This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

Maximum Eigenvalue Detection: Theory and Application

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***Abstract*—Channel sensing, i.e., detecting the presence of primary users, is a fundamental problem in cognitive radio. Energy detection is optimal for detecting independent and identically distributed (iid) signals, but not optimal for detecting correlated signals. In this paper, a method is proposed based on the sample covariance matrix calculated from a limited number of received signal samples. The maximum eigenvalue of the sample covariance matrix is used as the test statistic. Since the covariance matrix catches the correlations among the signal samples, the proposed method is better than the energy detection for correlated signals. For iid signals, the method approaches to the energy detection. The random matrix theory is used to analyze the method and set the threshold. Similar to energy detection, the methods do not need any information of the source signal and the channel as *a priori*. Also, no synchronization is needed. Simulations based on wireless microphone signals and iid signals are presented to verify the method.**

I. Introduction

Conventional fixed spectrum allocation policy leads to low spectrum usage in many of the frequency bands. Cognitive radio, first proposed in [1], is a promising technology to exploit the under-utilized spectrum in an opportunistic manner [2]. One application of cognitive radio is spectral reuse, which allows secondary networks/users to use the spectrum allocated/licensed to the primary users when they are not active [3]. To do so, the secondary users are required to frequently perform channel sensing, i.e., detecting the presence of the primary users. If the primary users are found to be inactive, the secondary users can use the spectrum for communications. On the other hand, whenever the primary users become active, the secondary users have to detect the presence of those users in high probability, and vacate the channel within certain amount of time. One communication system using the spectrum reuse concept is IEEE 802.22 wireless regional area networks (WRAN) [4], which operates on the VHF/UHF bands that are currently allocated for TV broadcasting services and other services such as wireless microphone.

Channel sensing is a fundamental task for cognitive radio. However, there are several factors which make channel sensing practically challenging. First, the signal to noise ratio (SNR) of the primary users may be very low. For example, the wireless microphones operating in TV bands only transmit signals with a power of about 50mW and a bandwidth of 200 kHz. If the secondary users are several hundred meters away from the microphone devices, the received SNR may be well below -20dB. Secondly, multipath fading and time dispersion of the wireless channels make the sensing problem more difficult. Multipath fading may cause the signal power to fluctuate as much as 30dB. On the other hand, coherent detection may not be reliable when the time dispersed channel is unknown, especially when the primary users are legacy systems which do not cooperate with the secondary users.

There have been several channel sensing methods, including the energy detection (ED) [5], [6], [7], [8], matched filtering (MF)-based methods [7], [8], [9] and cyclostationary detection (CSD) method [10], [11], [12], each of which has different requirements and advantages/disadvantages. MF-based methods require perfect knowledge of the channel responses from the primary user to the receiver and accurate synchronization (otherwise its performance will be reduced dramatically) [13], [9]. This may not be possible if there is no corporation between the primary and secondary users. Cyclostationary detection needs to know the cyclic frequencies of the primary users, which may not be realistic for many of the spectrum reuse applications. Furthermore, this method demands excessive analog to digital converter (ADC) requirement and signal processing capabilities [8]. Energy detection, unlike the other two methods, does not require any information of the source signal and is robust to unknown dispersed channel and fading. It is optimal for detecting independent and identically distributed (iid) signals [7], but not optimal for detecting correlated signals.

In this paper, we propose a new method based on the statistical covariances or auto-correlations of the received signal. The maximum eigenvalue of the sample covariance matrix is used as the test statistic. Since the covariance matrix catches the correlations among the signal samples, the proposed method is better than the energy detection for correlated signals. For iid signal, this method approaches to the energy detection. The method is a generalization of the energy detection since it includes the latter as a special case. The random matrix theory is used to analyze the method and set the threshold. Similar to energy detection, the methods do not need any information of the signal and the channel as *a priori*. Also, no synchronization is needed. Simulations based on wireless microphone signals and iid signals are presented to verify the method.

The rest of the paper is organized as follows. The detection algorithm is presented in Section II. Section III gives the detection probability and the threshold for the algorithm. Theoretical comparison with the energy detection is discussed in Section IV. Simulation results for two types of signals are given in Section V. Conclusions are drawn in Section VI.

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Some notations are used as follows: boldface capital and small letters are used to denote matrices and vectors, respectively, superscript $(\cdot)^T$ stands for transpose, $\mathbf{I}_q$ denotes the identity matrix of order q, and $\mathrm{E}[\cdot]$ stands for expectation operation.

II. Maximum Eigenvalue Detection

For signal detection, there are two hypothesizes: $\mathcal{H}_0$, signal does not exist; and $\mathcal{H}_1$, signal exists. The received signal samples under the two hypothesizes are given respectively as follows [7], [8], [9]:

$$\mathcal{H}_0: \quad x(n) = \eta(n), \tag{1}$$
$$\mathcal{H}_1: \quad x(n) = s(n) + \eta(n), \tag{2}$$

where $s(n)$ is the transmitted signal samples passed through a wireless channel consisting of path loss, multipath fading and time dispersion effects, and $\eta(n)$ is the white noise which is independent and identically distributed (iid), and with mean zero and variance σ_η^2. Note that $s(n)$ can be the superposition of the received signals from multiple primary users. No synchronization is required here.

Two probabilities are of interest for channel sensing: probability of detection, P_d, which defines, at the hypothesis $\mathcal{H}_1$, the probability of the sensing algorithm having detected the presence of the primary signal; and probability of false alarm, P_{fa}, which defines, at the hypothesis $\mathcal{H}_0$, the probability of the sensing algorithm claiming the presence of the primary signal.

Let us consider L consecutive samples and define the following vectors:

$$\mathbf{x}(n) = \left[\begin{array}{cccc} x(n) & x(n-1) & \cdots & x(n-L+1) \end{array}\right]^T, \tag{3}$$
$$\mathbf{s}(n) = \left[\begin{array}{cccc} s(n) & s(n-1) & \cdots & s(n-L+1) \end{array}\right]^T, \tag{4}$$
$$\boldsymbol{\eta}(n) = \left[\begin{array}{cccc} \eta(n) & \eta(n-1) & \cdots & \eta(n-L+1) \end{array}\right]^T, \tag{5}$$

where L is called the smoothing factor. Considering the statistical covariance matrices of the signal and noise defined as

$$\mathbf{R}_x = \mathrm{E}[\mathbf{x}(n)\mathbf{x}^T(n)], \tag{6}$$
$$\mathbf{R}_s = \mathrm{E}[\mathbf{s}(n)\mathbf{s}^T(n)], \tag{7}$$

we can verify that

$$\mathbf{R}_x = \mathbf{R}_s + \sigma_\eta^2 \mathbf{I}_L. \tag{8}$$

If the signal $s(n)$ is not present, $\mathbf{R}_s = 0$. If the signal $s(n)$ is present, $\mathbf{R}_s \neq 0$. Let λ_{max} and ρ_{max} be the maximum eigenvalue of $\mathbf{R}_x$ and $\mathbf{R}_s$, respectively. Then $\lambda_{max} = \rho_{max} + \sigma_\eta^2$. Obviously, $\rho_{max} = 0$ if and only if $\mathbf{R}_s = 0$. Hence, if signal is present, $\lambda_{max} > \sigma_\eta^2$, and if no signal, $\lambda_{max} = \sigma_\eta^2$. Thus, the maximum eigenvalue can be used to detect the presence of signal.

In practice, the statistical covariance matrix can only be calculated by using a limited number of signal samples. Define the sample auto-correlations of the received signal as

$$\lambda(l) = \frac{1}{N_s}\sum_{m=0}^{N_s-1} x(m)x(m-l), \; l = 0, 1, \cdots, L-1, \tag{9}$$

where N_s is the number of available samples. The statistical covariance matrix $\mathbf{R}_x$ can be approximated by the sample covariance matrix defined as

$$\hat{\mathbf{R}}_x(N_s) = \begin{bmatrix} \lambda(0) & \lambda(1) & \cdots & \lambda(L-1) \\ \lambda(1) & \lambda(0) & \cdots & \lambda(L-2) \\ \vdots & \vdots & \vdots & \vdots \\ \lambda(L-1) & \lambda(L-2) & \cdots & \lambda(0) \end{bmatrix}. \tag{10}$$

Note that the sample covariance matrix is symmetric and Toeplitz. Based on the sample covariance matrix, we propose the following signal detection method.

Algorithm 1: Maximum eigenvalue detection (MED)

Step 1. Compute the sample auto-correlations in (9) and form the sample covariance matrix defined in (10).

Step 2. Obtain the maximum eigenvalue $\hat{\lambda}_{max}(N_s)$ of the sample covariance matrix.

Step 3. Decision: if $\hat{\lambda}_{max}(N_s) > \gamma\sigma_\eta^2$, signal exists ("yes" decision); otherwise, signal does not exist ("no" decision), where $\gamma > 1$ is a threshold (to be given in the next section).

The method uses only the received signal and noise power to make a decision. When $L = 1$, the method turns to the energy detection. Hence, it can be treated as an extension of the energy detection.

III. Threshold and Probability of False Alarm

For a good detection algorithm, P_d should be high and P_{fa} should be low. The choice of the threshold γ is a compromise between the P_d and P_{fa}. Since we have no information on the signal (actually we even do not know if the signal is present), it is difficult to set the threshold based on the P_d. Hence, the threshold is selected based on the pre-defined P_{fa}. The steps are as follows. First we set a value for P_{fa}. Then we find a threshold γ to meet the required P_{fa}. To find the threshold based on the required P_{fa}, we can use either theoretical derivation or computer simulation. If simulation is used to find the threshold, we can generate white Gaussian noises as the input (no signal) and adjust the threshold to meet the P_{fa} requirement. If theoretical derivation is used, we need to find the probability distribution of the random variable $\hat{\lambda}_{max}(N_s)$. The eigenvalue distribution of $\hat{\mathbf{R}}_x(N_s)$ is very complicated [14], [15], [16], [17]. This makes the theoretical determination of the threshold very difficult. In this section, using random matrix theory, we will find an approximation for the distribution of this random variable and provide theoretical estimations for the threshold and P_{fa}.

When there is no signal, $\hat{\mathbf{R}}_x(N_s)$ turns to $\hat{\mathbf{R}}_\eta(N_s)$, the sample covariance matrix of the noise defined as,

$$\hat{\mathbf{R}}_\eta(N_s) = \frac{1}{N_s}\sum_{n=0}^{N_s-1} \boldsymbol{\eta}(n)\boldsymbol{\eta}^\dagger(n). \tag{11}$$

$\hat{\mathbf{R}}_\eta(N_s)$ is nearly a Wishart random matrix [14]. The study of the spectral (eigenvalue distributions) of a random matrix is a very hot topic in recent years in mathematics as well as communication and physics. The joint probability density function (PDF) of ordered eigenvalues of a Wishart random matrix has been known for many years [14]. However, since the expression of the PDF is very complicated, no closed form expression has been found for the marginal PDF of ordered eigenvalues. Recently, I. M. Johnstone and K. Johansson have found the distribution of the largest eigenvalue [15], [16] as described in the following theorem.

Theorem 1. Assume that the noise is real. Let

$$\mathbf{A}(N_s) = \frac{N_s}{\sigma_\eta^2}\hat{\mathbf{R}}_\eta(N_s),\ \mu = (\sqrt{N_s - 1} + \sqrt{L})^2 \tag{12}$$

and

$$\nu = (\sqrt{N_s - 1} + \sqrt{L})\left(\frac{1}{\sqrt{N_s - 1}} + \frac{1}{\sqrt{L}}\right)^{1/3}. \tag{13}$$

Assume that $\lim_{N_s \to \infty} \frac{L}{N_s} = y\ (0 < y < 1)$. Then $\frac{\lambda_{max}(\mathbf{A}(N_s)) - \mu}{\nu}$ converges (with probability one) to the Tracy-Widom distribution of order 1 (W_1) [18].

Based on the theorem, when N_s is large, the largest eigenvalue of $\hat{\mathbf{R}}_\eta(N_s)$ has mean $\frac{\sigma_\eta^2}{N_s}(\sqrt{N_s - 1} + \sqrt{L})^2$, and variance $\frac{\sigma_\eta^4}{N_s^2}\nu^2$. After normalization it has the Tracy-Widom distribution of order 1.

The Tracy-Widom distributions were found by Tracy and Widom (1996) as the limiting law of the largest eigenvalue of certain random matrices [18]. Let F_1 be the cumulative distribution function (CDF) (sometimes simply called distribution function) of the Tracy-Widom distribution of order 1. There is no closed form expression for the distribution function. The distribution function is defined as

$$F_1(t) = \exp\left(-\frac{1}{2}\int_t^\infty \left(q(u) + (u - t)q^2(u)\right) du\right), \tag{14}$$

where $q(u)$ is the solution of the nonlinear Painlevé II differential equation

$$q''(u) = uq(u) + 2q^3(u). \tag{15}$$

It is generally difficult to evaluate it. Fortunately, there have been tables for the functions [15] and Matlab codes to compute it [19]. Table I gives the values of F_1 at some points.

Using the theory, we are ready to analyze the algorithm. For simplicity, we replace $\sqrt{N_s - 1}$ by $\sqrt{N_s}$ because N_s is usually large. The probability of false alarm of the MED is

$$\begin{aligned} P_{fa} &= P\left(\lambda_{max}(\hat{\mathbf{R}}_\eta(N_s)) > \gamma\sigma_\eta^2\right) \\ &= P\left(\frac{\sigma_\eta^2}{N_s}\lambda_{max}(\mathbf{A}(N_s)) > \gamma\sigma_\eta^2\right) \\ &= P\left(\lambda_{max}(\mathbf{A}(N_s)) > \gamma N_s\right) \\ &= P\left(\frac{\lambda_{max}(\mathbf{A}(N_s)) - \mu}{\nu} > \frac{\gamma N_s - \mu}{\nu}\right) \\ &\approx 1 - F_1\left(\frac{\gamma N_s - \mu}{\nu}\right). \end{aligned} \tag{16}$$

This leads to

$$F_1\left(\frac{\gamma N_s - \mu}{\nu}\right) = 1 - P_{fa}, \tag{17}$$

or, equivalently,

$$\frac{\gamma N_s - \mu}{\nu} = F_1^{-1}(1 - P_{fa}). \tag{18}$$

From the definitions of μ and ν, we finally obtain the threshold

$$\begin{aligned} \gamma &= \frac{(\sqrt{N_s} + \sqrt{L})^2}{N_s} \\ &\cdot \left(1 + \frac{(\sqrt{N_s} + \sqrt{L})^{-2/3}}{(N_s L)^{1/6}} F_1^{-1}(1 - P_{fa})\right). \end{aligned} \tag{19}$$

Table I can be used to compute the $F_1^{-1}(y)$ at certain points. For example, $F_1^{-1}(0.9) = 0.45$, $F_1^{-1}(0.95) = 0.98$.

IV. Comparison with energy detection

Energy detection does not need any information of the signal to be detected and is robust to unknown dispersive channel. When the source signal is iid, it has been proved that energy detection is optimal [7]. Let $T_x(N_s)$ be the average energy of the received signals, that is,

$$T_x(N_s) = \frac{1}{N_s}\sum_{n=0}^{N_s - 1} |x(n)|^2. \tag{20}$$

It is easy to verify that

$$T_x(N_s) = \mathrm{Tr}(\hat{\mathbf{R}}_x(N_s))/L, \tag{21}$$

where $\mathrm{Tr}(\cdot)$ means the trace of a matrix. Since the trace of a matrix equals to the sum of its eigenvalues, $T_x(N_s)$ is the average value of the eigenvalues of $\hat{\mathbf{R}}_x(N_s)$. Energy detection simply compares $T_x(N_s)$ with the noise power to make a decision: if $T_x(N_s) > \gamma_1\sigma_\eta^2$, signal exists; otherwise, signal does not exists. For given P_{fa}, the threshold can be obtained as (for large N_s)

$$\gamma_1 = \sqrt{\frac{2}{N_s}}Q^{-1}(P_{fa}) + 1 \tag{22}$$

where

$$Q(t) = \frac{1}{\sqrt{2\pi}}\int_t^{+\infty} e^{-u^2/2}\mathrm{d}u. \tag{23}$$

When there is no signal, for large N_s, Theorem 1 tells us that $\hat{\lambda}_{max}(N_s)$ has mean $\frac{\sigma_\eta^2}{N_s}(\sqrt{N_s} + \sqrt{L})^2$ and variance $\frac{\sigma_\eta^4}{N_s^2}\nu^2$, while it is easy to show that $T_x(N_s)$ has mean σ_η^2 and variance $\frac{2\sigma_\eta^4}{N_s}$. This means that the expectations of the two variables are nearly the same and their variances are very small (approximately zeros).

When signal presents and the signal is iid, the analysis above is still valid if we replace σ_η^2 by $\sigma_s^2 + \sigma_\eta^2$, where σ_s^2 is the average signal energy. Hence, for iid signal, the MED approaches to energy detection (which is optimal) for large N_s.

t	-3.90	-3.18	-2.78	-1.91	-1.27	-0.59	0.45	0.98	2.02
$F_1(t)$	0.01	0.05	0.10	0.30	0.50	0.70	0.90	0.95	0.99

TABLE I
NUMERICAL TABLE FOR THE TRACY-WIDOM DISTRIBUTION OF ORDER 1

When the source signal is correlated, energy detection is no longer optimal. The proposed method can be much better than the energy detection. In fact, for highly correlated source signal, $\lambda_{max}(\mathbf{R}_s) \gg \mathrm{Tr}(\mathbf{R}_s)/L$. Hence,

$$\begin{aligned} \hat{\lambda}_{max}(N_s) &\approx \lambda_{max}(\mathbf{R}_s) + \sigma_\eta^2 \\ &\gg \mathrm{Tr}(\mathbf{R}_s)/L + \sigma_\eta^2 \approx T_x(N_s). \end{aligned} \quad (24)$$

For example, if the source signal is a constant A, we have $\mathbf{R}_s = A^2\mathbf{e}\mathbf{e}^T$, where $\mathbf{e}$ is a $L \times 1$ vector with all its elements being ones. Hence, $\lambda_{max}(\mathbf{R}_s) = LA^2$ while $\mathrm{Tr}(\mathbf{R}_s)/L = A^2$. For this case, MED boasts the signal energy by a factor L compared to energy detection.

In terms of the computational complexity, the energy detection needs about N_s multiplications and additions. The computational complexity of the MED comes from two parts: the computation of sample auto-correlations in (9), which requires LN_s multiplications and additions, and the computation of the maximum eigenvalue of $\hat{\mathbf{R}}_x(N_s)$ (size $L \times L$), which needs $\mathrm{O}(L^3)$ operations. Since $N_s \gg L$, the first part is dominant. Thus, the computational complexity of the proposed method is about L times that of the energy detection.

V. SIMULATIONS AND DISCUSSIONS

In this section, we will give some simulation results for two situations: iid signal and wireless microphone signal [20]. The smoothing factor is chosen as $L = 8$ (small to reduce complexity). SNR is defined as $\mathrm{SNR} = \mathrm{E}(|s(n)|^2)/\mathrm{E}(|\eta(n)|^2)$.

First, we simulate the probabilities of false alarm (P_{fa}) because the P_{fa} is not related to signal (at $\mathcal{H}_0$, there is no signal at all). We set the target $P_{fa} = 0.1$ and then obtain the thresholds using the formulae in Section III for given P_{fa}, L and N_s. Figure 1 gives the simulation results (average on 2000 Monte-Carlo realizations) for various sample sizes N_s. Comparing the target $P_{fa} = 0.1$ with the simulated result, we see that the theoretical threshold for the energy detection is very accurate, while that for the MED is a little bit lower than expected, which causes the P_{fa} to be slightly higher than 0.1.

Secondly, we fix the thresholds based on P_{fa} and simulate the probability of detection (P_d) for various cases. We consider two signal types as follows.

(1) Wireless microphone signal. In USA and some other countries, wireless microphone operates on some vacant TV channels. The wireless microphone signal is FM modulated and has a bandwidth less than 200KHz. For our simulation, wireless microphone signals (soft speaker) [20] are generated. The sampling rate at the receiver is 6 MHz (the same as the TV bandwidth in USA). Figure 2 gives the simulation results. The MED is much better than the energy detection (ED), which verifies our assertion in Section IV. The reason is that the source signal has narrow bandwidth (compared to 6MHz of a TV channel), and therefore its samples are highly correlated. The performances of the methods at different sample sizes (sensing times) and fixed SNR=-24 dB are given in Figure 3, with the corresponding P_{fa} shown in Figure 1. It is clear that the MED is much better than the energy detection.

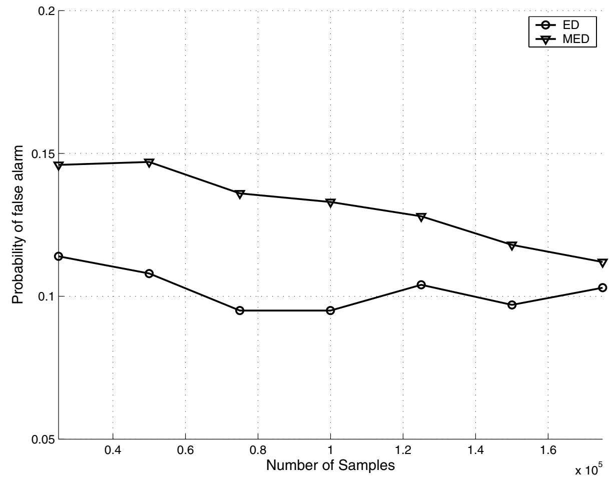

Fig. 1. Probability of false alarm

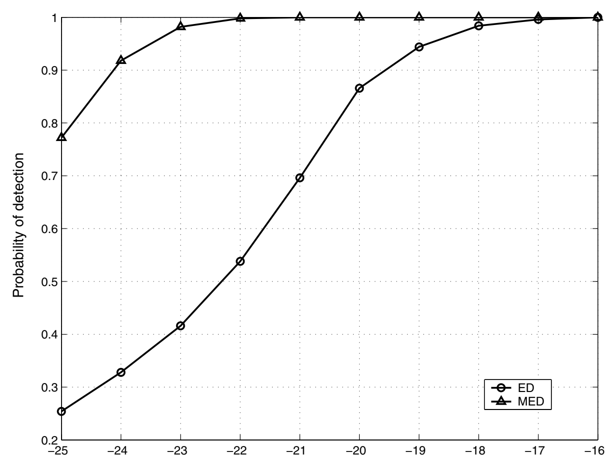

Fig. 2. P_d vs SNR for wireless microphone signal: $N_s = 100000$

(2) iid signal. We know that energy detection is optimal for detecting white noise signal. Here we want to see what is the performance of MED compared to the optimal detection. The input signal samples are iid with Gaussian distribution. Figure 4 and Figure 5 indicate that the MED degrades only slightly compared to the energy detection (for about 1 dB).

In summary, the simulations above show that the proposed

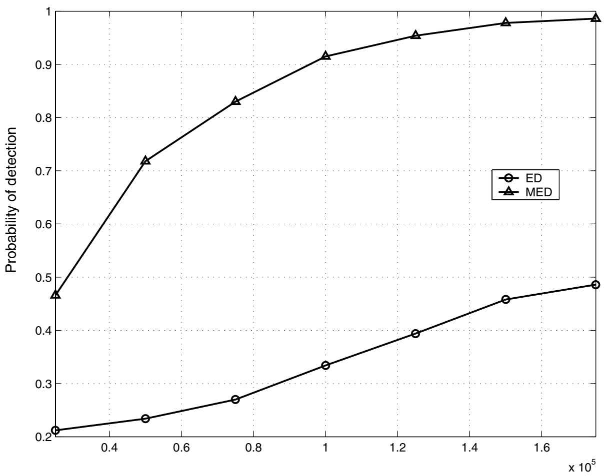

Fig. 3. P_d vs sample size for wireless microphone signal: $\mathrm{SNR} = -24\mathrm{dB}$

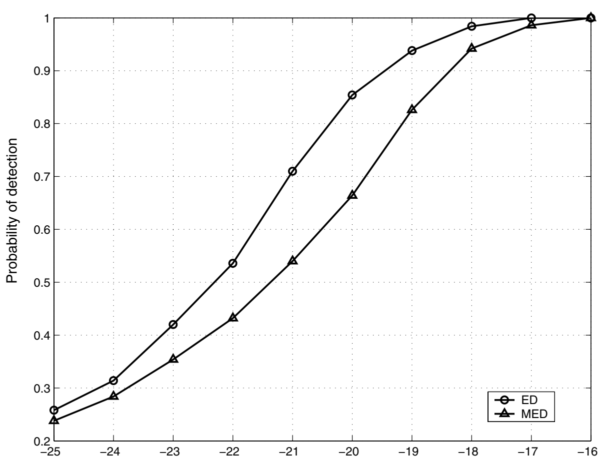

Fig. 4. P_d vs SNR for white noise signal: $N_s = 100000$

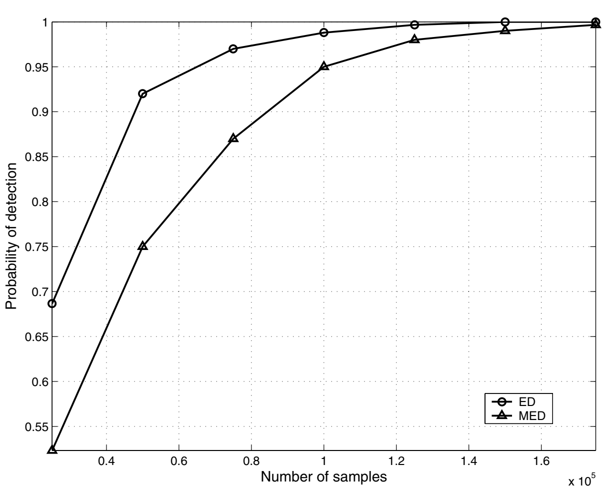

Fig. 5. P_d vs sample size for white noise signal: $\mathrm{SNR} = -18\mathrm{dB}$

method approaches to the energy detection for iid signals and is much better than the energy detection for highly correlated signals.

VI. Conclusions

In this paper, a sensing algorithm based on the sample covariance matrix of the received signal has been proposed. The random matrix theory has been used to set the threshold and obtain the probability of false alarm. Theoretical analysis and simulations have shown that the MED is much better than the ED for highly correlated signals and approaches to ED for iid signals. The methods can be used for various signal detection applications without knowledge of the source signal and the channel.

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