On the Energy Detection of Unknown Signals Over Fading Channels

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***Abstract*—This letter addresses the problem of energy detection of an unknown signal over a multipath channel. It starts with the no-diversity case, and presents some alternative closed-form expressions for the probability of detection to those recently reported in the literature. Detection capability is boosted by implementing both square-law combining and square-law selection diversity schemes.**

***Index Terms*—Diversity schemes, energy detection, fading channels, low-power applications, square-law detector, unknown signal detection.**

I. Introduction

THE PROBLEM of detecting an unknown deterministic signal over a flat bandlimited Gaussian noise channel was first addressed by Urkowitz [10]. In his proposal, the receiver consisted of an energy detector which measures the energy in the received waveform over an observation time window. This energy-detection problem has been revisited recently by Kostylev in [5] for signals operating over a variety of fading channels. Our contribution in this letter is twofold. First, we present an alternative analytical approach to the one presented in [5] and obtain closed-form expressions for the probability of detection over Rayleigh and Nakagami fading channels. Second, and more importantly, we quantify the improvement in detection capability (specially for relatively low-power applications) when low-complexity diversity schemes such as square-law combining (SLC) and square-law selection (SLS) are implemented. While diversity analysis is carried out for independent Rayleigh channels for the SLS scheme, both independent and correlated cases are considered for the SLC one. For more details, the reader is referred to [2, Ch. 9].

The rest of this letter is organized as follows. The system model is described in Section II. Conditional (on the fading) probability of detection P_d, and probability of a false alarm P_f, are evaluated in Section III over additive white Gaussian noise (AWGN) channels. While Section IV deduces these probabilities over Rayleigh and Nakagami fading channels, Section V studies the impact of diversity for Rayleigh channels. Finally, numerical examples are demonstrated in Section VI, and concluding remarks are offered in Section VII.

II. System Model and Notations

While our analysis applies to either low-pass (LP) or band-pass (BP) systems, we here focus on the BP representation. The received BP waveform can be represented as

$$r(t) = \begin{cases} \Re\{[hS_{\mathrm{LP}}(t) + n_{\mathrm{LP}}(t)]e^{j2\pi f_c t}\}, & H_1 \\ \Re\{n_{\mathrm{LP}}(t)e^{j2\pi f_c t}\}, & H_0 \end{cases} \tag{1}$$

where $\Re\{\cdot\}$ denotes the real part operation, $h = \alpha e^{j\theta}$ is a slow-fading channel with amplitude α and phase θ, f_c is the carrier frequency, both H_1 and H_0 refer to the two hypotheses of signal presence and signal absence, respectively, $S_{\mathrm{LP}}(t) = S_c(t) + jS_s(t)$ is an equivalent LP representation of the unknown signal with $s_c(t)$ and $s_s(t)$ denoting the in-phase (I) and quadrature (Q) components, respectively, and likewise, $n_{\mathrm{LP}}(t) = n_c(t) + jn_s(t)$ is an equivalent LP AWGN process with a zero mean and a known flat power spectral density (PSD). If N_0 denotes the one-sided noise PSD and W the signal bandwidth (i.e., positive spectrum support), the noise variance will, in turn, be N_0W. Also, I and Q components will be each confined to the frequency support $[-W/2, W/2]$ and the one-sided noise PSD of either $n_c(t)$ or $n_s(t)$ will be $2N_0$ (to reserve a noise variance of N_0W). $\gamma \triangleq (\alpha^2 E_s)/(N_0)$ shall denote the signal-to-noise ratio (SNR) where E_s is the signal energy.

The receiver structure can generally be described as follows. The received signal is first prefiltered by an ideal BP filter. Then, the output of this filter is squared and integrated over a time interval T to finally produce a measure of the energy of the received waveform. The output of the integrator denoted by y acts as a test statistic to test the two hypotheses H_0 and H_1. Under H_1, assuming a narrowband signal, and relying on the sampling theorem approximation[1] [2], y can be expressed as

$$\begin{aligned} y &\triangleq \frac{2}{N_0}\int_0^T r^2(t)\,dt \\ &\simeq \frac{1}{N_0W}\left[\sum_{i=1}^{N/2}(\alpha_c S_{ci} - \alpha_s S_{si} + n_{ci})^2 \right. \\ &\qquad \left. + \sum_{i=1}^{N/2}(\alpha_c S_{si} + \alpha_s S_{ci} + n_{si})^2\right] \end{aligned} \tag{2}$$

where $N/2$ is the number of samples per either I or Q components (we finally have N terms to sum over), $\alpha_c = \alpha\cos\theta$, $\alpha_s = \alpha\sin\theta$, and generally x_{ci} and x_{si}, respectively, denote the ith samples of $x_c(t)$ and $x_s(t)$, i.e., I and Q components. It then follows that under H_1, y has a noncentral chi-square distribution

[1] If a signal is assumed to be bandlimited and in the same time observed over a limited interval T, the sampling theorem will then yield an approximate expression of the signal in terms of its limited samples.

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with variance $\sigma^2 = 1$, noncentrality parameter[2] $\mu = 2\gamma$, and N degrees of freedom (DOFs)[3] . Likewise, given H_0, y will be central chi-square distributed. In the rest of the letter, we shall use the generic parameters σ^2 and $\mu = a\gamma$ (for some positive value a). The results can then be specialized to the presented case study for $\sigma^2 = 1$ and $a = 2$. Now, the probability density function (PDF) of y can be expressed as

$$f_Y(y) = \begin{cases} \frac{1}{\sigma^N 2^{N/2}\Gamma(N/2)} y^{N/2-1} e^{-\frac{y}{2\sigma^2}}, & H_0 \\ \frac{1}{2\sigma^2}\left(\frac{y}{a\gamma}\right)^{\frac{N-2}{4}} e^{-\frac{a\gamma+y}{2\sigma^2}} I_{N/2-1}\left(\frac{\sqrt{a\gamma y}}{\sigma^2}\right), & H_1 \end{cases} \quad (3)$$

where $\Gamma(\cdot)$ is the gamma function [3, Sec. 8.31] and $I_\nu(\cdot)$ is the νth-order modified Bessel function of the first kind [3, Sec. 8.43].

III. Detection and False Alarm Probabilities Over AWGN Channels

An approximate expression for the probability of detection P_d over AWGN channels was presented in [10]. In this section, we present exact closed-form expressions for both P_d and the probability of a false alarm P_f, which are defined as $P_d = \Pr(y > \lambda | H_1)$ and $P_f = \Pr(y > \lambda | H_0)$, respectively, where λ is a decision threshold. Based on the statistics of y, P_f can be evaluated as

$$P_f = \frac{\Gamma\left(N/2, \frac{\lambda}{2\sigma^2}\right)}{\Gamma(N/2)} \quad (4)$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [3]. This result matches the one obtained in [5, eq. (19)] for the aforementioned case with $\sigma^2 = 1$. Making use of [9, eq. (2.1-124)], the cumulative distribution function (CDF) of y can be obtained (for even N), and used to evaluate P_d as

$$P_d = Q_{N/2}\left(\sqrt{\frac{a\gamma}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right) \quad (5)$$

where $Q_{N/2}(\cdot,\cdot)$ is the generalized Marcum Q-function [8].

IV. Probability of Detection Over Fading Channels With No Diversity

In this section, we derive the average detection probability over Nakagami and Rayleigh fading channels. Notice that P_f in (4) in this case will remain the same, since it is independent of the SNR. The PDF of γ over a Nakagami channel is given by

$$f_{\text{Nak}}(\gamma) = \frac{1}{\Gamma(m)}\left(\frac{m}{\bar{\gamma}}\right)^m \gamma^{m-1} \exp\left(-\frac{m}{\bar{\gamma}}\gamma\right), \quad \gamma \geq 0 \quad (6)$$

where m is the Nakagami parameter and $\bar{\gamma}$ is the average SNR. With the aid of Appendix A, the average P_d for a Nakagami channel, $\bar{P}_{d,\text{Nak}}$, can be obtained as

$$\bar{P}_{d,\text{Nak}} = A_1 + \beta^m e^{-\frac{\lambda}{2\sigma^2}} \times \sum_{i=1}^{N/2-1} \frac{\left(\frac{\lambda}{2\sigma^2}\right)^i}{i!} {}_1F_1\left(m; i+1; \frac{\lambda(1-\beta)}{2\sigma^2}\right) \quad (7)$$

where $\beta = (2m\sigma^2)/(2m\sigma^2 + a\bar{\gamma})$, ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function ($\equiv \Phi(\cdot;\cdot;\cdot)$) [3, Sec. 9.2], and for integer m, we have

$$A_1 = e^{-\frac{\lambda\beta}{2m\sigma^2}}\left[\beta^{m-1} L_{m-1}\left(\frac{-\lambda(1-\beta)}{2\sigma^2}\right) + (1-\beta)\sum_{i=0}^{m-2} \beta^i L_i\left(\frac{-\lambda(1-\beta)}{2\sigma^2}\right)\right] \quad (8)$$

where $L_i(\cdot)$ is the Laguerre polynomial of degree i [3, eq. (8.970)]. As a by-product of the above result, the average P_d over a Rayleigh channel, $\bar{P}_{d,\text{Ray}}$, can be obtained by setting $m = 1$ in (7). Alternatively, starting with Rayleigh distribution [setting $m = 1$ in (6)] $\bar{P}_{d,\text{Ray}}$ can be obtained as[4]

$$\bar{P}_{d,\text{Ray}} = e^{-\frac{\lambda}{2\sigma^2}} \sum_{i=0}^{N/2-2} \frac{\left(\frac{\lambda}{2\sigma^2}\right)^i}{i!} + \left(\frac{2\sigma^2 + a\bar{\gamma}}{a\bar{\gamma}}\right)^{N/2-1} \times \left[e^{-\frac{\lambda}{2\sigma^2 + a\bar{\gamma}}} - e^{-\frac{\lambda}{2\sigma^2}} \sum_{i=0}^{N/2-2} \frac{\left(\frac{\lambda a\bar{\gamma}}{2\sigma^2(2\sigma^2 + a\bar{\gamma})}\right)^i}{i!}\right]. \quad (9)$$

V. Detection and False Alarm Probabilities With Diversity Reception

In this section, we look into the energy-detection performance when SLC and SLS diversity schemes are employed. While SLC is studied for independent and identically distributed (i.i.d.) as well as correlated Rayleigh fading channels, SLS is studied for the independent case only.

A. Square-Law Combining (SLC)

In this scheme, the outputs of the square-law devices (square-and-integrate operation per branch), denoted as $\{y_i\}_{i=1}^L$ where L is the number of diversity branches, are combined to yield a new decision statistic $y_{\text{SLC}} = \sum_{i=1}^L y_i$. Under H_0 and for AWGN channels, adding L i.i.d. central chi-square variates, each with N DOFs and variance σ^2, will result in another chi-square variate with LN DOFs and the same variance σ^2. Therefore, analogous to (4), we have

$$P_{f,\text{SLC}} = \frac{\Gamma\left(LN/2, \frac{\lambda}{2\sigma^2}\right)}{\Gamma(LN/2)}. \quad (10)$$

[2]The noncentrality parameter can be obtained from (2) after setting $n_{ci} = n_{si} = 0$ and using the sampling theory approximation $E_s \simeq (1/2) \cdot (1/W) \sum_{i=1}^{N/2}(S_{ci}^2 + S_{si}^2)$, with the $(1/2)$ factor reflecting the energy mapping from equivalent LP to BP signals.

[3]N can be approximated as either 2(TW) or 2(TW+1), depending on the first sample position (at 0 or 0^+).

[4]This expression in [1, eq. (16)] (with $\sigma^2 = 1$, $a = 2$, and $N/2 \equiv u$) has a typo (the last exponent i was missing). An alternative expression to that in (9) was also obtained in [4].

Likewise, under H_1, y_{SLC} will be a chi-square variate with LN DOFs, noncentrality parameter $\sum_{i=1}^{L} a\gamma_i \triangleq a\gamma_t$, and variance σ^2. Hence, P_d at the combiner output for AWGN channels, $P_{d,\text{SLC}}$, can be evaluated by analogy to (5) as

$$P_{d,\text{SLC}} = Q_{LN/2}\left(\sqrt{\frac{a\gamma_t}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right). \tag{11}$$

In the following, this probability of detection is averaged over both i.i.d. and correlated Rayleigh channels.

1) i.i.d. Rayleigh Channels: The PDF of γ_t for L i.i.d. Rayleigh branches is quite similar to that in (6), while replacing each m by L and each $\bar{\gamma}$ by $L\bar{\gamma}$. Hence, $\bar{P}_{d,\text{SLC,i.i.d.}}$ is equivalent to $\bar{P}_{d,\text{Nak}}$ in (7), after replacing each m by L, each $\bar{\gamma}$ by $L\bar{\gamma}$, and each N by LN.

2) Correlated Rayleigh Channels: For L correlated Rayleigh branches, the PDF of γ_t is given by [6, eq. (10-60)]

$$f(\gamma_t) = c_1 \sum_{i=1}^{L} c_{2i} e^{-\gamma_t/\eta_i} \tag{12}$$

where $c_1 = (1)/(\prod_{i=1}^{L} \eta_i)$, $c_{2i} = (1)/(\prod_{k=1,k\neq i}^{L}(1/\eta_k - 1/\eta_i))$, and η_i's are the eigenvalues of the covariance matrix $[\mathbf{\Lambda}]_{i_1 i_2} = \sqrt{\rho_{i_1 i_2}\bar{\gamma}_{i_1}\bar{\gamma}_{i_2}}$, $i_1, i_2 = 1, \ldots, L$, with $\rho_{i_1 i_2}$ denoting the power-correlation coefficient between γ_{i_1} and γ_{i_2}. Using the PDF expression in (12), which represents a weighted sum of exponential variates, and expressing $\bar{P}_{d,\text{Ray}}$ in (9) as $\bar{P}_{d,\text{Ray}}(\bar{\gamma}, N)$, the probability of detection in (11) can be averaged, yielding

$$\bar{P}_{d,\text{SLC,corr}} = c_1 \sum_{i=1}^{L} \eta_i c_{2i} \bar{P}_{d,\text{Ray}}(\eta_i, LN). \tag{13}$$

For the special case when $L = 2$, $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}$, and $\rho_{12} = \rho_{21} = \rho$, it follows that $\eta_1 = \bar{\gamma}(1 - \sqrt{\rho})$ and $\eta_2 = \bar{\gamma}(1 + \sqrt{\rho})$ [6]. In addition, the case of independent, but not identically distributed, branches can be deduced by simply having a diagonal $\mathbf{\Lambda}$ with $\bar{\gamma}_i$ being the ith diagonal entry.

B. Square-Law Selection (SLS)

In the SLS diversity scheme, the branch with maximum decision statistic $y_{\text{SLS}} = \max(y_1, \ldots, y_L)$ is to be selected [7]. Under H_0, and given i.i.d. $\{y_i\}_{i=1}^{L}$ variates, P_f for SLS, $P_{f,\text{SLS}}$, can be evaluated using the CDF of y_{SLS} given H_0, $F_{y_{\text{SLS}}}(y|H_0)$, yielding

$$\begin{aligned} P_{f,\text{SLS}} &= 1 - F_{y_{\text{SLS}}}(\lambda|H_0) \\ &= 1 - \left[1 - \frac{\Gamma\left(N/2, \frac{\lambda}{2\sigma^2}\right)}{\Gamma(N/2)}\right]^{L}. \end{aligned} \tag{14}$$

Similarly, and conditioning on γ_i (under H_1), P_d for SLS over AWGN channels, $P_{d,\text{SLS}}$, can be obtained as

$$P_{d,\text{SLS}} = 1 - \prod_{i=1}^{L}\left[1 - Q_{N/2}\left(\sqrt{\frac{a\gamma_i}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right)\right]. \tag{15}$$

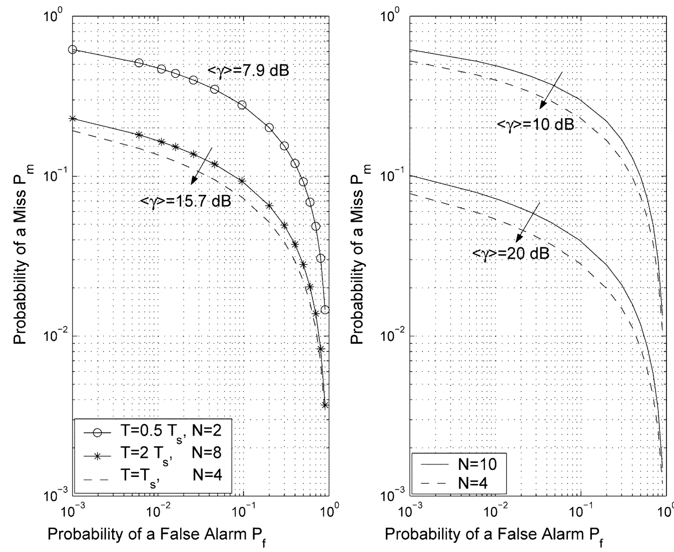

Fig. 1. Complementary ROC curves for a Rayleigh channel.

Averaging this $P_{d,\text{SLS}}$ over L independent Rayleigh branches, and using $\bar{P}_{d,\text{Ray}}(\bar{\gamma}, N)$ in (9) yields

$$\begin{aligned} &\bar{P}_{d,\text{SLS}} \\ &\quad = 1 - \int_0^{\infty} \cdots \int_0^{\infty} \prod_{i=1}^{L}\left[1 - Q_{N/2}\left(\sqrt{\frac{a\gamma_i}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right)\right] \\ &\qquad \times f(\gamma_i) d\gamma_i \\ &\quad = 1 - \prod_{i=1}^{L} \int_0^{\infty} \left[1 - Q_{N/2}\left(\sqrt{\frac{a\gamma_i}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right)\right] f(\gamma_i)\, d\gamma_i \\ &\quad = 1 - \prod_{i=1}^{L}(1 - \bar{P}_{d,\text{Ray}}(\bar{\gamma}_i, N)). \end{aligned} \tag{16}$$

VI. Numerical Examples

We quantify the receiver performance by depicting the receiver operating characteristic (ROC) (P_d versus P_f), or equivalently, complementary ROC (probability of a miss $P_m = 1 - P_d$, versus P_f) for different situations of interest. In the following examples, $\sigma^2 = 1$ and $a = 2$, as is the case from (2). Fig. 1 illustrates the complementary ROC over a Rayleigh channel. The right part of this figure asserts the fact that for the same signal energy, the fewer the samples, the better the performance, as is the case when E_s increases for a given N. The left part of this figure shows another scenario, in which the signal is time-limited over a T_s, interval and can be *approximated* as a bandlimited one. In particular, we study the effect of choosing T when T_s is unknown. Considering a raised cosine signal ($W \approx (1)/(2T_s)$), it is shown here that overdetermining T_s ($T = 2T_s$) implies better performance than underdetermining it ($T = Ts/2$). This asserts the expectation of higher performance sensitivity to a loss in signal energy, rather than to an increase in noise energy. Fig. 2 quantifies the performance gain as the Nakagami parameter increases for $\bar{\gamma} = 20$ dB. For example, there is roughly a gain of one order of magnitude from the P_m perspective for $m = 2$, compared with the Rayleigh case ($m = 1$).

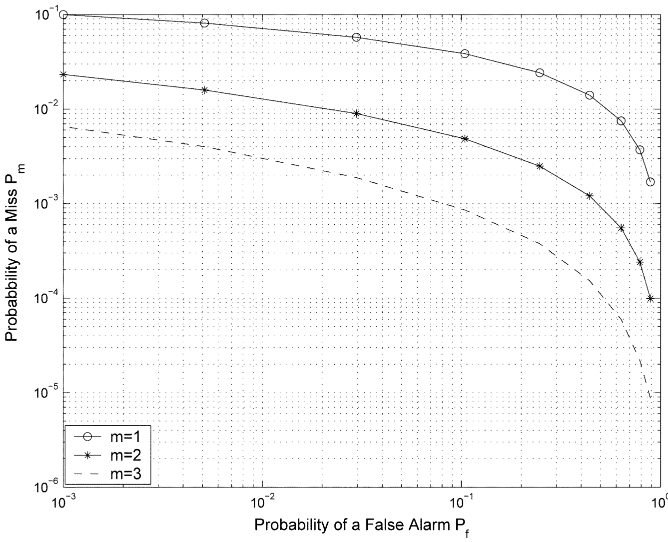

Fig. 2. Complementary ROC curves for a Nakagami channel ($\bar{\gamma} = 20$ dB, $N = 10$).

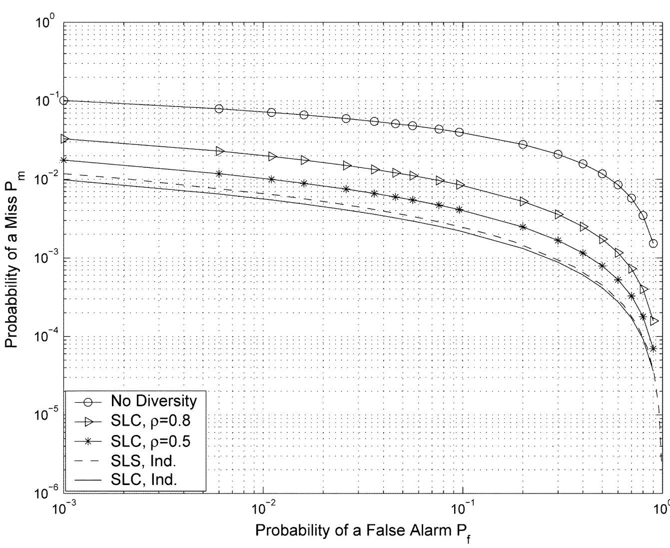

Fig. 3. Complementary ROC curves for dual-branch diversity systems over Rayleigh channels ($\bar{\gamma} = 20$ dB, $N = 10$).

Finally, the effect of diversity over Rayleigh branches is illustrated in Fig. 3 for $\bar{\gamma} = 20$ dB and $L = 2$. SLC and SLS schemes with i.i.d. branches provide mostly the same gain of at least one order of magnitude from the P_m perspective, compared with the no-diversity case. Approximately half of this gain is lost when employing the SLC scheme with two correlated branches of $\rho_{12} = \rho_{21} = \rho = 0.8$.

VII. Conclusion

Using a sampling theory-based approach, we studied the performance of an energy detector for an unknown transmit signal under both AWGN and fading channels. We also quantified the improvement in detection capability when receive diversity schemes are employed. The analysis of this energy detector is timely for emerging applications involving ultra-wideband and cognitive radio technologies.

Appendix A
Evaluation of $\bar{P}_{d,\mathrm{Nak}}$ in (7)

Averaging (5) over (6) while using the change of variable $x = \sqrt{(a\gamma)/(\sigma^2)}$ yields

$$\bar{P}_{d,\mathrm{Nak}} = \zeta \int_0^\infty x^u e^{-v^2 x^2/2} Q_M(b_1 x, b_2)\, dx \triangleq \zeta G_M \quad (17)$$

where $\zeta = (2/\Gamma(m))((m\sigma^2)/(a\bar{\gamma}))^m$, $u = 2m-1$, $v^2 = 2m\sigma^2/(a\bar{\gamma})$, $M = N/2$, $b_1 = 1$, and $b_2 = \sqrt{\lambda/\sigma^2}$.

For $u > -1$, G_M can be recursively evaluated with the aid of [8, eq. (29)], yielding

$$\begin{aligned} G_M &= G_{M-1} + D_{M-1}F_M \\ &= G_{M-2} + D_{M-2}F_{M-2} + D_{M-1}F_{M-1} \\ &\;\;\vdots \\ &= G_1 + \sum_{i=1}^{M-1} D_i F_{i+1} \end{aligned} \quad (18)$$

where

$$D_i = \frac{\Gamma\left(\frac{u+1}{2}\right)\left(\frac{b_2^2}{2}\right)^i e^{-b_2^2/2}}{2(i!)\left(\frac{v^2+b_1^2}{2}\right)^{\frac{u+1}{2}}} \quad (19)$$

$$F_i = {}_1F_1\left(\frac{u+1}{2}; i; \frac{b_1^2 b_2^2}{2(v^2+b_1^2)}\right) \quad (20)$$

and G_1 can be evaluated with the aid of [8, eq. (25)] for integer $(u+1)/2$ (i.e., integer m), and given that the first-order Marcum Q-function $Q_1(\cdot,\cdot) = Q(\cdot,\cdot)$.

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