MT105B

Summer 2020 online assessment guidance

MT105B Mathematics 2

The assessment will be an **open-book take-home online assessment within a 24-hour window**. The requirements for this assessment remain the same as the originally planned closed-book exam, **with an expected time/effort of 2 hours**.

Candidates should answer all **EIGHT** questions: all **SIX** questions of Section A (60 marks in total) and **BOTH** questions from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly**.

You may use *any* calculator for any appropriate calculations, but you may not use any computer software to obtain solutions. Credit will only be awarded if all workings are shown.

You should complete this paper using **pen and paper**. Please use **BLACK ink only**.

Handwritten work then needs to be scanned, converted to PDF and then uploaded to the VLE as **ONE individual file** including the coversheet. Each scanned sheet should have your **candidate number** written clearly at the top. Please **do not write your name anywhere** on any sheet.

The paper will be available at **12.00 midday (BST) on Monday 29 June 2020**.

You have **until 12.00 midday (BST) on Tuesday 30 June 2020** to upload your file into the VLE submission portal. However, you are advised not to leave your submission to the last minute. *A late penalty will be applied pro-rata of 5 percentage marks for every hour (or part) late outside of the 1 hour*.

If you think there is any information missing or any error in any question, then you should indicate this but proceed to answer the question stating any assumptions you have made.

The assessment has been designed with a duration of 24 hours to provide a more flexible window in which to complete the assessment and to appropriately test the course learning outcomes. As an open-book exam, the expected amount of effort required to complete all questions and upload your answers during this window is no more than 2 hours. Organise your time well and avoid working all night.

You are assured that there will be no benefit in you going beyond the expected 2 hours of effort. Your assessment has been carefully designed to help you show what you have learned in the hours allocated.

This is an open book assessment and as such you may have access to additional materials including but not limited to subject guides and any recommended reading. But the work you submit is expected to be 100% your own. Therefore, unless instructed otherwise, you must not collaborate or confer with anyone during the assessment. The University of London will carry out checks to ensure the academic integrity of your work. Many students that break the University of London's assessment regulations did not intend to cheat but did not properly understand the University of London's regulations on referencing and plagiarism. The University of London considers all forms of plagiarism, whether deliberate or otherwise, a very serious matter and can apply severe penalties that might impact on your award. The University of London 2019-20 Procedure for the Consideration of Allegations of Assessment offences is available online at:

https://london.ac.uk/sites/default/files/governance/assessment-offence-procedure-year-2019-2020.pdf

The University of London's Rules for Taking Online Timed Assessments have been included in an update to the University of London General Regulations and are available at:

https://london.ac.uk/sites/default/files/regulations/progregs-general-2019-2020.pdf

SECTION A

Answer all **six** questions from this section (60 marks in total).

1. The demand equation for a market is given by $p(q+3) = 15$ and, for some constant α, the supply equation is $q = \alpha p - 1$ where p is the price and q is the quantity.

   Given that the equilibrium price for this market is three, determine the equilibrium quantity and the value of α.

   Find the consumer and producer surpluses.

2. Expand as a power series, in terms up to x^6, $f(x) = \mathrm{e}^{1-\cos x}$.

3. Find the values of x, y and z that minimise $(x^4 + y^4 + z^4)^{1/4}$ subject to the constraint $x + 8y + 27z = 10$.

4. For what values of the constant a is the matrix

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 1 & 1 \\ 0 & 1 & a \end{pmatrix}$$

   invertible? For $a = 3$, find the inverse of A.

   What does the existence of this inverse tell you about the solution(s) to the system of equations $A\mathbf{x} = \mathbf{b}$? If $\mathbf{b} = (1, 1, 1)^T$, use the inverse that you have just found to find $\mathbf{x}$.

5. Find the function $y(x)$ that, for $x > -1$, satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{y\sqrt{1+x}}$$

   and the initial condition $y(0) = 1$.

6. For $t \geq 2$, the sequence x_t satisfies the difference equation

$$x_t - x_{t-1} + x_{t-2} = 10$$

   with $x_0 = 12$ and $x_1 = 11$. Find x_t and describe how it behaves as t increases.

SECTION B

Answer **both** questions from this section (20 marks each).

7. (a) **Use a matrix method** to find the relationship between the constants a and b for which the system of equations

$$\begin{aligned} u + 2v + 3w &= a, \\ u - v + w &= 1, \\ 2u + v + 4w &= b \end{aligned}$$

has solutions.

When a and b satisfy this relationship, find all of the solutions to this system in terms of a and write your answer in vector form.

(b) In a market, the price of a share t seconds after the market opens is $p(t)$ dollars. An analyst constructs a model which predicts that the price of this share should vary continuously with time according to the differential equation

$$\frac{\mathrm{d}p}{\mathrm{d}t} = p(t) - \mathrm{e}^{-t}.$$

If the initial price is \$1, solve this differential equation to find $p(t)$.

Show that, according to the model, $p(t)$ is increasing for all $t \geq 0$.

When will the price of a share reach \$1.25?

8. In a national economy, $I(t)$, $Y(t)$ and $C(t)$ denote (in trillions of dollars) the level of investment, income and consumption respectively at time t (in years) and these quantities are related by the equations

$$C(t) = 1 - Y'(t) \quad \text{and} \quad I(t) = \frac{1}{4}\Big(1 + 3Y(t) - 4Y''(t)\Big)$$

with $C(0) = 1/2$ and $I(0) = 7/2$.

If $Y(t) = C(t) + I(t)$, show that $Y(t)$ satisfies the differential equation

$$Y''(t) + Y'(t) + \frac{1}{4}Y(t) = \frac{5}{4}$$

and hence find $Y(t)$.

Deduce expressions for $C(t)$ and $I(t)$.

Describe the behaviour of each of these three functions as t increases.

END OF PAPER