

Spectrum-Sensing Algorithms for Cognitive Radio Based on Statistical Covariances

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Abstract—Spectrum sensing, i.e., detecting the presence of primary users in a licensed spectrum, is a fundamental problem in cognitive radio. Since the statistical covariances of the received signal and noise are usually different, they can be used to differentiate the case where the primary user’s signal is present from the case where there is only noise. In this paper, spectrum-sensing algorithms are proposed based on the sample covariance matrix calculated from a limited number of received signal samples. Two test statistics are then extracted from the sample covariance matrix. A decision on the signal presence is made by comparing the two test statistics. Theoretical analysis for the proposed algorithms is given. Detection probability and the associated threshold are found based on the statistical theory. The methods do not need any information about the signal, channel, and noise power *a priori*. In addition, no synchronization is needed. Simulations based on narrow-band signals, captured digital television (DTV) signals, and multiple antenna signals are presented to verify the methods.

Index Terms—Communication, communication channels, covariance matrices, signal detection, signal processing.

I. INTRODUCTION

CONVENTIONAL fixed spectrum-allocation policies lead to low spectrum usage in many frequency bands. Cognitive radio, which was first proposed in [1], is a promising technology for exploiting the underutilized spectrum in an opportunistic manner [2]–[5]. One application of cognitive radio is spectral reuse, which allows secondary networks/users to use the spectrum allocated/licensed to the primary users when they are not active [6]. To do so, the secondary users are required to frequently perform spectrum sensing, i.e., detecting the presence of the primary users. If the primary users are detected to be inactive, the secondary users can use the spectrum for communications. On the other hand, whenever the primary users become active, the secondary users have to detect the presence of those users in high probability and vacate the channel within a certain amount of time. One communication system using the spectrum reuse concept is the IEEE 802.22 wireless regional area networks [7], which operate on the very high-frequency/ultrahigh-frequency bands that are currently allocated for TV broadcasting services and other services, such as wireless microphones. Cognitive radio is also an emerging technology for vehicular devices. For example, in [8], cognitive

radio is proposed for underwater vehicles to fully use the limited underwater acoustic bandwidth, and in [9], it is used for autonomous vehicular communications.

Spectrum sensing is a fundamental task for cognitive radio. However, there are several factors that make spectrum sensing practically challenging. First, the signal-to-noise ratio (SNR) of the primary users may be very low. For example, the wireless microphones operating in TV bands only transmit signals with a power of about 50 mW and a bandwidth of 200 kHz. If the secondary users are several hundred meters away from the microphone devices, the received SNR may be well below –20 dB. Second, multipath fading and time dispersion of the wireless channels make the sensing problem more difficult. Multipath fading may cause signal power fluctuation of as large as 20–30 dB. On the other hand, coherent detection may not be possible when the time-dispersed channel is unknown, particularly when the primary users are legacy systems, which do not cooperate with the secondary users. Third, the noise/interference level may change with time, which yields noise uncertainty. There are two types of noise uncertainty: 1) receiver device noise uncertainty and 2) environment noise uncertainty. The receiver device noise uncertainty comes from [10]–[12] the nonlinearity of components and the time-varying thermal noise in the components. The environment noise uncertainty may be caused by the transmissions of other users, either unintentionally or intentionally. Because of noise uncertainty, in practice, it is very difficult to obtain accurate noise power.

There have been several sensing methods, including the likelihood ratio test (LRT) [13], energy detection method [10]–[15], matched filtering (MF)-based method [11], [13], [15], [16], and cyclostationary detection method [17]–[19], each of which has different requirements and advantages/disadvantages. Although LRT is proven to be optimal, it is very difficult to use, because it requires exact channel information and distributions of the source signal and noise. To use LRT for detection, we need to obtain the channels and signal and noise distributions first, which are practically intractable. The MF-based method requires perfect knowledge of the channel responses from the primary user to the receiver and accurate synchronization (otherwise, its performance will dramatically be reduced) [15], [16]. As mentioned earlier, this may not be possible if the primary users do not cooperate with the secondary users. The cyclostationary detection method needs to know the cyclic frequencies of the primary users, which may not be realistic for many spectrum reuse applications. Furthermore, this method demands excessive analog-to-digital (A/D) converter requirements and signal processing capabilities [11].

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Energy detection, unlike the two other methods, does not need any information of the signal to be detected and is robust to unknown dispersed channels and fading. However, energy detection requires perfect knowledge of the noise power. Wrong estimation of the noise power leads to an SNR wall and high probability of false alarm [10]–[12], [15], [20]. As pointed out earlier, the estimated noise power could be quite inaccurate due to noise uncertainty. Thus, the main drawback for the energy detection is its sensitiveness to noise uncertainty [10]–[12], [15]. Furthermore, while energy detection is optimal for detecting an independent and identically distributed (i.i.d.) signal [13], it is not optimal for detecting a correlated signal, which is the case for most practical applications.

In this paper, to overcome the shortcoming of energy detection, we propose new methods based on the statistical covariances or autocorrelations of the received signal. The statistical covariance matrices or autocorrelations of signal and noise are generally different. Thus, this difference is used in the proposed methods to differentiate the signal component from background noise. In practice, there are only a limited number of signal samples. Hence, the detection methods are based on the sample covariance matrix. The steps of the proposed methods are given as follows: First, the sample covariance matrix of the received signal is computed based on the received signal samples. Then, two test statistics are extracted from the sample covariance matrix. Finally, a decision on the presence of the signal is made by comparing the ratio of two test statistics with a threshold. Theoretical analysis for the proposed algorithms is given. Detection probability and the associated threshold for the decision are found based on the statistical theory. The methods do not need any information of the signal, channel, and noise power *a priori*. In addition, no synchronization is needed. Simulations based on narrow-band signals, captured digital television (DTV) signals, and multiple antenna signals are presented to evaluate the performance of the proposed methods.

The rest of this paper is organized as follows: The detection algorithms and theoretical analysis are presented in Section II. Section III gives the performance analysis and finds thresholds for the algorithms. A theoretical comparison with the energy detection is also discussed in this section. Simulation results for various types of signals are given in Section IV. Conclusions are drawn in Section V. Finally, a prewhitening technique is given in the Appendix.

Some of the notation we use is as follows: Boldface letters are used to denote matrices and vectors, superscript $(\cdot)^T$ stands for transpose, \mathbf{I}_q denotes the identity matrix of order q , and $E[\cdot]$ stands for expectation operation.

II. COVARIANCE-BASED DETECTIONS

Let $x_c(t) = s_c(t) + \eta_c(t)$ be the continuous-time received signal, where $s_c(t)$ is the possible primary user's signal and $\eta_c(t)$ is the noise. $\eta_c(t)$ is assumed to be a stationary process satisfying $E(\eta_c(t)) = 0$, $E(\eta_c^2(t)) = \sigma_\eta^2$, and $E(\eta_c(t)\eta_c(t+\tau)) = 0$ for any $\tau \neq 0$. Assume that we are interested in the frequency band with central frequency f_c and bandwidth W . We sample the received signal at a sampling rate f_s , where

$f_s \geq W$. Let $T_s = 1/f_s$ be the sampling period. For notation simplicity, we define $x(n) \triangleq x_c(nT_s)$, $s(n) \triangleq s_c(nT_s)$, and $\eta(n) \triangleq \eta_c(nT_s)$. There are two hypotheses: 1) \mathcal{H}_0 , i.e., the signal does not exist, and 2) \mathcal{H}_1 , i.e., the signal exists. The received signal samples under the two hypotheses are given by [11], [12], [15], and [16]

$$\mathcal{H}_0 : x(n) = \eta(n) \quad (1)$$

$$\mathcal{H}_1 : x(n) = s(n) + \eta(n) \quad (2)$$

respectively, where $s(n)$ is the transmitted signal samples that passed through a wireless channel consisting of path loss, multipath fading, and time dispersion effects; and $\eta(n)$ is the white noise, which is i.i.d., having mean zero and variance σ_η^2 . Note that $s(n)$ can be the superposition of the received signals from multiple primary users. No synchronization is needed here.

Two probabilities are of interest for spectrum sensing: 1) probability of detection P_d , which defines, at hypothesis \mathcal{H}_1 , the probability of the sensing algorithm having detected the presence of the primary signal, and 2) probability of false alarm P_{fa} , which defines, at hypothesis \mathcal{H}_0 , the probability of the sensing algorithm claiming the presence of the primary signal.

A. CAV Detection

Let us consider L consecutive samples and define the following vectors:

$$\mathbf{x}(n) = [x(n) \ x(n-1) \ \cdots \ x(n-L+1)]^T \quad (3)$$

$$\mathbf{s}(n) = [s(n) \ s(n-1) \ \cdots \ s(n-L+1)]^T \quad (4)$$

$$\boldsymbol{\eta}(n) = [\eta(n) \ \eta(n-1) \ \cdots \ \eta(n-L+1)]^T. \quad (5)$$

Parameter L is called the smoothing factor in the following. Considering the statistical covariance matrices of the signal and noise defined as

$$\mathbf{R}_x = E[\mathbf{x}(n)\mathbf{x}^T(n)] \quad (6)$$

$$\mathbf{R}_s = E[\mathbf{s}(n)\mathbf{s}^T(n)] \quad (7)$$

we can verify that

$$\mathbf{R}_x = \mathbf{R}_s + \sigma_\eta^2 \mathbf{I}_L. \quad (8)$$

If signal $s(n)$ is not present, $\mathbf{R}_s = 0$. Hence, the off-diagonal elements of \mathbf{R}_x are all zeros. If there is a signal and the signal samples are correlated, \mathbf{R}_s is not a diagonal matrix. Hence, some of the off-diagonal elements of \mathbf{R}_x should be nonzeros. Denote r_{nm} as the element of matrix \mathbf{R}_x at the n th row and m th column, and let

$$T_1 = \frac{1}{L} \sum_{n=1}^L \sum_{m=1}^L |r_{nm}| \quad (9)$$

$$T_2 = \frac{1}{L} \sum_{n=1}^L |r_{nn}|. \quad (10)$$

Then, if there is no signal, $T_1/T_2 = 1$. If the signal is present, $T_1/T_2 > 1$. Hence, ratio T_1/T_2 can be used to detect the presence of the signal.

In practice, the statistical covariance matrix can only be calculated using a limited number of signal samples. Define the sample autocorrelations of the received signal as

$$\lambda(l) = \frac{1}{N_s} \sum_{m=0}^{N_s-1} x(m)x(m-l), \quad l=0, 1, \dots, L-1 \quad (11)$$

where N_s is the number of available samples. Statistical covariance matrix \mathbf{R}_x can be approximated by the sample covariance matrix defined as

$$\hat{\mathbf{R}}_x(N_s) = \begin{bmatrix} \lambda(0) & \lambda(1) & \cdots & \lambda(L-1) \\ \lambda(1) & \lambda(0) & \cdots & \lambda(L-2) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda(L-1) & \lambda(L-2) & \cdots & \lambda(0) \end{bmatrix}. \quad (12)$$

Note that the sample covariance matrix is symmetric and Toeplitz. Based on the sample covariance matrix, we propose the following signal detection method:

Algorithm 1: Covariance Absolute Value (CAV) Detection Algorithm

- Step 1) Sample the received signal, as previously described.
- Step 2) Choose a smoothing factor L and a threshold γ_1 , where γ_1 should be chosen to meet the requirement for the probability of false alarm. This will be discussed in the next section.
- Step 3) Compute the autocorrelations of the received signal $\lambda(l)$, $l = 0, 1, \dots, L-1$, and form the sample covariance matrix.
- Step 4) Compute

$$T_1(N_s) = \frac{1}{L} \sum_{n=1}^L \sum_{m=1}^L |r_{nm}(N_s)| \quad (13)$$

$$T_2(N_s) = \frac{1}{L} \sum_{n=1}^L |r_{nn}(N_s)| \quad (14)$$

where $r_{nm}(N_s)$ are the elements of the sample covariance matrix $\hat{\mathbf{R}}_x(N_s)$.

- Step 5) Determine the presence of the signal based on $T_1(N_s)$, $T_2(N_s)$, and threshold γ_1 . That is, if $T_1(N_s)/T_2(N_s) > \gamma_1$, the signal exists; otherwise, the signal does not exist.

Remark: The statistics in the algorithm can directly be calculated from autocorrelations $\lambda(l)$. However, for better understanding and easing the mathematical derivation for the prewhitening later in the Appendix, here, we choose to use the covariance matrix expression.

B. Theoretical Analysis for the CAV Algorithm

The proposed method only uses the received signal samples. It does not need any information of the signal, channel, and noise power *a priori*. In addition, no synchronization is needed.

The validity of the proposed CAV algorithm relies on the assumption that the signal samples are correlated, i.e., \mathbf{R}_s is not a diagonal matrix. (Some of the off-diagonal elements of \mathbf{R}_s should be nonzeros.) Obviously, if signal samples $s(n)$ are i.i.d., then $\mathbf{R}_s = \sigma_s^2 \mathbf{I}_L$. In this case, the assumption is invalid, and the algorithm cannot detect the presence of the signal.

However, usually, the signal samples should be correlated due to three reasons.

- 1) The signal is oversampled. Let T_0 be the Nyquist sampling period of signal $s_c(t)$ and $s_c(nT_0)$ be the sampled signal based on the Nyquist sampling rate. Based on the sampling theorem, signal $s_c(t)$ can be expressed as

$$s_c(t) = \sum_{n=-\infty}^{\infty} s_c(nT_0)g(t - nT_0) \quad (15)$$

where $g(t)$ is an interpolation function. Hence, the signal samples $s(n) = s_c(nT_s)$ are only related to $s_c(nT_0)$. If the sampling rate at the receiver $f_s > 1/T_0$, i.e., $T_s < T_0$, then, $s(n) = s_c(nT_s)$ must be correlated. An example of this is a narrow-band signal, such as the wireless microphone signal. In a 6-MHz-bandwidth TV band, a wireless microphone signal only occupies about 200 kHz. When we sample the received signal with a sampling rate of not lower than 6 MHz, the wireless microphone signal is actually oversampled and, therefore, highly correlated.

- 2) The propagation channel has time dispersion; thus, the actual signal component at the receiver is given by

$$s_c(t) = \int_{-\infty}^{\infty} h(\tau)s_0(t - \tau)d\tau \quad (16)$$

where $s_0(t)$ is the original transmitted signal, and $h(t)$ is the response of the time-dispersive channel. Since sampling period T_s is usually very small, the integration (16) can be approximated as

$$s_c(t) \approx T_s \sum_{k=-\infty}^{\infty} h(kT_s)s_0(t - kT_s). \quad (17)$$

Hence

$$s_c(nT_s) \approx T_s \sum_{k=K_0}^{K_1} h(kT_s)s_0((n-k)T_s) \quad (18)$$

where $[K_0T_s, K_1T_s]$ is the support of channel response $h(t)$, i.e., $h(t) = 0$ for $t \notin [K_0T_s, K_1T_s]$. For the time-dispersive channel, $K_1 > K_0$; thus, the signal samples $s_c(nT_s)$ are correlated, even if the original signal samples $s_0(nT_s)$ could be i.i.d.

- 3) The original signal is correlated. In this case, even if the channel is a flat-fading channel and there is no oversampling, the received signal samples are correlated.

Another assumption for the algorithm is that the noise samples are i.i.d. This is usually true if no filtering is used. However, if a narrow-band filter is used at the receiver, the

noise samples will sometimes be correlated. To deal with this case, we need to prewhiten the noise samples or pretransform the covariance matrix. A method is given in the Appendix to solve this problem.

The computational complexity of the algorithm is given as follows: Computing the autocorrelations of the received signal requires about LN_s multiplications and additions. Computing $T_1(N_s)$ and $T_2(N_s)$ requires about L^2 additions. Therefore, the total number of multiplications and additions is about $LN_s + L^2$.

C. Generalized Covariance-Based Algorithms

Based on the same principle as CAV, generalized covariance-based methods can be designed to detect the signal. Let ψ_1 and ψ_2 be two nonnegative functions with multiple variables. Assume that

$$\begin{aligned}\psi_1(\mathbf{a}) &> 0, \quad \text{for } \mathbf{a} \neq \mathbf{0}, \quad \psi_1(\mathbf{0}) = 0 \\ \psi_2(\mathbf{b}) &> 0, \quad \text{for } \mathbf{b} \neq \mathbf{0}, \quad \psi_2(\mathbf{0}) = 0.\end{aligned}$$

Then, the following method can be used for signal detection:

Algorithm 2: Generalized Covariance-Based Detection

- Step 1) Sample the received signal, as previously described.
- Step 2) Choose a smoothing factor L and a threshold γ_2 , where γ_2 should be chosen to meet the requirement for the probability of false alarm.
- Step 3) Compute sample covariance matrix $\hat{\mathbf{R}}_x(N_s)$.
- Step 4) Compute

$$T_4(N_s) = \psi_2(r_{nn}(N_s), n = 1, \dots, L) \quad (19)$$

$$T_3(N_s) = T_4(N_s) + \psi_1(r_{nm}(N_s), n \neq m). \quad (20)$$

- Step 5) Determine the presence of the signal based on $T_3(N_s)$, $T_4(N_s)$, and threshold γ_2 . That is, if $T_3(N_s)/T_4(N_s) > \gamma_2$, the signal exists; otherwise, the signal does not exist.

Obviously, the CAV algorithm is a special case of the generalized method when ψ_1 and ψ_2 are absolute summation functions. As another example, we can choose $\psi_1(\mathbf{a}) = \mathbf{a}^T \mathbf{a}$ and $\psi_2(\mathbf{b}) = \mathbf{b}^T \mathbf{b}$. For this choice

$$T_3(N_s) = \frac{1}{L} \sum_{n=1}^L \sum_{m=1}^L |r_{nm}(N_s)|^2 \quad (21)$$

$$T_4(N_s) = \frac{1}{L} \sum_{n=1}^L |r_{nn}(N_s)|^2. \quad (22)$$

D. Spectrum Sensing Using Multiple Antennas

Multiple-antenna systems have widely been used to increase the channel capacity or improve the transmission reliability in wireless communications. In the following, we assume that there are $M > 1$ antennas at the receiver and exploit the received signals from these antennas for spectrum sensing. In this case, the received signal at antenna i is given by

$$\mathcal{H}_0 : x_i(n) = \eta_i(n) \quad (23)$$

$$\mathcal{H}_1 : x_i(n) = s_i(n) + \eta_i(n). \quad (24)$$

In hypothesis \mathcal{H}_1 , $s_i(n)$ is the signal component received by antenna i . Since all $s_i(n)$'s are generated from the same source signal, the $s_i(n)$'s are correlated for i . It is assumed that the $\eta_i(n)$'s are i.i.d. for n and i .

Let us combine all the signals from the M antennas and define the vectors in (25)–(27), as shown in the bottom of the page. Note that (3)–(5) are a special case ($M = 1$) of the preceding equations. Defining the statistical covariance matrices in the same way as those in (6) and (7), we obtain

$$\mathbf{R}_x = \mathbf{R}_s + \sigma_\eta^2 \mathbf{I}_{ML}. \quad (28)$$

Except for the different matrix dimensions, the preceding equation is the same as (8). Hence, the CAV algorithm and generalized covariance-based method previously described can directly be used for the multiple-antenna case.

Let $s_0(n)$ be the source signal. The received signal at antenna i is

$$s_i(n) = \sum_{k=0}^{N_i} h_i(k) s_0(n-k) + \eta_i(n), \quad i = 1, 2, \dots, M \quad (29)$$

where $h_i(k)$ is the channel responses from the source user to antenna i at the receiver. Define

$$\mathbf{h}(n) = [h_1(n), h_2(n), \dots, h_M(n)]^T \quad (30)$$

$$\mathbb{H} = \begin{bmatrix} \mathbf{h}(0) & \cdots & \cdots & \mathbf{h}(N) & \cdots & 0 \\ \ddots & & & \ddots & & \ddots \\ 0 & \cdots & \mathbf{h}(0) & \cdots & \cdots & \mathbf{h}(N) \end{bmatrix} \quad (31)$$

where $N = \max_i(N_i)$, and $h_i(n)$ is zero padded if $N_i < N$. Note that the dimension of \mathbb{H} is $ML \times (N+L)$. We have

$$\mathbf{R}_s = \mathbb{H} \mathbf{R}_{s_0} \mathbb{H}^T \quad (32)$$

$$\mathbf{x}(n) = [x_1(n) \ \cdots \ x_M(n) \ x_1(n-1) \ \cdots \ x_M(n-1) \ \cdots \ x_1(n-L+1) \ \cdots \ x_M(n-L+1)]^T \quad (25)$$

$$\mathbf{s}(n) = [s_1(n) \ \cdots \ s_M(n) \ s_1(n-1) \ \cdots \ s_M(n-1) \ \cdots \ s_1(n-L+1) \ \cdots \ s_M(n-L+1)]^T \quad (26)$$

$$\boldsymbol{\eta}(n) = [\eta_1(n) \ \cdots \ \eta_M(n) \ \eta_1(n-1) \ \cdots \ \eta_M(n-1) \ \cdots \ \eta_1(n-L+1) \ \cdots \ \eta_M(n-L+1)]^T \quad (27)$$

where $\mathbf{R}_{s_0} = E(\hat{\mathbf{s}}_0 \hat{\mathbf{s}}_0^T)$ is the statistical covariance matrix of the source signal, where

$$\hat{\mathbf{s}}_0 = [s_0(n) \ s_0(n-1) \ \cdots \ s_0(n-N-L+1)]^T. \quad (33)$$

Note that the received signals at different antennas are correlated. Hence, using multiple antennas, increase the correlations among the signal samples at the receiver and make the algorithms valid at all cases. In fact, at worst case, when all the channels are flat fading, i.e., $N_1 = N_2 = \cdots = N_M = 0$, and the source signal sample $s_0(n)$ is i.i.d., we have $\mathbf{R}_s = \sigma_s^2 \mathbb{H} \mathbb{H}^T$, where \mathbb{H} is an $ML \times L$ matrix, as previously defined. Obviously, \mathbf{R}_s is not a diagonal matrix, and the algorithms can work.

III. PERFORMANCE ANALYSIS AND THRESHOLD DETERMINATION

For a good detection algorithm, P_d should be high, and P_{fa} should be low. The choice of threshold γ is a compromise between P_d and P_{fa} . Since we have no information on the signal (we do not even know if there is a signal or not), it is difficult to set the threshold based on P_d . Hence, usually, we choose the threshold based on P_{fa} . The steps are given as follows: First, we set a value for P_{fa} . Then, we find a threshold γ to meet the required P_{fa} . To find the threshold based on the required P_{fa} , we can use either theoretical derivation or computer simulation. If simulation is used to find the threshold, we can generate white Gaussian noises as the input (no signal) and adjust the threshold to meet the P_{fa} requirement. Note that the threshold here is related to the number of samples used for computing the sample autocorrelations and the smoothing factor L but is not related to the noise power. If theoretical derivation is used, we need to find the statistical distribution of $T_1(N_s)/T_2(N_s)$, which is generally a difficult task. In this section, using the central limit theorem, we will find the approximations for the distribution of this random variable and provide the theoretical estimations for the two probabilities P_d and P_{fa} and the threshold associated with these probabilities.

A. Statistics Computation

Based on the symmetric property of the covariance matrix, we can rewrite $T_1(N_s)$ and $T_2(N_s)$ in (13) and (14) as

$$T_1(N_s) = \lambda(0) + \frac{2}{L} \sum_{l=1}^{L-1} (L-l) |\lambda(l)| \quad (34)$$

$$T_2(N_s) = \lambda(0). \quad (35)$$

Define

$$\mathbf{X}_l = [x(N_s - 1 - l) \ \cdots \ x(-l)]^T \quad (36)$$

$$\mathbf{S}_l = [s(N_s - 1 - l) \ \cdots \ s(-l)]^T \quad (37)$$

$$\boldsymbol{\eta}_l = [\eta(N_s - 1 - l) \ \cdots \ \eta(-l)]^T. \quad (38)$$

Let the normalized correlation among the signal samples be

$$\alpha_l = E[s(n)s(n-l)] / \sigma_s^2 \quad (39)$$

where σ_s^2 is the signal power, i.e., $\sigma_s^2 = E[s^2(n)]$. $|\alpha_l|$ defines the correlation strength among the signal samples; here, $0 \leq |\alpha_l| \leq 1$. Based on the notations, we have

$$\begin{aligned} \lambda(l) &= \frac{1}{N_s} \mathbf{X}_0^T \mathbf{X}_l = \frac{1}{N_s} (\mathbf{S}_0^T + \boldsymbol{\eta}_0^T) (\mathbf{S}_l + \boldsymbol{\eta}_l) \\ &= \frac{1}{N_s} (\mathbf{S}_0^T \mathbf{S}_l + \mathbf{S}_0^T \boldsymbol{\eta}_l + \boldsymbol{\eta}_0^T \mathbf{S}_l + \boldsymbol{\eta}_0^T \boldsymbol{\eta}_l). \end{aligned} \quad (40)$$

Obviously

$$E(\lambda(0)) = \sigma_s^2 + \sigma_\eta^2 \quad (41)$$

$$E(\lambda(l)) = \alpha_l \sigma_s^2, \quad l = 1, 2, \dots, L-1. \quad (42)$$

Now, we need to find the variance of $\lambda(l)$. Since

$$\begin{aligned} \lambda^2(l) &= \frac{1}{N_s^2} (\mathbf{S}_0^T \mathbf{S}_l + \mathbf{S}_0^T \boldsymbol{\eta}_l + \boldsymbol{\eta}_0^T \mathbf{S}_l + \boldsymbol{\eta}_0^T \boldsymbol{\eta}_l)^2 \\ &= \frac{1}{N_s^2} \left[(\mathbf{S}_0^T \mathbf{S}_l)^2 + (\mathbf{S}_0^T \boldsymbol{\eta}_l)^2 + (\boldsymbol{\eta}_0^T \mathbf{S}_l)^2 + (\boldsymbol{\eta}_0^T \boldsymbol{\eta}_l)^2 \right. \\ &\quad + 2(\mathbf{S}_0^T \mathbf{S}_l)(\mathbf{S}_0^T \boldsymbol{\eta}_l) + 2(\mathbf{S}_0^T \mathbf{S}_l)(\boldsymbol{\eta}_0^T \mathbf{S}_l) \\ &\quad + 2(\mathbf{S}_0^T \boldsymbol{\eta}_l)(\boldsymbol{\eta}_0^T \boldsymbol{\eta}_l) + 2(\mathbf{S}_0^T \boldsymbol{\eta}_l)(\boldsymbol{\eta}_0^T \mathbf{S}_l) \\ &\quad \left. + 2(\boldsymbol{\eta}_0^T \boldsymbol{\eta}_l)(\boldsymbol{\eta}_0^T \boldsymbol{\eta}_l) + 2(\boldsymbol{\eta}_0^T \boldsymbol{\eta}_l)(\boldsymbol{\eta}_0^T \mathbf{S}_l) \right] \end{aligned} \quad (43)$$

it can be verified that

$$E((\mathbf{S}_0^T \boldsymbol{\eta}_l)^2) = E((\boldsymbol{\eta}_0^T \mathbf{S}_l)^2) = N_s \sigma_s^2 \sigma_\eta^2 \quad (44)$$

$$E((\boldsymbol{\eta}_0^T \boldsymbol{\eta}_0)^2) = (N_s^2 + 2N_s) \sigma_\eta^4 \quad (45)$$

$$E((\boldsymbol{\eta}_0^T \boldsymbol{\eta}_l)^2) = N_s \sigma_\eta^4, \quad l = 1, \dots, L-1 \quad (46)$$

$$\begin{aligned} E((\mathbf{S}_0^T \mathbf{S}_l)(\mathbf{S}_0^T \boldsymbol{\eta}_l)) &= E((\mathbf{S}_0^T \mathbf{S}_l)(\boldsymbol{\eta}_0^T \mathbf{S}_l)) \\ &= E((\mathbf{S}_0^T \boldsymbol{\eta}_l)(\boldsymbol{\eta}_0^T \mathbf{S}_l)) \\ &= E((\boldsymbol{\eta}_0^T \mathbf{S}_l)(\boldsymbol{\eta}_0^T \boldsymbol{\eta}_l)) = 0 \end{aligned} \quad (47)$$

$$E((\mathbf{S}_0^T \mathbf{S}_0)(\boldsymbol{\eta}_0^T \boldsymbol{\eta}_0)) = N_s^2 \sigma_s^2 \sigma_\eta^2 \quad (48)$$

$$E((\mathbf{S}_0^T \mathbf{S}_l)(\boldsymbol{\eta}_0^T \boldsymbol{\eta}_l)) = 0, \quad l = 1, 2, \dots, L-1 \quad (49)$$

$$E((\mathbf{S}_0^T \boldsymbol{\eta}_l)(\boldsymbol{\eta}_0^T \mathbf{S}_l)) = \alpha_{2l}(N_s - l) \sigma_s^2 \sigma_\eta^2 \quad (50)$$

$$E(\mathbf{S}_0^T \mathbf{S}_l) = \alpha_l \sigma_s^2. \quad (51)$$

Based on these results, we can easily obtain the following two lemmas.

Lemma 1: When there is no signal, we have

$$E(\lambda(0)) = \sigma_\eta^2 \quad \text{var}(\lambda(0)) = \frac{2}{N_s} \sigma_\eta^4 \quad (52)$$

$$E(\lambda(l)) = 0 \quad \text{var}(\lambda(l)) = \frac{1}{N_s} \sigma_\eta^4, \quad l = 1, \dots, L-1. \quad (53)$$

Lemma 2: When there is a signal, we have

$$E(\lambda(0)) = \sigma_s^2 + \sigma_\eta^2 \quad (54)$$

$$\text{var}(\lambda(0)) = \text{var}\left(\frac{1}{N_s} \mathbf{S}_0^T \mathbf{S}_0\right) + \frac{2\sigma_\eta^2}{N_s} (2\sigma_s^2 + \sigma_\eta^2) \quad (55)$$

$$E(\lambda(l)) = \alpha_l \sigma_s^2 \quad (56)$$

$$\begin{aligned} \text{var}(\lambda(l)) &= \text{var}\left(\frac{1}{N_s} \mathbf{S}_0^T \mathbf{S}_l\right) \\ &\quad + \frac{\sigma_\eta^2}{N_s} \left(\sigma_\eta^2 + 2\sigma_s^2 + \frac{2(N_s - l)\alpha_{2l}}{N_s} \sigma_s^2 \right), \\ l &= 1, \dots, L-1. \end{aligned} \quad (57)$$

Note that $\text{var}((1/N_s) \mathbf{S}_0^T \mathbf{S}_l), l = 0, 1, \dots, L-1$ depends on the signal properties.

For simplicity, we denote $E(\lambda(l))$ by Θ_l and $\text{var}(\lambda(l))$ by Δ_l . Note that, usually, N_s is very large. Based on the central limit theorem, $\lambda(l)$ can be approximated by the Gaussian distribution.

Lemma 3: When the signal is not present, we have

$$E(|\lambda(l)|) = \sqrt{\frac{2}{\pi N_s}} \sigma_\eta^2, \quad l = 1, 2, \dots, L-1. \quad (58)$$

When the signal is present, we have

$$\begin{aligned} E(|\lambda(l)|) &= \sqrt{\frac{2\Delta_l}{\pi}} \left(2 - e^{-\frac{\Theta_l^2}{2\Delta_l}} \right) \\ &\quad + |\Theta_l| \left(1 - \sqrt{\frac{2}{\pi}} \int_{|\Theta_l|/\sqrt{\Delta_l}}^{+\infty} e^{-\frac{u^2}{2}} du \right) \\ l &= 1, 2, \dots, L-1. \end{aligned} \quad (59)$$

For a large N_s and a low SNR

$$\begin{aligned} E(|\lambda(l)|) &\approx \sqrt{\frac{2}{\pi N_s}} (\sigma_s^2 + \sigma_\eta^2) \left(2 - e^{-\frac{\tau_l^2}{2}} \right) \\ &\quad + |\Theta_l| \left(1 - \sqrt{\frac{2}{\pi}} \int_{\tau_l}^{+\infty} e^{-\frac{u^2}{2}} du \right), \quad l = 1, 2, \dots, L-1 \end{aligned} \quad (60)$$

where

$$\tau_l = \frac{|\alpha_l| \text{SNR} \sqrt{N_s}}{1 + \text{SNR}}, \quad \text{SNR} = \frac{\sigma_s^2}{\sigma_\eta^2}. \quad (61)$$

Proof: Based on the central limit theorem, we have

$$\begin{aligned} E(|\lambda(l)|) &= \frac{1}{\sqrt{2\pi\Delta_l}} \int_{-\infty}^{+\infty} |u| e^{-\frac{(u-\Theta_l)^2}{2\Delta_l}} du \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} |\sqrt{\Delta_l}u + \Theta_l| e^{-\frac{u^2}{2}} du \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\Theta_l/\sqrt{\Delta_l}} (-\sqrt{\Delta_l}u - \Theta_l) e^{-\frac{u^2}{2}} du \\ &\quad + \frac{1}{\sqrt{2\pi}} \int_{-\Theta_l/\sqrt{\Delta_l}}^{+\infty} (\sqrt{\Delta_l}u + \Theta_l) e^{-\frac{u^2}{2}} du \\ &= \sqrt{\frac{2\Delta_l}{\pi}} \int_0^{+\infty} ue^{-\frac{u^2}{2}} du + \sqrt{\frac{2\Delta_l}{\pi}} \int_{-\Theta_l/\sqrt{\Delta_l}}^0 ue^{-\frac{u^2}{2}} du \\ &\quad + \frac{2\Theta_l}{\sqrt{2\pi}} \int_{-\Theta_l/\sqrt{\Delta_l}}^0 e^{-\frac{u^2}{2}} du \\ &= \sqrt{\frac{2\Delta_l}{\pi}} \left(2 - e^{-\frac{\Theta_l^2}{2\Delta_l}} \right) + \frac{2\Theta_l}{\sqrt{2\pi}} \int_{-\Theta_l/\sqrt{\Delta_l}}^0 e^{-\frac{u^2}{2}} du \\ &= \sqrt{\frac{2\Delta_l}{\pi}} \left(2 - e^{-\frac{\Theta_l^2}{2\Delta_l}} \right) \\ &\quad + |\Theta_l| \left(1 - \sqrt{\frac{2}{\pi}} \int_{|\Theta_l|/\sqrt{\Delta_l}}^{+\infty} e^{-\frac{u^2}{2}} du \right). \end{aligned} \quad (62)$$

For a large N_s and a low SNR

$$\Delta_l \approx \frac{(\sigma_s^2 + \sigma_\eta^2)^2}{N_s} \quad \frac{|\Theta_l|}{\sqrt{\Delta_l}} \approx \tau_l.$$

Thus, we obtain (60).

When there is no signal, $\Theta_l = 0$, and $\Delta_l = (1/N_s)\sigma_\eta^4$. Hence, we obtain (58). ■

Theorem 1: When there is no signal, we have

$$E(T_1(N_s)) = \left(1 + (L-1) \sqrt{\frac{2}{\pi N_s}} \right) \sigma_\eta^2 \quad (63)$$

$$E(T_2(N_s)) = \sigma_\eta^2 \quad (64)$$

$$\text{var}(T_2(N_s)) = \frac{2}{N_s} \sigma_\eta^4. \quad (65)$$

When there is a signal and for a large N_s , we have

$$\begin{aligned} E(T_1(N_s)) &\approx \sigma_s^2 + \sigma_\eta^2 \\ &\quad + \frac{2\sigma_s^2}{L} \sum_{l=1}^{L-1} (L-l) |\alpha_l| \left(1 - \sqrt{\frac{2}{\pi}} \int_{\tau_l}^{+\infty} e^{-\frac{u^2}{2}} du \right) \\ &\quad + \frac{2(\sigma_s^2 + \sigma_\eta^2)}{L} \sum_{l=1}^{L-1} (L-l) \sqrt{\frac{2}{\pi N_s}} \left(2 - e^{-\frac{\tau_l^2}{2}} \right) \end{aligned} \quad (66)$$

$$E(T_2(N_s)) = \sigma_s^2 + \sigma_\eta^2 \quad (67)$$

$$\text{var}(T_2(N_s)) = \text{var}\left(\frac{1}{N_s} \mathbf{S}_0^T \mathbf{S}_0\right) + \frac{2\sigma_\eta^2}{N_s} (2\sigma_s^2 + \sigma_\eta^2). \quad (68)$$

For a large N_s , $T_1(N_s)$ and $T_2(N_s)$ approach Gaussian distributions.

Proof: Equations (63)–(68) are direct results from Lemmas 1–3. Noting that $\lambda(l)$ is a summation of N_s random variables, when N_s is large, based on the central limit theorem, it can be approximated by Gaussian distributions. From the definition of $T_1(N_s)$ and $T_2(N_s)$, we know that they also approach Gaussian distributions. ■

B. Detection Probability and the Associated Threshold

From the preceding theorem, we have

$$\lim_{N_s \rightarrow \infty} E(T_1(N_s)) = \sigma_s^2 + \sigma_\eta^2 + \frac{2\sigma_s^2}{L} \sum_{l=1}^{L-1} (L-l)|\alpha_l|. \quad (69)$$

For simplicity, we denote

$$\Upsilon_L \triangleq \frac{2}{L} \sum_{l=1}^{L-1} (L-l)|\alpha_l| \quad (70)$$

which is the overall correlation strength among the consecutive L samples. When there is no signal, we have

$$\begin{aligned} T_1(N_s)/T_2(N_s) &\approx E(T_1(N_s))/E(T_2(N_s)) \\ &= 1 + (L-1)\sqrt{\frac{2}{\pi N_s}}. \end{aligned} \quad (71)$$

Note that this ratio approaches 1 as N_s approaches infinity. In addition, note that the ratio is not related to the noise power (variance). On the other hand, when there is a signal (the signal-plus-noise case), we have

$$\begin{aligned} T_1(N_s)/T_2(N_s) &\approx E(T_1(N_s))/E(T_2(N_s)) \\ &\approx 1 + \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\eta^2} \Upsilon_L \end{aligned} \quad (72)$$

$$= 1 + \frac{\text{SNR}}{\text{SNR} + 1} \Upsilon_L. \quad (73)$$

Here, the ratio approaches a number that is larger than 1 as N_s approaches infinity. The number is determined by the correlation strength among the signal samples and the SNR. Hence, for any fixed SNR, if there are a sufficiently large number of samples, we can always differentiate if there is a signal or not based on the ratio.

However, in practice, we have only a limited number of samples. Thus, we need to evaluate the performance at fixed N_s . First, we analyze the P_{fa} at hypothesis \mathcal{H}_0 . The probability of false alarm for the CAV algorithm is

$$\begin{aligned} P_{\text{fa}} &= P(T_1(N_s) > \gamma_1 T_2(N_s)) \\ &= P\left(T_2(N_s) < \frac{1}{\gamma_1} T_1(N_s)\right) \\ &\approx P\left(T_2(N_s) < \frac{1}{\gamma_1} \left(1 + (L-1)\sqrt{\frac{2}{N_s \pi}}\right) \sigma_\eta^2\right) \end{aligned}$$

$$\begin{aligned} &= P\left(\frac{T_2(N_s) - \sigma_\eta^2}{\sqrt{\frac{2}{N_s} \sigma_\eta^2}} < \frac{\frac{1}{\gamma_1} \left(1 + (L-1)\sqrt{\frac{2}{N_s \pi}}\right) - 1}{\sqrt{2/N_s}}\right) \\ &\approx 1 - Q\left(\frac{\frac{1}{\gamma_1} \left(1 + (L-1)\sqrt{\frac{2}{N_s \pi}}\right) - 1}{\sqrt{2/N_s}}\right) \end{aligned}$$

where

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{+\infty} e^{-u^2/2} du. \quad (74)$$

For a given P_{fa} , the associated threshold should be chosen such that

$$\frac{\frac{1}{\gamma_1} \left(1 + (L-1)\sqrt{\frac{2}{N_s \pi}}\right) - 1}{\sqrt{2/N_s}} = -Q^{-1}(P_{\text{fa}}). \quad (75)$$

That is

$$\gamma_1 = \frac{1 + (L-1)\sqrt{\frac{2}{N_s \pi}}}{1 - Q^{-1}(P_{\text{fa}})\sqrt{\frac{2}{N_s}}}. \quad (76)$$

Note that, here, the threshold is not related to noise power and SNR. After the threshold is set, we now calculate the probability of detection at various SNRs. For the given threshold γ_1 , when the signal is present

$$\begin{aligned} P_d &= P(T_1(N_s) > \gamma_1 T_2(N_s)) \\ &= P\left(T_2(N_s) < \frac{1}{\gamma_1} T_1(N_s)\right) \\ &\approx P\left(T_2(N_s) < \frac{1}{\gamma_1} E(T_1(N_s))\right) \\ &= P\left(\frac{T_2(N_s) - \sigma_s^2 - \sigma_\eta^2}{\sqrt{\text{var}(T_2(N_s))}} < \frac{\frac{1}{\gamma_1} E(T_1(N_s)) - \sigma_s^2 - \sigma_\eta^2}{\sqrt{\text{var}(T_2(N_s))}}\right) \\ &= 1 - Q\left(\frac{\frac{1}{\gamma_1} E(T_1(N_s)) - \sigma_s^2 - \sigma_\eta^2}{\sqrt{\text{var}(T_2(N_s))}}\right). \end{aligned} \quad (77)$$

For a very large N_s and a low SNR

$$\begin{aligned} \text{var}(T_2(N_s)) &\approx \frac{2\sigma_\eta^2}{N_s} (2\sigma_s^2 + \sigma_\eta^2) \approx \frac{2(\sigma_s^2 + \sigma_\eta^2)^2}{N_s} \\ E(T_1(N_s)) &\approx \sigma_s^2 + \sigma_\eta^2 + \sigma_s^2 \Upsilon_L. \end{aligned}$$

Hence, we have a further approximation, i.e.,

$$\begin{aligned} P_d &\approx 1 - Q\left(\frac{\frac{1}{\gamma_1} + \frac{\Upsilon_L \sigma_s^2}{\gamma_1 (\sigma_s^2 + \sigma_\eta^2)} - 1}{\sqrt{2/N_s}}\right) \\ &= 1 - Q\left(\frac{\frac{1}{\gamma_1} + \frac{\Upsilon_L \text{SNR}}{\gamma_1 (\text{SNR}+1)} - 1}{\sqrt{2/N_s}}\right). \end{aligned} \quad (78)$$

TABLE I
PROBABILITIES OF FALSE ALARM

method	EG-2 dB	EG-1.5 dB	EG-1 dB	EG-0.5 dB	EG-0dB	CAV
P_{fa}	0.497	0.496	0.490	0.470	0.102	0.099

Obviously, P_d increases with the number of samples N_s , the SNR, and the correlation strength among the signal samples. Note that γ_1 is also related to N_s , as previously shown, and $\lim_{N_s \rightarrow \infty} \gamma_1 = 1$. Hence, for fixed SNR, P_d approaches 1 when N_s approaches infinity.

For a target pair of P_d and P_{fa} , based on (78) and (76), we can find the required number of samples as

$$N_c \approx \frac{2(Q^{-1}(P_{fa}) - Q^{-1}(P_d) + (L-1)/\sqrt{\pi})^2}{(\Upsilon_L \text{SNR})^2}. \quad (79)$$

For fixed P_d and P_{fa} , N_c is only related to the smoothing factor L and the overall correlation strength Υ_L . Hence, the best smoothing factor is

$$L_{\text{best}} = \min_L \{N_c\} \quad (80)$$

which is related to the correlation strength among the signal samples.

C. Comparison With Energy Detection

Energy detection is the basic sensing method, which was first proposed in [14] and further studied in [10]–[12] and [15]. It does not need any information of the signal to be detected and is robust to unknown dispersive channels. Energy detection compares the average power of the received signal with the noise power to make a decision. To guarantee a reliable detection, the threshold must be set according to the noise power and the number of samples [10]–[12]. On the other hand, the proposed methods do not rely on the noise power to set the threshold [see (76)] while keeping the other advantages of energy detection.

Accurate knowledge on the noise power is then the key of the energy detection. Unfortunately, in practice, noise uncertainty is always present. Due to noise uncertainty [10]–[12], the estimated (or assumed) noise power may be different from the real noise power. Let the estimated noise power be $\hat{\sigma}_\eta^2 = \alpha\sigma_\eta^2$. We define the noise uncertainty factor (in decibels) as

$$B = \sup \{10 \log_{10} \alpha\}. \quad (81)$$

It is assumed that α (in decibels) is evenly distributed in an interval $[-B, B]$ [11]. In practice, the noise uncertainty factor of a receiving device normally ranges from 1 to 2 dB [11], [20]. Environment/interference noise uncertainty can be much higher [11]. When there is noise uncertainty, the energy detection is not effective [10]–[12], [20]. The simulation presented in the next section also shows that the proposed method is much better than the energy detection when noise uncertainty is present. Hence, here, we only compare the proposed method with ideal energy detection (without noise uncertainty).

To compare the performances of the two methods, we first need a criterion. By properly choosing the thresholds, both methods can achieve any given P_d and $P_{fa} > 0$ if a sufficiently

large number of samples are available. The key point is how many samples are needed to achieve the given P_d and $P_{fa} > 0$. Hence, we choose this as the criterion for comparing the two algorithms. For energy detection, the required number of samples is approximately [11]

$$N_e = \frac{2(Q^{-1}(P_{fa}) - Q^{-1}(P_d))^2}{\text{SNR}^2}. \quad (82)$$

Comparing (79) and (82), if we want $N_c < N_e$, we need

$$\Upsilon_L > 1 + \frac{L-1}{\sqrt{\pi}(Q^{-1}(P_{fa}) - Q^{-1}(P_d))}. \quad (83)$$

For example, if $P_d = 0.9$ and $P_{fa} = 0.1$, we need $\Upsilon_L > 1 + (L-1/4.54)$. In conclusion, if the signal samples are highly correlated such that (83) holds, CAV detection is better than ideal energy detection; otherwise, ideal energy detection is better.

In terms of the computational complexity, the energy detection needs about N_s multiplications and additions. Hence, the computational complexity of the proposed methods is about L times that of the energy detection.

IV. SIMULATION AND DISCUSSION

In this section, we will give some simulation results for three situations: 1) narrow-band signals; 2) captured DTV signals [21]; and 3) multiple antenna received signals.

First, we simulate the probabilities of false alarm P_{fa} , because P_{fa} is not related to the signal. (At \mathcal{H}_0 , there is no signal at all.) We set the target $P_{fa} = 0.1$ and choose $L = 10$ and $N_s = 50\,000$. We then obtain the thresholds based on P_{fa} , L , and N_s . The threshold for energy detection is given in [11]. Table I gives the simulation results for various cases, where, and in the following, “EG- x dB” means the energy detection with x -dB noise uncertainty. The P_{fa} ’s for the proposed method and energy detection without noise uncertainty meet the target, but the P_{fa} for the energy detection with noise uncertainty (even as low as 0.5 dB) far exceeds the limit. This means that the energy detection is very unreliable in practical situations with noise uncertainty.

Second, we fix the thresholds based on P_{fa} and simulate the probability of detection P_d for various cases. We consider two signal types.

- 1) *Narrow-band signals:* A frequency-modulated wireless microphone signal is used here (soft speaker) [22]. The central frequency is $f_c = 100$ MHz. The sampling rate at the receiver is 6 MHz (the same as the TV bandwidth in the USA). Fig. 1 shows the simulation results. (*The corresponding P_{fa} is shown in Table I.*) Note that “CAV-theo” means the theoretical results given in Section III-B. Due to some approximations, the theoretical results do not exactly match the simulated results.

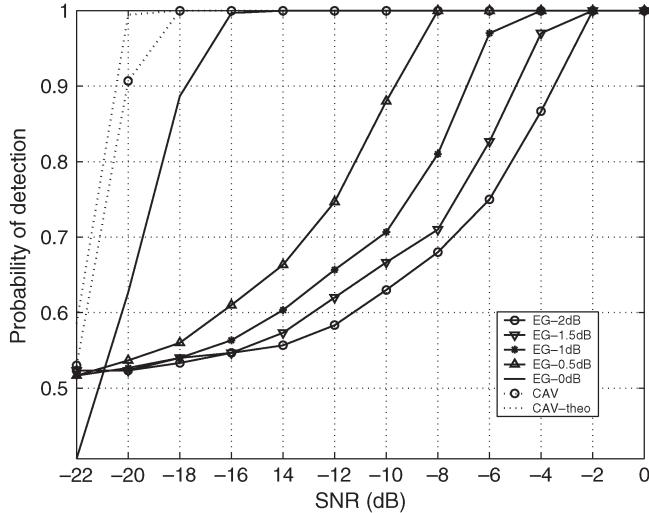


Fig. 1. Probability of detection for a wireless microphone signal: $N_s = 50\,000$.

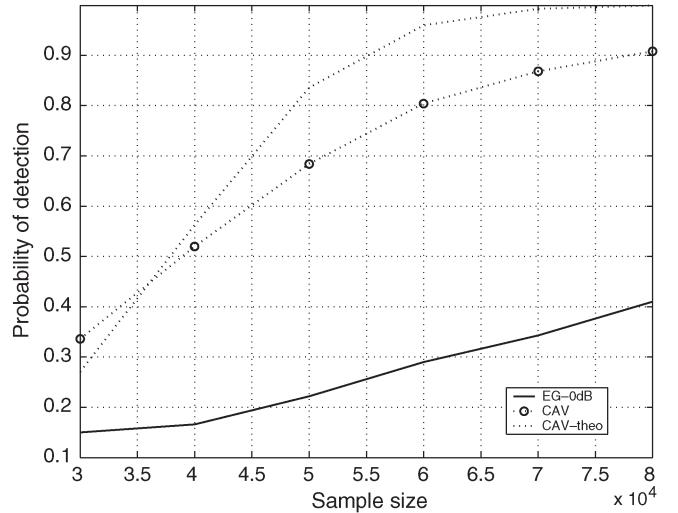


Fig. 3. P_d versus sample size N_s for a wireless microphone signal: $P_{fa} = 0.01$, and SNR = -20 dB.

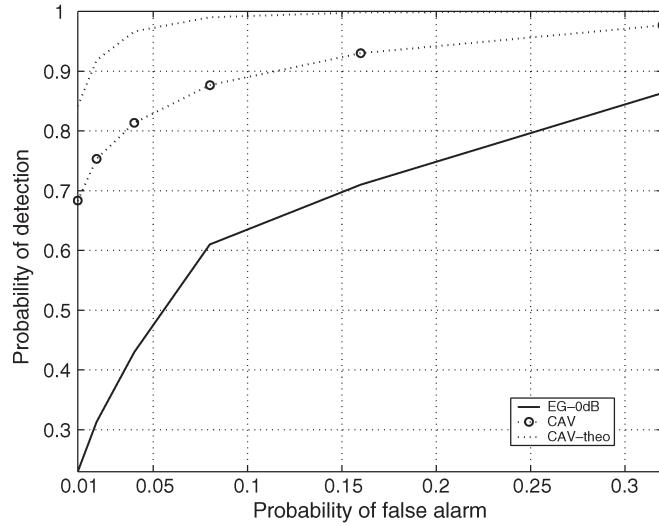


Fig. 2. P_d versus P_{fa} for a wireless microphone signal: $N_s = 50\,000$, and SNR = -20 dB.

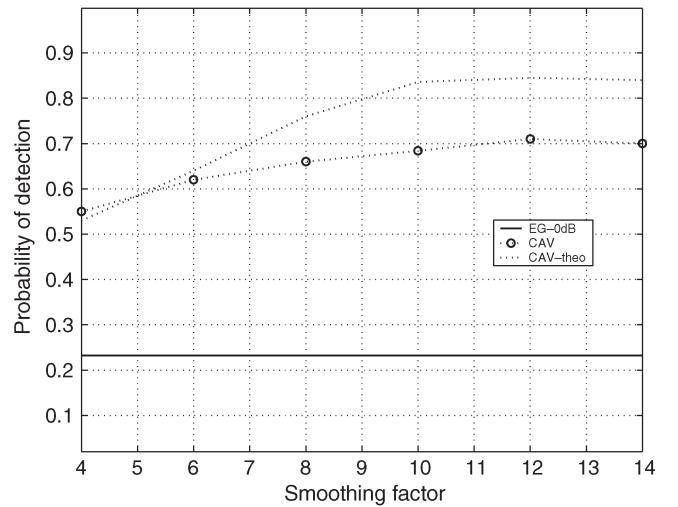


Fig. 4. P_d versus the smoothing factor for a wireless microphone signal: $P_{fa} = 0.01$, $N_s = 50\,000$, and SNR = -20 dB.

CAV detection is better than ideal energy detection (without noise uncertainty), which verifies our assertion in Section III-C. The reason is that, as we pointed out in Section II-B, the source signal is a narrow-band signal; therefore, their samples are highly correlated. As shown in the figure, if there is noise uncertainty, the P_d of the energy detection is much worse than that of the proposed method. Fig. 2 shows the receiver operating characteristic curve (P_d versus P_{fa}) at fixed SNR = -20 dB. The performances of the methods at different sample sizes (sensing times) are given in Fig. 3. It is clear that CAV detection is always better than ideal energy detection.

To test the impact of the smoothing factor, we fix SNR = -20 dB, $P_{fa} = 0.01$, and $N_s = 50\,000$ and vary the smoothing factor L from 4 to 14. Fig. 4 shows the results for P_d . We see that P_d is not very sensitive to the smoothing factor for $L \geq 8$. Noting that a smaller L means lower complexity, in practice, we can choose

a relatively small L . However, it is very difficult to choose the best L , because it is related to the signal property (unknown). Note that energy detection is not affected by L .

- 2) *Captured DTV signals.* The real DTV signals (field measurements) are collected at Washington, DC. The data rate of the vestigial sideband DTV signal is 10.762 Msample/s. The recorded DTV signals were sampled at 21.524476 Msample/s and downconverted to a low central intermediate frequency of 5.381119 MHz (one fourth of the sampling rate). The A/D conversion of the radio-frequency signal used a 10-bit or a 12-bit A/D converter. Each sample was encoded into a 2-B word (signed int16 with two's complement format). The multipath channel and SNR of the received signal are unknown. To use the signals for simulating the algorithms at a very low SNR, we need to add white noise to obtain various SNR levels [23].

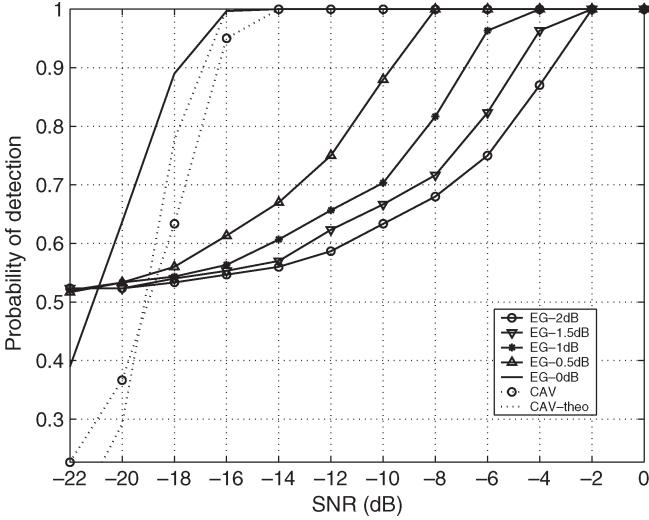


Fig. 5. Probability of detection for the DTV signal WAS-051/35/01: $N_s = 50\,000$.

Fig. 5 shows the simulation results based on the DTV signal file WAS-051/35/01. (The receiving antenna is outside and located 20.29 mi from the DTV station; the antenna height is 30 ft.) [21]. The corresponding P_{fa} is shown in Table I. If the noise variance is exactly known ($B = 0$), the energy detection is better than the proposed method. However, as discussed in [10]–[12], noise uncertainty is always present. Even if the noise uncertainty is only 0.5 dB, the P_d of the energy detection is much worse than that of the proposed method.

In summary, all the preceding simulations show that the proposed method works well without using the information about the signal, channel, and noise power. The energy detection is not reliable (i.e., with low probability of detection and high probability of false alarm) when there is noise uncertainty.

Third, we simulate the proposed algorithms with multiple antennas/receivers. We consider a system with four receiving antennas. Assume that the antennas are well separated (with the separation larger than a half-wavelength) such that their channels are independent. This assumption is only for simplicity. In fact, the proposed algorithms perform better if the channels are correlated. Assume that each multipath channel $h_i(k)$ has five taps ($N_i = 4$) and that all the channel taps are independent with equal power. The channel taps are generated as Gaussian random numbers and different for different Monte Carlo realizations. Source signal $s_0(n)$ is i.i.d. and binary phase-shift keying modulated. The received signal at antenna i is defined in (29). The smoothing factor is $L = 8$, and the number of samples at each antenna is $N_s = 25\,000$. We fix the $P_{fa} = 0.1$ at all cases. Fig. 6 shows the P_d for three cases. From the figure, we see that, when only one antenna's signal is used ($M = 1$), the method still works. This verifies our assertion in Section II-B that the method is valid, even if the inputs are i.i.d., but the channel is dispersive. When the signals of two ($M = 2$) or four ($M = 4$) antennas are combined based on the method in Section II-D, P_d is much better. The more the antennas used, the better the P_d . The optimal LRT [13] detection for $M = 4$ is also included as an upper bound for any detection methods.

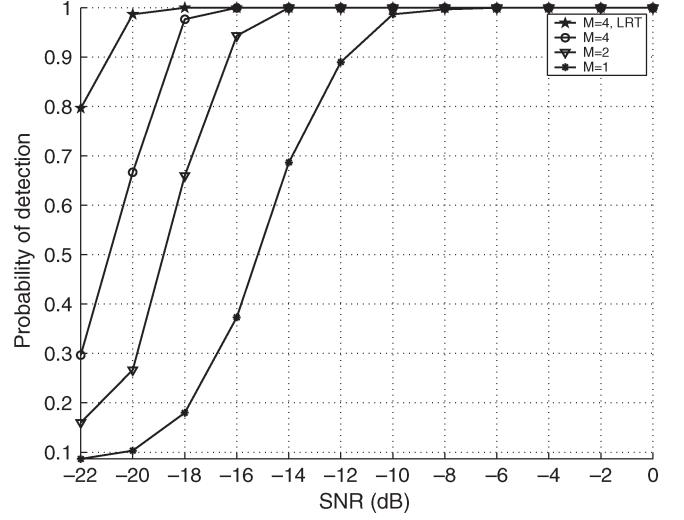


Fig. 6. Probability of detection using multiple antennas: $P_{fa} = 0.1$, and $N_s = 25\,000$, with one source signal.

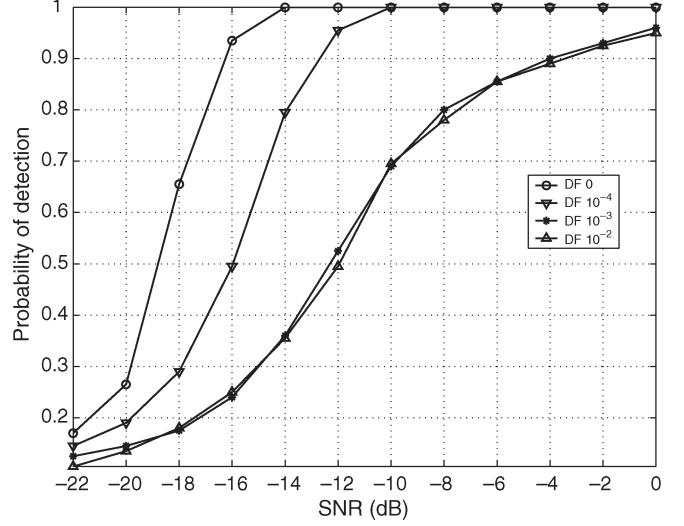


Fig. 7. Probability of detection for time variant channels: $M = 2$, $P_{fa} = 0.1$, and $N_s = 25\,000$.

To check the performance of the methods for time-variant channels, we give a simulation result here. The time-variant channel is generated based on the simplified Jake's model. Let f_d be the normalized maximum Doppler frequency (DF). The time-variant channel for simulation is defined as

$$h_i(n, k) = \exp \left(j 2\pi n \frac{6-k}{5} f_d \right) h_i(k), \quad k = 1, \dots, 5 \quad (84)$$

where $h_i(k)$ is the time-invariant channel previously defined. For different DFs ranging from 0 to 10^{-2} , the simulation result is shown in Fig. 7 for $M = 2$. For fast time-variant channels, the performance of the proposed methods will degrade.

We also simulate the situation when there are multiple source signals. Fig. 8 shows the results for the case of three source signals. Compared with Fig. 6, here, the results do not change much. Hence, the proposed method is valid when there are multiple source signals.

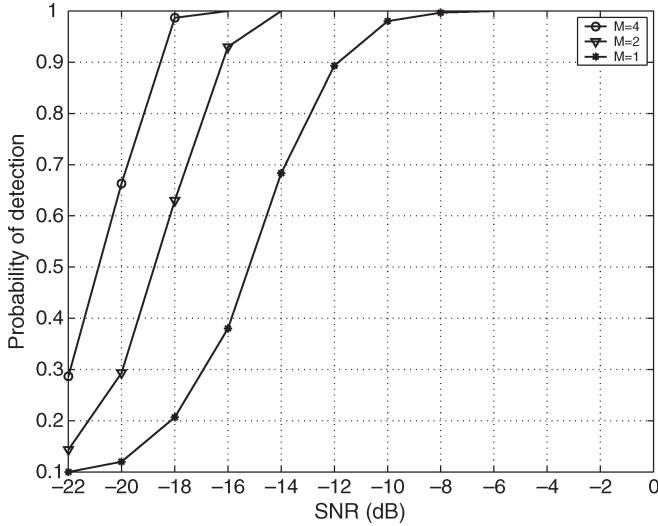


Fig. 8. Probability of detection using multiple antennas: $P_{fa} = 0.1$, and $N_s = 25\,000$, with three source signals.

V. CONCLUSION

In this paper, sensing algorithms based on the sample covariance matrix of the received signal have been proposed. Statistical theories have been used to set the thresholds and obtain the probabilities of detection. The methods can be used for various signal detection applications without knowledge of the signal, channel, and noise power. Simulations based on the narrow-band signals, captured DTV signals, and multiple antenna signals have been carried out to evaluate the performance of the proposed methods. It is shown that the proposed methods are, in general, better than the energy detector when noise uncertainty is present. Furthermore, when the received signals are highly correlated, the proposed method is better than the energy detector, even if the noise power is perfectly known.

APPENDIX

At the receiving end, the received signal is sometimes filtered by a narrow-band filter. Therefore, the noise embedded in the received signal is also filtered. Let $\eta(n)$ be the noise samples before the filter, which are assumed to be i.i.d. Let $f(k)$, $k = 0, 1, \dots, K$ be the filter. After filtering, the noise samples turn to

$$\tilde{\eta}(n) = \sum_{k=0}^K f(k)\eta(n-k), \quad n = 0, 1, \dots \quad (85)$$

Consider L consecutive outputs, and define

$$\tilde{\eta}(n) = [\tilde{\eta}(n), \dots, \tilde{\eta}(n-L+1)]^T. \quad (86)$$

The statistical covariance matrix of the filtered noise becomes

$$\tilde{\mathbf{R}}_\eta = E(\tilde{\eta}(n)\tilde{\eta}(n)^T) = \sigma_\eta^2 \mathbf{F}\mathbf{F}^T \quad (87)$$

where \mathbf{F} is an $L \times (L + K)$ matrix defined as

$$\mathbf{F} = \begin{bmatrix} f(0) & \cdots & f(K-1) & f(K) & \cdots & 0 \\ \ddots & & & & \ddots & \ddots \\ 0 & \cdots & f(0) & & \cdots & f(K) \end{bmatrix}. \quad (88)$$

Let $\mathbf{G} = \mathbf{F}\mathbf{F}^T$. If an analog filter or both analog and digital filters are used, matrix \mathbf{G} should be defined based on those filter properties. Note that \mathbf{G} is a positive-definite symmetric matrix. It can be decomposed to

$$\mathbf{G} = \mathbf{Q}^2 \quad (89)$$

where \mathbf{Q} is also a positive-definite symmetric matrix. Hence, we can transform the statistical covariance matrix into

$$\mathbf{Q}^{-1} \tilde{\mathbf{R}}_\eta \mathbf{Q}^{-1} = \sigma_\eta^2 \mathbf{I}_L. \quad (90)$$

Note that \mathbf{Q} is only related to the filter. This means that we can always transform the statistical covariance matrix \mathbf{R}_x in (6) (by using a matrix obtained from the filter) such that (8) holds when the noise has passed through a narrow-band filter. Furthermore, since \mathbf{Q} is not related to signal and noise, we can precompute its inverse \mathbf{Q}^{-1} and store it for later usage.

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