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Literature Review on Correlations of the Non-Darcy Coefficient

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Abstract

Darcy's law can not describe fluid flow accurately when the flow rate is high. In most cases in the recovery process, fluid flow is governed by Darcy's law. But when the flow rate is very high, for an instance, near the wellbore, Darcy's law is inadequate to describe fluid flow.

In 1901, Forchheimer put forward a classical equation, known as the Forchheimer equation, to make up the deficiency encountered by Darcy's law at high flow rates. He added a non-Darcy term into the Darcy flow equation. The non-Darcy term is the multiplication of the non-Darcy coefficient, fluid density, and the second power of velocity. One of the most important aspects in determining the non-Darcy effect is to estimate the non-Darcy coefficient as accurately as possible.

In this paper, theoretical and empirical correlations of the non-Darcy coefficient in one-phase and multi-phase cases in the literature are reviewed. Most researchers have agreed that the non-Darcy effect is not due to turbulence but to inertial effect. The non-Darcy coefficient in wells is usually determined by analysis of multi-rate pressure test results, but such data are not available in many cases. So, people have to use correlations obtained from the literature. This paper summarizes many correlations in the literature, and will provide a good reference for those who are interested in the investigation of the non-Darcy effect in the recovery process.

Introduction

In most cases (not near the well-bore) in recovery processes, the flow pattern is governed by Darcy's law, which describes a linear relationship between pressure gradient and velocity as follows,

$$u = -\frac{K}{\mu}\frac{\partial p}{\partial x} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where u is superficial velocity, K is permeability, p is pressure, μ is viscosity, and x is dimension in x direction.

Forchheimer[1] found that the pressure gradient required to maintain a certain flow rate through porous media was higher than that predicted by Darcy's law. He added a non-Darcy term to Darcy's law to account for this discrepancy, and the flow equation became

$$-\frac{dp}{dx} = \frac{\mu}{K}u + \beta\rho u^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where ρ is fluid density, and β is called the non-Darcy coefficient in this paper. From equation (2), we see that the non-Darcy term is a multiplication of the second power of velocity, fluid density, and β. There have been many names for β. β was called: the turbulence factor by Cornell and Katz,[2] and Tek et al.;[3] the coefficient of inertial resistance by Geertsma,[4] and Al-Rumhy et al.;[5] the velocity coefficient by Firoozabadi;[6] the non-Darcy flow coefficient by Civan and Evans,[7] Liu et al.,[8] Grigg and Hwang,[9] Narayanaswamy et al.,[10] and Li et al.;[11] the Forchheimer coefficient by Ruth and Ma;[12] Inertial Coefficient by Ma and Ruth;[13] the beta factor by Milton-Taylor;[14] the non-Darcy coefficient by Thauvin and Mohanty,[15] Cooper et al.,[16] and Li et al..[11] Equation (2) is called the Forchheimer equation by Ruth and Ma,[12] Milton-Taylor,[14] Ma and Ruth,[13] Civan and Evans,[17] Thauvin and Mohanty,[15] Coles and Hartman,[18] Cooper et al.,[16] and Li et al..[11]

When flow rate is very high, Darcy's law is not adequate to describe flow pattern. High-velocity gas flow occurs in the near-well-bore region and condensate reservoirs. Non-Darcy effect is important in these regions according to Kalaydljian et al..[19]

Cornell and Katz[2] attributed the non-Darcy effect, in another word, the nonlinearity between pressure gradient and velocity, to turbulence. But now many researchers, such as Bear,[20] Scheidegger,[21] Barak,[22] Ruth and Ma,[12] Whitaker[23] have agreed that the nonlinearity is not due to turbulence but to inertial effects. Bear[20] systematically gave three reasons to exclude turbulence as the cause for the non-Darcy effect:

1) in turbulent flow through pipes, the linear term in equation (2) does not exist;
2) in flow through pipes, the transition from laminar to turbulent flow is not gradual but rather sharp;

3) the critical Reynolds number Re at which the transition starts is several orders of magnitude higher than that at which the non-Darcy effect begins.

For the above third reason, Ruth and Ma[12] gave an example to illustrate the idea that non-Darcy flow does not necessarily imply a high microscopic Reynolds number. They argue that, in a straight tube model, non-Darcy effects will not become evident until true turbulence sets in at Re ≈ 2000, while, in a bent tube model, microscopic inertial effects will become important when Re ≈ 1. They conclude that the non-Darcy effect occurs because microscopic inertial effects alter the velocity and pressure fields. The above example implies that tortuosity should be one of the key factors in determining the non-Darcy coefficient.

The non-Darcy coefficient in wells is usually determined by analysis of multi-rate pressure test results, but such data are not always available. So, people have to use correlations obtained from the literature. In this paper, correlations in the literature will be reviewed.

Literature Review

There are many correlations in the literature. Both empirical correlations and theoretical equations of the non-Darcy coefficient will be reviewed. For the empirical correlations, reviews will be made on one-phase and two-phase bases. No theoretical equations in two or more phases have been found so far, so for theoretical equations, review will be limited to one-phase case.

Theoretical Equations. Some investigators use capillaric models to describe fluid flow through porous media. The models can be classified into two groups: parallel type models and serial type models. Scheidegger[21,24] and Bear[20] summarized many researchers' work on capillaric models.

Parallel Type Models. Parallel type models assume that a porous medium is made up of a bundle of straight, parallel capillaries of uniform diameter, D_c. The model is illustrated in Fig. 1.

Ergun and Orning[25] argue that the total energy loss for fluid flow through porous media comprises two parts: viscous energy and kinetic energy. Based on the parallel type model, by combining the Poiseuille equation and the equation similar to the one introduced by Brillouin[26] for capillary flow, Ergun and Orning developed a theoretical equation to describe the non-linear laminar flow,

$$-\frac{dp}{dx} = 2\alpha'\frac{(1-\phi)^2}{\phi^3}S_{gv}^2\mu u + \frac{\beta'}{8}\frac{1-\phi}{\phi^3}S_{gv}\rho u^2 \quad \text{……(3)}$$

where α' and β' are correction factors, ϕ is porosity, and S_{gv} is specific surface defined as the solid surface area divided by the solid volume.

Based on the same model, Irmay[27] derived theoretically Darcy and Forchheimer equations from the dynamic Navier-Stokes equations,

$$\begin{aligned} & \boldsymbol{u}_t - \boldsymbol{u}\times curl\boldsymbol{u} + grad(u^2/2) \\ & = -grad(gh) - \mu curlcurl\boldsymbol{u} \end{aligned} \quad \text{……….(4)}$$

where **u** = velocity vector, $\mathbf{u}_t = \partial\mathbf{u}/\partial t$, h = piezometric head, g = acceleration of gravity, and t = time. When the inertial term **u** × curl**u** in the Navier-stokes equations is assumed to be zero (neglected), Irmay derived the Darcy equation. When the inertial term is not zero, Irmay derived the following equation,

$$-\frac{dp}{dx} = \frac{\beta''(1-\phi)^2\mu}{\phi^3 D_c^2}u + \frac{\alpha''(1-\phi)\rho}{\phi^3 D_c}u^2 + \frac{1}{\phi}u_t \quad \text{…..(5)}$$

where α'' and β'' are correction factors. The last term in the right-hand side of equation (5) is caused by unsteady flow and can be neglected according to Polubarinova-Kochina.[28] Comparing equations (3) and (5) with the Forchheimer equation, we can find an equation for the non-Darcy coefficient β as follows,

$$\beta = \frac{c}{K^{0.5}\phi^{1.5}} \quad \text{……………………(6)}$$

where c is a constant.

Serial Type Models. A drawback of the parallel models is that all the pores are assumed to go from one face of the porous media to the other.

Scheidegger[21,24] put forward a serial model in which all the pore space is serially lined up, so that each particle of fluid would have to enter at one pinhole at one side of a porous medium, go through very tortuous channels through all the pores, and then emerge at only one pinhole at the other face of the porous medium. This kind of model is called serial type model[21,24] because capillaries of different pore diameter are aligned in series. This model is illustrated in Fig. 2.

Assuming a model of length x where there are n capillaries per unit area in each dimensional direction of a pore diameter δ and length s, Scheidegger[21,24] derived an equation to describe the non-Darcy flow,

$$\begin{aligned} gradp = {} & u\frac{3c\tau^2}{\phi}\mu\left(\int_{\delta_R}^{\infty}\frac{\alpha(\delta)d\delta}{\delta^6}\right)\left(\int\delta^2\alpha(\delta)d\delta\right)^2 \\ & + u^2\frac{9c'\tau^3}{\phi^2}\rho\left(\int_0^{\delta_R}\frac{\alpha(\delta)d\delta}{\delta^7}\right)\left(\int\delta^2\alpha(\delta)d\delta\right)^3 \end{aligned} \quad \text{…….(7)}$$

where δ is pore diameter, τ is tortuosity, δ_R is critical pore diameter that separates non-Darcy region from Darcy region, c = 32, c' = 1/2, and α(δ) is the differential pore size distribution function, which is defined as the fraction of the total pore space has a 'pore diameter' between δ and δ+dδ. α(δ) has the property,

$$\int_0^{\infty}\alpha(\delta)d\delta = 1 \cdot \quad \text{…………………..(8)}$$

Comparing equation (7) with the Forchheimer equation, we obtain

$$\beta = \frac{c''\tau}{K\phi} \quad \text{.............................(9)}$$

where c'' is a constant related to pore size distribution.

Empirical Correlations. Permeability is the key parameter in the correlations for predicting the non-Darcy coefficient developed by many researchers such as Ergun,[29] Janicek and Katz,[30] Cooke,[31] MacDonald et al.,[32] Kutasov,[33] Liu et al.,[8] Coles and Hartman,[18] and Thauvin and Mohanty.[15]

One-Phase. Based on the theoretical equation developed by Ergun and Orning,[25] Ergun[29] found an empirical equation by analyzing the data from 640 experiments conducted by himself, and the data found in the literature, which involved various-sized spheres, sand, pulverized coke, and the following gases: carbon dioxide, nitrogen, methane, and hydrogen. According to Thauvin and Mohanty's review,[15] the comparison of Ergun's empirical flow equation with the Forchheimer equation leads to,

$$\beta = ab^{-1/2}(10^{-8}K)^{-1/2}\phi^{-3/2} \quad \text{................(10)}$$

where a = 1.75, b = 150, K is expressed in Darcy and β in 1/cm. MacDonald et al.[32] analyzed equation (3) for particles of different roughness and found that b = 180, in the meantime, a ranged from 1.8 to 4.

Janicek and Katz[30] proposed to use the equation,

$$\beta = 1.82\times10^{8}K^{-5/4}\phi^{-3/4} \quad \text{................(11)}$$

to predict the non-Darcy coefficient for natural porous media. In equation (11), K is expressed in md and β in 1/cm.

Cooke[31] studied non-Darcy flow for brines, reservoir oils, and gases in propped fractures and used only permeability to predict the non-Darcy coefficient,

$$\beta = bK^{-a} \quad \text{..........................(12)}$$

where a and b are constants determined by experiments based on proppant type. Equation (12) is simple and applicable to different types of proppants.

By processing the data measured from unconsolidated media and consolidated media, Geertsma[4] found that equation (6) was not applicable to consolidated materials but to unconsolidated media. After he analyzed the data obtained for unconsolidated sandstones, consolidated sandstones, limestones, and dolomites from his experiments, and by Green and Duwez[34] and Cornell and Katz,[2] and performed dimensional analysis, he reached an empirical correlation,

$$\beta = \frac{0.005}{K^{0.5}\phi^{5.5}} \quad \text{........................(13)}$$

where K is in cm^2 and β in 1/cm.

Pascal et al.[35] proposed a mathematical model to estimate the fracture length and the non-Darcy coefficient. By using the model and the data from variable rate tests from low permeability hydraulically fractured wells, they calculated the non-Darcy coefficients. Based on their results, they proposed an empirical correlation,

$$\beta = \frac{4.8\times10^{12}}{K^{1.176}} \quad \text{.......................(14)}$$

where K is in md and β in 1/m.

Jones[36] carried out experiments on 355 sandstone and 29 limestone cores, which were in different core types: vuggy limestone, crystalline limestone, and fine-grained sandstone. By analyzing the data from his experiments, he arrived at a correlation to estimate the non-Darcy coefficient,

$$\beta = \frac{6.15\times10^{10}}{K^{1.55}} \quad \text{.....................(15)}$$

where K is expressed in md and β in 1/ft.

Liu et al.[8] plotted equation (13) developed by Geertsma (1974) against the data obtained respectively by Cornell and Katz,[2] Geertsma,[4] Evans and Evans,[37] Evans et al.,[38] and Whitney.[39] They found that equation (13) was inaccurate. By considering the effect of τ, the tortuosity of the porous medium, on the non-Darcy coefficient, and by analyzing those data, they got a better regression fit correlation,

$$\beta = 8.91\times10^{8}K^{-1}\phi^{-1}\tau \quad \text{.................(16)}$$

where β is expressed in 1/ft and K in md.

Thauvin and Mohanty[15] developed a pore-level network model to describe high velocity flow. They input pore size distributions and network coordination numbers into the model, and obtained outputs such as permeability, non-Darcy coefficient, tortuosity, and porosity. After analyzing all the data they collected, they obtained a correlation,

$$\beta = \frac{1.55\times10^{4}\tau^{3.35}}{K^{0.98}\phi^{0.29}} \quad \text{....................(17)}$$

where β is expressed in 1/cm and K in Darcy.

While suspecting tortuosity may influence the non-Darcy coefficient, by using two different methods to process data from measurements on limestone and sandstone samples, Coles and Hartman[18] proposed two equations to calculate the non-Darcy coefficient,

$$\beta = \frac{1.07\times10^{12}\times\phi^{0.449}}{K^{1.88}} \text{(Same porosity method).....(18)}$$

and

$$\beta = \frac{2.49\times10^{11}\phi^{0.537}}{K^{1.79}} \text{ (Simultaneous equations)} \ldots\ldots(19)$$

where β is expressed in 1/ft and K in md. Comparing equations (18) and (19) with correlations developed by other investigators, the exponents for porosity in equations (18) and (19) are positive instead of being negative in other equations.

Cooper et al.[16] conducted non-Darcy flow studies in anisotropic porous media with a microscopic model. They also included tortuosity in predicting the non-Darcy coefficient,

$$\beta = \frac{10^{-3.25}\tau^{1.943}}{K^{1.023}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(20)$$

where β is expressed in 1/cm and K in cm^2.

Li et al.[11] incorporated non-Darcy effect into a reservoir simulator and simulated the wafer non-Darcy flow experiments, where nitrogen was injected at various flow rates, in several different directions, into a wafer-shaped Berea sandstone core sample with a 3-in. diameter and a 3/8-in. thickness. Comparing differential pressures from simulations with their counterparts from experiments, they found a β correlation for Berea sandstone,

$$\beta = \frac{11500}{K\phi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(21)$$

where K is in Darcy and β in 1/cm.

All the above equations for predicting the non-Darcy coefficient are valid only for one-phase case. Some researchers conducted non-Darcy flow experiments in multi-phase systems, and they obtained some empirical equations to predict the non-Darcy coefficient.

Multi-Phase. In addition to the one-phase equation (13), Geertsma[4] proposed a β correlation in a two-phase system. He was the first researcher to make this effort. He argued that, in the two-phase system, the permeability in equation (13) would be replaced by the gas effective permeability at a certain water saturation, while the porosity would be replaced by the void fraction occupied by the gas. Therefore, in the two-phase system, where the fluid was immobile, the β correlation became

$$\beta = \frac{0.005}{K^{0.5}\phi^{5.5}}\cdot\frac{1}{(1-S_{wr})^{5.5}K_{rel}^{0.5}} \ldots\ldots\ldots\ldots..(22)$$

where S_{wr} is the residual water saturation (or called immobile liquid saturation) and K_{rel} is the gas relative permeability. The units of K and ϕ in equation (22) are the same as their counterparts in equation (13). Equation (22) shows that the presence of the liquid phase increases the non-Darcy coefficient.

Wong[40] found that β increased by eight times when liquid saturation increased from 40 to 70 percent. Evans et al.[38], Grigg and Hwang[9], Coles and Hartman[18] also found that the non-Darcy coefficient increased with increased liquid saturation.

Based on experimental and analytical investigations, Kutasov[33] found that equation (23) could estimate β for both situations: with a mobile liquid saturation and with an immobile liquid saturation.

$$\beta = \frac{1432.6}{K_g^{0.5}[\phi(1-S_w)]^{1.5}} \ldots\ldots\ldots\ldots\ldots\ldots(23)$$

where β is in 1/cm, K_g is gas effective permeability in Darcy, and S_w is water saturation.

Frederick and Graves[41] obtained 407 data points from their experiments, where permeability varied from 0.00197 md to 1230 md, and data obtained by Cornell and Katz,[2] Geertsma,[4] and Evans et al..[42] By using two different regression methods to analyze the data and considering the water saturation effect, Frederick and Graves found two empirical correlations:

$$\beta = \frac{2.11\times10^{10}}{K_g^{1.55}\left[\phi(1-S_w)\right]} \ldots\ldots\ldots\ldots\ldots\ldots..(24)$$

and

$$\beta = \frac{1}{\left[\phi(1-S_w)\right]^2}e^{45-\sqrt{407+81\times\ln\{K_g/[\phi(1-S_w)]\}}} \ldots\ldots\ldots\ldots(25)$$

where β is in 1/ft and K_g is gas effective permeability in md. Although equations (24) and (25) were achieved from systems with immobile liquid saturations, Frederick and Graves found the two equations can be used in systems with mobile liquid saturations.

Coles and Hartman[18] conducted non-Darcy experiments with nitrogen and paraffin. They found that β increased with paraffin saturation. When the paraffin saturation was less than 20%, they found that equation (26) fitted their measurements.

$$\beta = \beta_{dry}\exp(6.265S_p) \ldots\ldots\ldots\ldots\ldots(26)$$

where β_{dry} is the one phase non-Darcy coefficient and S_p is the paraffin saturation.

Results and Discussions

From theoretical equations (6) and (9), which are respectively for the parallel type model and the serial type model, we see that they are very different from each other. This observation is one of the reasons that the empirical correlations of the non-Darcy coefficient vary. Suppose theoretical equations (6) and (9) are fundamentally correct and there is a rock sample with pore geometry illustrated by Fig. 1. If multi-rate flow experiments are conducted and the direction of fluid flow is parallel to the pore channels, then the result will be similar to equation (6) after the experimental data are analyzed. If multi-rate flow experiments are carried out, and the direction of fluid flow is perpendicular to the pore channels, then a correlation similar to equation (9) will be obtained after the experimental data are processed. So the relativity of flow direction to pore channels can affect the correlation of the non-Darcy coefficient.

The second reason causing the diversity of the empirical correlations is that a different number of parameters are considered in developing correlations. Permeability, porosity, and tortuosity are the three major players in the correlations. However, they are related to each other. A general correlation that agrees with most previous work is,

$$\beta = \frac{c_1 \tau}{K^{c_2} \phi^{c_3}} \quad \text{.....................(27)}$$

where c_1, c_2, and c_3 are constants for a specific formation. The Carman-Kozeny equation is given by,

$$K = \frac{\phi}{(k_z \tau) S_{pv}{}^2} \quad \text{.......................(28)}$$

where k_z is Kozeny's constant, and S_{pv} is internal surface area of the pores per unit of pore volume. If we use average properties of a formation, S_{pv} can be treated as a constant. Then, we have

$$K = \frac{c_4 \phi}{\tau} \quad \text{........................(29)}$$

where c_4 is a constant for a certain formation. The combining of equations (27) and (29) leads to,

$$\beta = \frac{c_1 c_4}{K^{1+c_2} \phi^{c_3 - 1}} \quad \text{. ....(30)}$$

Equations (27) and (30) show that the exponent of permeability in equation (27) is very different from its counterpart in equation (30). A similar conclusion can be reached for exponents of porosity.

The third reason leading to different empirical correlations comes from the difference of lithology. Although c_1, c_2, c_3, and c_4 are constants for a specific formation, for another formation, they will be another set of constants inducing different correlations of the non-Darcy coefficient even if they are in the same formats indicated by equations (27) and (30).

Based on the above analysis, some guidelines are given below for picking a correlation from the literature when a correlation is needed.

1. Determine the lithology of the formation. Lithology can be found by comparing measurements from neutron and density logs or using other methods.
2. Determine what parameters are known or can be found. If permeability, porosity, and tortuosity are known, then use the correlations with all the three parameters; otherwise use the correlations with fewer parameters.
3. Determine the pore geometry of the formation and the relativity of flow direction to pore channels. If flow direction is parallel to the dominant direction of pore channels, a correlation in form of equation (30) that is similar to equation (6) should be used. If flow direction is perpendicular to the dominant direction of pore channels, a correlation in form of equation (27) that is similar to equation (9) should be used. If a correlation is in the form of equation (27), the procedures illustrated above can be taken to simplify it. If flow direction is neither parallel nor perpendicular to the pore channel, some discretion needs to be taken to find a right correlation.

Fig. 3 shows procedures to choose a right one-phase correlation of the non-Darcy coefficient. In some cases, the taking of the procedures may not lead to a single choice; however, the procedures will greatly aid in eliminating unsuitable choices. After a right one-phase correlation is found, if necessary, it can be converted to a multi-phase correlation by following Geertsma's method[4] illustrated in equation (22).

Conclusions

Some conclusions can be reached for this paper:

1. This paper summarizes both theoretical equations and many empirical correlations of the non-Darcy coefficient in one-phase and multi-phase cases in the literature. Empirical correlations vary considerably.
2. Three main reasons lead to the diversity of the empirical correlations of the non-Darcy coefficient: pore geometry, number of known parameters, and lithology.
3. This paper will provide a good reference for researchers who are interested in investigating the non-Darcy effect. Some guidelines given in the paper will aid readers in finding a right correlation for the formation of interest.

Nomenclature

English Symbols

a	=	constant
b	=	constant
c	=	constant
c'	=	constant
c''	=	constant
D	=	diameter, L
h	=	piezometric head, L
g	=	acceleration of gravity, L/T^2
K	=	permeability, L^2; relative permeability
k	=	Kozeny's constant
n	=	number of capillaries per unit area in each dimensional direction
p	=	pressure, $M/(T^2L)$
s	=	length of pore channel, L
S	=	specific surface, 1/L; saturation
t	=	time, T
u	=	superficial velocity, L/T
$\mathbf{u}$	=	velocity vector, L/T
x	=	dimension in x direction

Greek Symbols

α	=	pore size distribution function
α'	=	correction factor
α''	=	correction factor
β	=	non-Darcy coefficient, 1/L
β'	=	correction factor
β''	=	correction factor
δ	=	pore size
ϕ	=	porosity
μ	=	viscosity, M/(LT)
ρ	=	fluid density (M/L^3)
τ	=	tortuosity

Subscripts

c	=	capillary
dry	=	one-phase, no liquid
g	=	gas
gv	=	grain volume, solid volume
p	=	paraffin
R	=	critical
r	=	residual, immobile
rel	=	relative
t	=	$\partial/\partial t$
w	=	water, liquid
z	=	Kozeny
1	=	index for a constant in a β correlation
2	=	index for a constant for the exponent of permeability
3	=	index for a constant for the exponent of porosity
4	=	index for a constant in the K-τ correlation

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SI Metric Conversion Factors

ft	× 3.048	E-01 = m
md	× 9.869 233	E-04 = μm^2

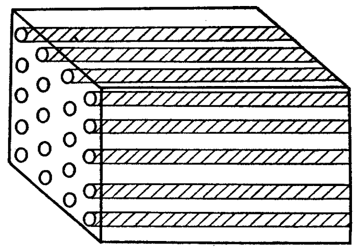

Fig. 1–Parallel type model (after Scheidegger[21]).

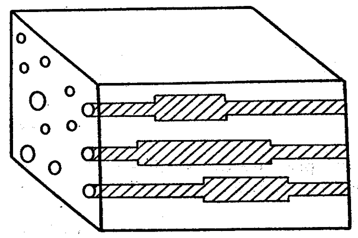

Fig. 2–Serial type model (after Scheidegger[21]).

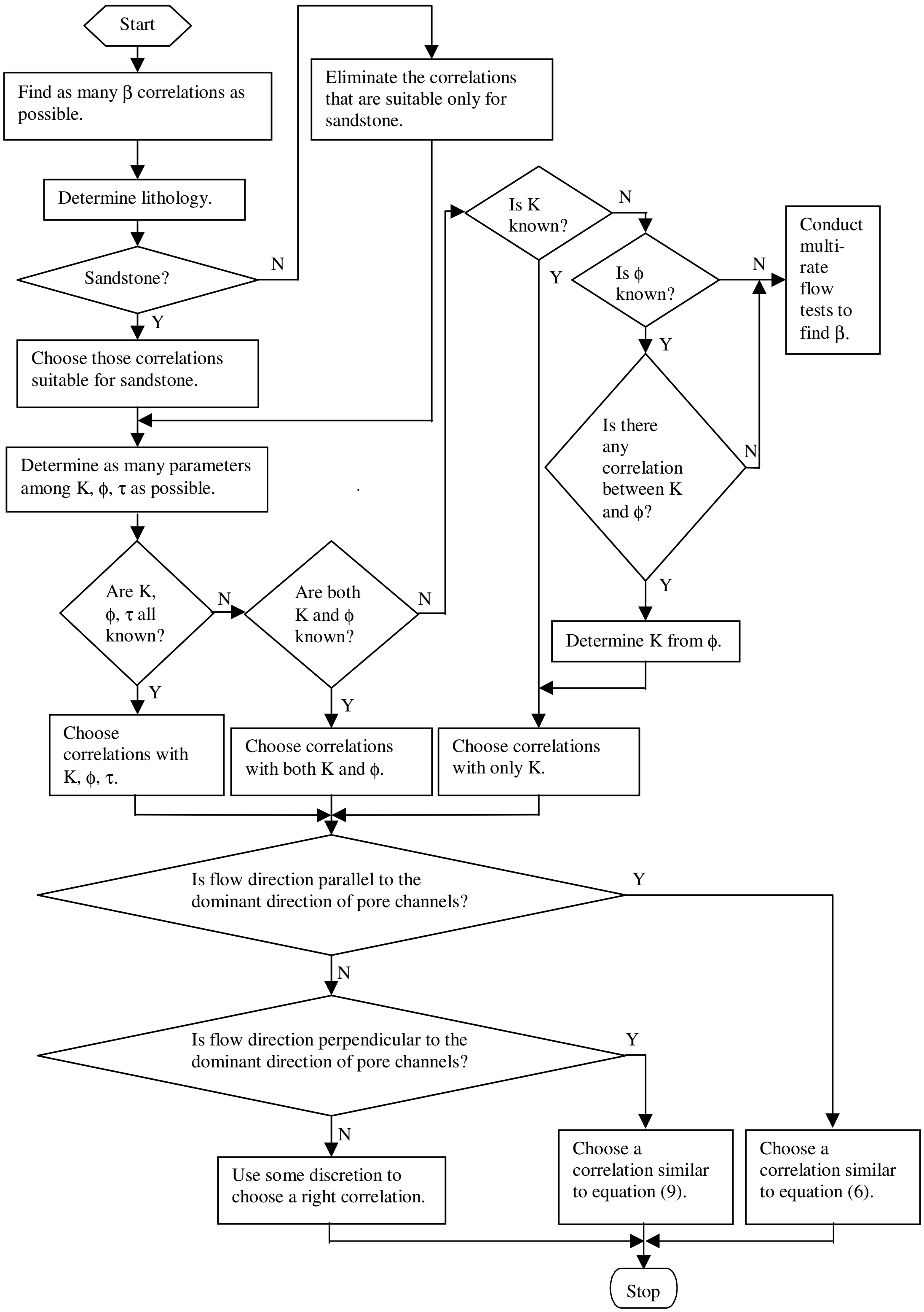

Fig. 3–Procedures to choose a right one-phase correlation of the non-Darcy coefficient.