Energy Detection Technique for Adaptive Spectrum Sensing

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***Abstract*—The increasing scarcity in the available spectrum for wireless communication is one of the current bottlenecks impairing further deployment of services and coverage. The proper exploitation of white spaces in the radio spectrum requires fast, robust, and accurate methods for their detection. This paper proposes a new strategy to detect adaptively white spaces in the radio spectrum. Such strategy works in cognitive radio (CR) networks whose nodes perform spectrum sensing based on energy detection in a cooperative way or not. The main novelty of the proposal is the use of a cost-function that depends upon a single parameter which, by itself, contains the aggregate information about the presence or absence of primary users. The detection of white spaces based on this parameter is able to improve significantly the deflection coefficient associated with the detector, as compared to other state-of-the-art algorithms. In fact, simulation results show that the proposed algorithm outperforms by far other competing algorithms. For example, our proposal can yield a probability of miss-detection 20 times smaller than that of an optimal soft-combiner solution in a cooperative setup with a predefined probability of false alarm of 0.1.**

***Index Terms*—Cognitive radio, cooperative spectrum sensing, energy detection, adaptive signal processing, deflection coefficient.**

I. Introduction

COGNITIVE RADIO (CR) emerged as a promising technology to deal with the current inefficient usage of limited spectrum [1], [2]. In a CR scenario, users who have no spectrum licenses, also known as secondary users (SUs), are allowed to take advantage of vacant spectrum while primary users (PUs) do not demand its use. The first step of the CR process, coined as *spectrum sensing* (SS), consists of discovering the available portions of the wireless spectrum or white spaces to be employed by SUs.

In the SS context, energy-detection (ED) algorithms are widely employed in the literature since they do not require prior information about PU signals and offer implementation simplicity [3]–[7]. Their main drawback is that the corresponding detection threshold highly depends on the environment conditions. Cooperative strategies are usually employed to increase the detection reliability in either centralized or decentralized modes [8]. Examples of cooperative schemes include soft-combining techniques, which are based on linear combinations of energy estimates acquired by distinct SUs to decide whether there are white spaces available in the spectrum. Several optimization strategies, such as maximal ratio combining [9], Neyman-Pearson criterion [9], [10], modified deflection-coefficient maximization [11], [12], adaptive exponential-weighting windows [13], or mean-squared error (MSE) minimization [14]–[17], are employed to find the optimal weights for the soft-combiner. The decision rule associated with those techniques is based on the comparison between the output of the soft-combiner and its corresponding detection threshold.

The performance of the aforementioned cooperative strategies is strongly affected by the random fluctuations present at the output of the linear combiner [18]. Those undesirable fluctuations occur due to the inherent random nature of the energy estimates acquired by each SU. Working with more stable statistics based on energy estimates would be, therefore, rather desirable.

This work proposes an efficient and robust ED-based technique which performs the SS process based on an adaptive parameter instead of a soft-combiner of energy estimates. To do that, an MSE minimization problem of a new cost-function has been solved through the least mean squares (LMS) algorithm. The cost-function is defined to perform both single-node and weighted cooperative SS using an adaptive test statistic that depends on pre-processed inputs and desired signals chosen adequately to improve the well-known deflection coefficient [11], [19], [20] of the test statistic. As a result, the proposed method significantly increases the deflection coefficient compared with that achieved in conventional ED methods. By using this simple, yet very effective strategy, the detection performance in terms of probability of miss-detection is considerably improved as compared to other soft-combiner ED-based techniques [11], [15], [16], even in the case where a single node is employed. Furthermore, apart from assessing the algorithm with various weighting strategies of the literature [21], [22], a new weighting proposal is presented.

The paper is organized as follows. Section II is split into two subsections: first we introduce the system model for ED of PU

Manuscript received June 30, 2014; revised November 4, 2014 and January 9, 2015; accepted January 11, 2015. Date of publication January 21, 2015; date of current version March 13, 2015. This work has been financially supported in part by the Brazilian research councils FAPERJ, CAPES, and CNPq, as well as by the University of the Basque Country UPV/EHU (UFI 11/30), by the Department of Education, Universities and Research of the Basque Government, by the Basque Government (IT-683-13 and SAIOTEK), and by the Spanish Ministry of Economy and Competitiveness under the project HEDYT-GBB (TEC2012-33302). The associate editor coordinating the review of this paper and approving it for publication was H. Dai.

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Digital Object Identifier 10.1109/TCOMM.2015.2394436

signals and then we present the motivations that led to the new ED proposal. Section II-B starts with the proposal of an adaptive algorithm for both single-node and cooperative SS taking into account the test statistic through the deflection coefficient. Then, the proposed detection test based on a new adaptive parameter is described. The derivation of a new detection threshold for the adaptive ED parameter is presented in Section III. To assess the performance of our proposal, we consider two viewpoints in Section IV. Firstly, by analyzing and comparing the deflection coefficient of the proposal with that corresponding to a common ED method. Secondly, by evaluating the performance of the proposed technique in terms of probability of miss-detection *versus* a desired probability of false alarm in both non-cooperative and cooperative scenarios. Finally, conclusions are drawn in Section V.

Notation: The operator $\mathrm{diag}\{\cdot\}$ places the elements of a given vector in the diagonal of a square matrix, and $[\cdot]^{\mathrm{T}}$ is the transpose operator. Vectors and matrices are represented in boldface lowercase and uppercase, respectively. The operators $\mathrm{E}[\cdot]$ and $\mathrm{Var}[\cdot]$ are the expectation and variance of a given random value, respectively.

II. Adaptive ED-Based Spectrum Sensing

A. System Model

Let us consider a CR network with SUs spatially distributed. Each SU employs an energy detector to sense the environment under the hypotheses H_0 (absence of PU signal) and H_1 (presence of PU signal), such that the received signal at the mth SU is modeled as

$$x_m(n) = \begin{cases} v_m(n) & \text{if } H_0 \text{ holds} \\ v_m(n) + h_m s(n) & \text{if } H_1 \text{ holds,} \end{cases} \tag{1}$$

where the PU signal at discrete-time instant n has been represented as $s(n)$. This signal is affected by a channel gain/attenuation h_m and corrupted by a local zero-mean additive white Gaussian noise (AWGN) $v_m(n)$ with variance σ_m^2. It is assumed that the channel gain h_m is constant during the period of detection of spectrum vacancies.

In a CR scenario, both hypotheses H_0 and H_1 last at least for some minimum time interval during which just one hypothesis holds. The energy detector provides energy estimates corresponding to that hypothesis. Let us assume that each spatially anchored node generates a local energy estimate $y_{m,k}$ based on N received samples, as follows:

$$y_{m,k} = \sum_{n=0}^{N-1} |x_m(n+kN)|^2, \tag{2}$$

which can be shared with its neighbors in the CR network. Note that k consists of the discrete-time index associated with the output data of the energy estimator. As shown in [3], [23], for each fixed k, $y_{m,k}$ follows a Chi-squared distribution of degree N, which can be approximated to a normal distribution with mean $\mathrm{E}[y_{m,k}]$ and variance $\mathrm{Var}[y_{m,k}]$, as long as N is large enough [24], in which

$$\mathrm{E}[y_{m,k}] = \begin{cases} N\sigma_m^2 & \text{if } H_0 \text{ holds} \\ [N+\eta_{m,k}]\sigma_m^2 & \text{if } H_1 \text{ holds,} \end{cases} \tag{3}$$

where $\eta_{m,k}$ is defined as N times the signal-to-noise ratio (SNR) at the mth SU node, i.e.,

$$\eta_{m,k} = \frac{|h_{m,k}|^2}{\sigma_m^2} \sum_{n=0}^{N-1} |s_m(n+kN)|^2. \tag{4}$$

Despite the fact that $h_{m,k}$ is considered time-invariant during the period of detection of spectrum vacancies, which may take several values of k, we shall explicitly denote the dependency of the channel gain on variable k since $h_{m,k}$ can assume different values for different periods of detection. In addition, the variance of $y_{m,k}$ is given as

$$\mathrm{Var}[y_{m,k}] = \begin{cases} 2N\sigma_m^4 & \text{if } H_0 \text{ holds} \\ 2[N+2\eta_{m,k}]\sigma_m^4 & \text{if } H_1 \text{ holds.} \end{cases} \tag{5}$$

To decide whether the observations were made under H_0 and H_1, the energy estimate $y_{m,k}$ is compared with a predetermined detection threshold γ as follows:

$$y_{m,k} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma. \tag{6}$$

The threshold γ can be chosen according to a desired probability of false alarm $P_f = \Pr(y_{m,k} > \gamma | H_0)$. Considering the Gaussian approximation for $y_{m,k}$, P_f can be rewritten as

$$P_f = Q\left(\frac{\gamma - \mathrm{E}[y_{m,k}]_{H_0}}{\sqrt{\mathrm{Var}[y_{m,k}]_{H_0}}}\right), \tag{7}$$

where $Q(z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{x^2}{2}} \mathrm{d}x$ is the Q-function.

As a final remark, we assume that each node is able to estimate the noise variance σ_m^2 and share it along with its energy estimates $y_{m,k}$ with its neighbors of the CR network.

B. Motivations and Proposal

The first and second-central moments of the energy estimates convey all the information required to detect which hypothesis holds at a given period of time. For example, based on (3), if one knew that $\mathrm{E}[y_{m,k}] > N\sigma_m^2$, then one could state that H_1 certainly holds true. However, one can only have access to estimates of $\mathrm{E}[y_{m,k}]$. In fact, ED techniques usually employ raw energy estimates $y_{m,k}$ to perform the detection process. As described in (2), these estimates are computed based on signal samples acquired at a sampling rate which is high enough to allow us to assume that the channel is time-invariant during the corresponding N-sample sensing window. Nevertheless, the energy estimates can vary drastically from one sensing window to another, depending on the channel conditions at the particular sensing window, thus affecting the detection performance. An improvement of the detection performance could be achieved if the energy estimates were as close to their average value as possible, i.e., the variances of the energy estimates were as small as possible. In this context, one could conceive the following single-node-based cost-function that helps us motivate our proposals:

$$J(\omega_m) = \mathrm{E}\left[(d_{m,k} - \omega_m y_{m,k})^2\right], \tag{8}$$

where $d_{m,k}$ is the desired signal of a node m at instant k, ideally equal to $\mathrm{E}[y_{m,k}]$, $y_{m,k}$ is the corresponding energy estimate, and ω_m is a node-dependent control parameter.

Adaptive algorithms can be a solution to soften erratic behavior or abrupt variations of energy estimates due to channel impairments. Indeed, it is well-known that adaptive filters implicitly implement an averaging process as long as their related step-size parameters are small enough [25], [26]. For that reason, other SS techniques such as [15], [16] have adopted adaptive solutions for the detection process based on soft linear combiners of the energy estimates. In this context, we could employ $\omega_m y_{m,k}$ to perform SS, where the parameter ω_m in (8) can be computed using standard adaptive algorithms that search for the minimum of either the cost-function $J(\omega_m)$ or a deterministic approximation of it. If we examine (8) closely, we see that $\omega_m y_{m,k}$ is a product of scalars and the resulting ω_m will bear memory corresponding to previous energy estimates due to its adaptive nature. In addition, it is worth noting that minimizing (8) is a strict convex optimization problem under two hypotheses (H_0 and H_1), thus implying that there exists one optimal solution for each hypothesis, which means that the detection task employing ω_m is a well-posed problem. Consequently, if the energy detector can provide several energy estimates of the same hypothesis during various consecutive timeslots k, the control parameter ω_m can filter out possible abrupt variations present in current signal $y_{m,k}$. We, therefore, propose using ω_m as the detection parameter instead of the raw estimates $y_{m,k}$ or the product $\omega_m y_{m,k}$ since it is less sensitive to random fluctuations of the measurements which do not arise from actual changes of hypothesis, and it also allows us to perform some modifications in the cost-function to improve the test statistic. Note that SS based on the variables $y_{m,k}$ or ω_m does not lead to the same performance for both detection problems, since the statistics of the detection parameters plays a key role in the overall problem.

To evaluate different detection tests, one can employ the deflection coefficient, which is a meaningful figure of merit defined as [19], [27]

$$\delta^2 = \frac{(\mathrm{E}[T]_{H_1} - \mathrm{E}[T]_{H_0})^2}{\mathrm{Var}[T]_{H_0}}, \tag{9}$$

where T is a given test statistic. When Gaussian assumptions are introduced,[1] the maximization of deflection leads to similar behavior as the likelihood ratio receiver [19]. As a result, this figure of merit has been used in the literature to characterize the performance of ED in SS [11], [12], [20]. Large values of δ^2 leads to easier distinction between the corresponding two hypotheses and, as a result, to a better detection performance. In addition, we can infer from (9) that, if one could modify the test statistic, for instance, by increasing the distance between the means or reducing the variance of the test, then one could improve the detection process. In this context, the detection test based on ω_m can be considerably improved by choosing/changing adequately the inputs and the desired signal in (8). For that purpose, we propose the following cost-function:

$$\tilde{J}(\omega_m) = \mathrm{E}\left[(\tilde{d}_{m,k} - \omega_m \tilde{u}_{m,k})^2\right], \tag{10}$$

where $\tilde{u}_{m,k} = |y_{m,k} - \gamma|$, $\tilde{d}_{m,k} = d_{m,k} - \gamma$, and γ is the threshold over the test statistic $y_{m,k}$ for a predefined P_f (see (7)), i.e.

$$\gamma = \mathrm{E}[y_{m,k}]_{H_0} + Q^{-1}(P_f)\sqrt{\mathrm{Var}[y_{m,k}]_{H_0}}. \tag{11}$$

Solving (10) allows us to achieve a deflection coefficient based on ω_m much larger than that using directly the energy estimates $y_{m,k}$. This occurs due to the significant reduction of the denominator of the test statistics when using ω_m instead of $y_{m,k}$, which intuitively stems from the reduction of variance of $\tilde{u}_{m,k} = |y_{m,k} - \gamma|$ as compared with the variance of $y_{m,k}$ under the H_0 hypothesis. Indeed, we can write $\mathrm{Var}[\tilde{u}_{m,k}] = \mathrm{Var}[u_{m,k}] + (\mathrm{E}[u_{m,k}]^2 - \mathrm{E}[\tilde{u}_{m,k}]^2)$. We also know that $u_{m,k} = y_{m,k} - \gamma \leq |y_{m,k} - \gamma| = \tilde{u}_{m,k}$, thus implying that $\mathrm{E}[u_{m,k}] \leq \mathrm{E}[\tilde{u}_{m,k}]$. In addition, as $\mathrm{Var}[y_{m,k}] = \mathrm{Var}[y_{m,k} - \gamma] = \mathrm{Var}[u_{m,k}]$, then we can conclude that

$$\mathrm{Var}[\tilde{u}_{m,k}] \leq \mathrm{Var}[y_{m,k}]. \tag{12}$$

Actually, we observed in all of our experiments that $\mathrm{Var}[\tilde{u}_{m,k}] \ll \mathrm{Var}[y_{m,k}]$. Nonetheless, it is worth mentioning that numerator and denominator of the deflection coefficient $\delta^2(\omega_{m,k})$ are respectively smaller than numerator and denominator of $\delta^2(y_{m,k})$, but the ratio keeps larger in comparison to $\delta^2(y_{m,k})$, as illustrated in Fig. 1, which depicts the estimated probability density functions (PDFs) of $y_{m,k}$ and ω_m for hypotheses H_0 and H_1. These results consider a single-node spectrum sensing setup that will be detailed in Section IV-B. This example highlights how the Gaussian-like PDFs related to the H_0 and H_1 hypotheses are more separated for ω_m than for $y_{m,k}$.

To showcase the improvements that come from using ω_m as test statistics, an additional simulation analysis has been conducted for several SNRs $\eta_{m,k}$. Fig. 2 depicts the deflection coefficient for the interval probabilities of false alarm, $P_f \in [0.01, 0.99]$, and four different values of SNR, i.e. $\eta_{m,k} = 0$, 3, 6 and 9 dB, considering two test statistics, namely: using directly the energy estimates $y_{m,k}$ and using ω_m in (10). From the figure, one can observe that the deflection coefficient of ω_m is larger than that of $y_{m,k}$ for the interesting (in practical terms) range of P_f, i.e., low values of P_f. In conventional ED techniques in which energy estimates are directly used in the detection process, δ^2 is the same for all predefined P_f, as can be seen in detail in Fig. 2. The proposed method modifies the statistics of the test as a function of P_f due to the inclusion of γ in (10) (γ depends on P_f following (11)). Thus, the deflection coefficient, and ultimately the detection performance itself, will change according to the desired P_f.

In a CR network where several nodes are affected by the same PU signal, a binary hypothesis test can be solved with a single parameter. Based on this fact, we generalize (10) for cooperative SS by stating an analogous minimization problem

[1] Most of the ED-based techniques approximate the probability distribution function of the decision parameter, under both hypotheses, to Gaussian distributions due to the validity of the central limit theorem when N is large enough [24], [28].

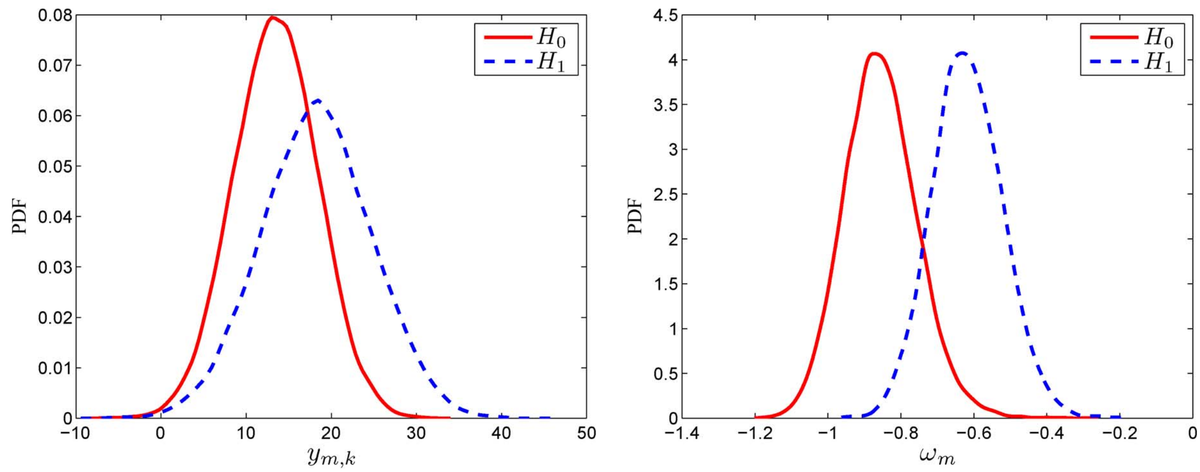

Fig. 1. Estimated PDFs of $y_{m,k}$ (left) and ω_m (right) under both hypotheses H_0 and H_1.

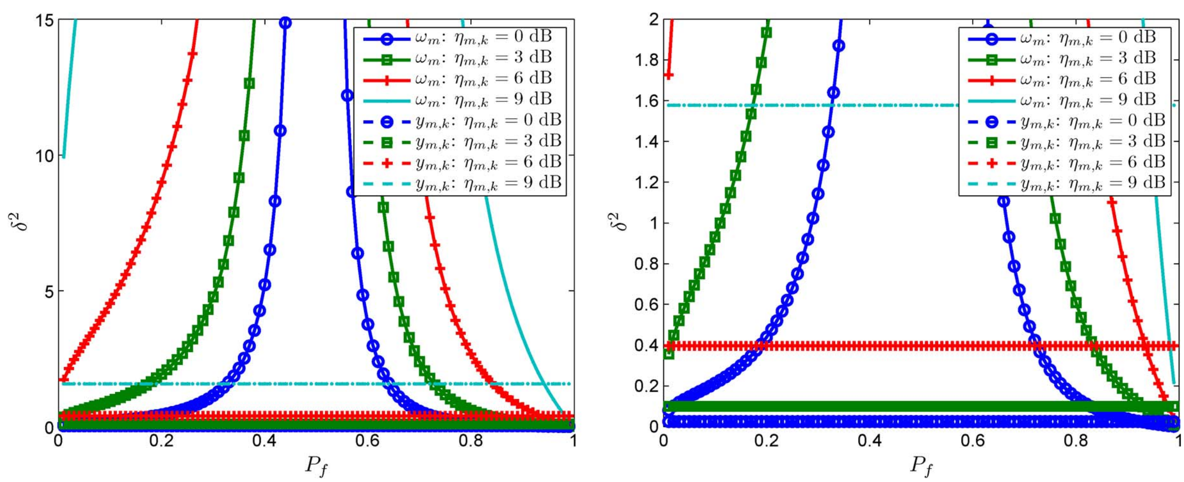

Fig. 2. Deflection coefficient, δ^2, *versus* predefined probability of false alarm, P_f, from 0.01 to 0.99 for different SNRs and a fixed noise variance of $\sigma_m^2 = 1.2$ obtained through a test statistic $y_{m,k}$. The figure on the right-hand side zooms in for small values of δ^2.

that relies on a single adaptive parameter. Let us assume that the neighborhood of a node i of the CR network, denoted by $\mathcal{N}_i$, is formed by M_i secondary users linked to each other. To perform a collaborative SS among neighbors in $\mathcal{N}_i$, we propose the following cost-function:

$$\bar{J}(\omega_i) = \sum_{m\in\mathcal{N}_i} c_m \mathrm{E}\left[(\tilde{d}_{m,k} - \omega_i \tilde{u}_{m,k})^2\right]. \tag{13}$$

The coefficients c_m allow us to perform a weighted cooperation strategy as in [22], which we will discuss in Section IV. Note that the single-node case of (10) is included as a particular solution of (13). As the results presented in Section IV indicate, this cooperative proposal clearly outperforms conventional soft-combiner based ED techniques.

As we are interested in practical online algorithms to minimize (13), a stochastic gradient algorithm is employed to iteratively approximate its solution. In this context, a reasonable approximation can be obtained through an LMS-like solution [25], as follows:

$$\omega_{i,k+1} = \omega_{i,k} + \mu_i \sum_{m\in\mathcal{N}_i} c_m \varepsilon_{m,k} \tilde{u}_{m,k}, \tag{14}$$

where μ_i is the step-size for the ith node and the output-error coefficient $\varepsilon_{m,k}$ is computed as

$$\varepsilon_{m,k} = \tilde{d}_{m,k} - \omega_{m,k} \tilde{u}_{m,k}. \tag{15}$$

The coefficients c_m must satisfy $\sum_{m\in\mathcal{N}_i} c_m = 1$ and are chosen to perform a weighted cooperation as a function of parameters such as noise variance and number of linked nodes [22]. This paper also presents a new proposal based on a relative degree of SNR that can be employed whenever the SUs are able to estimate and share their local SNRs. We propose to compute the coefficients c_m at node i as

$$c_m = \frac{\eta_{m,k}}{\sum\limits_{n\in\mathcal{N}_i} \eta_{n,k}}, \quad \text{for each } m \in \mathcal{N}_i. \tag{16}$$

As previously discussed, a good choice for the desired signals $d_{m,k}$ would be the average value of the energy estimates at each node m, i.e. $d_{m,k} = \mathrm{E}[y_{m,k}]$. Nevertheless, those means are not achievable in practice since we do not have perfect knowledge about which hypothesis holds at each moment. Assume that the duration in which the hypotheses H_0 and H_1 do not change is considered long enough compared with the

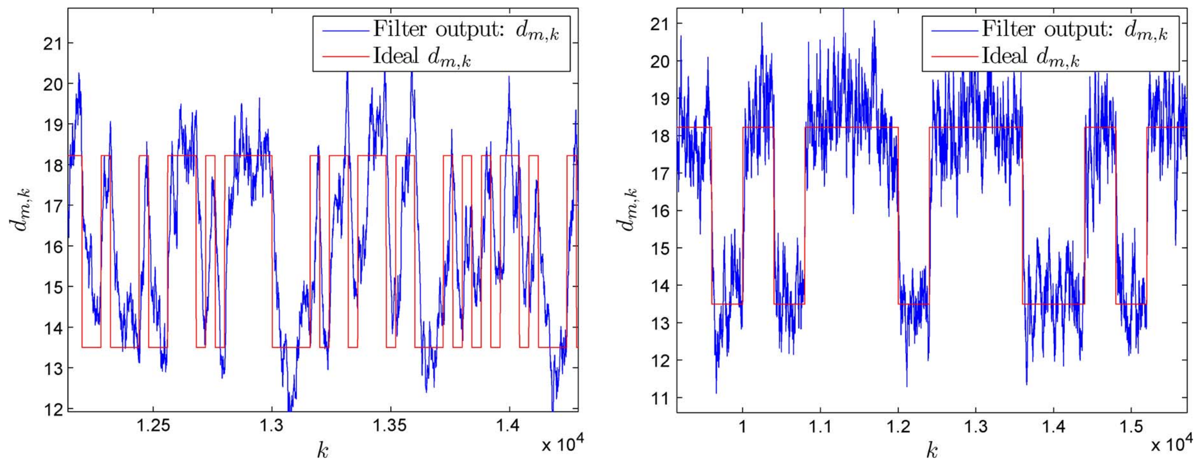

Fig. 3. Comparison between the filter output $d_{m,k}$ and the ideal desired signal considering each hypothesis constant for at least 40 (on the left) and 400 (on the right) iterations.

time taken by the sensing process, then, the use of a first-order filter of the energy estimates gives us a good approximation of $\mathrm{E}[y_{m,k}]$, since the filter output tends to the mean of the energy estimates at each hypothesis after some iterations. Thus, the desired signal can be provided at each instant k as

$$d_{m,k} = (1-\alpha)y_{m,k} + \alpha d_{m,k-1}, \tag{17}$$

where α is a scalar close to but less than one.

It is worth pointing out that the sampling rate associated with the discrete-time signal $y_{m,k}$ corresponds to the same sampling rate of the recursive filter that computes $d_{m,k}$. This underlying sampling rate must be sufficiently large to provide this filter with the capability of tracking any change of hypothesis. The proposed detection algorithm measures the output of the filter during both transient and steady states. Therefore, if the sampling rate is adequately chosen to observe several times the same hypothesis, then the transient state will have a negligible effect on the overall performance, even if the PU changes during the detection process. Fig. 3 depicts a comparison between the filter output and the ideal desired signal considering each hypothesis constant in at least 40 and 400 samples (at the filter sampling rate). We can observe that, for the case of 40 samples, it would be desirable to increase the sampling rate of the energy detector since the transient-state duration is not negligible as compared with the steady-state duration. In the case of 400 samples, we can clearly observe the convergence of the filter.

As a result, the use of (14) allows us to sense in an adaptive way through the following detection test:

$$\omega_{i,k} \underset{H_0}{\overset{H_1}{\gtrless}} \tilde{\gamma}_i \tag{18}$$

where $\tilde{\gamma}_i$ is the new detection threshold for the neighborhood $\mathcal{N}_i$. Let us motivate the rationale behind the detection test (18). Note that, as $\omega_{i,k}$ in (14) is an approximation to the solution of the problem described in (13), and since this problem can be regarded as a strict convex optimization problem assuming that one hypothesis holds, then the actual values of $\omega_{i,k}$ fluctuate around the optimal solutions of (13) for each hypothesis, following a certain probability distribution, which tends to be close to Gaussian in steady state. Therefore, the detection problem consists of separating those two Gaussian-like distributions, which can be achieved through a unique threshold $\tilde{\gamma}_i$. Taking that into account, we will present below a possible threshold to decide between both hypotheses.

As we shall see in Section IV, we can assume that $\omega_{i,k}$ follows a behavior whose distribution is close to Gaussian. Consequently, the probability of false alarm P_f, which is defined as the probability of detecting H_1 when H_0 holds, can be expressed as

$$P_f = Q\left(\frac{\tilde{\gamma}_i - \mathrm{E}[\omega_{i,k}]_{H_0}}{\sqrt{\mathrm{E}\left[\omega_{i,k}^2\right]_{H_0} - \mathrm{E}[\omega_{i,k}]_{H_0}^2}}\right). \tag{19}$$

Therefore, from (19), we can obtain a $\tilde{\gamma}_i$ for a predefined P_f, as follows:

$$\tilde{\gamma}_i = \mathrm{E}[\omega_{i,k}]_{H_0} + Q^{-1}(P_f)\sqrt{\mathrm{Var}[\omega_{i,k}]_{H_0}}. \tag{20}$$

The computation of the statistics of $\omega_{i,k}$ is not a trivial task and it is carried out in the next section.

III. Detection-Threshold Derivation

Considering that $y_{m,k}$ is asymptotically normally distributed, we can say that $u_{m,k} = y_{m,k} - \gamma_m$ is also asymptotically normally distributed with mean

$$\mathrm{E}[u_{m,k}] = \begin{cases} -Q^{-1}(P_f)\sqrt{2N}\sigma_m^2 & \text{if } H_0 \text{ holds} \\ \eta_{m,k}\sigma_m^2 - Q^{-1}(P_f)\sqrt{2N}\sigma_m^2 & \text{if } H_1 \text{ holds} \end{cases} \tag{21}$$

and variance $\mathrm{Var}[u_{m,k}] = \mathrm{Var}[y_{m,k}]$ as in (5).

If we examine $\omega_{i,k}$, we can observe from (14) that its value is calculated using an adaptive filter whose inputs are the absolute values of normally distributed variables. Therefore, we must use a folded normal distribution for the inputs $\tilde{u}_{m,k}$ to yield the statistics of $\omega_{i,k}$.

Using the general expressions for the mean and variance of a folded normal distribution [29], we can express the first and second order statistics of $\tilde{u}_{m,k}$ as

$$E[\tilde{u}_{m,k}] = \mathrm{E}[u_{m,k}]\left[1-2\Phi\left(\frac{-\mathrm{E}[u_{m,k}]}{\sqrt{\mathrm{Var}[u_{m,k}]}}\right)\right] + \sqrt{\mathrm{Var}[u_{m,k}]}\sqrt{\frac{2}{\pi}}\exp\left(-\frac{\mathrm{E}[u_{m,k}]^2}{2\mathrm{Var}[u_{m,k}]}\right), \quad (22)$$

where $\Phi(z) = \int_{-\infty}^{z}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{x^2}{2}}\mathrm{d}x$ is the cumulative distribution function (CDF) of the standard normal distribution, and

$$\mathrm{Var}[\tilde{u}_{m,k}] = \mathrm{E}[u_{m,k}]^2 + \mathrm{Var}[u_{m,k}] - \mathrm{E}[\tilde{u}_{m,k}]^2. \quad (23)$$

To compute $\mathrm{E}[\omega_{i,k}]$ and $\mathrm{Var}[\omega_{i,k}]$, let us define the following vectors/matrix: $\tilde{\mathbf{u}}_{i,k} = [\tilde{u}_{1,k}\ldots\tilde{u}_{M_i,k}]^{\mathrm{T}}$, $\tilde{\mathbf{d}}_{i,k} = [\tilde{d}_{1,k}\ldots\tilde{d}_{M_i,k}]^{\mathrm{T}}$, $\mathbf{c}_i = [c_1\ldots c_{M_i}]^{\mathrm{T}}$, and $\mathbf{C}_i = \mathrm{diag}\{\mathbf{c}_i\}$. We can rewrite (14) as

$$\omega_{i,k+1} = \omega_{i,k}\left(1-\mu_i\tilde{\mathbf{u}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right) + \mu_i\tilde{\mathbf{d}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}. \quad (24)$$

By taking the expected values of (24) and by assuming that $\omega_{i,k}$ and $\tilde{\mathbf{u}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}$ are uncorrelated random sequences, then we obtain

$$\mathrm{E}[\omega_{i,k+1}] = \mathrm{E}[\omega_{i,k}]\left(1-\mu_i\mathrm{E}\left[\tilde{\mathbf{u}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right]\right) + \mu_i\mathrm{E}\left[\tilde{\mathbf{d}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right]. \quad (25)$$

Assuming that the step-size μ_i is properly chosen, in the steady-state one has $\mathrm{E}[\omega_{i,k+1}] \approx \mathrm{E}[\omega_{i,k}]$ and the reference signals tend to the average values of the energy estimates, i.e., $\tilde{\mathbf{d}}_{i,k} \approx [\mathrm{E}[u_{1,k}]\ldots\mathrm{E}[u_{M_i,k}]]^{\mathrm{T}}$. Therefore, we can write the mean of the cooperative detection parameter $\omega_{i,k}$ as

$$\mathrm{E}[\omega_{i,k}] = \frac{\tilde{\mathbf{d}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\mathrm{E}[\tilde{\mathbf{u}}_{i,k}]}{\mathrm{E}\left[\tilde{\mathbf{u}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right]} = \frac{\sum_{m\in\mathcal{N}_i} c_m\tilde{d}_{m,k}\mathrm{E}[\tilde{u}_{m,k}]}{\sum_{m\in\mathcal{N}_i} c_m\mathrm{E}\left[\tilde{u}_{m,k}^2\right]}. \quad (26)$$

It is worth noticing that (26) also corresponds to the Wiener solution of (13).

To obtain the variance of $\omega_{i,k}$, we will analyze the second order moment of $\omega_{i,k}$. By applying the expectation operation to the squared value of (24) and taking into account the previous assumptions of uncorrelated variables, we arrive after some operations at

$$\begin{aligned}\mathrm{E}\left[\omega_{i,k+1}^2\right] &= \mathrm{E}\left[\omega_{i,k}^2\right] \times \Bigg(1 + \underbrace{\mu_i^2\mathrm{E}\left[\tilde{\mathbf{u}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\tilde{\mathbf{u}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right] - 2\mu_i\mathrm{E}\left[\tilde{\mathbf{u}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right]}_{A}\Bigg)\\ &+ \mathrm{E}[\omega_{i,k}]\underbrace{2\mu_i\mathrm{E}\left[\left(1-\mu_i\tilde{\mathbf{u}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right)\tilde{\mathbf{d}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right]}_{B}\\ &+ \underbrace{\mu_i^2\mathrm{E}\left[\tilde{\mathbf{d}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\tilde{\mathbf{d}}_{i,k}^{\mathrm{T}}\mathbf{C}_i\tilde{\mathbf{u}}_{i,k}\right]}_{C}. \end{aligned} \quad (27)$$

In the steady-state, we assume that $\mathrm{E}[\omega_{i,k+1}^2] \approx \mathrm{E}[\omega_{i,k}^2]$. Therefore, by examining (27), we can infer by using this approximation that $\mathrm{E}[\omega_{i,k}^2]$ can be written as

$$\mathrm{E}\left[\omega_{i,k}^2\right] = \frac{-\mathrm{E}[\omega_{i,k}]B - C}{A}, \quad (28)$$

whose expanded expression is (considering the previously presented assumptions to obtain $\mathrm{E}[\omega_{i,k}]$) is given in (29), shown at bottom of the page.

It is worth noting that, apart from the mean and the second order moments, we need the third and fourth order moments of the folded normally distributed $\tilde{u}_m$. Based on [29], these moments can be calculated as follows:

$$\mathrm{E}\left[\tilde{u}_{m,k}^3\right] = \mathrm{E}[u_{m,k}]\mathrm{Var}[u_{m,k}]\left[1-2\Phi\left(\frac{-\mathrm{E}[u_{m,k}]}{\sqrt{\mathrm{Var}[u_{m,k}]}}\right)\right] + \left(\mathrm{E}[u_{m,k}]^2 + 2\mathrm{Var}[u_{m,k}]\right)\mathrm{E}[\tilde{u}_{m,k}] \quad (30)$$

and

$$\mathrm{E}\left[\tilde{u}_{m,k}^4\right] = \mathrm{E}[u_{m,k}]^4 + 6\mathrm{E}[u_{m,k}]^2\mathrm{Var}[u_{m,k}] + 3\mathrm{Var}[u_{m,k}]^2. \quad (31)$$

To obtain the detection threshold, we substitute (26) and (29) into (20) using the statistics of hypothesis H_0, which can be easily obtained through the shared information of the noise variances σ_m^2.

$$\mathrm{E}\left[\omega_{i,k}^2\right] = \frac{2\mu_i\mathrm{E}[\omega_{i,k}]\left(\sum\limits_{m\in\mathcal{N}_i}c_m^2\tilde{d}_{m,k}\mathrm{E}\left[\tilde{u}_{m,k}^3\right] + \sum\limits_{m\in\mathcal{N}_i}c_m\mathrm{E}\left[\tilde{u}_{m,k}^2\right]\sum\limits_{\substack{n\in\mathcal{N}_i\\ n\neq m}}c_n\tilde{d}_{n,k}\mathrm{E}[\tilde{u}_{n,k}]\right)}{\mu_i\left(\sum\limits_{m\in\mathcal{N}_i}c_m^2\mathrm{E}\left[\tilde{u}_{m,k}^4\right] + \sum\limits_{m\in\mathcal{N}_i}c_m\mathrm{E}\left[\tilde{u}_{m,k}^2\right]\sum\limits_{\substack{n\in\mathcal{N}_i\\ n\neq m}}c_n\mathrm{E}\left[\tilde{u}_{n,k}^2\right]\right) - 2\sum\limits_{m\in\mathcal{N}_i}c_m\mathrm{E}\left[\tilde{u}_{m,k}^2\right]} - \frac{2\mathrm{E}[\omega_{i,k}]\sum\limits_{m\in\mathcal{N}_i}\tilde{d}_{m,k}c_m\mathrm{E}[\tilde{u}_{m,k}] + \mu_i\left(\sum\limits_{m\in\mathcal{N}_i}c_m^2\tilde{d}_{m,k}^2\mathrm{E}\left[\tilde{u}_{m,k}^2\right] + \sum\limits_{m\in\mathcal{N}_i}c_m\tilde{d}_{m,k}\mathrm{E}[\tilde{u}_{m,k}]\sum\limits_{\substack{n\in\mathcal{N}_i\\ n\neq m}}c_n\tilde{d}_{n,k}\mathrm{E}[\tilde{u}_{n,k}]\right)}{\mu_i\left(\sum\limits_{m\in\mathcal{N}_i}\sum c_m^2\mathrm{E}\left[\tilde{u}_{m,k}^4\right] + \sum\limits_{m\in\mathcal{N}_i}c_m\mathrm{E}\left[\tilde{u}_{m,k}^2\right]\sum\limits_{\substack{n\in\mathcal{N}_i\\ n\neq m}}c_n\mathrm{E}\left[\tilde{u}_{n,k}^2\right]\right) - 2\sum\limits_{m\in\mathcal{N}_i}c_m\mathrm{E}\left[\tilde{u}_{m,k}^2\right]} \quad (29)$$

TABLE I
P_f VALUES FOR WHICH $\delta^2(y_{m,k}) = \delta^2(\omega_{m,k})$ CONSIDERING SINGLE-NODE SPECTRUM SENSING

For any noise variance within the interval $[0.8, 2]$.								
$\eta_{m,k}$ (dB)	-5	-3	-1	1	3	5	7	9
P_f with $N = 15$	7.4×10^{-5}	6.4×10^{-5}	5.0×10^{-5}	3.4×10^{-5}	1.8×10^{-5}	6.4×10^{-6}	1.0×10^{-6}	4.1×10^{-8}
P_f with $N = 20$	1.7×10^{-4}	1.5×10^{-4}	1.3×10^{-4}	9.5×10^{-5}	5.8×10^{-5}	2.6×10^{-5}	6.3×10^{-6}	5.2×10^{-7}

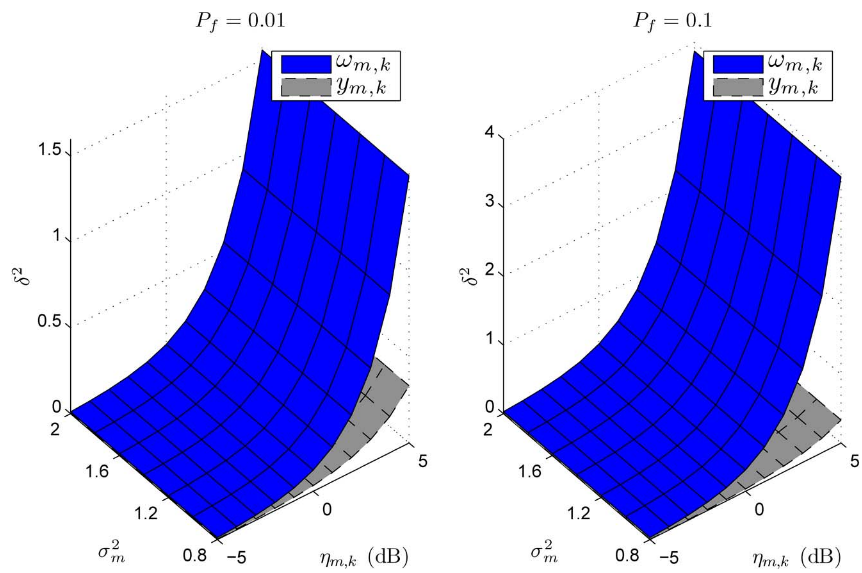

Fig. 4. Deflection-coefficient analysis for different SNR values and noise variances considering single-node spectrum sensing.

Now that we have computed analytically the optimal value for the detection threshold, we can proceed with the analysis of the proposed algorithm.

IV. ALGORITHM EVALUATION

A. Deflection-Coefficient Analysis

As discussed in previous sections, the deflection coefficient is a useful figure of merit that can be employed to assess the performance of binary detection processes. To analyze the performance of the proposed detector, we will compare it with a conventional energy detector of the literature [7]. Considering a single-node energy detector based on the energy estimates $y_{m,k}$, we can obtain the deflection coefficient taking (3) and (5) and substituing them into (9). Thus, the value of $\delta^2(y_{m,k})$ is given by

$$\delta^2(y_{m,k}) = \frac{\eta_{m,k}^2}{2N}. \quad (32)$$

One can see that this value is constant and independent of the predefined P_f which is chosen to tune the performance of the detector.

By using the results based on $\omega_{m,k}$ of Section III, if we compute the expressions (26) for H_0 and H_1 and (29) for H_0 and substitute them into (9), we obtain the expression given by (33), shown at bottom of the next page, where we have also substituted the desired signal $\tilde{d}_{m,k}$ by their values for H_0 and H_1, i.e. $\mathrm{E}[u_{m,k}]_{H_0}$ and $\mathrm{E}[u_{m,k}]_{H_1}$, respectively. Note that we also take into account the following equalities: $\mathrm{E}[\tilde{u}_{m,k}^2] = \mathrm{E}[u_{m,k}^2]$ and $\mathrm{E}[\tilde{u}_{m,k}^4] = \mathrm{E}[u_{m,k}^4]$.

In practical detection processes, small values of P_f are chosen to compute the detection threshold. Consequently, we will analyze for convenience the behavior of the deflection coefficient in the interval $0 \le P_f \le 0.5$. Let us first analyze what occurs in the bounds of the interval. On one hand, we can verify that the deflection coefficient $\delta^2(\omega_{m,k})$ is zero for $P_f = 0$, as it is proved in the Appendix of this paper. On the other hand, if we compute the deflection coefficient for $P_f = 0.5$, which corresponds to $Q^{-1}(P_f) = 0$, we can observe after calculating the corresponding moments in (33) that $\delta^2(\omega_{m,k}) = \infty$. As a consequence of these two results along with the continuity of δ^2 with respect to $P_f \in [0, 0.5]$, *we can assure that there always exists a range of values of* P_f *in which* $\delta^2(\omega_{m,k}) > \delta^2(y_{m,k})$.

As we have seen in Fig. 2, the behavior of the deflection coefficient $\delta^2(\omega_{m,k})$ is monotonically increasing in the interval $0 \le P_f \le 0.5$. We should therefore find the point where $\delta^2(\omega_{m,k}) = \delta^2(y_{m,k})$ to find the lower bound of the interval $P_f^{\text{lower}} < P_f \le 0.5$ in which $\delta^2(\omega_{m,k}) > \delta^2(y_{m,k})$. However, the equality does not have a closed-form solution for P_f and, as a result, we have carried out a numerical analysis for a range of interesting values of number of samples N, SNRs, and noise variances.

Table I shows the P_f values for which $\delta^2(y_{m,k}) = \delta^2(\omega_{m,k})$, considering a range of SNRs and noise variances present in the literature [11], [16]. One can see that, for fixed values of SNR and N, the values of P_f are almost constant for any noise variance $\sigma_m^2 \in [0.8, 2]$. Besides, the deflection-coefficient equality is satisfied for very small values of P_f. The performance of ED techniques in terms of probability of detection, P_d, is rather poor ($P_d \approx 0$) for those P_f values [11], [16]. Consequently, the range of predefined P_f values which offer a reasonable trade-off between P_f and P_d is above those values that appear in Table I. If we consider that the desired P_f is greater than those described in Table I, the deflection coefficient associated with our proposal will be greater than that of a simple ED detector [11], thus enhancing the detection performance.

In Fig. 4, a numerical comparison between both deflection coefficients (based on $y_{m,k}$ *versus* based on ω_m) is shown for several values of SNR and noise variance, fixing the P_f to 0.01 and to 0.1. The number of samples has been chosen as $N = 15$. One can observe that the deflection coefficient $\delta^2(\omega_{m,k})$ always outperforms $\delta^2(y_{m,k})$ in this scenario.

B. Performance Analysis

The performance of the proposed algorithm has been assessed numerically for a CR network of 5 SUs with noise variances $\sigma_m^2 = \{0.9, 1.3, 1.0, 2.0, 1.8\}$ and local SNRs in dB $\eta_{m,k} = \{7.2, 5.1, 0.8, -1.2, 3.6\}$. The number of samples in (2) has been fixed to $N = 15$ for all nodes. The number of independent energy estimates has been set to 10^5. We have considered that H_0 and H_1 are equally likely and we assume that each hypothesis is kept fixed during at least 400 iterations of the adaptive filter. The step-size μ_i should be chosen roughly between zero and the inverse of the input-signal power [25]. Thus, we have chosen one fifth of the upper bound, i.e., $\mu_i = \frac{1}{5} \min_{m \in \mathcal{N}_i} (\mathrm{E}[\tilde{u}_{m,k}^2]_{H_0}^{-1})$, which has been shown to be robust in terms of stability. The forgetting factor in (17) is $\alpha = 0.95$.

Figs. 5 and 6 depict the complementary receiver operating characteristic (C-ROC) curves, which represent the probability of miss-detection, P_m, (probability of failing the detection of H_1 when a PU is present) *versus* the probability of false alarm, P_f, for both single-node and cooperative SS. One can see that the proposed method clearly outperforms that proposed in [11] in both single-node and cooperative SS. Although not presented in this work, it is worth mentioning that the proposal also outperforms other recent techniques based on soft-combiners of energy estimates (cf. [14]–[16]). Due to the fact that the statistical nature of the test is invariant with P_f for the conventional ED techniques (see Fig. 2), a monotonic behavior of the C-ROC curves is expected when the decision threshold is shifted according to a desired P_f, as we can observe in Figs. 5 and 6. Note that in our proposal that monotonicity property is lost,

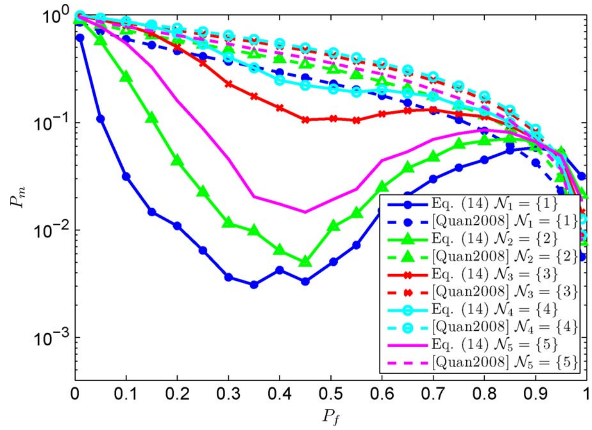

Fig. 5. C-ROC curves for single-node spectrum sensing. Comparison between the proposed algorithm and the simple ED technique proposed in [11].

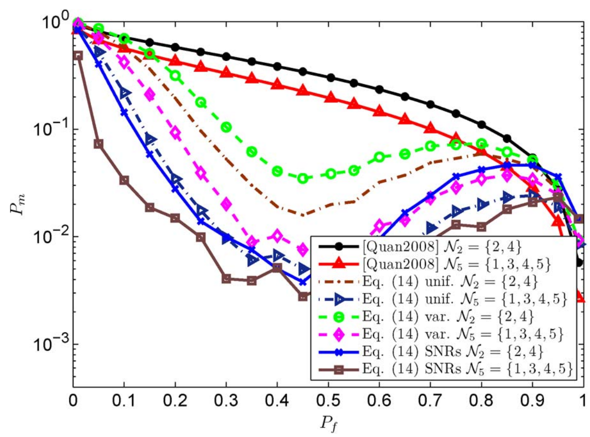

Fig. 6. C-ROC curves for cooperative spectrum sensing. Comparison between the proposed algorithm and the optimal linear solution of [11].

which turns out beneficial because the deflection coefficient based on $\omega_{i,k}$ can vary as a function of P_f, as we have seen in Fig. 2. The unstable behavior of P_m around the value $P_f = 0.5$ stems from the fact that the related deflection coefficient tends to infinity in this region, consequently, P_m tends to zero during the steady states of $\omega_{m,k}$. As C-ROC curves are computed considering the detection performance in both transient and steady states, a smoothless behavior appears close to $P_f = 0.5$ due to the fact that the value of P_m during the transient state is dominant compared with a much smaller value during the steady state. We can also observe that this new ED-based method in single-node SS can achieve better performance than the optimal linear cooperation with several nodes.

$$\delta^2(\omega_{m,k}) = \frac{\left(\frac{\mathrm{E}[u_{m,k}]_{H_1} \mathrm{E}[\tilde{u}_{m,k}]_{H_1}}{\mathrm{E}\left[u_{m,k}^2\right]_{H_1}} - \frac{\mathrm{E}[u_{m,k}]_{H_0} \mathrm{E}[\tilde{u}_{m,k}]_{H_0}}{\mathrm{E}\left[u_{m,k}^2\right]_{H_0}} \right)^2}{\frac{2\mu_m \frac{\mathrm{E}[\tilde{u}_{m,k}]_{H_0} \mathrm{E}[u_{m,k}]_{H_0}^2 \mathrm{E}\left[\tilde{u}_{m,k}^3\right]_{H_0}}{\mathrm{E}\left[u_{m,k}^2\right]_{H_0}} - 2 \frac{\mathrm{E}[u_{m,k}]_{H_0}^2 \mathrm{E}[\tilde{u}_{m,k}]_{H_0}^2}{\mathrm{E}\left[u_{m,k}^2\right]_{H_0}} + \mu_m E[u_{m,k}]_{H_0}^2 \mathrm{E}\left[u_{m,k}^2\right]_{H_0}}{\mu_m \mathrm{E}\left[u_{m,k}^4\right]_{H_0} - 2\mathrm{E}\left[u_{m,k}^2\right]_{H_0}} - \left(\frac{\mathrm{E}[u_{m,k}]_{H_0} \mathrm{E}[\tilde{u}_{m,k}]_{H_0}}{\mathrm{E}\left[u_{m,k}^2\right]_{H_0}} \right)^2} \quad (33)$$

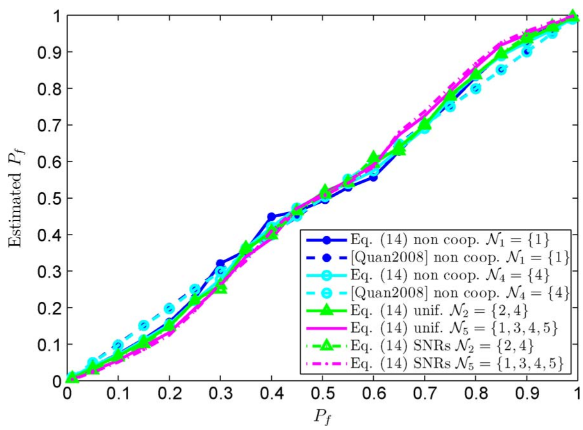

Fig. 7. Estimated P_f *versus* predefined P_f for different setups.

The performance of the proposed algorithm can be improved through the use of weighting strategies. In Fig. 6, we have compared the optimal linear combiner of [11] with several weighted methods for the presented adaptive algorithm in (14): a uniform weighting [21], the relative degree-variance proposed in [22], and the new *relative SNR-degree* proposal in (16). Paying attention to the analyzed weighting strategies, we can conclude that the *relative SNR-degree* proposal provides a better performance, even for low P_f values. Specifically, one can observe that the *relative SNR-degree* proposal provides a 20 times reduction of P_m compared with the optimal soft-combiner of [11] for $P_f = 0.1$ in the neighborhood of node 5, i.e., $\mathcal{N}_5$.

Note that the proposed ED technique reaches smaller values of P_m around $P_f = 0.5$ due to the fact that $\delta^2(\omega_m)$ tends to infinity in $P_f = 0.5$. The case of operating close to a pre-defined P_f around 0.5 leads to a low rate of opportunistic access to the channel. As a result, we consider that an interesting operating range would be $0.05 < P_f < 0.3$ since harmless interference rates against PUs can be achieved along with acceptable opportunistic access rate for SUs.

Furthermore, Fig. 7 presents the estimated P_f against the predefined P_f to demonstrate the validity of the Gaussian approximation on the distribution of $\omega_{i,k}$. We can observe that the actual value of P_f is quite close to those predefined values that have been included in (20) as a target. Consequently, (20) can be considered valid in both single-node and cooperative SS.

V. Conclusion

This paper presented a new adaptive algorithm for spectrum-sensing applications in cognitive-radio networks. The sensing was performed through energy detection implemented by each cognitive user. The main contributions of the paper were: (i) a new cost-function that defines a new test statistic based on an adaptive single parameter for single-node and collaborative scenarios, (ii) a new form of aggregating the information from different neighboring nodes that depends on the normalized SNRs of the nodes and, (iii) the derivation of the statistics for the new detection test. The adaptive algorithm employs pre-processed information based on the energy estimates and noise variances coming from different neighbors in such a way that the deflection coefficient of the new test statistic is improved compared with that achieved in conventional ED techniques, where energy estimates are used directly. This key feature turned out to be very effective in the detection of white spaces in the radio spectrum, as indicated by the simulation results. Indeed, the proposed solutions outperformed state-of-the art techniques with respect to the probability of miss-detection for predefined values of probability of false alarm.

Appendix

The aim of this appendix is to prove that the deflection coefficient $\delta^2(\omega_{m,k})$ is zero for $P_f = 0$.

For P_f very small, we can make the following approximations: $\mathrm{E}[\tilde{u}_{m,k}] \approx -\mathrm{E}[u_{m,k}]$ and $\mathrm{E}[\tilde{u}^3_{m,k}] \approx 2\mathrm{E}[u_{m,k}]^3 - 3\mathrm{E}[u_{m,k}]\mathrm{E}[u^2_{m,k}]$. Therefore, (33) can be expressed after some manipulations as in (34), shown at bottom of the page.

Since all moments of $u_{m,k}$ are functions of $Q^{-1}(P_f)$, this approximation can be expressed as a polinomial quotient taking $Q^{-1}(P_f)$ as the variable x. Rewriting the moments of $u_{m,k}$ that appear in (34) as

$$\mathrm{E}[u_{m,k}]_{H_0} = ax \tag{35}$$

$$\mathrm{E}\left[u^2_{m,k}\right]_{H_0} = a^2x^2 + c \tag{36}$$

$$\mathrm{E}\left[u^4_{m,k}\right]_{H_0} = a^4x^4 + 6a^2cx^2 + 3c^2 \tag{37}$$

$$\mathrm{E}[u_{m,k}]_{H_1} = ax + b \tag{38}$$

$$\mathrm{E}\left[u^2_{m,k}\right]_{H_1} = a^2x^2 + 2abx + b^2 + d \tag{39}$$

where $a = -\sqrt{2N}\sigma^2_m$, $b = \eta_{m,k}\sigma^2_m$, $c = \mathrm{Var}[u_{m,k}]_{H_0}$, and $d = \mathrm{Var}[u_{m,k}]_{H_1}$ are considered constant, we express the numerator of (34) as

$$\mathrm{Num}\left(\delta^2(\omega_{m,k})\right) = \left(\frac{a^2x^2 + 2abx + b^2}{a^2x^2 + 2abx + b^2 + d} - \frac{a^2x^2}{a^2x^2 + c}\right)^2 \tag{40}$$

$$\delta^2(\omega_{m,k}) \approx \frac{\left(\dfrac{\mathrm{E}[u_{m,k}]^2_{H_1}}{\mathrm{E}\left[u^2_{m,k}\right]_{H_1}} - \dfrac{\mathrm{E}[u_{m,k}]^2_{H_0}}{\mathrm{E}\left[u^2_{m,k}\right]_{H_0}}\right)^2}{\dfrac{6\mu_m \mathrm{E}[u_{m,k}]^4_{H_0} - 4\mu_m \dfrac{\mathrm{E}[u_{m,k}]^6_{H_0}}{\mathrm{E}\left[u^2_{m,k}\right]_{H_0}} - 2\dfrac{\mathrm{E}[u_{m,k}]^4_{H_0}}{\mathrm{E}\left[u^2_{m,k}\right]_{H_0}} + \mu_m \mathrm{E}[u_{m,k}]^2_{H_0}\mathrm{E}\left[u^2_{m,k}\right]_{H_0}}{\mu_m \mathrm{E}\left[u^4_{m,k}\right]_{H_0} - 2\mathrm{E}\left[u^2_{m,k}\right]_{H_0}} - \left(\dfrac{\mathrm{E}[u_{m,k}]^2_{H_0}}{\mathrm{E}\left[u^2_{m,k}\right]_{H_0}}\right)^2} \tag{34}$$

and after some manipulations, the denominator as

$$\mathrm{Den}\left(\delta^2(\omega_{m,k})\right) = \frac{2\mu_m a^8x^8 + 5\mu_m ca^6x^6 + 6\mu_m c^2a^4x^4 + \mu_m c^3a^2x^2}{(\mu_m(a^4x^4 + 6ca^2x^2 + 3c^2) - 2(a^2x^2 + c))\,(a^2x^2 + c)^2}. \tag{41}$$

Taking limits in (40) and (41) when P_f tends to zero, that is $Q^{-1}(P_f) \to \infty$, we have

$$\lim_{x\to\infty} \mathrm{Num}\left(\delta^2(\omega_{m,k})\right) = 0, \lim_{x\to\infty} \mathrm{Den}\left(\delta^2(\omega_{m,k})\right) = 2, \tag{42}$$

thus implying that

$$\lim_{x\to\infty} \delta^2(\omega_{m,k}) = 0. \tag{43}$$

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