega)) = 1$.
- ii) $\Re_{V} < 1$ if and only if $\rho(\Phi_{F_1-V_1}(\omega)) < 1$.
- iii) $\Re_V > 1$ if and only if $\rho(\Phi_{F_1-V_1}(\omega)) > 1$.

Hence, if \Re_{V} < 1 then the mosquito-free equilibrium is locally asymptotically stable.

Theorem 3.1. The mosquito-free equilibrium $H^+ = (0, 0)$ is globally attractive if $\Re_V < 1$.

Proof. For all $t \ge 0$, since $J(t) \le K(t)$, we have $1 - \frac{J(t)}{K(t)} \le 1$. Thus, system (9) becomes

$$\begin{cases} \dot{J}(t) \leq b(t)N_{\nu}(t) - (r_{\nu}(t) + d_{J}(t))J(t), \\ \\ \dot{N}_{\nu}(t) = r_{\nu}(t)J(t) - d_{\nu}(t)N_{\nu}(t). \end{cases}$$

Let us consider the following auxiliary system:

$$\begin{cases} \dot{\tilde{J}}(t) = b(t)N_{\nu}(t) - (r_{\nu}(t) + d_{J}(t))J(t), \\ \dot{\tilde{N}}_{\nu}(t) = r_{\nu}(t)J(t) - d_{\nu}(t)N_{\nu}(t). \end{cases}$$

For the convenience, we will rewrite it as follows

$$\dot{\tilde{R}}_{1}(t) = (F_{1}(t) - V_{1}(t))\tilde{R}_{1}(t), \text{ with } \tilde{R}_{1}(t) = (\tilde{J}(t), \tilde{N}_{V}(t))^{T}.$$
(14)

If $\Re_{V}<1$, then from Lemma 3.5, $\rho(\Phi_{F_1-V_1}(\omega))<1$. By Lemma 3.1, there exists a positive ω -periodic function v(t) such that $\tilde{R}_1(t)=v(t)e^{rt}$ with $r=\frac{1}{\omega}\ln\rho(\Phi_{F_1-V_1}(\omega))$. Since the function v(t) is bounded and $\rho(\Phi_{F_1-V_1}(\omega))<1$ then, we have r<0 and $\tilde{R}_1(t)\to 0$ as $t\to\infty$. By applying the comparison theorem [19] on system (14), we get

$$\lim_{t\to+\infty}(J(t),N_{\nu}(t))=(0,0).$$

It then follows that the mosquito-free equilibrium H^+ is globally attractive which complete the proof.

3.2.4 Persistence of mosquitoes

Let us define the following sets:

$$\mathcal{X} := \mathbb{R}^{2}_{+},$$
 $\mathcal{X}_{0} := \{(J, N_{v}) \in \mathcal{X} : J > 0, N_{v} > 0\},$
 $\partial \mathcal{X}_{0} := \{(E, N_{v}) \in \mathcal{X} : JN_{v} = 0\}.$

Let $\Phi(t)$ be the periodic semiflow generated by periodic the system (9) and $S_1: \mathcal{X} \longrightarrow \mathcal{X}$ the Poincaré map associated with system (9), namely:

$$\begin{array}{lcl} S_1(\varphi_1) & = & \varPhi(\omega)\varphi_1 = u_1(\omega,\varphi_1), \ \forall \varphi_1 \in \mathcal{X}. \\ S_1^m(\varphi_1) & = & \varPhi(m\omega)\varphi_1 = u_1(m\omega,\varphi_1), \ \forall m \geq 0. \end{array}$$

It then follows that the study of the dynamics (9) reduces to that of the discrete dynamical system $\{S_1^m\}$ on \mathcal{X} . Otherwise, note that for all $t \ge 0$ and $\varphi_1 \in \mathcal{X}_0$,

$$\Phi(t)(\varphi_1)=u_1(t,\varphi_1)\in \mathfrak{X}_0.$$

Thus,

$$\Phi(t)(\mathfrak{X}_0)\subset\mathfrak{X}_0,\ \forall t\geq 0.$$

So, X_0 and ∂X_0 are positively invariant. Therefore, Lemma 3.1 implies that discrete-time system S_1 is point dissipative.

Lemma 3.6. *If* $\Re_{V} > 1$, *there exists* $\delta > 0$ *such that when*

$$\|\varphi_1 - H^+\| \le \delta$$
, for any $\varphi_1 \in \mathcal{X}_0$,

one has

$$\limsup_{k\to\infty}d\left(S_1^k(\varphi_1),H^+\right)\geq\delta.$$

Proof. Suppose by contradiction that

$$\limsup_{k\to\infty} d(S_1^k(\varphi_1), H^+) < \delta \text{ for some } \varphi_1 \in \mathcal{X}_0.$$
 (15)

Then, there exists an integer k_2 such that for all $k \ge k_2$, $d(S_1^k(\varphi_1), H^+) < \delta$. By the continuity of the solution $u_1(t, \varphi_1)$, we have

$$\left\|u_1\left(t,S_1^k(\varphi_1)\right)-u_1(t,H^+)\right\|\leq \alpha, \forall t\geq 0 \text{ and } \alpha>0.$$

Let $t = k\omega + t_1$, where $t_1 \in [0, \omega]$ and $k = \left[\frac{t}{\omega}\right]$. $\left[\frac{t}{\omega}\right]$ is the greatest integer less than or equal to $\frac{t}{\omega}$. If $\|\varphi_1 - H^+\| \le \delta$, then we get

$$\begin{aligned} \|u_1(t,\varphi_1) - u_1(t,H^+)\| &= \|u_1(t_1 + k\omega,\varphi_1) - u_1(t_1 + k\omega,H^+)\| \\ &= \|u_1(t_1,S_1^k(\varphi_1)) - u_1(t_1,H^+)\| \le \alpha, \text{ for any } t \ge 0. \end{aligned}$$

It then follows that for all $t \ge 0$, $0 \le J(t) \le \alpha$. Thus, there exists $\alpha^* > 0$ such that

$$1 - \frac{J(t)}{K(t)} \ge 1 - \alpha^*.$$

Furthermore, the system (9) can be rewritten as follows:

$$\begin{cases}
\dot{J}(t) \ge b(t)(1 - \alpha^*)N_{\nu}(t) - (r_{\nu}(t) + d_{J}(t))J(t), \\
\dot{N}_{\nu}(t) = r_{\nu}(t)J(t) - d_{\nu}(t)N_{\nu}(t).
\end{cases}$$
(16)

Let us consider the following auxiliary linear system

$$\dot{p}(t) = M_{\alpha^*}(t)p(t),\tag{17}$$

where $p(t) = (J(t), N_v(t))^T$ and $M_{\alpha^*}(t)$ is a 2 × 2 matrix defined by

$$M_{\alpha^*}(t) = \begin{pmatrix} -(r_{\nu}(t) + d_J(t)) & b(t)(1-\alpha^*) \\ r_{\nu}(t) & -d_{\nu}(t) \end{pmatrix}.$$

Since $\lim_{\alpha^* \to 0^+} \Phi_{M_{\alpha^*}}(\omega) = \Phi_{F_1 - V_1}(\omega)$ and by the continuity of the spectral radius, we have $\lim_{\alpha^* \to 0^+} \rho\left(\Phi_{M_{\alpha^*}}(\omega)\right) = \rho(\Phi_{F_1 - V_1}(\omega))$. From Lemma 3.5 if $\mathcal{R}_{V} > 1$, then $\rho(\Phi_{F_1 - V_1}(\omega)) > 1$ and $\lim_{\alpha^* \to 0^+} \rho\left(\Phi_{M_{\alpha^*}}(\omega)\right) > 1$. It then follows that there exists $\alpha_1 > 0$ such that $\rho\left(\Phi_{M_{\alpha^*}}(\omega)\right) > 1$, for all $\alpha \in [0,\alpha_1]$. From Lemma 3.1, there exists a positive ω -periodic function v(t) such that $p(t) = v(t)e^{rt}$ with $r = \frac{1}{\omega} \ln \rho\left(\Phi_{M_{\alpha^*}}(\omega)\right)$. Since $\rho\left(\Phi_{M_{\alpha^*}}(\omega)\right) > 1$ and the function v(t) is bounded, then r > 0 and $p(t) \to \infty$ as $t \to \infty$. By applying the comparison theorem on system (17), we get

$$\lim_{t\to\infty}|(J(t),N_{\nu}(t))|=\infty,$$

which is in contradiction with (15).

Theorem 3.2. If $\Re_{\nu} > 1$, there exists $\eta_1 > 0$ such that any solution $(J(t), N_{\nu}(t))$ with initial condition $\varphi_1 \in \Im_0$ satisfies

$$\liminf_{t\to\infty}J(t)\geq\eta_1,\ \liminf_{t\to\infty}N_{\nu}(t)\geq\eta_1$$

and system (9) has at least one positive periodic solution.

Proof. Denote

$$M_{\partial} = \{ \varphi_1 \in \partial \mathcal{X}_0 : S_1^k(\varphi_1) \in \partial \mathcal{X}_0, k \geq 0 \}.$$

At first, we prove that $M_{\partial} = \{(0, 0)\}.$

It is clear that $\{(0,0)\}\subset M_{\partial}$. So, we only need to show that $M_{\partial}\subset\{(0,0)\}$, which means that for any initial condition $\varphi_1\in\partial\mathcal{X}_0$,

$$I(k\omega) = 0$$
, and $N_{\nu}(k\omega) = 0$, $\forall k \ge 0$.

Let $\varphi_1 \in \partial \mathcal{X}_0$. Suppose by contradiction that there exists an integer $k_1 \ge 0$ such that $(J(k_1\omega), N_v(k_1\omega))^T > 0$. Using the constant variation formula, it yields that

$$J(t) = \exp\left(-\int_{0}^{t} \pi(s)ds\right) \left[J(0) + \int_{0}^{t} b(s)N_{\nu}(s) \exp\left(\int_{0}^{s} \pi(\tau)d\tau\right)ds\right], \tag{18}$$

$$N_{V}(t) = \exp\left(-\int_{0}^{t} d_{V}(s)ds\right) \left[N_{V}(0) + \int_{0}^{t} r_{V}(s)J(s)\exp\left(-\int_{0}^{s} d_{V}(\tau)d\tau\right)ds\right], \tag{19}$$

where

$$\pi(t) = \frac{b(t)}{K(t)} N_{\nu}(t) + r_{\nu}(t) + d_{J}(t), \text{ for all } t \ge 0.$$

Thus, taking $k_1\omega$ as the initial time in (18) – (19), we get

$$J(t) > 0$$
 and $N_{\nu}(t) > 0$, for all $t > k_1 \omega$,

which contradicts the fact that $\partial \mathcal{X}_0$ is positively invariant so $M_{\partial} \subset \{(0,0)\}$. Hence, it then follows that $M_{\partial} = \{(0,0)\}$.

The equality $M_{\partial} = \{(0,0)\}$ implies that H^+ is a fixed point of S_1 and acyclic in M_{∂} . Thus, every solution in M_{∂} approaches to H^+ . Moreover, Lemma 3.6 implies that H^+ is an isolate invariant set in \mathcal{X} and $W^s(H^+) \cap \mathcal{X}_0 = \mathcal{X}_0$

 \emptyset . Therefore, by Lemma 3.2, we finally obtain that S_1 is uniformly persistent with respect to $(\mathfrak{X}_0, \partial \mathfrak{X}_0)$. So, the periodic semiflow $\Phi(t)$ is also uniformly persistent. It then follows that there exists $\eta_1 > 0$ such that any solution $(J(t), N_V(t))$ with initial condition $(J(0), N_V(0)) \in \mathfrak{X}_0$ satisfies

$$\liminf_{t\to\infty}J(t)\geq\eta_1,\ \liminf_{t\to\infty}N_{\nu}(t)\geq\eta_1.$$

Besides, thanks to Theorem 1.3.6 in [51], system (9) has at least one periodic solution $u_1^*(t, \varphi_1^*)$ with $\varphi_1^* = (J^*(0), N_v^*(0)) \in \mathcal{X}_0$. Since $\varphi_1^* \in \mathcal{X}_0$, then from equations (18)–(19) we obtain that $J^*(t) > 0$ and $N_v^*(t) > 0$ for all $t \ge 0$. Thus, the ω -periodic solution $u_1^*(t, \varphi_1^*)$ is positive.

Theorem 3.3. If $\mathcal{R}_{\nu} > 1$, then the positive ω -periodic solution $(J^{*}(t), N_{\nu}^{*}(t))$ is globally attractive for system (9) in $\Delta_{1} \setminus (0, 0)$.

Proof. Let $(h_1(t), h_2(t))$ and $(\tilde{h}_1(t), \tilde{h}_2(t))$ be two solutions of system (9) with initial conditions $(h_1(0), h_2(0))$ and $(\tilde{h}_1(0), \tilde{h}_2(0))$ respectively, such that $(h_1(0), h_2(0)) \ge (\tilde{h}_1(0), \tilde{h}_2(0))$. Then thanks to the comparison theorem for cooperative ordinary differential systems we have $(h_1(t), h_2(t)) \ge (\tilde{h}_1(t), \tilde{h}_2(t))$ for all $t \ge 0$, that is $\Phi(t)$ is monotone for each $t \ge 0$.

Now, we need to show that $\Phi(t)$ is strongly monotone for each $t \ge 0$. Indeed, let $(h_1(0),h_2(0)) > (\tilde{h}_1(0),\tilde{h}_2(0))$. Then, the comparison theorem for cooperative ordinary differential systems implies that $(h_1(t),h_2(t)) > (\tilde{h}_1(t),\tilde{h}_2(t))$.

(i) Suppose $h_1(0) > \tilde{h}_1(0)$. Let

$$f_{1,1}(t,z(t)) = b(t) \left[1 - \frac{z(t)}{K(t)}\right] \tilde{h}_2(t) - (d_J(t) + r_v(t))z(t)$$

and

$$f_{1,2}(t,z(t)) = r_v(t)\tilde{h}_1(t) - d_v(t)z(t).$$

Since

$$\begin{split} \dot{h}_1(t) &= b(t) \left[1 - \frac{h_1(t)}{K(t)} \right] h_2(t) - (d_J(t) + r_v(t)) h_1(t) \\ &\geq b(t) \left[1 - \frac{h_1(t)}{K(t)} \right] \tilde{h}_2(t) - (d_J(t) + r_v(t)) h_1(t) = f_{1,1}(t, h_1(t)), \end{split}$$

and

$$\begin{split} \dot{h}_2(t) &= r_v(t)\tilde{h}_1(t) - d_v(t)h_2(t) \\ &\geq r_v(t)\tilde{h}_1(t) - d_v(t)h_2(t) = f_{1,2}(t, h_1(t)), \end{split}$$

then we have

$$\dot{h}_1(t) - f_{1,1}(t, h_1(t)) \ge 0 = \dot{\tilde{h}}_1(t) - f_{1,1}(t, \tilde{h}_1(t)), \ \forall t \ge 0,$$
(20)

$$\dot{h}_2(t) - f_{1,2}(t, h_2(t)) \ge 0 = \dot{\tilde{h}}_2(t) - f_{1,2}(t, \tilde{h}_2(t)), \ \forall t \ge 0.$$
 (21)

Since $h_1(0) > \tilde{h}_1(0)$ and $h_2(0) \ge \tilde{h}_2(0)$, then by applying Walter's theorem [44] on equations (20) and (21), we obtain $h_1(t) > \tilde{h}_1(t)$ and $h_2(t) > \tilde{h}_2(t)$, $\forall t \ge 0$.

(ii) If $h_2(0) > \tilde{h}_2(0)$, then the similar arguments as above are applicable. Finally, we conclude that $\Phi(t)$ is strongly monotone for each t > 0.

Moreover, for any initial condition $l = (l_1, l_2) \in \mathcal{X}_0$ and $\lambda \in [0, 1]$, let $\phi(t, l)$ and $\phi(t, \lambda l)$ be the solutions of system (9) satisfying $\phi(0) = l$ and $\phi(0) = \lambda l$ respectively. Denote $\mu_1(t) = \lambda \phi(t, l)$ and $\mu_2(t) = \phi(t, \lambda l)$, we have

$$\dot{\mu}_1(t) = \lambda \dot{\phi}(t, l) = \lambda f_1(t, \phi(t, l)) \le f_1(t, \lambda \phi(t, l)).$$

Since $\mu_1(0) = \mu_2(0) = \lambda l$, then by the comparison principle, we have $\lambda \phi(t, l) \le \phi(t, \lambda l)$, $\forall t \ge 0$. Thus, $\Phi(t)$ is subhomogeneous. In addition, it is easy to see that Φ is strictly subhomogeneous. By the continuity and

differentiability of solutions with respect to initial conditions, it follows that Φ is differentiable at zero and the Frechét derivative $D\Phi(0) = \Phi$.

Thanks to Theorem 2.3.4 in [51], the system admits a unique positive ω -periodic solution $(J^*(t), N_v^*(t))$, which is globally attractive in $\Delta_1 \setminus \{(0,0)\}$ if $\rho(D\phi(0)) > 1$. Since $\rho(D\phi(0)) - 1$ have the same sign as $\mathcal{R}_v - 1$ (see Theorem 2.2 in [46]), then we can conclude.

3.3 Analysis of transmission model

The previous results (see Theorem 3.1 and Theorem 3.3) indicate that the mosquito population will die out if $\mathcal{R}_{\nu} < 1$, and if $\mathcal{R}_{\nu} > 1$, the mosquito population persists and the system admits a unique positive ω -periodic solution ($f^{\star}(t)$, $N^{\star}_{\nu}(t)$) which is globally asymptotically stable. Consequently, when the parasite is introduced in the human population, the dynamics of virus transmission model due to the interaction between humans and mosquitoes is given by the following limiting system:

$$\begin{cases}
\dot{S}_{h}(t) = \Lambda + \gamma R_{h}(t) - (d_{h} + k_{h}(t))S_{h}(t), \\
\dot{E}_{h}(t) = k_{h}(t)S_{h}(t) - (d_{h} + \alpha)E_{h}(t), \\
\dot{I}_{h}(t) = \alpha E_{h}(t) - (d_{h} + d_{p} + r_{h})I_{h}(t), \\
\dot{R}_{h}(t) = r_{h}I_{h}(t) - (d_{h} + \gamma)R_{h}(t), \\
\dot{E}_{v}(t) = k_{v}(t)N_{v}^{*}(t) - k_{v}(t)I_{v}(t) - (v_{v} + d_{v}(t) + k_{v}(t))E_{v}(t), \\
\dot{I}_{v}(t) = v_{v}E_{v}(t) - d_{v}(t)I_{v}(t).
\end{cases} (22)$$

System (22) can be written as follows:

$$\begin{cases}
\dot{Z}_{2}(t) = f_{2}(t, Z_{2}(t)), \\
Z_{2}(0) > 0,
\end{cases}$$
(23)

where

$$Z_2(t) = (S_h(t), E_h(t), I_h(t), R_h(t), E_v(t), I_v(t))^T.$$

3.3.1 Positivity and boundedness of solutions

Lemma 3.7. For any positive initial condition $\varphi_2 = (S_h(0), E_h(0), I_h(0), R_h(0), E_v(0), I_v(0))$, system (22) admits a unique positive solution $u_2(t, \varphi_2)$ for all $t \ge 0$. Moreover, the closed set

$$\Delta_2 = \left\{ (S_h, E_h, I_h, R_h, E_v, I_v) \in \mathbb{R}^6_+ : N_h \leq \frac{\Lambda}{d_h}, E_v + I_v \leq \frac{\hat{r}_v}{\bar{d}_v} \hat{K} \right\}$$

is positively invariant and attracting under the flow described by system (22).

Proof. For any positive initial condition φ_2 , the function $f_2(t, \varphi_2)$ is continuous in (t, φ_2) and Lipschitzian in φ_2 . So, thanks to Theorem 2.2.1 and 2.2.3 of Hale and Verduyn Lunel [11], system (22) has a unique solution $u_2(t, \varphi_2)$ on its maximal interval $[0, t_{max})$ of existence.

In addition, for all $t \ge 0$, let

$$e'(t) = \min \{S_h(t), E_h(t), I_h(t), R_h(t), E_v(t), I_v(t)\}.$$

Suppose that there exists $t_1 > 0$ such that $e'(t_1) \notin \mathbb{R}_+^*$ and e'(t) > 0 for all $t \in [0, t_1)$.

If $e'(t) = S_h(t)$ then, from the second equation of system (22), we have

$$\dot{E}_h(t) > -(d_h + \alpha)E_h(t).$$

It then follows that

$$E_h(t_1) > E_h(0) \exp \left[-(d_h + \alpha)t_1 \right] > 0,$$

which leads to a contradiction.

If $e'(t) = E_h(t)$ then, from the third equation of system (22), we have

$$\dot{I}_h(t) > -(d_h + d_p + r_h)E_h(t).$$

It then follows that

$$I_h(t_1) > I_h(0) \exp \left[-(d_h + d_p + r_h)t_1 \right] > 0,$$

which leads to a contradiction.

If $e'(t) = I_h(t)$ then, from the fourth equation of system (22), we have

$$\dot{R}_h(t) > -(d_h + \gamma)R_h(t).$$

It then follows that

$$R_h(t_1) > I_h(0) \exp \left[-(d_h + \gamma)t_1 \right] > 0,$$

which leads to a contradiction.

Similar contradictions can be obtained if $e'(t) = I_v(t)$ and $e'(t) = E_v(t)$. Hence, the solution $u_2(t, \varphi_2)$ is positive. Moreover, let

$$Q_{3} = \left\{ (S_{h}, E_{h}, I_{h}, R_{h}, E_{v}, I_{v}) \in \mathbb{R}^{6} : S_{h} + E_{h} + I_{h} + R_{h} \leq \frac{\Lambda}{d_{h}} \right\},$$

$$Q_{4} = \left\{ (S_{h}, E_{h}, I_{h}, R_{h}, E_{v}, I_{v}) \in \mathbb{R}^{6} : E_{v} + I_{v} \leq \frac{\hat{r}_{v}}{\bar{d}_{v}} \hat{K} \right\}$$

and

$$G_3(S_h, E_h, I_h, R_h, E_v, I_v) = S_h + E_h + I_h + R_h, \quad G_4(S_h, E_h, I_h, R_h, E_v, I_v) = E_v + I_v.$$

It is clear that G_3 and G_4 are differentiable and we have:

$$\nabla G_3(S_h, E_h, I_h, R_h, E_V, I_V) = (1, 1, 1, 1, 0, 0)$$
 and $\nabla G_4(S_h, E_h, I_h, R_h, E_V, I_V) = (0, 0, 0, 0, 1, 1)$.

Moreover, let

$$\Gamma_3 = \left\{ (S_h, E_h, I_h, R_h, E_v, I_v) \in \mathbb{R}^6 : S_h + E_h + I_h + R_h = \frac{\Lambda}{d_h} \right\}$$

and

$$\Gamma_4 = \left\{ (S_h, E_h, I_h, R_h, E_{\nu}, I_{\nu}) \in \mathbb{R}^6 : E_{\nu} + I_{\nu} = \frac{\hat{r}_{\nu}}{\bar{d}_{\nu}} \hat{K} \right\}.$$

We have:

$$\begin{split} \left\langle f_2(t,x),\nabla G_3(x)\right\rangle &= \Lambda - d_h N_h - d_p I_h \\ &\leq \Lambda - d_h N_h. \\ \left\langle f_2(t,x),\nabla G_4(x)\right\rangle &= k_\nu(t)(N_\nu^\star(t) - E_\nu - I_\nu) - d_\nu(t)(E_\nu + I_\nu) \\ &\leq k_\nu(t) \left(\frac{\hat{r}_\nu}{\hat{d}_\nu} \hat{K} - (E_\nu + I_\nu)\right) - d_\nu(t)(E_\nu + I_\nu). \end{split}$$

Thus, if:

- $x \in \Gamma_3$ then, $\langle f_2(t,x), \nabla G_3(x) \rangle \leq 0$,
- $x \in \Gamma_4$ then, $\langle f_2(t,x), \nabla G_4(x) \rangle \leq 0$.

So, we conclude that the compact set Δ_2 is positively invariant and then all the solutions are positive and bounded (see [6]).

3.3.2 Threshold dynamics: the basic reproduction ratio, \mathcal{R}_0

Using the same method as mentioned above, we derive the basic reproduction associated to system (22). Hence, by linearizing system (22) at the disease-free equilibrium $W_0 = (S_h^*, 0, 0, 0, 0, 0, 0)$, we obtain the following system:

$$\begin{cases}
\dot{E}_{h}(t) = c_{\nu h}\beta(t)I_{\nu}(t) - (d_{h} + \alpha)E_{h}(t), \\
\dot{I}_{h}(t) = \alpha E_{h}(t) - (d_{h} + d_{p} + r_{h})I_{h}(t), \\
\dot{R}_{h}(t) = r_{h}I_{h}(t) - (d_{h} + \gamma)R_{h}(t), \\
\dot{E}_{\nu}(t) = c_{h\nu}\beta(t)\frac{N_{\nu}^{*}(t)}{S_{h}^{*}}I_{h}(t) + \bar{c}_{h\nu}\beta(t)\frac{N_{\nu}^{*}(t)}{S_{h}^{*}}R_{h}(t) - (\nu_{\nu} + d_{\nu}(t))E_{\nu}(t), \\
\dot{I}_{\nu}(t) = \nu_{\nu}E_{\nu}(t) - d_{\nu}(t)I_{\nu}(t).
\end{cases} \tag{24}$$

System (24) can be written as follows:

$$\dot{U}_2(t) = (F_2(t) - V_2(t)) U_2(t)$$

with

$$U_2(t) = (E_h(t), I_h(t), R_h(t), E_v(t), I_v(t))^T$$

and the matrix $F_2(t)$, $V_2(t)$ defined as follows:

$$V_2(t) = \begin{pmatrix} d_h + \alpha & 0 & 0 & 0 & 0 \\ -\alpha & d_h + d_p + r_h & 0 & 0 & 0 \\ 0 & -r_h & d_h + \gamma & 0 & 0 \\ 0 & 0 & 0 & \nu_v + d_v(t) & 0 \\ 0 & 0 & 0 & -\nu_v & d_v(t) \end{pmatrix}.$$

Let us assume that $\tilde{Y}(t, s)$, $t \ge s$ is the matrix solution of the linear ω -periodic system

$$\begin{cases} \dot{\tilde{Y}}(t,s) = -V_2(t)\tilde{Y}(t,s), \ \forall t \ge s, \\ \tilde{Y}(s,s) = I, \end{cases}$$
 (25)

where *I* is the 5×5 identity matrix.

Let C_{ω} be the ordered Banach space of all ω -periodic functions from \mathbb{R} to \mathbb{R}^5 which is equipped with the maximum norm $\|.\|$ and the positive cone $C_{\omega}^+ := \{ \phi \in C_{\omega} : \phi(t) \ge 0, \forall t \in \mathbb{R} \}$. Then, we can define a linear operator $\tilde{\mathcal{L}}: C_{\omega} \longrightarrow C_{\omega}$ by

$$(\tilde{\mathcal{L}}\phi)(t) = \int_{0}^{\infty} \tilde{Y}(t, t-a)F_{2}(t-a)\phi(t-a)da, \quad \forall t \in \mathbb{R}, \ \phi \in C_{\omega}.$$
 (26)

 $\tilde{\mathcal{L}}$ is the next infection operator, and the basic reproduction ratio is $\Re_0 = \rho(\tilde{\mathcal{L}})$, the spectral radius of $\tilde{\mathcal{L}}$.

Lemma 3.8. [41] Let $\tilde{\mathbb{R}}_0$ be the basic reproduction ratio associated to the temperature-dependent biting rate, $\tilde{\beta}(t)$ and κ be a positive real. Then we have

$$\tilde{\beta}(t) = \kappa \beta(t) \Longrightarrow \tilde{\mathcal{R}}_0 = \kappa \mathcal{R}_0.$$

3.3.3 Global stability of disease-free equilibrium

Lemma 3.9. [27]

i) $\Re_0 = 1$ if and only if $\rho(\Phi_{F_2-V_2}(\omega)) = 1$.

ii) $\Re_0 < 1$ if and only if $\rho(\Phi_{F_2-V_2}(\omega)) < 1$.

iii) $\Re_0 > 1$ if and only if $\rho(\Phi_{F_2-V_2}(\omega)) > 1$.

Hence, if $\Re_0 < 1$ then the disease-free equilibrium, W_0 is locally asymptotically stable.

Theorem 3.4. The disease-free equilibrium W_0 is globally attractive if $\Re_0 < 1$ and $d_p = 0$.

Proof. If $d_p = 0$, then from equation (6), there exists a period $\omega(\epsilon) > 0$ such that

$$N_h(t) \geq S_h^{\star} - \epsilon, \ \forall t \geq \omega(\epsilon).$$

Thus, for all $t \ge T(\epsilon)$, we have

$$k_{\nu}(t)N_{\nu}^{\star}(t) \leq c_{h\nu}\beta(t)\frac{N_{\nu}^{\star}(t)}{S_{h}^{\star}-\epsilon}I_{h}(t) + \bar{c}_{h\nu}\beta(t)\frac{N_{\nu}^{\star}(t)}{S_{h}^{\star}-\epsilon}R_{h}(t).$$

Hence, from system (22) we obtain the following system:

$$\begin{cases}
\dot{E}_{h}(t) & \leq c_{\nu h} \beta(t) I_{\nu}(t) - (d_{h} + \alpha) E_{h}(t), \\
\dot{I}_{h}(t) & = \alpha E_{h}(t) - (d_{h} + r_{h}) I_{h}(t), \\
\dot{R}_{h}(t) & = r_{h} I_{h}(t) - (d_{h} + \gamma) R_{h}(t), \\
\dot{E}_{\nu}(t) & \leq c_{h \nu} \beta(t) \frac{N_{\nu}^{*}(t)}{S_{h}^{*} - \epsilon} I_{h}(t) + \bar{c}_{h \nu} \beta(t) \frac{N_{\nu}^{*}(t)}{S_{h}^{*} - \epsilon} R_{h}(t) - (\nu_{\nu} + d_{\nu}(t)) E_{\nu}(t), \\
\dot{I}_{\nu}(t) & = \nu_{\nu} E_{\nu}(t) - d_{\nu}(t) I_{\nu}(t).
\end{cases} \tag{27}$$

Now, let us consider the following auxiliary system,

$$\tilde{R}_2(t) = A_{\epsilon}(t)\tilde{R}_2(t)$$

with

$$\tilde{R}_2(t) = (\tilde{E}_h(t), \tilde{I}_h(t), \tilde{R}_h(t), \tilde{E}_v(t), \tilde{I}_v(t))^T$$

and

$$A_{\varepsilon}(t) = \begin{pmatrix} -(d_h + \alpha) & 0 & 0 & 0 & c_{vh}\beta(t) \\ \alpha & -(d_h + r_h) & 0 & 0 & 0 \\ 0 & r_h & -(d_h + \gamma) & 0 & 0 \\ 0 & c_{hv}\beta(t)\frac{N_v^*(t)}{S_h^* - \varepsilon} & \bar{c}_{hv}\beta(t)\frac{N_v^*(t)}{S_h^* - \varepsilon} & -(\nu_v + d_v(t)) & 0 \\ 0 & 0 & 0 & \nu_v & -d_v(t) \end{pmatrix}.$$

By continuity of spectral radius, we have $\lim_{\epsilon \to 0^+} \rho \left(\Phi_{A_\epsilon}(\omega) \right) = \rho \left(\Phi_{F_2 - V_2}(\omega) \right)$. In addition, from Lemma 3.9, if $\Re_0 < 1$, then $\rho(\Phi_{F_2 - V_2}(\omega)) < 1$, thus, $\lim_{\epsilon \to 0^+} \rho \left(\Phi_{A_\epsilon}(\omega) \right) < 1$. It then follows that there exists $\epsilon_1 > 0$ such that $\rho \left(\Phi_{A_\epsilon}(\omega) \right) < 1$ for all $\epsilon \in [0, \epsilon_1]$. From Lemma 3.1, there exists a positive ω -periodic function v(t) such that $\tilde{R}_2(t) = v(t)e^{rt}$ is a solution of (28). Since $\rho \left(\Phi_{A_\epsilon}(\omega) \right) < 1$, so r < 0. Therefore, the ω -periodic function v(t) is bounded, hence $\tilde{R}_2(t) \to 0$ as $t \to \infty$. Applying comparison theorem on system (27), we get

$$\lim_{t\to\infty} (E_h(t), I_h(t), R_h(t), E_v(t), I_v(t)) = (0, 0, 0, 0, 0).$$

Moreover, we have

$$\lim_{t\to\infty} (S_h(t) - S_h^{\star}) = \lim_{t\to\infty} \left(N_h(t) - E_h(t) - I_h(t) - R_h(t) - S_h^{\star} \right) = 0.$$

Hence, we have $\lim_{t\to\infty} S_h(t) = S_h^*$.

It then follows that the disease-free equilibrium W_0 is globally attractive which complete the proof. \Box

3.3.4 Persistence disease transmission

Let us consider the following sets:

$$\begin{array}{ll} \mathcal{Y} & := & \mathbb{R}^6_+, \\ \mathcal{Y}_0 & := & \left\{ (S_h, E_h, I_h, R_h, E_v, I_v) \in \mathcal{Y} : E_h > 0, I_h > 0, R_h > 0, E_v > 0, I_v > 0 \right\}, \\ \partial \mathcal{Y}_0 & := & \left\{ (S_h, E_h, I_h, R_h, E_v, I_v) \in \mathcal{Y} : E_h I_h R_h E_v I_v = 0 \right\}. \end{array}$$

Let $S_2: \mathcal{Y} \longrightarrow \mathcal{Y}$ be the Poincaré map associated with system (22).

Lemma 3.10. The sets y_0 and ∂y_0 are positively invariant with respect to system (22).

Proof. For any initial condition $\varphi_2 \in \mathcal{Y}_0$, by solving system (22) we obtain, for all t > 0,

$$S_h(t) = \exp\left(-\int_0^t H_2(\tau)d\tau\right) \left[S_h(0) + \int_0^t H_1(\tau) \exp\left(\int_0^\tau H_2(s)ds\right)d\tau\right] > 0, \tag{28}$$

$$E_h(t) = \exp\left(-(d_h + \alpha)t\right) \left[E_h(0) + \int_0^t k_h(\tau) S_h(\tau) \exp\left((d_h + \alpha)\tau\right) d\tau \right] > 0, \tag{29}$$

$$I_h(t) = \exp\left(-(d_h + d_p + r_h)t\right) \left[I_h(0) + \int_0^t \alpha E_h(\tau) \exp\left((d_h + d_p + r_h)\tau\right) d\tau\right] > 0$$
 (30)

$$R_h(t) = \exp\left(\left(-(d_h + \gamma)t\right)\right) \left[R_h(0) + \int_0^t r_h I_h(\tau) \exp\left((d_h + \gamma)\tau\right) d\tau\right] > 0, \tag{31}$$

$$E_{\nu}(t) = \exp\left(-\int_{0}^{t} H_{4}(\tau)d\tau\right) \left[E_{\nu}(0) + \int_{0}^{t} H_{3}(\tau) \exp\left(\int_{0}^{\tau} H_{4}(s)ds\right)d\tau\right] > 0$$
 (32)

$$I_{\nu}(t) = \exp\left(-\int_{0}^{t} d_{\nu}(\tau)d\tau\right) \left[I_{\nu}(0) + \int_{0}^{t} \nu_{\nu} E_{\nu}(\tau) \exp\left(\int_{0}^{\tau} d_{\nu}(s)ds\right)\right] > 0, \tag{33}$$

with

$$\begin{array}{lcl} H_1(t) & = & \Lambda + I_h(t) + \gamma R_h(t), & H_2(t) = k_h(t) + d_h, \\ H_3(t) & = & k_v(t) \big(N_v^{\star}(t) - I_v(t) \big), & H_4(t) = k_v(t) + d_v(t) + v_v. \end{array}$$

So, it then follows that y_0 is positively invariant. Therefore, ∂y_0 is relatively closed in y, then, ∂y_0 is also positively invariant with respect to system (22).

Moreover, the compact set Δ_2 attracts all positive orbits in \mathcal{Y} , which implies that the discrete-time system $S_2:\mathcal{Y}\longrightarrow\mathcal{Y}$ is point dissipative.

Lemma 3.11. *If* $\Re_0 > 1$, *there exists* $\xi > 0$ *such that when*

$$\|\varphi_2 - W_0\| \le \xi$$
, for any $\varphi_2 \in \mathcal{Y}_0$,

one has

$$\limsup_{n\to\infty} d(S_2^n(\varphi_2), W_0) \geq \xi.$$

Proof. Suppose by contradiction that

$$\limsup_{n\to\infty} d(S_2^n(\varphi_2), W_0) < \xi \text{ for some } \varphi_2 \in \mathcal{Y}_0.$$

Then, it follows that there exists an integer $N \ge 1$ such that for all $n \ge N$ we have

$$d(S_2^n(\varphi_2), W_0) < \xi.$$

By the continuity of solution $u_2(t, \varphi_2)$, we have

$$||u_2(t, S_2^n(\varphi_2)) - u_2(t, W_0)|| \le \varepsilon, \forall t \ge 0, \ \varepsilon > 0.$$

Let $t = n\omega + t'$, with $t' \in [0, \omega]$ and $n = \left[\frac{t}{\omega}\right]$. $\left[\frac{t}{\omega}\right]$ is the greatest integer less than or egal to $\frac{t}{\omega}$. Then, we have

$$||u_2(t, \varphi_2) - u_2(t, W_0)|| = ||u_2(t', S_2^n(\varphi_2)) - u(t', W_0)|| < \varepsilon.$$

Hence, it follows that

$$S_h^{\star} - \varepsilon \leq S_h(t) \leq S_h^{\star} + \varepsilon$$
.

Thus, there exists $\zeta(\varepsilon) > 0$ such that

$$\frac{S_h(t)}{N_h(t)} \ge 1 - \zeta(\varepsilon) \text{ and } \frac{N_v^{\star}(t)}{N_h(t)} \ge \frac{N_v^{\star}(t)}{S_h^{\star}}.$$

From (22) we have:

$$\begin{cases} \dot{E}_{h}(t) \geq c_{vh}\beta(t)(1 - \zeta(\varepsilon))I_{v}(t) - (d_{h} + \alpha)E_{h}(t), \\ \dot{I}_{h}(t) = \alpha E_{h}(t) - (d_{p} + d_{h} + r_{h})I_{h}(t), \\ \dot{R}_{h}(t) = r_{h}I_{h}(t) - (d_{h} + \gamma)R_{h}(t), \\ \dot{E}_{v}(t) \geq \beta(t)\frac{N_{v}^{*}(t)}{S_{h}^{*}} \left(c_{hv}I_{h}(t) + \bar{c}_{hv}R_{h}(t)\right) - (v_{v} + d_{v}(t))E_{v}(t), \\ \dot{I}_{v}(t) = v_{v}E_{v}(t) - d_{v}(t)I_{v}(t). \end{cases}$$

Now, let us consider the following auxiliary linear system

$$\dot{\tilde{R}}_3(t) = N_{\zeta(\varepsilon)}(t)\tilde{R}_3(t) \tag{34}$$

with

$$\tilde{R}_3(t) = (\tilde{E}_h(t), \tilde{I}_h(t), \tilde{R}_h(t), \tilde{E}_v(t), \tilde{I}_v(t))^T$$

and

$$N_{\zeta(\varepsilon)}(t) = \left(\begin{array}{ccccc} -(d_h + \alpha) & 0 & 0 & 0 & (1 - \zeta(\varepsilon))c_{\nu h}\beta(t) \\ \alpha & -(d_h + r_h) & 0 & 0 & 0 \\ 0 & r_h & -(d_h + \gamma) & 0 & 0 \\ 0 & c_{h\nu}\frac{N_{\nu}^*(t)}{S_h^*}\beta(t) & \bar{c}_{h\nu}\frac{N_{\nu}^*(t)}{S_h^*}\beta(t) & -(\nu_{\nu} + d_{\nu}(t)) & 0 \\ 0 & 0 & 0 & \nu_{\nu} & -d_{\nu}(t) \end{array} \right).$$

Once again if $\Re_0 > 1$, there exists $\sigma > 0$ such that $\rho(\Phi_{N_{\zeta(\varepsilon)}}(\omega)) > 1$ for all $\zeta(\varepsilon) \in [0, \sigma]$. In this case r > 0 and from Lemma 3.1 we get $\tilde{R}_3(t) \to \infty$ as $t \to \infty$. Thus, by applying the theorem of comparison, we get

$$\lim_{t\to\infty} |(E_h(t), I_h(t), R_h(t), E_V(t), I_V(t))| = \infty.$$

This leads to a contradiction.

Theorem 3.5. If $\Re_0 > 1$, then system (22) is uniformly persistent and there exists at least one positive periodic solution.

Proof. Let us consider the set

$$M_{\partial} = \{ \varphi_2 \in \partial \mathcal{Y}_0 : S_2^n(\varphi_2) \in \partial \mathcal{Y}_0, \text{ for any } n \ge 0 \}.$$

We aim to show that

$$M_{\partial} = \{ (S_h, 0, 0, 0, 0, 0) \in \mathcal{Y} : S_h \ge 0 \}. \tag{35}$$

Indeed, it is obvious that

$$M_{\partial} \supset \{(S_h, 0, 0, 0, 0, 0) \in \mathcal{Y} : S_h \ge 0\}.$$

So we only need to show that

$$M_{\partial} \subset \{(S_h, 0, 0, 0, 0, 0) \in \mathcal{Y} : S_h \ge 0\},\$$

that means for any initial condition $\varphi_2 \in \partial \mathcal{Y}_0$, we have

$$E_h(n\omega) = 0$$
 or $I_h(n\omega) = 0$ or $R_h(n\omega) = 0$ or $E_v(n\omega) = 0$ or $I_v(n\omega) = 0$ $\forall n \ge 0$.

Let $\varphi_2 \in \partial \mathcal{Y}_0$. Suppose by contradiction that there exists and integer $n_1 \ge 0$ such that $E_h(n_1\omega) > 0$, $I_h(n_1\omega) > 0$, $R_h(n_1\omega) > 0$, $E_v(n_1\omega) > 0$ and $I_v(n_1\omega) > 0$. Thus, by replacing the initial time t = 0 by $t = n_1\omega$ in (28)–(33) we get $S_h(t) > 0$, $E_h(t) > 0$, $I_h(t) > 0$, $R_h(t) > 0$, $E_v(t) > 0$, $I_v(t) > 0$. That contradict the fact that $\partial \mathcal{Y}_0$ is positively invariant. Hence, $M_\partial \subset \left\{ (S_h, 0, 0, 0, 0, 0, 0) \in \mathcal{Y} : S_h \ge 0 \right\}$ and then (35) holds.

The equality (35) implies that W_0 is a fixed point of S_2 and acyclic in M_{∂} . Thus, every solution in M_{∂} approaches to W_0 . Moreover, Lemma 3.11 implies that W_0 is an isolate invariant set in \mathcal{Y} and $W^s(W_0) \cap \mathcal{Y}_0 = \emptyset$. By Lemma 3.2, it then follows that S_2 is uniformly persistent with respect to $(\mathcal{Y}_0, \partial \mathcal{Y}_0)$. So, system (22) is uniformly persistent. It yields that the system (22) has at least one periodic solution $u_2^*(t, \varphi_2^*)$ with $\varphi_2^* \in \mathcal{Y}_0$. We further claim that $u_2^*(t, \varphi_2^*)$ is positive for all $t \ge 0$.

Suppose that $u_2^{\star}(0, \varphi_2^{\star}) = 0$. Then, from equations (28)-(33) we have $u_2^{\star}(t, \varphi_2^{\star}) > 0$ for all t > 0. By the periodicity of the solution, we have $u_2^{\star}(0 + n\omega, \varphi_2^{\star}) = u_2^{\star}(n\omega, \varphi_2^{\star}) = u_2^{\star}(0, \varphi_2^{\star}) = 0$ for all $n \ge 1$. That leads to a contradiction.

3.4 Analysis of the global model

Now, we focus on the global behavior of system (5). Thus, using the theory of chain transitive set [51], we lift the threshold type result for system (22) to system (5). Note that system (5) is well defined on the set $\Delta = \Delta_1 \times \Delta_2$.

3.4.1 Positivity, boundedness of solutions and Disease-free equilibria of system (5)

Lemma 3.12. For any positive initial condition $\varphi = (\varphi_1, \varphi_2)$, system (5) has a unique positive bounded solution $u(t, \varphi)$ on $[0, +\infty)$ with $u(0) = \varphi$. Moreover, the closed set Δ is positively invariant for system (5).

Proof. For any φ , we have $f(t, \varphi) = (f_1(t, \varphi_1), f_2(t, \varphi_2))^T$ is continuous in (t, φ) and lipschitzian in φ in each compact of Ω . So, thanks to Theorems 2.2.1 and 2.2.3 in [11], system (5) has a unique solution $u(t, \varphi)$ on its maximal interval $[0, \sigma_{\varphi})$ of existence. Since Δ_1 and Δ_2 are both respectively positively invariant under (9) and (22), then Ω is positively invariant for (5). Hence, the solution $u(t, \varphi)$ is bounded. It then follows that $\sigma_{\varphi} = +\infty$.

Remark 3.1. In view of the results of Section 3.2 and Section 3.3, we note that system (5) has two disease-free equilibria:

- (i) a trivial disease-free equilibrium, $(0, 0, 0, 0, S_h^*, 0, 0, 0)$,
- (ii) a disease-free periodic equilibrium, $(J^*(t), N_{\nu}^*(t), 0, 0, S_h^*, 0, 0, 0)$ if $\Re_{\nu} > 1$.

3.4.2 Global stability of disease-free equilibria

In this part of the paper, we focus on the global stability of disease-free equilibria of system (5).

Theorem 3.6. If $\Re_V < 1$, then the disease-free equilibrium $(0, 0, 0, 0, S_h^*, 0, 0, 0)$ is globally attractive for system (5).

Proof. Let $\tilde{\Phi}(t)$ be the periodic semiflow generated by periodic system (5) and *S* the Poincaré map associated with system (5), namely:

$$S(\varphi) = \tilde{\Phi}(\omega)\varphi = u(\omega, \varphi),$$

$$S^{n}(\varphi) = \tilde{\Phi}(n\omega)\varphi = u(n\omega, \varphi), \forall n \ge 0.$$

Let $\mathbb O$ be the omega limit set of the discrete-time orbit $\{\tilde{\Phi}^n(\varphi)\}_{n\geq 0}$. Then, from Lemma 1.2.1 in [51], $\mathbb O$ is an internally chain transitive set for $\tilde{\Phi}^n$ on Δ . If $\mathcal R_{\mathcal V}<1$, then thanks to Theorem 3.1, we have

$$\lim_{n\to\infty} (J(n\omega,\varphi_1), N_{\nu}(n\omega,\varphi_1), E_{\nu}(n\omega,\varphi_2), I_{\nu}(n\omega,\varphi_2)) = (0,0,0,0).$$

Then, there exists $\mathcal{O}^{'} \subset \mathbb{R}^{4}$ such that $\mathcal{O} = \{(0,0,0,0)\} \times \mathcal{O}^{'}$. For any $x = (x_{i})_{1 \leq i \leq n} \in \mathcal{O}$, there exists a sequence $n_{k} \to \infty$ such that $u(n_{k}\omega, \varphi) \to x$ as $k \to \infty$. Since,

$$0 \le S_h(n_k\omega, \varphi_2) \le N_h^*, \quad 0 \le E_h(n_k\omega, \varphi_2) \le N_h^*,$$

$$0 \le I_h(n_k\omega, \varphi_2) \le N_h^*, \quad 0 \le R_h(n_k\omega, \varphi_2) \le N_h^*,$$

then $\mathfrak{O}' \subset \Delta_2$. Moreover, it is obvious that

$$\tilde{\Phi}^n_{|\mathcal{O}}(0,0,0,0,x_5,x_6,x_7,x_8) = \{(0,0,0,0)\} \times \Psi^n_{|\mathcal{O}'}(x_5,x_6,x_7,x_8)$$

where $\{\Psi(t)\}_{t\geq 0}$ is the semiflow associated to the following system:

$$\begin{cases}
\dot{S}_h(t) = \Lambda + \gamma R_h(t) - d_h S_h(t), \\
\dot{E}_h(t) = -(d_h + \alpha) E_h(t), \\
\dot{I}_h(t) = \alpha E_h(t) - (d_h + d_p + r_h) I_h(t), \\
\dot{R}_h(t) = r_h I_h(t) - (d_h + \gamma) R_h(t),
\end{cases} (36)$$

Since \mathcal{O} is internally chain transitive set for $\tilde{\Phi}^n$, it then follows that \mathcal{O}' is internally chain transitive set for $\tilde{\Psi}^n$. In addition, the from second equation of system (36), we have:

$$\lim_{t\to\infty} E_h(t) = \lim_{t\to\infty} e^{-(d_h+\alpha)t} = 0.$$

Thus, we obtain the following system:

$$\begin{cases} \dot{I}_h(t) & \leq \alpha E_h(t), \\ \dot{R}_h(t) & \leq r_h I_h(t). \end{cases}$$
(37)

Using the comparison theorem, on system (37) we have:

$$\lim_{t\to\infty} \left(I_h(t), R_h(t)\right) = (0, 0).$$

It yields from the first equation of system (36) that $\lim_{t\to\infty} S_h(t) = \frac{\Lambda}{d_h} = S_h^*$. Hence, $(S_h^*, 0, 0, 0, 0)$ is globally attractive for system (36). So, from the Theorem 1.2.1 in [51], we have $O' = (S_h^*, 0, 0, 0, 0)$ and then $O = (0, 0, 0, 0, S_h^*, 0, 0, 0, 0)$.

Theorem 3.7. If $d_p = 0$, $\Re_v > 1$ and $\Re_0 < 1$, then the disease-free periodic equilibrium $(J^*(t), N_v^*(t), 0, 0, S_h^*, 0, 0, 0)$ is globally attractive for system (5).

Proof. If $\Re_{V} > 1$, then thanks to Theorem 3.2, we have

$$\lim_{n\to\infty} (J(n\omega,\varphi_1),N_{\nu}(n\omega,\varphi_1)) = (J^{\star}(0),N_{\nu}^{\star}(0)).$$

Then, there exists $O'' \subset \mathbb{R}^6$ such that $O = \{(J^*(0), N_{\nu}^*(0))\} \times O''$.

For any $y = (y_i)_{1 \le i \le 10} \in \mathcal{O}$, there exists a sequence $n_l \to \infty$ such that $v(n_l \omega, \varphi) \to y$ as $l \to \infty$. Since,

$$0 \leq S_h(n_l \omega, \varphi_2) \leq N_h^{\star}, \quad 0 \leq E_h(n_l \omega, \varphi_2) \leq N_h^{\star}, \quad 0 \leq I_h(n_l \omega, \varphi_2) \leq N_h^{\star},$$

$$0 \leq R_h(n_l \omega, \varphi_2) \leq N_h^{\star}, \quad 0 \leq E_v(n_l \omega, \varphi_2) \leq N_v^{\star}(n_l \omega, \varphi_2), \quad 0 \leq I_v(n_l \omega, \varphi_2) \leq N_v^{\star}(n_l \omega, \varphi_2),$$

then $O'' \subset \Delta_2$. Moreover, let $y' = (y_5, y_6, y_7, y_8, y_9, y_{10})$. It is then obvious that

$$\tilde{\Phi}^{n}_{|(\cdot)}(J^{*}(0), N_{\nu}^{*}(0), y^{'}) = \{(J^{*}(0), N_{\nu}^{*}(0))\} \times \Psi^{n}_{|(\cdot)''}(y^{'})$$

where $\{\Psi(t)\}_{t\geq 0}$ is the semiflow associated to system (22). Hence, from Theorem 3.4, if $\mathcal{R}_0 < 1$ and $d_p = 0$ then, the equilibrium W_0 is globally attractive. So, from the Theorem 1.2.1 in [51], we have $\mathcal{O}'' = (0, 0, S_h^*, 0, 0, 0)$ and then $\mathcal{O} = (J^*(0), N_{\nu}^*(0), 0, 0, S_h^*, 0, 0, 0)$.

3.4.3 Existence of positive periodic solutions

Theorem 3.8. If $\Re_{V} > 1$ and $\Re_{0} > 1$, then system (5) admits at least one positive periodic solutions $(J^{\star}(t), N_{V}^{\star}(t), E_{V}^{\star}(t), I_{V}^{\star}(t), E_{h}^{\star}(t), E_{h}^{\star}(t), I_{h}^{\star}(t), R_{h}^{\star}(t))$.

Proof. If $\Re_{V} > 1$, then thanks to Theorem 3.3, we have

$$\lim_{n\to\infty} (J(n\omega,\varphi_1), N_{\nu}(n\omega,\varphi_1)) = (J^*(0), N_{\nu}^*(0)).$$

Then, there exists $\mathfrak{I} \subset \mathbb{R}^6$ such that $\mathfrak{O} = \{(J^*(0), N_{\nu}^*(0))\} \times \mathfrak{I}$. Thus, from Hirsch *et al.* [12] (see Theorem 3.1),

$$\mathcal{T} = \left(E_{\nu}^{\star}(0), I_{\nu}^{\star}(0), S_{h}^{\star}(0), E_{h}^{\star}(0), I_{h}^{\star}(0), R_{h}^{\star}(0)\right) \text{ or } \mathcal{T} = (0, 0, 0, 0, 0, 0).$$

Let us suppose that $\mathfrak{T}=(0,0,0,0,0,0)$. Then, it yields that $u_2(t,\varphi_2)\to 0$ as $t\to\infty$. Hence, there exists a period ω_1 such that for any $\varepsilon^*>0$ and $t\geq\omega_1$,

$$\|(J(t), N_{\nu}(t)) - (J^{\star}(t), N_{\nu}^{\star}(t))\| < \epsilon^{\star}.$$

Hence, from system (22) we have

$$\begin{cases} \dot{E}_{h}(t) & \geq c_{vh}\beta(t)(1-e^{*})I_{v}(t) - (d_{h} + \alpha)E_{h}(t), \\ \dot{I}_{h}(t) & = \alpha E_{h}(t) - (d_{p} + d_{h} + r_{h})I_{h}(t), \\ \dot{R}_{h}(t) & = r_{h}I_{h}(t) - (d_{h} + \gamma)R_{h}(t), \\ \dot{E}_{v}(t) & \geq \beta(t)\frac{N_{v}^{*}(t)}{S_{h}^{*}}\left(c_{hv}I_{h}(t) + \bar{c}_{hv}R_{h}(t)\right) - (v_{v} + d_{v}(t))E_{v}(t), \\ \dot{I}_{v}(t) & = v_{v}E_{v}(t) - d_{v}(t)I_{v}(t). \end{cases}$$

Let us consider the following auxiliary system:

$$\dot{\tilde{R}}_{4}(t) = A_{\varepsilon^{*}}(t)\tilde{R}_{4}(t), \text{ with } \tilde{R}_{4}(t) = (\tilde{E}_{h}^{*}(0), \tilde{I}_{h}^{*}(0), \tilde{R}_{h}^{*}(0), \tilde{E}_{v}^{*}(0), \tilde{I}_{v}^{*}(0))^{T}.$$
(38)

If $\mathcal{R}_0 > 1$ then, system (38) admits a positive fixed point $(\tilde{E}_h^*(0), \tilde{I}_h^*(0), \tilde{E}_h^*(0), \tilde{E}_{\nu}^*(0), \tilde{I}_{\nu}^*(0))$ for all φ_2 in \mathcal{Y}_0 . So, by applying the comparison principle, we get

$$\liminf_{n\to\infty} \left(E_h(n\omega,\varphi_2), I_h(n\omega,\varphi_2), R_h(n\omega,\varphi_2), E_v(n\omega,\varphi_2), I_v(n\omega,\varphi_2) \right) \gg 0,$$

which leads to a contradiction. It then follows that $\mathfrak{T} = (E_{\nu}^{\star}(0), I_{\nu}^{\star}(0), S_{h}^{\star}(0), E_{h}^{\star}(0), I_{h}^{\star}(0), R_{h}^{\star}(0)).$

4 Numerical simulations

In this section, we perform some numerical simulations to support our theoretical analysis.

4.1 Estimation of the parameters

We suppose that the life expectancy of humans for Burkina Faso in 2011 is 55 years. Thus, the human natural death rate d_h can be calculated as follows [20]

$$d_h = \frac{1}{12 \times 55} = 0.00152$$
 per month.

In addition, in Burkina Faso, the total size of human population in 2011 was 16, 248, 558, then the recruitment rate Λ is determined by

$$\Lambda = d_h \times 16,248,558 = 24698$$
 humans per month.

Further, values of some constant parameters for the malaria transmission model (5) are listed in Table 2 with the references therein. Now, we estimate the periodic coefficients of the model. Thus, using the reported monthly mean temperature, for ASECNA in Burkina Faso, from January 2011 to December 2015, we can calculate the average monthly temperature for Burkina Faso as follows (Table 3).

Moreover, assuming that the average number of mosquito bites depends on their gonotrophic cycle which is also a function of temperature (θ), then the temperature-dependent biting rate of mosquitoes reads as follows [47]:

$$\beta(\theta) = 0.00609\theta(\theta - 11.7)\sqrt{42.3 - \theta}$$
 per month.

Similarly, the temperature-dependent death rate of adult mosquitoes and larvae reads respectively as follows [20]:

$$d_{\nu}(\theta) = 3 + 29.564 \exp\left(\frac{5 - \theta}{2.7035}\right) \text{ per month,}$$

$$d_{J}(\theta) = \frac{30}{-4.4 + 1.31\theta - 0.03\theta^{2}} \text{ per month.}$$

Table 2: Values for constant parameters of the malaria model.

Parameters	Values	References	Dimensions
Λ	24, 698	see text	/month
d_h	0.00152	see text	/month
d_p	0.0028	[20]	/month
α	3.04	[20]	/month
r_h	0.0833	[20]	/month
γ	0.0146	[20]	/month
$ u_{ u}$	2.523	[20]	/month
c_{vh}	0.022	[8]	-
c_{hv}	0.48	[8]	-
\bar{c}_{hv}	0.048	[8]	-

Table 3: Monthly mean temperature for Burkina Faso (in C).

Month	January	February	March	April	May	June
Temperature	25.8	28.77	31.11	30.75	29.54	27.50
Month	July	August	September	October	November 28.33	December
Temperature	25.95	25.04	25.57	27.32		25.36

According to Rubel et al. (see [47]), the birth rates of juvenile and adult mosquitoes can be given by

$$b(\theta) = 2.325\beta(\theta) \text{ and } r_{\nu}(\theta) = 0.2325\beta(\theta).$$

Furthermore, assuming that the temperature varies as a function of time, we use the CFTOOL of MATLAB to get the best estimation of periodic functions $\beta(t)$, $d_I(t)$, $d_V(t)$ for Burkina Faso as follows, [47]

$$\beta(t) = a_0 + \sum_{n=1}^{3} \left[a_n \sin\left(\frac{n\pi t}{6}\right) + \tilde{a}_n \cos\left(\frac{n\pi t}{6}\right) \right] \text{ per month}$$

with

$$a_0 = 10.16, \ a_1 = 1.289, \ a_2 = -0.06678, \ a_3 = 0.3301,$$

$$\tilde{a}_1 = 0.3614, \ \tilde{a}_2 = -1.097, \quad \tilde{a}_3 = -0.4723.$$

$$d_{J}(t) = b_{0} + \sum_{n=1}^{3} \left[b_{n} \sin\left(\frac{n\pi t}{6}\right) + \tilde{b}_{n} \cos\left(\frac{n\pi t}{6}\right) \right] \text{ per month}$$

with

$$b_0 = 3.443, \ b_1 = 0.3287, \ b_2 = 0.03792, \ b_3 = -0.004983,$$
 $\tilde{b}_1 = 0.08803, \ \tilde{b}_2 = -0.2627, \ \tilde{b}_3 = -0.1211.$

$$d_{v}(t) = c_0 + \sum_{n=1}^{3} \left[c_n \sin\left(\frac{n\pi t}{6}\right) + \tilde{c}_n \cos\left(\frac{n\pi t}{6}\right) \right] \text{ per month}$$

with

$$c_0 = 3.008$$
, $c_1 = -0.004631$, $c_2 = 0.0009815$, $c_3 = -0.0026$, $\tilde{c}_1 = -0.001408$, $\tilde{c}_2 = 0.004167$, $\tilde{c}_3 = 0.001817$.

Besides, we suppose that K(t) = K = 80,000,000.

4.2 Numerical results

Now, we use the MATLAB technical computing software with the fourth-order Runge-Kutta method to validate our analytical results. Thus, using the values of Table 2 and the initial conditions: J(0) = 60,000; $N_V(0) = 24,000$; $S_h(0) = 15,000,000$; $E_h(0) = 10,000$; $I_h(0) = 50,000$; $R_h(0) = 1,188,558$; $E_V(0) = 10,000$; $I_V(0) = 50,000$, we obtain the following results: Figure 3, 4 and 5.

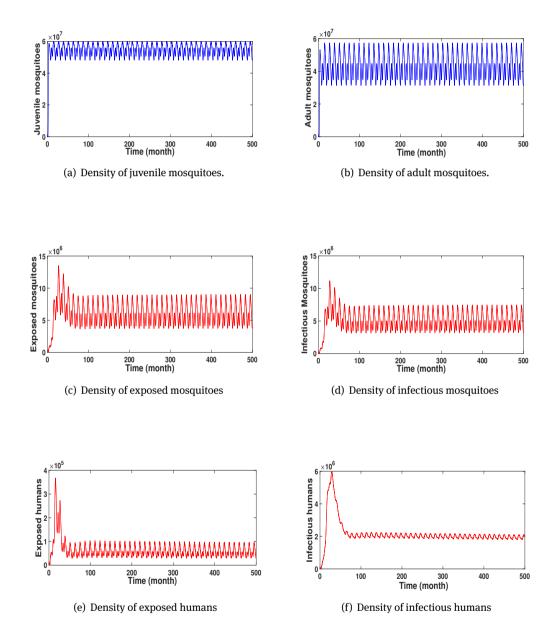


Figure 3: Global behavior of system (5) for $\Re_{\nu} > 1$ and $\Re_{0} > 1$.

Numerical calculations of the basic reproduction ratio and vector reproduction ratio give $\Re_{\nu} = 3.2$ and $\Re_0 = 1.35$. Moreover, Figure 3 (see 3 (a)–3 (f)) indicates that mosquito population and disease tend towards periodic oscillation. It yields that system (5) admits a positive periodic solution, which is in accordance with the result of our Theorem 3.8.

Now, we suppose that people became more conscious about malaria and developed some strategies to fight against the disease transmission. Such strategies include first, the reduction of juvenile mosquitoes

breeding sites through the ecological cleaning of environment during the rainy season or by drying them. Secondly, humans used some methods to avoid the bites of mosquitoes through the use of insecticide-treated bed net or any other suitable method.

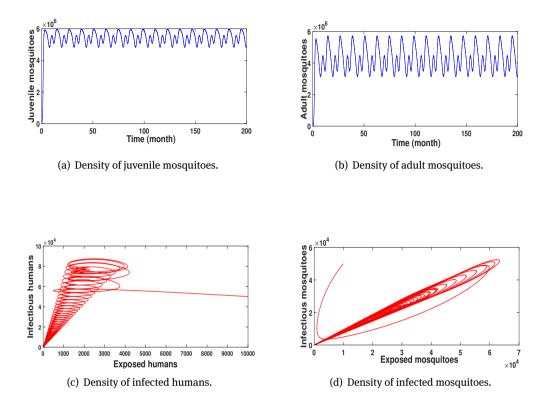
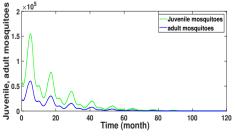
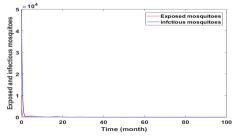


Figure 4: Global behavior of system (5) for $\Re_{\nu} > 1$, $\Re_{0} < 1$.

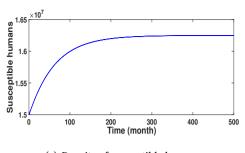
Here, we focus on the reduction of the breeding sites of mosquitoes by human intervention that consist for instance to dry them. Assuming that 10% of this reduction has been achieved ($\tilde{K}(t) = 0.1K(t)$), then numerical calculations of basic reproduction ratio and vector reproduction ratio give $\Re_V = 3.2$ and $\Re_0 = 0.90$. We remark that the vector density decrease and despite the persistence of the mosquito population (see Figure 4(a)–4(b)), the disease will dies out to both populations (see Figure 4(c)–4(d)). It then follows that system (5) converges to the periodic disease-free equilibrium which is globally asymptotically stable. That numerical result confirms the result of our Theorem 3.7. Moreover, we note that the reduction of breeding sites has a double effect. First, it increases the death rate of immature stage because nutrients are necessary for the development at that stage.

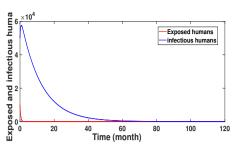


(a) Density of juvenile and adult mosquitoes.



(b) Density of infected mosquitoes.





(c) Density of susceptible humans.

(d) Density of infected humans.

Figure 5: Global behavior of system (5) for $\Re_{V} < 1$ and $\tilde{b}(t) = 0.49b(t)$.

Now, we suppose that the two strategies mentioned above are applied. However, if human intervention is more accentuated on vector reduction, In this case, numerical calculations of vector reproduction ratio give $\Re_V = 0.97$. Figure 5 shows that mosquitoes and malaria die out in the long run. This numerical result illustrates the result of our Theorem 3.6. These results prove that the eradication of the disease can be achieved if the mosquito population is eradicated or if the parameter \Re_V is less than unity. That threshold parameter may provide conditions in order to control the proliferation of vector population.

5 Conclusion

In this paper, a dynamics behaviors of malaria model in periodic environment has been investigated. The model has been constructed by coupling two subsystems. The first subsystem governs the dynamics of vector population and the second subsystem describes the dynamics of malaria virus transmission due to the interaction between vector and human hosts. The model has taken into account the temperature effects in the vector population dynamics and the intrinsic incubation period in mosquitoes and human hosts is incorporated. In order to get qualitative dynamics, we have first studied the subsystem describing the dynamics of vector population. Then, we have used the limiting system and the theory of chain transitive to obtain the information for the whole model. The threshold dynamics associated to each subsystem is derived, and we have proved that the global behavior of the whole model is determined by these two thresholds. Numerical simulations have been performed to illustrate the obtained results.

Moreover, since the disease transmission is directly linked to the basic reproduction ratio, then controlling malaria implies the reduction of the value of this threshold. Our study has shown that the carrying capacity of juvenile mosquitoes K(t), the temperature-dependent biting rate of mosquitoes $\beta(t)$ and the vector reproduction ratio \Re_V have a great impact on the basic reproduction ratio. Indeed, the larger these parameters are, the larger \Re_0 becomes. Hence, these results prove the importance of these parameters in the fight against malaria transmission. In that case, they must not be neglected in the modeling of the disease. Thus, adding the juvenile control to the prevention control will allow to develop more optimal strategies in order to eliminate the disease transmission.

In our model, the juvenile mosquitoes have been grouped in a single compartment. However, female *Anopheles* undergo four distinct metamorphic stages (egg, larva, pupae, adult) during a lifetime. So, the efficiency of juvenile control depends highly on the stage at which it is applied. For instance the use of larvicides like BTI (*Bacillus Thuringensis Israelensis*) allows to reduce larvae but it has no effect on eggs. So, in order to have a qualitative dynamics of malaria virus transmission, it is important to include the four distinct stages. That will be planned in our future researches.

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