Exercise 1

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1 Linear algebra

1.1 Matrix multiplication

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$A * B = \begin{pmatrix} -1 & 0 \\ 10 & 14 \end{pmatrix}$$

1.2 Matrix Inversion

$$C = A * B = \begin{pmatrix} -1 & 0 \\ 10 & 14 \end{pmatrix}, \quad C^{-1} = \begin{pmatrix} -1 & 0 \\ 0.71428571 & 0.07142857 \end{pmatrix}$$

1.3 Eigen value and eigen vector

Let a matrix C be given by

$$C = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$$

Its Eigen value are :

$$\lambda_1 = 1, \quad \lambda_2 = 0$$

2 Calculus

2.1 Gradient

Let

$$f_1(x) = x^2 + 2y^2 - xy \tag{1}$$

the gradient of f_1 is given by

$$\nabla f_1 = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] = [2x - y, 4y - x]$$

2.2 Minimum, maximum

Find the single point (x^*, y^*) that minimises f_1 given by equation (1). (x^*, y^*) is the solution of

$$\nabla f_1 = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] = [2x - y, 4y - x] = 0$$

and

$$(x^*, y^*) = (0, 0)$$

2.3 Lagrange Multiplier

We want dind the value of x, y and λ to minimise

$$f_1 = x^2 + 2y^2 - xy$$

subjected to the constraint

$$g = 2x + y - 22.$$

To find (x, y, λ) we define the Lagrangian by

$$L(x, y, \lambda) = x^2 + 2y^2 - xy - \lambda(2x + y - 22) \tag{2}$$

and solve

$$\begin{aligned} \frac{\partial L}{\partial x} &= 0 \\ \frac{\partial L}{\partial y} &= 0 \\ \frac{\partial L}{\partial \lambda} &= 0 \end{aligned} \tag{3}$$

for (x, y, λ). The solution is given by

$$x = 2.75, \quad , y = 16.5, \quad \lambda = -5.5$$

3 Probability

3.1 Conditional probability

$$P(\text{card is king}|\text{card is not ace}) = \frac{P(\text{card is king and card is not ace})}{P(\text{card is ace})} \tag{4}$$

In a 52 cards deck we have 4 aces and 4 kings.

$$p(\text{card is king}) = \frac{4}{52} \tag{5}$$

$$p(\text{card is ace}) = \frac{4}{52} \tag{6}$$

$$p(\text{card is not ace}) = \frac{52}{52} - \frac{4}{52} = \frac{48}{52} \tag{7}$$

$$p(\text{card is king and card is not ace}) = \frac{4}{52}\frac{4}{48} \tag{8}$$

$$P(\text{card is king}|\text{card is not ace}) = \frac{4}{52} \tag{9}$$

3.2 Rolling a dice

the variance of rolling a dice with six sides is $\frac{35}{12}$ and the mean is 3.5. By definition, in a uniform distribution finite number of value are equally likely to be observe. If you throw a fair dice, you are equally likely to get 1,2,3,4,5,6, that is why throwing a fair dice have a discrete uniform distribution

4 Machine learning

4.1 Well posed machine learning problem

4.1.1 Grocery store problem

The problem can be formulated as follow: Given a product and a location (zip code) approximate the revenue for that product. In this case:

- Task T: Regression (supervise learning)
- Performance P: Root Mean Square Error
- Experience E: Historical data of product with corresponding location and revenue

4.1.2 Oil drilling problem

The problem can be formulated as follow: Given a drilling technology, approximate the output oil production.

- Task T: Regression
- Performance P: Root Mean Square Error
- Experience E: historical data of output oil production per drilling technology

4.1.3 Self driving car

In this case we have multiple tasks. One of then could be

- Tasks T: Object recognition(Classification)
- Performance P: F_β score, Mathew correlation coefficient,
- Experience E: object to identify with corresponding labels

4.2 K-Nearest -Neighbours

4.2.1 Cat-dog problem

$K = 3$, corresponds to a cats and $k = 9$ corresponds to a cats

4.2.2 Iris data set

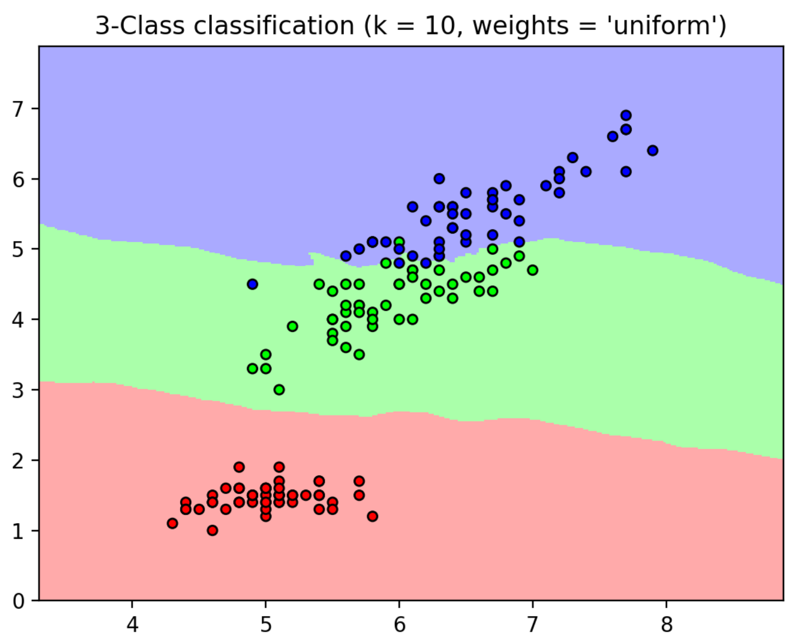

Figure 1: Classification with Iris data set. $K = 10$

The classification result gives :

- red class (y=0) 50

- green class (y=1) 50
- blue class (y=2) 50

Correct classification should be :

- red class (y=0) 50
- green class (y=1) 50 - 5 +1 = 46
- blue class (y=2) 50 -1 +5 = 54

4.2.3 Limitation of KNN

The KNN algorithm uses a distance metric for classification. This works well for features with numerical values, such as petal length, petal width and so on. For categorical features such as gender (Male, Female, Trans-sexual) it can be challenging to evaluate distance.