EXERCISE 1

MAT260, SPRING 2016

Problem 1.

a): Write the system of differential equations

$$\begin{aligned} u''' &= t^2uu'' - uv' \\ v'' &= tvv' + 4u' \end{aligned}$$

as a system of first-order differential equations $\mathbf{y}' = \mathbf{f}(t, \mathbf{y})$

b): Determine the Jacobian matrix $\mathbf{f}_{\mathbf{y}}(t, \mathbf{y})$ of the system in a)

c): Assume $t \in [0, 1]$ and $||\mathbf{y}||_1 \leq 1$ on this interval. Determine a Lipschitz constant, L, for the system in b)

d): Write a matlab script which find an approximate solution to $u(1)$ and $v(1)$ using Euler's method. Take $u(0) = 1,\ u'(0) = 0;\ u''(0) = 0,\ v(0) = v'(1) = 1$ as initial values.
Try with different values of h and plot u and v.

Problem 2. Exercise 1.2 from the textbook.
(*Remember: A symmetric implies it has an eigendecomposition $A = QDQ^T$ where Q is orthogonal and D is a diagonal matrix containing the eigenvectors and eigenvalues of A, respectively. If you need a refresh see: Appendix A.1.5 and A.2.3*)

Problem 3. As documented in the notes `calculus_and_zombies.pdf` you may be able to save the life of a chemistry professor by applying the *circular pursuit problem*. When applying this technique the zombie will following a trajectory similar to the one in Figure 3, which is described by a set of two coupled ODEs. As stated, these can not be solved analytically. Solve them by Euler's method and plot the solution in a figure similar to Figure 3.

Our unit of length is chosen by setting $R = 1$. Likewise we may set $\omega = 1$. However, with these choices s_z have to chosen with care. Run your simulation for $t \in [0, 8\pi]$ and try different initial values $(x_z(0), y_z(0)) = (a, -a)$ where $0 < a < \sqrt{1/2}$.