# Financial Intermediation with Discretion over Liability Value: A Dynamic Model of Stablecoin Issuers and Regulations

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August 20, 2025

**Abstract**

Stablecoin issuers are financial intermediaries that retain discretion to devalue their liabilities through depegging, even in the absence of redemption pressure. Our model characterizes the trade-offs underlying this depegging decision and uncovers an "instability trap." Lower risk in reserve assets can make depegging more likely—a "risk paradox" that challenges conventional views on regulation. Capital requirements take on a distinct role: by curbing the issuer's reliance on procyclical seigniorage, they enhance stability and eliminate the risk paradox. Finally, our analysis highlights the importance of stablecoin issuers' access to equity financing and provides a quantitative framework for valuing their equity.

**Keywords**: Stablecoin, depegging, regulations, financial intermediation, financial instability, intermediary asset pricing, production-based asset pricing

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Stablecoins are digital assets pegged to fiat currencies, typically circulating on blockchain-based platforms. Their rapid growth in market capitalization and regulatory momentum in major economies have drawn attention. Governments view stablecoins as instruments to expand the international reach of their currencies, while major financial institutions are examining the competitive threat posed by stablecoin issuers. Yet, fundamental questions remain. What, precisely, is a stablecoin? How do issuers manage stablecoins? In what ways do their behaviors differ from those of traditional financial intermediaries? What mechanisms sustain stability or cause instability? And, how should stablecoin issuers be regulated?

According to the contracts that the major issuers offer to stablecoin users, a stablecoin is neither a definitive claim on the issuers or their assets that presumably back the stablecoin. An issuer can change the redemption terms "without prior notice". This differentiates stablecoin issuers from other financial intermediaries. Banks commit to redeem deposits at par on a first-come, first-served basis, provided they have the liquid resources. Money market mutual funds follow NAV-based redemption rules. Such commitments underpin classic instability mechanisms, such as runs.[1] In this paper, we analyze stablecoin issuers that are not legally bound to maintain the peg. As they represent a new category of financial intermediaries, analyses based on analogies to traditional financial institutions can be misleading.

In our model, stablecoin issuers possess the discretion to devalue their liabilities and may do so even in the absence of redemption pressure from stablecoin users.[2] A novel mechanism emerges that leads to a persist instability spiral and seemingly paradoxical relationships between the issuer's reserve-asset risk and stablecoin depegging. Our continuous-time model characterizes the issuer's trade-offs involved in its depegging decision and other aspects of its optimal strategy, including the fees charged to stablecoin users and payout policy.

While we derive the equilibrium dynamics analytically, we calibrate the model to real-world data for quantitative implications on key policy and market issues—such as the severity

[1]The issuer does not engage in liquidity transformation. Thus, our model stands in contrast to models of stablecoins that emphasize the run mechanism. The commitment to first-come-first-serve redemption offered by banks and mutual funds and illiquidity of their assets are the two key ingredients behind the run mechanism: under first-come-first-serve, depositors to front-run one another (a coordination failure) as early withdrawal leads to asset liquidation costs that are born by those that do not withdraw. Neither of the two ingredients appears in our model.

[2]In general—and not only in the financial sector—it is rare for non-financial firms to have the discretion to alter the promised value of their liabilities without undergoing bankruptcy or a formal restructuring process.

and persistence of depegging, the benefits of granting issuers access to equity-market financing, and the valuation of stablecoin issuers' equity. Popular regulatory proposals play new roles in our setting. These questions lie at the heart of the ongoing debate over stablecoins.

**Depegging and instability trap.** On the asset side of the issuer's balance sheet is the reserve assets. The liability side (or the funding sources) includes the stablecoins and its net worth. The issuer in our model earns a spread between the return on reserve assets and cost of issuing stablecoins, including operating costs and stablecoin users' required return (their discount rate). The users derive utility from holding the stablecoin, which we interpret as a form of convenience yield often tied to the payment function of monetary assets. Such utility reduces the stablecoin users' required return on their stablecoin holdings, generating the issuer's *seigniorage revenues* that are essentially a funding-cost reduction. The users' monetary utility decreases when depegging happens (the stablecoin price falls below one and fluctuates). What triggers depegging are the negative shocks to the issuer's reserve assets.[3]

The issuer covers the costs of issuing stablecoins with revenues from its reserve assets. It must maintain net worth above a certain level that is endogenously determined in our model. Once its net worth falls below it, denoted by $\underline{n}$, the issuer's revenues would not be enough to cover the costs, and as a result, the issuer faces a permanent lack of profitability that translates into a zero franchise value—that is, the issuer's value function is permanently zero. The existence of this absorbing state of zero value generates concavity of the issuer's value function. Therefore, the issuer becomes effectively risk-averse, even though its objective is to maximize the present value of consumption flows with a risk-neutral preference.

When negative shocks reduce its net worth, its effective risk aversion rises endogenously. To reduce risk exposure and preserve net worth against subsequent shocks, the issuer faces two options, deleveraging and depegging. When deleveraging, the issuer reduces the outstanding amount of stablecoin liabilities, selling reserve assets and using the proceeds to repurchase stablecoins out of circulation (which we call *quantity adjustment*). This is the preferred option if the issuer's net worth is above a threshold, denoted by $\widetilde{n}$; otherwise, the

[3] In reality, some stablecoin issuers hold safer assets than others, but it is impossible to completely avoid risk. For quantitative implications, we calibrate our model parameters, including the risk-return profile of reserve assets, to data on Tether, the issuer of USDT (the largest stablecoin by market capitalization).

issuer chooses to depeg the stablecoin, engaging in *quality adjustment*. Below $\widetilde{n}$, the further its net worth falls, the more severe depegging becomes, which takes place through the issuer supplying more stablecoins albeit at increasingly lower prices. In our calibrated model, such "crisis dynamics" results in the stablecoin price falling to 0.7 (30% below the peg).

The stationary density of the issuer's net worth increases as net worth approaches $\underline{n}$, which implies that, once the system enters this region of low net worth, it gets trapped here for a long time. What lies at the heart of this *instability trap* is the *procyclicality of seigniorage*. The seigniorage is from the quantity of stablecoin issued and, per unit of stablecoin, the users' monetary utility. In response to negative reserve-asset shocks, the issuer either reduces the stablecoin quantity (deleverages) or quality (depegs the stablecoin), causing its seigniorage to decline. This in turn slows down the issuer's rebuild of net worth, resulting in persistently high effective risk aversion and thus making depegging necessary.

The *procyclical seigniorage* earned by the stablecoin issuer stands in contrast with the countercyclical risk premia earned by traditional financial intermediaries (e.g., He and Krishnamurthy, 2013). While our modeling methodology is related to the literature on intermediary asset pricing, the cyclicality of equilibrium dynamics differs. When risk premia rise in bad times, intermediaries enjoy an increase in the expected growth rate of net worth in models of intermediary asset pricing and thereby faces a prospect of accelerated net worth recovery. In our model, the stablecoin issuer, who intermediates between its stablecoin and reserve assets, suffers from a decrease in seigniorage profits in bad times.

While depegging causes seigniorage to decline in bad times, they are necessary risk-management tools. Specifically, depegging works as a risk-sharing mechanism between the stablecoin issuer and users. Depegging allows the issuer's liabilities (the stablecoin) to decrease in value as it loses reserve assets due to negative shocks, thus preserving its net worth. When the issuer's risk tolerance is below that of the stablecoin users, it is efficient for the issuer to offload risk to the users. We show that the equilibrium price of stablecoin can be intuitively expressed as a conditional expectation of paths of risk sharing, i.e., the percentage of the issuer's risk exposure in its net worth offloaded to the users via depegging.

The issuer's depegging decision is disciplined by the users. We show that when the users' monetary utility has a sufficiently high sensitivity to stablecoin price fluctuation, the issuer refrains from depegging at all values of net worth and focuses on quantity adjustment (deleveraging) as the only risk-management tool. Our model characterizes the parameter condition under which, even in a laissez-faire environment where reserve assets are risky, stablecoins can still be perfectly stable in its price (though its quantity can vary significantly).

**Risk Paradox.** Interestingly, when the riskiness of reserve assets rises, the parameter region, where depegging never happens, enlarges. This *risk paradox* is due to the issuer's precaution against the instability trap. Offloading risk to the stablecoin users reduces their monetary utility. When the reserve-asset risk is higher, reducing stablecoin quality (depegging) as a way to control risk exposure causes a greater reduction of seigniorage. As a result, the issuer would prefer quantity adjustment (deleveraging) for managing risk exposure of its net worth.

In addition, we show that in the parameter region where depegging happens, when the issuer's net worth falls below the threshold $\widetilde{n}$, the highest level of stablecoin volatility is *decreasing* in the riskiness of reserve assets once the asset riskiness surpasses a certain level. This is again due to the issuer's precaution against the instability trap. Admittedly, when the reserve assets are perfectly safe, the stablecoin does not exhibit any volatility, as the issuer does not need to offload risk to the stablecoin users. Therefore, in the parameter region where depegging happens, we characterize an inverse-U shaped relationship between the riskiness of reserve assets and the highest level of stablecoin volatility.

**Narrow banking and asset-choice flexibility.** Since the root of the problem lies in reserve-asset risk, a natural question is: what if the issuer is regulated on its risk-taking? Our results on the risk paradox show that the relationship between reserve-asset risk and stablecoin volatility is non-monotonic and can even be inverse, as long as some risk—even at a low level—exists in the issuer's reserves. What if a stablecoin issuer were instead required to hold only perfectly safe assets? Moreover, the GENIUS Act recently passed in the U.S. requires stablecoin issuers to maintain positive net worth at all times. This regulatory framework reflects the principle of narrow banking, where payment instruments must be fully backed by safe assets. Our calibrated model shows that, relative to the laissez-faire benchmark, such a narrow-banking framework reduces the supply of stablecoins by 73–88% and lowers users' welfare by 60–70%, depending on the issuer's net worth.

Therefore, allowing the issuer to take some risk and thereby earn higher expected returns on reserve assets can be beneficial, as it not only incentivizes the issuer's intermediation (issuing stablecoins to fund reserve assets) but also allows the issuer to rely more on the reserve-asset returns rather than fees charged on the stablecoin users as sources of revenues.[4]

We show that more flexibility in the issuer's asset choice is beneficial in general. In an extension, we allow the issuer to dynamically adjust the riskiness of reserve assets. The issuer takes more risk to earn a higher expected return when it is well-capitalized, and in response to negative shocks, it de-risks the asset portfolio as a preferred way to reduce risk exposure before engaging in stablecoin depegging (offloading risk to the stablecoin users).

**A new role of capital requirement.** The GENIUS Act combines a capital requirement (non-negative net worth) and restriction on reserve-asset risk. Next, we focus on the capital requirement: the issuer is required to maintain net worth above a level that is higher than $\underline{n}$—the laissez-faire bound of permanent zero franchise value—but is allowed to hold risky assets. A unique role of capital requirement emerges for regulating stablecoin issuers that can devalue their liabilities. The mechanism is absent for traditional financial institutions.

Capital requirement essentially regulates the issuer's profit composition between the return on reserve assets and seigniorage from stablecoin issuances. As previously discussed, the procyclicality of seigniorage leads to the instability trap. Reserve assets are funded by the issuer's net worth (equity) and stablecoin issuances. Increasing equity reduces the issuer's reliance on seigniorage—the liability side of balance sheet—as a source of profits and increases the share of profits from the equity-funded reserve assets. Reserve-asset return is exogenous to the issuer's depegging decision and does not suffer from the feedback loop behind the procyclicality of seigniorage (i.e., comovement between seigniorage and net worth).

By reducing the issuer's reliance on procyclical seigniorage, capital requirement weakens the force of instability trap. As a result, the system does not spend as much time trapped in the region of low issuer net worth and persistent depegging as in the laissez-faire environment. As the stationary probability density shifts towards the high net worth region, the long-run average stablecoin supply increases (quantity improves) and the probability of depegging

[4] We acknowledge that in the presence of bank runs, which are beyond the scope of our analysis, a narrow banking approach can offer additional benefits.

decreases (quality improves). While the stablecoin users' welfare increases, imposing capital requirement reduces the issuer's franchise value, resulting in a wealth transfer.

Another distinct feature of capital requirement is that it changes how the system responds to changes in reserve-asset risk. Specifically, it eliminates the risk paradox in the parameter region where depegging happens: as the riskiness of reserve asset increases, both the depegging probability and the highest level of stablecoin volatility increase. Previously, we discussed the risk paradox in the laissez-faire environment that emerges from the issuer's precaution against the instability trap. Under capital requirement, the force of instability trap is significantly weakened, and such precaution is no longer needed; in other words, capital requirement already mandates precaution against relying too much on procyclical seigniorage as a profit source (the root cause of the instability trap), and this mandated precaution substitutes the issuer's voluntary precaution against the instability trap.

**Stochastically growing demand.** Finally, we extend our model to account for stochastically growing demand for the stablecoin. Demand evolves stochastically in response to technology changes, competition, and growth or contraction of the broader community that transacts on blockchains. Some stablecoin issuers hold cryptocurrencies in their reserve portfolio. The growth of cryptocurrency adoption and prices tend to be correlated with stablecoin demand. We find that such correlation amplifies the procyclicality of seigniorage and exacerbates the instability trap. Capital requirement dampens this harmful impact by reducing the issuer's reliance on procyclical seigniorage. This result reveals a distinctive role of capital requirement in the presence of randomly evolving stablecoin demand.

Extending our model to incorporate the growth of stablecoin demand brings important insights into our model mechanism. Demand growth improves stability by imputing a trend in the issuer's seigniorage profits that counterbalances the procyclical fluctuation along the trend. As a result, for any given level of stablecoin liabilities, the issuer maintains a lower net worth when the growth rate of demand is higher. Intuitively, as demand growth fosters stability, the issuer finds it less necessary to maintain net worth as a risk buffer.

Currently, the majority of stablecoin issuers are private companies. The recent IPO of Circle, the issuer of stablecoin USDC, has brought enormous attention to stablecoin issuers' access to equity-market financing. We show that being able to raise external equity significantly improves the stability of stablecoin because equity issuance essentially allows the stablecoin issuer to share risk with external equity investors, adding a risk-management tool beyond deleveraging and depegging. A major hurdle to public listing is the ambiguity in the valuation of the issuer's equity and, in particular, its connection with the growing trend of stablecoin adoption. By incorporating a rich set of modeling ingreidents—such as, the issuer's profits from both reserve-asset returns and stablecoin seigniorage, its dynamic decision to deleverage and depeg, its payout (consumption) policy, and dynamic fees charged on stablecoin users—our model provides a valuation framework that goes beyond the discounted cash-flow analysis. Our model is calibrated to Tether. Under different projections of demand growth, we map out the implied market capitalization of Tether's equity shares.

**Related Literature.** Our paper contributes to the growing literature on stablecoins (see, e.g., Gorton and Zhang (2021) for an overview). Much of the theoretical literature draws parallels with traditional financial instruments like bank deposits and money market fund shares, emphasizing instability driven by bank-run dynamics—coordination failures and liquidity transformation by the issuer (Routledge and Zetlin-Jones, 2022; Uhlig, 2022; Gorton, Klee, Ross, Ross, and Vardoulakis, 2022; Ma, Zeng, and Zhang, 2023; Bertsch, 2023; Ahmed, Aldasoro, and Duley, 2024).d'Avernas, Maurin, and Vandeweyer (2022) analyze a stablecoin issuer facing uncertain demand, exploring the role of commitment.

In our framework, the issuer functions much like a central bank, conducting open market operations to manage supply and stabilize price, with net worth determining its capacity to do so. Crucially, and consistent with practice, the issuer faces no legally binding redemption requirement that constrains its strategy. The analogy between stablecoin issuers and central banks is also explored by Bolt, Frost, Shin, and Wierts (2024), who highlight the importance of central bank net worth in a historical context. Unlike prior models, we endogenize net worth as a key state variable, rather than treating it as fixed or exogenous. This enables us to (i) calibrate the model to major stablecoin issuers (e.g., for valuation purposes), and (ii) analyze the dynamic relationship between net worth and price stability. A further innovation of our model is its ability to capture both reserve and demand risk—potentially correlated—which is especially relevant for issuers like Tether.

Our paper contributes to the growing literature on digital currencies and tokens (Cong, Li, and Wang, 2021; Chod and Lyandres, 2021; Gryglewicz, Mayer, and Morellec, 2021; Biais, Bisiere, Bouvard, Casamatta, and Menkveld, 2018; Sockin and Xiong, 2020, 2023; Brunnermeier and Payne, 2023). For broader overviews, see Brunnermeier, James, and Landau (2019); John, O'Hara, and Saleh (2022); John, Kogan, and Saleh (2023). Most closely related, Jermann and Xiang (2024) study the optimal design of fee and monetary policies by a token issuer, focusing on steady-state outcomes and the role of commitment. In contrast, we incorporate financial constraints, risky reserve assets, uncertain demand, and user preferences for price stability — key features of stablecoin issuers, giving rise to both stability and instability regimes and price fluctuations.

More generally, our analysis shows that net worth functions as the stablecoin issuer's productive capital, determining its capacity to issue tokens, maintain stability, and generate seigniorage, which in turn affects valuation. This echoes the core idea of production-based asset pricing, where asset values reflect firms' optimal investment decisions and the marginal value of capital (see, e.g., Cochrane (1991); Jermann (1998); Gomes, Kogan, and Zhang (2003), and Kogan and Papanikolaou (2012) for a review). Relatedly, the intermediary asset pricing literature (Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2013) highlights how intermediary net worth influences asset prices. We contribute by modeling stablecoin issuers as a novel class of financial intermediaries with distinct institutional features, focusing on how their net worth influences the pricing of their own liabilities, while abstracting from broader general equilibrium effects on other assets.

We also relate to the dynamic liquidity management and financial contracting literature (e.g., DeMarzo and Sannikov (2006); Décamps, Mariotti, Rochet, and Villeneuve (2011); Bolton, Chen, and Wang (2011); Hugonnier, Malamud, and Morellec (2015); Hartman-Glaser, Mayer, and Milbradt (2025)) and highlight two key distinctions between stablecoin issuers and traditional firms. First, while stablecoin issuers manage reserve assets analogous to liquidity in corporate finance, their liquidity—captured by net worth—directly affects seigniorage-based revenues. Second, we frame stablecoin issuance as a security design problem, solving for the optimal issuance and fee structure. Unlike standard liabilities such as debt, stablecoins provide payment convenience, whose value depends endogenously on their return volatility. This combination of endogenous convenience yield and design flexibility sets our framework apart from corporate finance models of dynamic debt issuance (e.g., Abel (2018); DeMarzo and He (2021); Hu, Varas, and Ying (2021); Malenko and Tsoy (2025)).

# 1 Model

Consider an infinite-horizon economy in continuous time. An issuer supplies stablecoins to users who derive utility from their stablecoin holdings. This utility, modeled in line with the literature on monetary assets, generates a demand curve that evolves dynamically driven by the endogenous risk of depegging and fees charged by the issuer. The issuer's proceeds from stablecoin issuance and fees are invested in reserve assets, which the issuer uses as sources of funds for managing the stablecoin supply (e.g., repurchasing stablecoins out of circulation). The issuer also decides on its consumption (i.e., pays itself a dividend). In the following, we first introduce the demand for stablecoins and then set up the issuer's problem.

## 1.1 Stablecoin demand

We model a unit mass of representative stablecoin users. The generic consumption goods ("dollars") are the numeraire in this economy. Let $p_t$ denote the price of one unit of stablecoin in dollars. When pegged to the numeraire, $p_t = 1$; otherwise, $p_t < 1$ represents depegging.

A unit mass of atomic users take as given the equilibrium process of stablecoin price,

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t \,, \tag{1}$$

where $\mu_t^p$ and $\sigma_t^p$ in the drift and diffusion terms, respectively, are determined after we solve the issuer's optimal strategy, and $dZ_t$ is a Brownian shock to the issuer's reserve assets, which is the only risk in our baseline model.[5] In an extended model, we consider shocks to $K_t$ that scales the users' demand and allow such demand shocks to be correlated with $Z_t$.

[5] Appendix G.2 shows that implementing a continuous price path is in fact optimal for the issuer.

The representative user derives the following utility from stablecoin holdings:

$$U(X_t) := \frac{1}{\xi} K_t^{1-\xi} X_t^{\xi} - X_t \eta |\sigma_t^p| = \underbrace{\left(\frac{1}{\xi} x_t^{\xi} - x_t \eta |\sigma_t^p|\right)}_{u(x_t)} K_t, \tag{2}$$

where the parameter $\xi$ is a constant in $(0, 1)$, $K_t$ is the demand scaler, $X_t$ is the numeraire value of stablecoin holdings, and $x_t = X_t/K_t$.[6] Quantity variables are homogeneous of degree one in $K_t$ (i.e., the model is "scale-invariant"). In our baseline model, we consider a constant demand scaler, $K_t = K$. When calibrating the model for quantitative analysis, we set $K$ to one billion so that the quantities variables match their empirical counterparts. In Section 6, we extend our model to allow $K_t$ to evolve stochastically over time.

The first component of $u(x_t)$ is increasing in the numeraire value of stablecoin holdings in line with the modeling approach of real balance-in-utility in monetary economics (e.g., Baumol, 1952; Tobin, 1956; Feenstra, 1986; Freeman and Kydland, 2000).[7] The second component is decreasing in the stablecoin price's shock sensitivity, $|\sigma_t^p|$. The user's safety preference is captured by $\eta$ ($> 0$). Such safety preference can be motivated, for example, by the role of stablecoins as means of payment. As a transaction medium, an asset must be information-insensitive and thereby deters private information acquisition, preventing asymmetric information on the payment instrument between trade counterparties (e.g., Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999). Therefore, safety preference is defined on the absolute value of $\sigma_t^p$ as any loading on the issuer's asset shock (the source of "information" in the baseline model), whether positive or negative, generates information sensitivity.

The user has a quasi-linear instantaneous utility over $dt$ that constitutes the utility from stablecoin holdings and her consumption, $Ku(x_t)dt + dY_t^u$, where $Y_t^u$ is the cumulative (undiscounted) consumption process with superscript "$u$" for users. The user is risk-neutral in the consumption streams and discount them at the risk-free rate $r$ ($> 0$). The user chooses

[6]There always exists a trivial equilibrium where $p_t$ is a constant and equal to zero and $X_t = 0$ (implying $u(X_t) = 0$). We focus on the equilibrium where $p_t > 0$.

[7]This approach of modeling money demand functions has received empirical support (e.g., Poterba and Rotemberg, 1986; Lucas and Nicolini, 2015; Nagel, 2016; Krishnamurthy and Li, 2022).

$x_t$ at any $t \in [0, \infty)$ to maximize the life-time utility

$$\max_{\{x_t\}_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-rt}\big[dY_t^u + Ku(x_t)dt\big]\right]. \tag{3}$$

Let $N_t^u$ denote a representative user's wealth. The user faces the budget constraint:

$$dN_t^u = rN_t^u dt + Kx_t(\mu_t^p dt + \sigma_t^p dZ_t - f_t dt - rdt) - dY_t^u \tag{4}$$

and the transversality condition, $\lim_{s\to\infty} e^{-r(s-t)}\mathbb{E}_t[N_s^u] = 0$. The user allocates wealth between a risk-free asset and stablecoins. The stablecoin holdings, $X_t = Kx_t$, earns an excess return $dp_t/p_t - f_t dt - rdt$ where $dp_t/p_t = \mu_t^p dt + \sigma_t^p dZ_t$ and $f_t$ is the fees charged by the issuer.

**Proposition 1 (Stablecoin demand).** *The users' demand is given by $X_t = Kx_t$, where*

$$x_t = \left(\frac{1}{r - \mu_t^p + \eta|\sigma_t^p| + f_t}\right)^{\frac{1}{1-\xi}}. \tag{5}$$

The user's demand has several intuitive properties. It is increasing in the expected return from price change, $\mu_t^p$, decreasing in the fees, $f_t$, and the riskiness of the stablecoin, $\sigma_t^p$, and the user's discount rate $r$, which is the prevailing risk-free rate in the economy. If at time $t$ the stablecoin is pegged to one dollar, then its price does not fluctuation, that is $\mu_t^p = 0$ and $\sigma_t^p = 0$, which implies a downward-sloping demand curve: $X_t = K\left(\frac{1}{r+f_t}\right)^{\frac{1}{1-\xi}}$, where $r + f_t$ is essentially the carry cost (the forgone interest-rate difference) for holding stablecoins.

In equilibrium, the stablecoin market clears:

$$X_t = S_t p_t, \tag{6}$$

where $S_t$ is the total outstanding units of stablecoin (i.e., the aggregate nominal supply). Next, we set up the issuer' s problem with supply as part of the issuer's optimal strategy.

## 1.2 The stablecoin issuer

The issuer chooses a strategy, $\mathcal{G}$, that involves the processes of stablecoin supply $(S_t)_{t\geq 0}$, fees, $(f_t)_{t\geq 0}$, and consumption, $(Y_t)_{t\geq 0}$ (or equivalently, incremental consumption, $dY_t$), i.e.,

$$\mathcal{G} := (S_t, f_t, dY_t)_{t\geq 0}\,. \tag{7}$$

We assume $dY_t \geq 0$ to capture restricted access to external equity financing, as, for example, in Bolton, Chen, and Wang (2011).[8] In Section 3.3, we allow the issuer to raise equity at a cost. At $t = 0$, the issuer is endowed with net worth, $N_0$ (i.e., the initial equity position).

At time $t$, the liability side of the issuer's balance sheet includes its net worth, $N_t$, and outstanding stablecoins, $p_t S_t$, so on the asset side, its total reserve assets are $A_t = N_t + p_t S_t$. We assume $A_t \geq 0$—that is, the issuer does not take short position in the reserve assets. The issuer's assets generate return $\mu dt + \sigma dZ_t$ over $dt$, where $dZ_t$ is a standard Brownian shock, the only source of risk in our baseline model. As previously discussed, we introduce shocks to users' demand in Section 6. In our quantitative analysis, we calibrate $\mu$ and $\sigma$ based on the composition and risk-return profiles of major stablecoin issuers' reserve assets.

In Appendix C, we derive the following law of motion of $N_t$, the issuer's net worth:

$$dN_t = (N_t + p_t S_t)(\mu dt + \sigma dZ_t) - p_t S_t(\mu_t^p dt + \sigma_t^p dZ_t) + p_t S_t f_t dt - p_t S_t \kappa dt - dY_t. \tag{8}$$

The first term is the return on its reserve assets. The second term is the price appreciation of stablecoin liabilities ($dp_t/p_t = \mu_t^p dt + \sigma_t^p dZ_t$). The third term represents the fee revenues. The operating cost, $\kappa dt$ per numeraire value of stablecoins, broadly reflects the issuer's expenses for sustaining the stablecoin's utility or function for its users, for example, by supporting and promoting it as a means of payment. And, the last term is the issuer's consumption. Note that when issuing more stablecoins and investing the proceeds in reserve assets, the issuer simultaneously expands liabilities and assets, which does not change the net worth, so $dN_t$ only depends on the relative appreciation or depreciation of *existing* assets and liabilities (i.e., the first two terms), fee revenues, operating costs, and consumption.

[8] As in Brunnermeier and Sannikov (2014), negative consumption is equivalent to equity issuance.

In the law of motion (8), the right side contains the current state, $N_t$, the stablecoins supply, $S_t$, the stablecoin price, $p_t$ (and its rate of change over $dt$, i.e., $dp_t/p_t$), the fees, $f_t$, and the issuer's consumption, $dY_t$. Note that once the supply process is given, the price process is determined by the market-clearing condition under the stablecoin demand characterized in Proposition 1. Therefore, $dN_t$ essentially depends on $N_t$ and the strategy $\mathcal{G} = (S_t, f_t, dY_t)_{t\geq 0}$.

We introduce a parameter condition:

$$\lambda := r + \kappa - \mu > 0 \,. \tag{9}$$

Consider the hypothetical scenario where users' utility from stablecoin holdings is absent, i.e., $u(x_t) = 0$. Then the users requires a return of $r$ to hold stablecoins, and on top of that, the issuer also covers the operational cost, $\kappa$. Under $r + \kappa - \mu > 0$, such pure financial intermediation—that is, the issuer raises funds via stablecoin issuances and invests in the reserve assets without providing any utility to users—is not profitable. Therefore, the condition (9) states that the users' utility, $u(x_t)$, from stablecoin holdings is the source of *seigniorage* for the issuer and ultimately justifies a positive amount of stablecoin supply.

The issuer chooses a strategy $\mathcal{G}$ to maximize the present value of its lifetime consumption:

$$V_0 := \max_{\mathcal{G}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dY_t\right], \tag{10}$$

subject to the law of motion of $N_t$ in (8), as well as $dY_t \geq 0$ and $N_t + S_t p_t = A_t \geq 0$. The issuers chooses the optimal strategy at $t = 0$, as, for instance, in Jermann and Xiang (2024). Such commitment to a long-term strategy can be relaxed: we discuss in Appendix G.1 that short-term commitment over $dt$ period is sufficient. We assume $\rho > \mu$, which is standard in dynamic corporate finance and macro-finance models with financial constraints, e.g., Décamps et al. (2011), Bolton, Chen, and Wang (2011), Brunnermeier and Sannikov (2014) among others. Impatience induces consumption; otherwise, the issuer never consumes and always accumulates financial slack (net worth) so that eventually the restriction on external financing (i.e., $dY_t \geq 0$) no longer matters. Since $\rho$ determines the issuer's willingness to grow net worth (rather than consume it), we calibrate $\rho$ by matching data on stablecoin issuers' net worth. Finally, we impose standard regularity conditions in

Appendix A.

# 2 Model Solution

In this section, we characterize the solution to the issuer's problem and provide analytical results describing the model dynamics. The next section focuses on quantitative analysis. Specifically, we analyze the issuer's optimal strategy, including its decisions on stablecoin depegging, issuances, fees, and consumption. We also characterize the issuer's franchise value through the value function and to what extent the stablecoin can be under-collateralized (i.e., the issuer's net worth can turn negative). The model reveals several distinct phenomena about stablecoins, such as the risk paradox and instability trap.

## 2.1 State variable, strategy space, and value function

In the following, we start our analysis by simplifying the issuer's strategy space. As previously discussed, we divide all quantity variables by $K$, the stablecoin-demand scaler, and use $n_t$, $s_t$, and $dy_t$ to denote, respectively, the $K$-scaled net worth ($N_t/K$), stablecoin supply ($S_t/K$), and consumption ($dY_t/K$). We simply refer to $n_t$, which is the state variable in our model, as the issuer's net worth, $s_t$ as supply, and $dy_t$ as consumption. The next lemma summarizes the law of motion of $n_t$. It shows that the strategy space of three stochastic processes, i.e., $(S_t)_{t\geq 0}, (f_t)_{t\geq 0}$, and $(dY_t)_{t\geq 0}$, can be transformed into a more tractable form.

**Lemma 1** (**State variable law of motion**). *The issuer's net worth, $n_t$, evolves as follows:*

$$dn_t = \underbrace{[\mu(n_t + x_t) - x_t(r - \zeta_t) - x_t\kappa]}_{\mu_n(n_t)}dt + \underbrace{[\sigma(n_t + x_t) - x_t\sigma_t^p]}_{\sigma_n(n_t)}dZ_t - dy_t, \tag{11}$$

*where $\zeta_t$ is the stablecoin user's marginal utility from holding stablecoins:*

$$\zeta_t = u'(x_t) = x_t^{\xi-1} - \eta|\sigma_t^p|. \tag{12}$$

Let $V(n_t)$ denote the issuer's value function at time $t$, i.e., the continuation value:

$$V(n_t) = \mathbb{E}\left[\int_t^{\infty} e^{-\rho s} dY_s\right] = \mathbb{E}\left[\int_t^{\infty} e^{-\rho s} dy_s\right] K, \tag{13}$$

and $v(n_t) = V(n_t)/K$, the $K$-scaled value function, which we simply refer to as value function. Three variables control the law of motion of $n_t$: 1) the value or quantity of stablecoins, $x_t$; 2) the instantaneous volatility of stablecoin (price) returns, $\sigma_t^p$; 3) the issuer's consumption, $dy_t$. Once they are determined as functions of $n_t$, the dynamics given by (11) give an autonomous law of motion. Note that $\mathcal{G}$ affects the dynamics of $n_t$ only through $(x_t, \sigma_t^p, dy_t)_{t\geq 0}$. Therefore, instead of characterizing optimal processes for stablecoin supply, fees, and the issuer's consumption (i.e., the optimal strategy $\mathcal{G} = (S_t, f_t, dY_t)_{t\geq 0}$), we solve an auxiliary problem of optimization over $(x_t, \sigma_t^p, dy_t)_{t\geq 0}$. We then show that this auxiliary problem indeed solves the problem given by (10), in that the optimal choice of $(x_t, \sigma_t^p, dy_t)_{t\geq 0}$ can be implemented via a strategy $\mathcal{G}$ (see Appendix D for details on the solution).

The law of motion of $n_t$ given by (11) has several intuitive properties. In the drift, $\mu_n(n_t)$, the first term represents the expected return on reserve assets funded by the issuer's net worth and stablecoin issuances. The second term is the cost of issuing stablecoins: the issuer must compensate the users their required rate of return, $r$, minus the user's marginal utility from holding stablecoins, $\zeta_t$, which is the marginal *seigniorage* earned by the issuer. The third term is the operating cost. The diffusion, $\sigma_n(n_t)$, includes the risk in the reserve-asset return, and the second part captures the risk borne by stablecoin holders. Under $\sigma_t^p > 0$, the issuer effectively offloads risk to users. Under these circumstances, the stablecoin price and value of the issuer's stablecoin liabilities decline following a negative shock to reserve assets. This effect reduces the risk exposure of the issuer's net worth. We will show that this risk sharing mechanism is key for understanding the stablecoin issuer's incentive to depeg the stablecoin.

Next, we characterize the issuer's optimal strategy and how it affects the law of motion of $n_t$, starting with the issuer's consumption choice. The next proposition shows that the issuer consumes, $dy_t > 0$, only when $n_t$ reaches $\overline{n}$, an endogenous upper boundary of $n_t$.

**Proposition 2** (**The issuer's consumption**). *There exists $\overline{n}$, a reflecting upper bound of $n_t$ with the following properties: 1) the issuer consumes variations of $n_t$ that move $n_t$*

*beyond $\overline{n}$: when $n_t$ reaches $\overline{n}$, $dy_t = dn_t$ if $dn_t > 0$, and $dy_t = 0$ if $dn_t \leq 0$; 2) consumption optimality implies that the value function satisfies the following two boundary conditions:*

$$v'(\overline{n}) - 1 = 0, \text{ and, } v''(\overline{n}) = 0. \tag{14}$$

The issuer does not consume (i.e., $dy_t = 0$ in the law of motion (11)) if $n_t < \overline{n}$. Thus, for $n < \overline{n}$, the value function, $v(n)$, satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho v(n) = \max_{\sigma^p, x} \left\{ v'(n) \Big[ \mu(n+x) - rx + x^{\xi} - \eta x |\sigma^p| - \kappa x \Big] + \frac{v''(n)}{2} \Big[ \sigma(x+n) - x\sigma^p \Big]^2 \right\}. \tag{15}$$

We suppress the time subscripts to simplify the notations. The following Proposition summarizes the properties of value function and introduces the issuer's effective risk aversion.

**Proposition 3** (**Value function**). *The value function, $v(n)$, is strictly increasing and concave in $n$ for $n < \overline{n}$, i.e., $v'(n) > 0$ and $v''(n) < 0$. The issuer's effective risk aversion based on the value function is defined as*

$$\gamma(n) = -\frac{v''(n)}{v'(n)} > 0, \tag{16}$$

*for $n < \overline{n}$. $\gamma(n)$ is strictly decreasing in $n$, i.e., $\gamma'(n) < 0$. At $n = \overline{n}$, $\gamma(n) = 0$.*

The issuer's effective risk aversion arises from its financial constraint—that is, it cannot raise external funds and must rely its own net worth for managing the stablecoin. The wedge between $v'(n)$, the marginal value of net worth ("retained earnings" accumulated up to time $t$), and 1, the marginal value of consumption ("payout"), reflects how tight the financial constraint is. At the consumption (upper) boundary of $n_t$, the issuer is effectively unconstrained with $v'(\overline{n}) = 1$ (see the boundary condition (14)) and not risk-averse, i.e., $\gamma(\overline{n}) = 0$. At any $n < \overline{n}$, we have $v'(n) > 1$ (implied by the concavity of $v(n)$ and $v'(\overline{n}) = 1$), and $v'(n)$ is higher when $n$ falls further below $\overline{n}$, indicating that the issuer is more financially constrained and thus more risk-averse (i.e., $\gamma'(n) < 0$). In Section 3.3 where we allow the issuer to raise external financing at a cost, this pattern remains. In the next subsection, we show that $\gamma(n)$ is key for understanding the issuer's decision to depeg the stablecoin.

Finally, to complete the characterization of the state space and value function, we characterize the lower bound of $n_t$ in the following proposition, denoted by $\underline{n}$. In the Appendix, we provide the closed-form expression for $\underline{n}$ (see (C.4)).

**Proposition 4** (**State variable lower bound**). *There exists $\underline{n}$ ($< \overline{n}$), an absorbing bound of $n_t$ that is never reached in the equilibrium (i.e., $n_t > \underline{n}$). The value function is zero at $\underline{n}$:*

$$v(\underline{n}) = 0\,. \tag{17}$$

The intuition behind $\underline{n}$ as an absorbing lower bound is as follows. The condition $n_t = a_t - x_t \geq \underline{n}$ states that the issuer needs to maintain adequate reserves, $a_t = \frac{A_t}{K}$, relative to its stablecoin liabilities, $x_t$. If its reserves fall short, the issuer lacks the revenues from reserve assets to cover the costs of its stablecoin liabilities and thus cannot generate profits to grow net worth.[9] Once $n_t$ falls to $\underline{n}$, the issuer has no prospect of recovery: both the drift and diffusion of $N$ fall to zero. This permanent lack of profitability translates to a zero continuation value, which the issuer seeks to avoid. In order to maintain $n_t$ above $\underline{n}$, the issuer may have to depeg the stablecoin, which we discuss in the next subsection.

In summary, the issuer's value function solves (15), a differential equation for $v(n)$, subject to the boundary conditions (14) and (17), and its net worth evolves according to (11) in $(\underline{n}, \overline{n}]$.

## 2.2 Depegging, risk paradox, and instability trap

The next proposition lays out when and how depegging happens in equilibrium.

**Proposition 5** (**Stablecoin quality and optimal depegging**). *Under the condition,*

$$\mu - (1-\xi)(r+\kappa) - \eta\sigma < 0\,, \tag{18}$$

*the issuer optimally sets $\sigma^p(n) = 0$ and the stablecoin price is pegged to one, i.e., $p(n) = 1$. If the condition (18) does not hold, there exists a unique $\widetilde{n} < \overline{n}$ that separates two regions:*

- *For $n \geq \widetilde{n}$, the peg holds, i.e., $p(n) = 1$ and $\sigma^p(n) = 0$;*

[9] As previously discussed, the issuer must compensate the users their required return, $r$, minus their marginal utility from holding stablecoins, $\zeta_t$, and incurs the operating cost, $\kappa$.

- *For $n < \widetilde{n}$, depegging happens, i.e., $p(n) < 1$, and the stablecoin price comoves with the issuer's reserve-asset shocks, i.e., $p'(n) > 0$ and $\sigma^p(n) > 0$, with $\sigma^p(n)$ decreasing in $n$.*

As previously discussed and specified in (2), the quality or "moneyness" of stablecoin is inversely related to $\sigma_t^p = \sigma^p(n_t)$. The proposition above shows that the issuer always maintains the peg if the condition (18) holds; otherwise, depegging happens once the issuer's net worth falls below a threshold. In the following, we first explain the intuition behind the condition (18) and then discuss depegging when the condition does not hold.

According to Proposition 3, the issuer's effective risk aversion, $\gamma(n)$, is highest when $n$ approaches $\underline{n}$, where the issuer's risk-bearing capacity is exhausted. To reduce risk exposure, the issuer can either reduce stablecoin issuance, i.e., deleverage (the quantity margin), or offload asset risk to the users through depegging and allowing the stablecoin price to fluctuate with asset shocks, i.e., $\sigma^p(n) > 0$ (the quality margin). The latter option—that is, issuing a marginal unit of stablecoin, investing the proceeds in reserve assets, and offloading the reserve-asset risk to the stablecoin users—generates a marginal profit of $\mu-(1-\xi)(r+\kappa)-\eta\sigma$. Here, $\mu-(1-\xi)(r+\kappa)$ is akin to a net interest margin: the marginal asset funded by stablecoin issuance earns $\mu$, but the issuer must compensate the users' required return, $r$, and cover the operating cost, $\kappa$, facing an overall cost of $(1-\xi)(r+\kappa)$, where $(1-\xi)$ is tied to the users' demand elasticity (see (5)). The last term, $\eta\sigma$, is the reduction of users' marginal utility from holding stablecoins (i.e., the issuer's seigniorage given by (12)). Therefore, if the condition (18) holds, i.e., $\mu-(1-\xi)(r+\kappa)-\eta\sigma < 0$, adjusting quality is not profitable, and the issuer would prefer adjusting quantity to reduce risk exposure. Since $\gamma(n)$ is the highest and the issuer's incentive to depeg strongest as $n$ approaches $\underline{n}$, if adjusting the quality margin is not profitable near $\underline{n}$, the issuer would not depeg when $n$ is higher. Therefore, if the condition (18) holds, depegging does not happen in the whole range of $n$.

If the condition (18) does not hold, the issuer depegs the stablecoin and offloads risk to the users by setting $\sigma^p(n) > 0$, once its net worth falls below the threshold $\widetilde{n}$ and it becomes sufficiently risk-averse. As shown in Proposition 3, $\gamma(n)$ rises as $n$ falls. Therefore, the issuer shares more risk with the users by increasing $\sigma^p(n)$ when $n$ falls further below $\widetilde{n}$. Under $p'(n) > 0$ and $\sigma^p(n) > 0$, when the issuer's reserve assets are hit by negative shocks, its liabilities—the value of stablecoins—decline as well, mitigating the impact on its net worth.

An increase in $\eta$—the parameter for users' stability preference—enlarges the parameter region where depegging does not happen. Under a higher $\eta$, the reduction of seigniorage triggered by the issuer's depegging is larger, thus making the quality adjustment less attractive as a way to control risk exposure (the issuer focuses on the quantity adjustment).

Interestingly, increasing $\sigma$ (reserve-asset riskiness) while holding other parameters fixed makes the condition (18) more likely to hold, which implies a greater parameter region where a perfectly safe stablecoin can be sustained. In contrast, reducing $\sigma$—that is, the issuer's reserve assets are safer—enlarges the parameter region where depegging happens. The following proposition summarizes this result of risk paradox.

**Corollary 1** (**Risk paradox: parameter region**). *Higher reserve-asset risk, $\sigma$, shrink the parameter region where the condition (18) does not hold and depegging happens.*

When reserve assets are riskier, the parameter region where the condition (18) fails and depegging happens actually shrinks, and the region without depegging expands. This seemingly counterintuitive result is due to the users' discipline on the issuer's depegging decision. As shown in (5), offloading risk to users dampens demand. When $\sigma$ is higher, reducing stablecoin quality as a way to control risk exposure (i.e., sharing risk with the users) causes a larger reduction of seigniorage. As a result, the issuer would prefer quantity adjustment (i.e., reducing stablecoin issuance or deleveraging) when its net worth is low. As shown below by our results on the "instability trap", once depegging happens, it causes persistent demand destruction. Under a higher $\sigma$, such demand destruction is more significant.

In the parameter region where depegging can happen (i.e., the condition (18) does not hold), we observe the following self-reinforcing dynamics, which we call the "instability trap." In this parameter region, the depegging happens and the price fluctuates, when the issuer's net worth, $n$, falls below the critical threshold (i.e., $p(n) < 1$ and $\sigma^P(n) > 0$ for $n < \widetilde{n}$ in Proposition 5). Depegging reduces the seigniorage revenues.[10] The further $n$ declines, the lower the issuer's profits from the seigniorage is as depegging becomes more severe and $\sigma^P(n)$ increases. Profit reduction slows net-worth rebuild, trapping the system in the low-$n$ region

[10] The seigniorage revenues are given by $x_t\zeta_t$. Specifically, when $n_t$ falls below $\widetilde{n}$, a further decline raises $\sigma^p(n_t)$, so the seigniorage per dollar value of stablecoin, $\zeta_t = \zeta(n_t)$ in Lemma 1), decreases. Moreover, we will show in Proposition 9, $x_t = x(n_t)$ is a constant for $n_t < \widetilde{n}$. Therefore, $x_t\zeta_t$ is decreasing in $n_t$.

where depegging becomes persistent. Put differently, seigniorage revenues are *procyclical*, comoving with the issuer's net worth. This procyclicality underlies the instability trap.

The next proposition characterizes the stationary distribution of state variable that describes the amount of time the system spends at each level of the issuer's net worth, $n$. The instability trap manifests itself into the rising probability density of the stationary distribution as $n$ falls, which reflects that the lower boundary $\underline{n}$ is absorbing.

**Proposition 6** (**Instability trap**)**.** *If the condition (18) does not hold and thus depegging happens at $n < \widetilde{n}$, where $\widetilde{n}$ is defined in Proposition 5, the stationary density of state variable $n$, denoted by $g(n)$, is strictly decreasing in $n$, i.e., $g'(n) < 0$, for $n < \widetilde{n}$.*

Next, we introduce a new aspect of risk paradox in our model. The next proposition describes what happens within the parameter region where the condition (18) does not hold and depegging happens once the issuer's net worth, $n$, falls below $\widetilde{n}$ given by Proposition 5. In this parameter region, the highest level of volatility is *inverted U-shaped* in $\sigma$.

**Proposition 7** (**Risk paradox: endogenous volatility**)**.** *When the condition (18) does not hold (depegging happens when $n < \widetilde{n}$ in Proposition 5), stablecoin volatility satisfies*

$$\sigma^p(n) \leq \sup_n \{\sigma^p(n)\} = \sigma \left( \frac{\mu - (1-\xi)(r+\kappa) - \eta\sigma}{\xi(\mu - \eta\sigma)} \right), \tag{19}$$

*where the supremum is first increasing in $\sigma$ and, when $\sigma$ is sufficiently high, decreasing in $\sigma$.*

Intuitively, volatility starts at zero when $\sigma = 0$, i.e., $\sigma^P(n) = 0$ for any $n$ under $\sigma = 0$. As $\sigma$ increases, the issuer starts sharing risk with the users in the low-$n$ region, so the highest level of $\sigma^p(n)$ rises. This is the scaling effect of $\sigma$ on the right side of (19).

The risk paradox emerges once $\sigma$ passes a threshold—that is, a higher $\sigma$ leads to a reduction in the maximum level of stablecoin-price volatility. The scaling effect is dominated by the issuer's precaution against the instability trap triggered by quality adjustment (i.e., risk-sharing with the users). The instability trap is more potent a force under a higher $\sigma$. Instead of quality adjustment, the issuer relies more on the quantity margin (reduces stablecoin issuance) to control risk exposure when $\sigma$ is higher.

Having summarized the dynamics of stablecoin price, we introduce the next proposition that provides an intuitive representation of stablecoin price.

**Proposition 8** (**Stablecoin price and risk-sharing**). *Stablecoin price can be written as*

$$p(n) = \exp\left(-\int_{n}^{\widetilde{n}} \frac{\sigma^p(\nu)}{\sigma_n(\nu)} d\nu\right), \tag{20}$$

*in the parameter region where depegging happens (i.e., when the condition (18) does not hold), where the threshold $\widetilde{n}$ is given by Proposition 5.*

As shown in Proposition 5, $p(n) = 1$ for any $n$ if the condition (18) holds; otherwise, the proposition above shows that the price of stablecoin reflects the extent to which the issuer shares reserve-asset risk with the users. The ratio $\frac{\sigma^p(\nu)}{\sigma_n(\nu)}$ measures the fraction of risk exposure in the issuer's net worth that has been offloaded to the users. Since only when $n < \widetilde{n}$, the issuer shares risk (sets $\sigma^p(n) > 0$), the upper limit of the integral is $\widetilde{n}$. Intuitively, the more risk the issuer offloads to the users, the lower the stablecoin price is.

## 2.3 Stablecoin supply dynamics

So far, our analysis focuses on stablecoin price or "quality". The next proposition summarizes the quantity dynamics, i.e., how the value of stablecoins outstanding evolves.

**Proposition 9** (**Stablecoin issuance dynamics**). *If the condition (18) holds, the issuer supplies stablecoins worth $x(n)$ that is strictly increasing in its net worth $n$, i.e., $x'(n) > 0$. If the condition (18) does not hold, $x'(n) > 0$ if $n > \widetilde{n}$, and for $n < \widetilde{n}$, $x(n) = \underline{x}$, a constant.*

A key message from the proposition above is that the issuer creates more stablecoins when it accumulates net worth. Supplying stablecoins backed by the reserve assets requires risk-taking capacity that is inversely tied to the issuer's effective risk aversion, $\gamma(n)$, and from Proposition 3, $\gamma'(n) < 0$. Increasing $x(n)$, the value of stablecoins issued, allows the issuer to earn the spread between the reserve assets and stablecoins as sources of funds. The stablecoin funding cost is $r - \zeta_t$ as shown in $n_t$'s law of motion (11). In Lemma 1, we show that $\zeta_t$ is the users' marginal utility from holding stablecoins, which reduces the issuer's funding cost and is essentially a form of *seigniorage* for the money supplier. However, a higher $x(n)$ also means the issuer bears more risk, as shown by the diffusion term in (11), unless the issuer allows the stablecoin price to fluctuate and thereby share risk with the users. Sharing risk, i.e., increasing $\sigma^p(n)$, reduces the seigniorage per dollar of stablecoins issued, as shown in (12). Therefore, the issuer faces a trade-off between seigniorage profits and risk exposure when determining the value of stablecoins issued, $x(n)$.

The next corollary derives the dynamics of $s(n) = x(n)/p(n)$, the nominal supply or units of stablecoins outstanding from the results in Proposition 5 and 9.

**Corollary 2** (**Stablecoin supply**). *If the condition (18) holds, we have $s'(n) > 0$; otherwise, $s'(n) > 0$ for $n > \widetilde{n}$, where $\widetilde{n}$ is defined in Proposition 5, and $s'(n) < 0$ for $n < \widetilde{n}$.*

If the condition (18) holds, the stablecoin is always pegged (see Proposition 5), so under $p(n) = 1$, $s(n) = x(n)/p(n) = x(n)$. Therefore, $s(n) = x(n)$ is strictly increasing in $n$ (see Proposition 9). If the condition (18) does not hold, we have $s(n) = x(n)$ under $p(n) = 1$ for $n \geq \widetilde{n}$ (see Proposition 9) with $s'(n) = x'(n) > 0$ (see Proposition 5) and, for $n < \widetilde{n}$, we have $p'(n) > 0$ (see Proposition 9) and $x(n)$ being a constant (see Proposition 5), so $s(n) = x(n)/p(n)$ is decreasing in $n$. The units of stablecoins supplied, $s(n)$, is *U-shaped* in $n$. The issuer follows a pecking-order strategy when negative shocks erode its net worth: first, when $n$ is still above the critical threshold $\widetilde{n}$, it deleverages by reducing the value and units of stablecoins supplied, i.e., $x'(n) = s'(n) > 0$, while maintaining the peg; second, once $n$ falls below $\widetilde{n}$, the dollar value of stablecoins is held constant at $\underline{x}$, and the issuer offloads risk to the users through depegging, reducing the quality of stablecoins as shown in Proposition 5. In addition, further its net worth falls, the more units of stablecoins it issues albeit at lower prices as depegging intensifies.

In summary, our approach to solve the issuer's optimization was as follows. Proposition 2 shows the issuer's optimal consumption, $dy_t$. In Lemma 1, we show that the law of motion of the state variable, $n_t$, depends on $(S_t, f_t)$ only via $(x_t, \sigma_t^p)$, whose optimal choices are given by Propositions 5 and 9. Finally, We characterize the supply, $s_t$, in Corollary 2 and, in Appendix D.5, the fees, $f_t$ that implement optimal $(x_t, \sigma_t^p)$.

## 2.4 Under-collateralization

Finally, we turn to a widely debated question about stablecoins: can under-collateralization be sustained? The next proposition confirms that this is indeed the case: the lower bound for $n_t$, the issuer's net worth—the difference between the value of reserve assets and that of outstanding stablecoins—is negative, which gives the maximum level of under-collateralization.

**Corollary 3** (**Under-collateralization**). *The lower bound* $\underline{n}$ *in Proposition 4 satisfies closed form expression* (C.4)*. Further, it is negative and has the following properties: (1)* $\partial \underline{n}/\partial r > 0$*; (2) when the condition (18) holds (i.e., depegging does not happen),* $\underline{n}$ *does not depend on* $\sigma$*; otherwise,* $\partial \underline{n}/\partial \sigma > 0$*.*

The issuer maintains its net worth above the absorbing bound $\underline{n}$ where the franchise value falls to zero. The bound is negative, which implies that the stablecoins can be under-collateralized. When users' required return increases, $\underline{n}$ becomes less negative, implying a smaller room for under-collateralization. The issuer has to deliver a higher return to the users, so its seigniorage and franchise value decline, and its net worth, $n$, cannot fall too far below zero before the franchise value (or value function) reaches zero.

If the condition (18) holds, the issuer always maintains the peg, as shown in Proposition 5, and as a result, the riskiness of its reserve asset, $\sigma$, does not affect the lower bound of its net worth. However, when the condition (18) does not hold and depegging happens in the low-net worth region, the users absorb the issuer's asset risk. Therefore, the riskier the issuer's asset is, the more risk the users absorb, which either dampens their stablecoin demand or, to sustain demand, leads to less fees to the issuer (i.e., fee reduction as users' risk compensation). Both forces reduce the issuer's franchise value, and the second force requires the issuer to hold more reserve assets. These mechanisms raise the lower bound, $\underline{n}$.

One may argue that under-collateralization is infeasible because over-collateralization is necessary to meet the users' withdrawal. Such argument ignores the fact that any withdrawal can be met if the issuer is willing to depeg the stablecoin. As shown in Proposition 5, maintaining the peg or not is the issuer's choice, which is different from the traditional corporate finance settings where debt repayments are legally binding and failing to repay leads to bankruptcy. In other words, the value of the issuer's liabilities—stablecoins—is chosen by the issuer itself.[11] Such choice is priced in through the users' expectation of price volatility as shown in (see (5)). Expectation of depegging weakens the users' demand and reduces the issuer's seigniorage—that is, the stablecoin issuer faces a trade-off between preserving net worth by offloading asset risk to the users (i.e., maintaining $n_t > \underline{n}$) and sustaining the users' demand and seigniorage (i.e., $\zeta_t$ in Lemma 1).

A "run-like" phenomenon can emerge in our model if the condition (18) does not hold. Following negative shocks to the issuer's assets, its net worth declines. As shown in Proposition 5, the price of stablecoin falls and volatility rises, which dampens the users' demand (see Proposition 1). Such phenomenon of depegging and withdrawal (i.e., a decrease of users' demand) resembles a run but is not due to coordination failure that is seen in Diamond and Dybvig (1983) among others; instead, it is due to the issuer's optimal decision to dynamically share reserve-asset risk with the users. As previously discussed and formalized in Proposition 6, such run-like behavior is self-reinforcing, resulting in the instability trap.[12]

# 3 Quantitative Analysis

We calibrate the model, conduct quantitative analysis of equilibrium dynamics, and evaluate several regulatory proposals. Our numerical solution is obtained by solving the differential equation for $v(n)$ (the HJB equation (15)) with the boundary conditions (14) and (17).

## 3.1 Model calibration

We calibrate our model to Tether, the largest stablecoin issuer. Its U.S. dollar stablecoin is USDT. Tether's report as of December 31, 2024 states that Tether holds reserve assets of \$143.7 billion, with liabilities (stablecoins outstanding) of \$136.6 billion, resulting in equity of \$7.1 billion (Tether Transparency Report). In our model, all quantity variables are scaled by $K$, which we set to 1 billion. We map the balance-sheet status of Tether to $n_t = \overline{n}$, i.e., the level (upper bound) of issuer's net worth that triggers payout, which is in line with the

[11] We also want to highlight that the issuer maintains a positive continuation value—that is, $n_t > \underline{n}$ and $v(n_t) > 0$. Thus, it does not voluntarily shut down the operation. Therefore, the users' withdrawal cannot force the issuer into bankrupt, and the issuer is not willing to declare bankruptcy on its own.

[12] The absorbing lower bound $\underline{n}$ is never reached so the stationary distribution in Proposition 6 exists.

fac that Tether made a sizable payout in 2024. Therefore, we have $x(\overline{n}) = 136.6$ and $\overline{n} = 7.1$. We adjust the composite parameter $\lambda$ (defined in (9)) and the issuer's discount rate $\rho$ to match these two numbers. As a result, obtain $\lambda = 0.0282$ and $\rho = 0.1371$.

Next, we calibrate $\xi$ that governs the users' demand elasticity (see 5). As shown in Proposition 5, the stablecoin is pegged at $p(\overline{n}) = 1$ at this level of the issuer's net worth, which is in line with the robust peg of USDT around 2024 year end. At $\overline{n}$, $\sigma^p(\overline{n}) = 0$, so the users' marginal utility from stablecoin holdings is $\zeta(\overline{n}) = x(\overline{n})^{\xi-1} = 136.6^{\xi-1}$ (see (12)), which we calibrate to the marginal convenience yield of USDT to obtain $\xi = 0.441$. In Appendix H, we provide details on how to measure the convenience yield of USDT.

We set $\mu$ and $\sigma$ to 13.4% and 7%, respectively, based on Tether's disclosure of reserve assets. In Appendix H, we provide details on the decomposition of reserve assets into different asset classes and how we compute the returns and volatilities of each asset class. Note that in our model, $r$ and $\kappa$ appear in equilibrium conditions together with $\mu$ as part of the composite parameter, $\lambda = r + \kappa - \mu$. Once $\lambda$ and $\mu$ are set, we no longer need to pin down $r$ and $\kappa$.

Finally, for the parameter $\eta$ that represents the stablecoin users' risk sensitivity, we consider a value that is sufficiently low so the condition (18) for perfect stability does not hold and depegging is possible. Given the parameter values above, the upper bound for $\eta$ is 0.61. For the lack a direct empirical counterpart, we set $\eta$ to 0.25 and report results of comparative statics across different values of $\eta$, such as $\eta = 0.1$ and $\eta = 0.4$.

Our calibrated model sheds light on Tether's payouts. According to Proposition 2, the issuer pays out $dn_t > 0$ at $n_t = \overline{n}$, where $dn_t$ has the drift and diffusion components. We have a drift $\mu_n(\overline{n}) = 5.84$, which is \$ 5.84 billion as quantity variables are scaled by $K = \$1$ billion. The diffusion component is random, loading on the Brownian shocks. Tether reported a \$10 billion payout for 2024. Through the lens of our model, it includes the \$5.84 billion expected component and \$4.16 billion due to unexpected gains from reserve-asset shocks. In Section 6, we extend our model to allow the demand-scaler $K$ to evolve stochastically over time and discuss how our model can be used to value stablecoin issuers' equity.

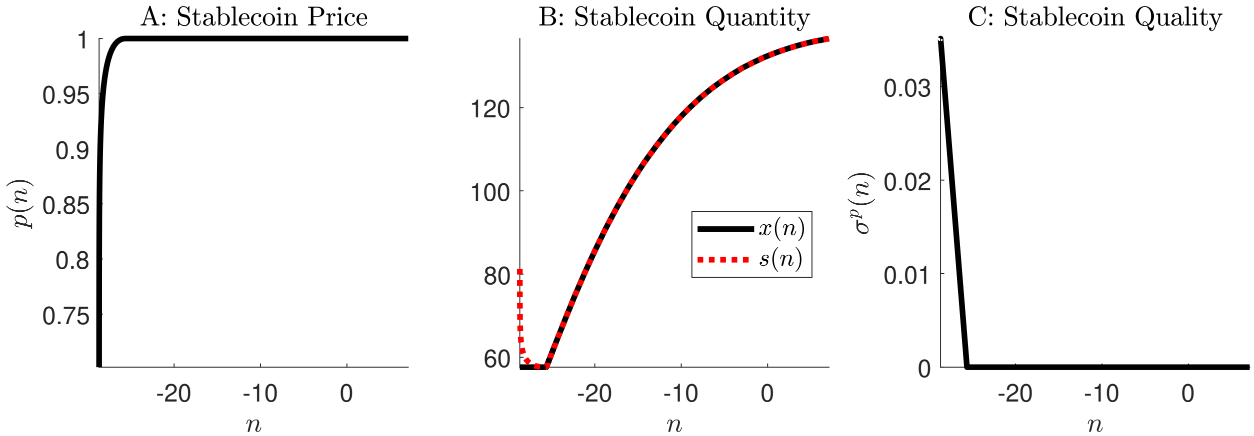


Figure 1: **Stablecoin Price and Quantity Dynamics.**

## 3.2 Model Dynamics

Figure 1 illustrates the model dynamics under the calibrated parameters. In each panel, we plot the endogenous variable against the issuer's net worth, $n$. In Panel A, we plot the stablecoin price, $p(n)$. The peg holds until the issuer's net worth falls below $\widetilde{n}$—the critical threshold in Proposition 5. This threshold is significantly below zero. Near $\underline{n}$, the maximum deviation from the peg is about 30%. In Panel B, we plot the instantaneous volatility of $dp/p$ (see (1)). As depegging takes place in the low-$n$ region, the (annualized) volatility ranges from 0% to above 3.5%.

In Panel B of Figure 1, we plot the value of stablecoins supplied by the issuer (solid line). It is strictly increasing in its net worth for $n > \widetilde{n}$, reaching \$136.6 billion at $\overline{n}$, which is one of our calibration targets. The value of stablecoins supplied drops by almost 60% once $n$ falls below $\widetilde{n}$. As shown in Proposition 9, the value of stablecoin supplied is a constant if $n < \widetilde{n}$. Thus, as the price declines when $n$ falls further below $\widetilde{n}$, more units of stablecoins are issued. In Panel C, we plot the units of stablecoin or the nominal supply in the dotted line. It shows a sharp upturn as $n$ approaches $\underline{n}$, which, in practice, is often viewed as the stablecoin issuer desperately trying to raise revenues by expanding supply in spite of a rapidly falling price.

Panel C of Figure 1 is the key for understanding the mechanism behind depegging. As discussed in the previous section, depegging happens because the issuer wants to offload risk to the users, which manifests into a rising $\sigma^p(n)$ as $n$ falls below $\widetilde{n}$. As shown in (11), a

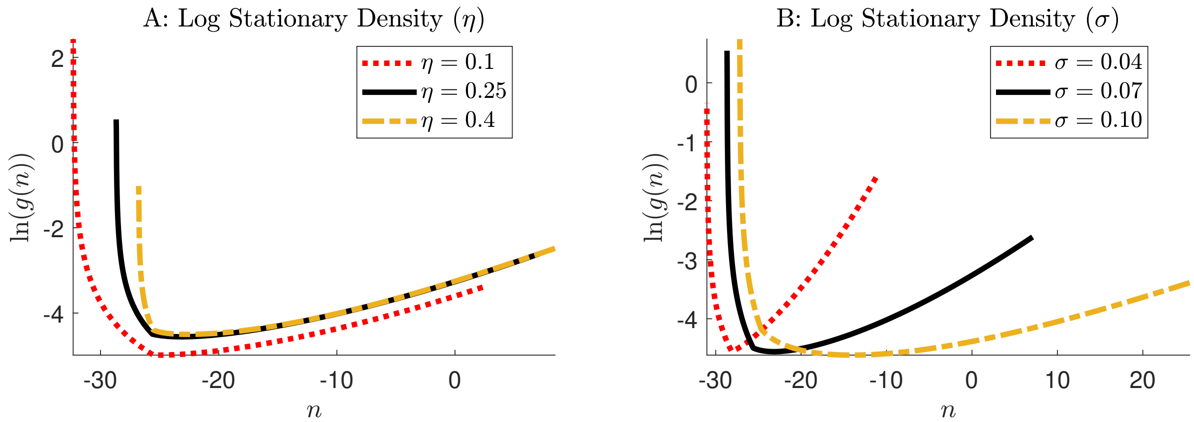


Figure 2: **The Stationary Probability Density and Instability Trap.**

higher $\sigma^p(n)$ reduces the issuer's risk exposure—that is, when shocks hit the issuer's assets, the value of its stablecoin liabilities adjusts as well. As shown in (5), risk-sharing through depegging dampens demand and results in lower revenues for the issuer.

Therefore, once the issuer's net worth falls below $\widetilde{n}$, it faces a difficult trade-off: risk-sharing through depegging is necessary for net worth preservation but the resultant demand destruction and decline of profits (i.e., the reduction of seigniorage $\zeta_t$ given by (12)) slows down the rebuild of net worth. The endogenous procyclicality of seigniorage revenues leads to the instability trap in Proposition 6. In Figure 2, we plot the logarithm of stationary probability density of $n$. The stationary density shows the amount of time the system spends at different levels of $n$. In Panel A, we plot it for different values of the stablecoin users' risk sensitivity, $\eta$, and in Panel B, we plot the stationary density for different levels of reserve-asset riskiness, $\sigma$. Across different values of $\eta$ and $\sigma$, the pattern is consistent: the instability trap emerges and is represented by the rising density as $n$ falls below $\widetilde{n}$.

Panel A of Figure 2 shows that instability trap emerges across different values of $\eta$. It also delivers an interesting message: as $\eta$ increases, the system spends less time in the region near $\underline{n}$—that is, the force of instability trap weakens. When the users become more averse to fluctuation in the stablecoin price, the issuer is more "disciplined" in its decision to share risk with the users through depegging, so the downward spiral of depegging, demand and profit destruction, and persistently low net worth is less likely to be triggered.

Panel B of Figure 2 echoes the intuition behind our results on risk paradox in Section 2.2.

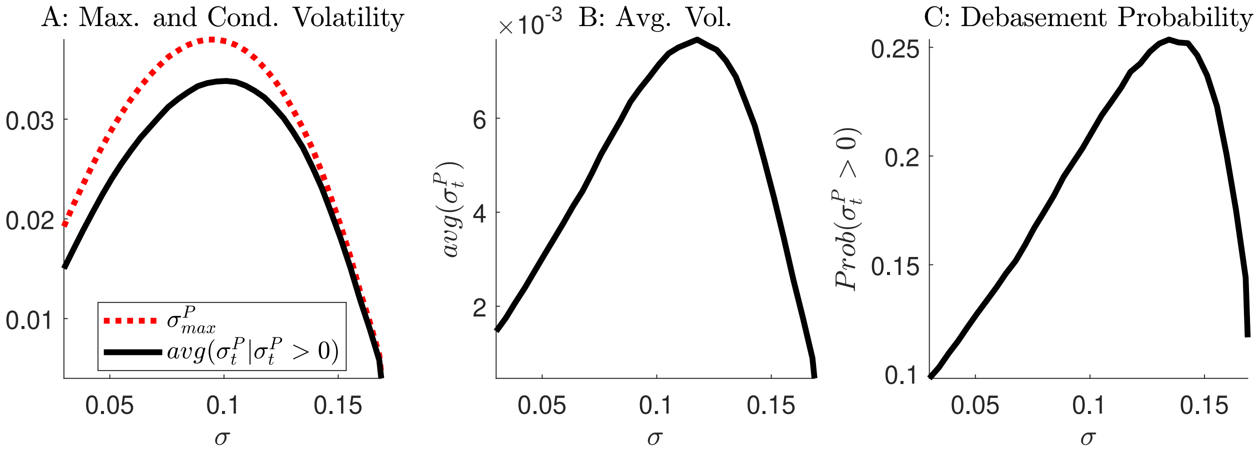


Figure 3: **Risk Paradox.**

As $\sigma$ increases, the instability trap becomes more potent a force, represented by a sharper upturn in the stationary density as $n$ approaches $\underline{n}$. As a result, the issuer becomes more precautionary, and through its efforts to avoid triggering the instability trap, two things happen. First, an increase in $\sigma$ enlarges the parameter region where the issuer maintains a perfect peg (see Proposition 1). Second, in the parameter region where depegging happens, the highest level of stablecoin-price volatility decreases as $\sigma$ increases (see Proposition 7)—that is, endogenous price volatility declines as exogenous risk in the issuer's asset rises.

In Figure 3, we demonstrate the risk paradox. Note that there are two aspects of risk paradox. The first aspect is about the parameter region where a perfect peg is maintained (Proposition 1). This result is straightforward as one can simply inspect how $\sigma$ affects the condition (18). Figure 3 is not about this aspect of risk paradox. The figure is about the second aspect, i.e., the results in Proposition 7: in the parameter region where depegging happens, an inverted-U shaped relationship exists between the highest level of stablecoin-price volatility and the riskiness of the issuer's reserve assets, which is illustrated by the dotted line in Panel A. We also plot the average volatility against $n$, where the average is computed from the stationary probability density of $n$ conditional on depegging (i.e., $p(n) < 1$, or equivalently, $\sigma^p(n) > 0$). A similar inverted-U shape emerges.

As previously discussed in Section 2.2, the inverted-U shape results from two forces. First, under $\sigma = 0$, the issuer does not need to share any risk with the users, so the stablecoin price does not fluctuate. As $\sigma$ increases, the need for risk sharing arises, which leads to depegging and stablecoin-price fluctuation. The second force is about the issuer's endogenous precaution against the instability trap, and it becomes the dominant force once $\sigma$ surpasses a certain level. In Panel B of Figure 3, we show the inverted U-shaped relationship between the average volatility and $\sigma$. Different from the solid line in Panel A, the average volatility here is computed over the full state space (not conditional on $n < \widetilde{n}$, i.e., being in the depegging region). In Panel C, we plot the probability of $n < \widetilde{n}$ (depegging) in the long run using the stationary distribution of $n$ and demonstrate a similar inverted-U shape.

Overall, our results on the risk paradox is about the transmission of exogenous risk in the issuer's reserve assets to the endogenous fluctuation of stablecoin price. One may argue that we can legally force stablecoin issuers to hold reserves in perfectly safe assets. However, as we will show next, such restriction significantly reduces stablecoin supply and the users' welfare. Moreover, in reality, perfectly safe assets are rare. Even for the U.S. Treasuries, once the maturity goes beyond one year, significant risk emerges due to changes in inflation, the stand of monetary policy, and currency exchange rates for foreign investors.

### 3.3 Equity issuance

We extend our model by allowing the stablecoin issuer to raise equity. In practice, equity financing may come from traditional sources, such as public offerings and venture capital investments, or the issuance of "governance tokens" that resemble equity.[13] We introduce issuance costs following the literature on dynamic corporate finance (e.g., Riddick and Whited, 2009; Décamps, Mariotti, Rochet, and Villeneuve, 2011; Bolton, Chen, and Wang, 2011). Specifically, the issuer faces a fixed cost $F$. As standard in the literature, the issuer raises equity to a targeted level, denoted by $n^E$.

Since the equity investors are competitive (i.e., they require shares worth of one dollar for each dollar contributed), it is optimal for the issuer to raise equity as long as the marginal value, $v'(n)$, is greater than one, once the issuance cost is incurred. As shown in Proposition 3, the value function is strictly concave, so the marginal value of equity, $v'(n)$, is greater than one for any $n < \overline{n}$, and equal to one at $\overline{n}$. Therefore, we have $n^E = \overline{n}$. Appendix F.3

[13] One example of governance token is MKR, the governance token of the MakerDAO protocol that issues DAI, the fourth largest stablecoin by market capitalization.

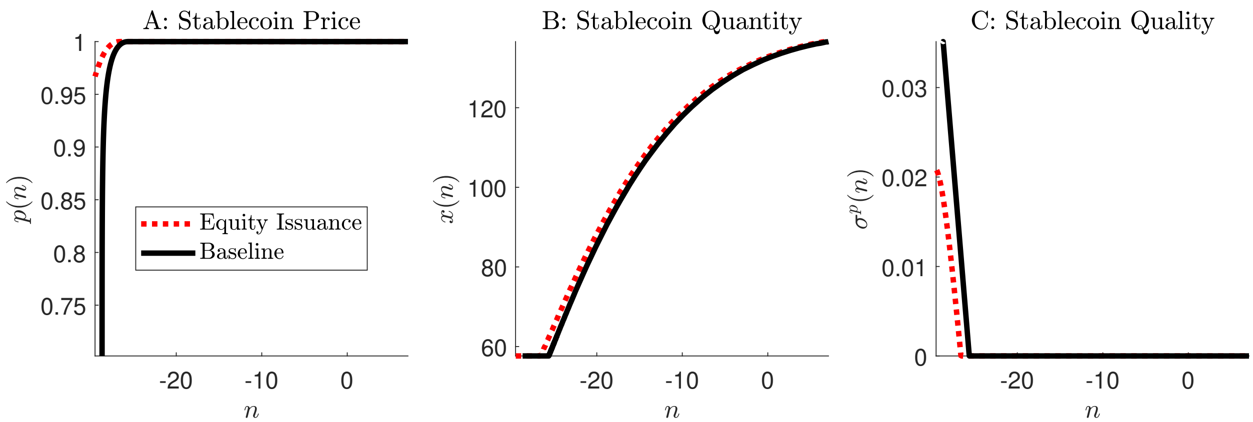


Figure 4: **Equity Issuance.**

provides details on the solution. We set $f = 6$ in our numerical solution.[14]

In Figure 4, we plot the stablecoin price (Panel A), the value of stablecoin issued (Panel B), and the price volatility or "stablecoin quality" (Panel C) against the issuer's net worth $n$, where we use solid lines for the baseline model and dotted lines for the extended model with equity issuance. The ability to raise equity allows the issuer to share risk with equity investors, albeit at a cost. Therefore, the issuer relies less on the stablecoin users for risk sharing. In particular, the ability to raise equity financing allows the issuer to limit depegging. Overall, the stablecoin becomes more stable, and the issuer supplies a higher value of stablecoins. Our findings indicate that granting stablecoin issuers access to equity markets—as exemplified by the recent IPO of Circle that issues USDC, the second-largest stablecoin by market capitalization–enhances both the stability and supply of stablecoins.

# 4 Capital Requirement

The U.S. GENIUS Act (Guiding and Establishing National Innovation for U.S. Stablecoins Act), signed into law on July 18, 2025, requires stablecoin issuers to hold reserve assets backing stablecoins on an at least 1-to-1 basis, (i.e., the issuer's net worth cannot be negative, $n_t \geq 0$). Similar regulations have been adopted in other regions, such as the EU's Markets

[14] Given that the amount of equity raised at the lower boundary is close to 40, this implies an issuance cost of 15% of money raised, which is conservative relative to the underpricing of Circle (the issuer of stablecoin USDC) implied by the immediate market response after the initial public offering.

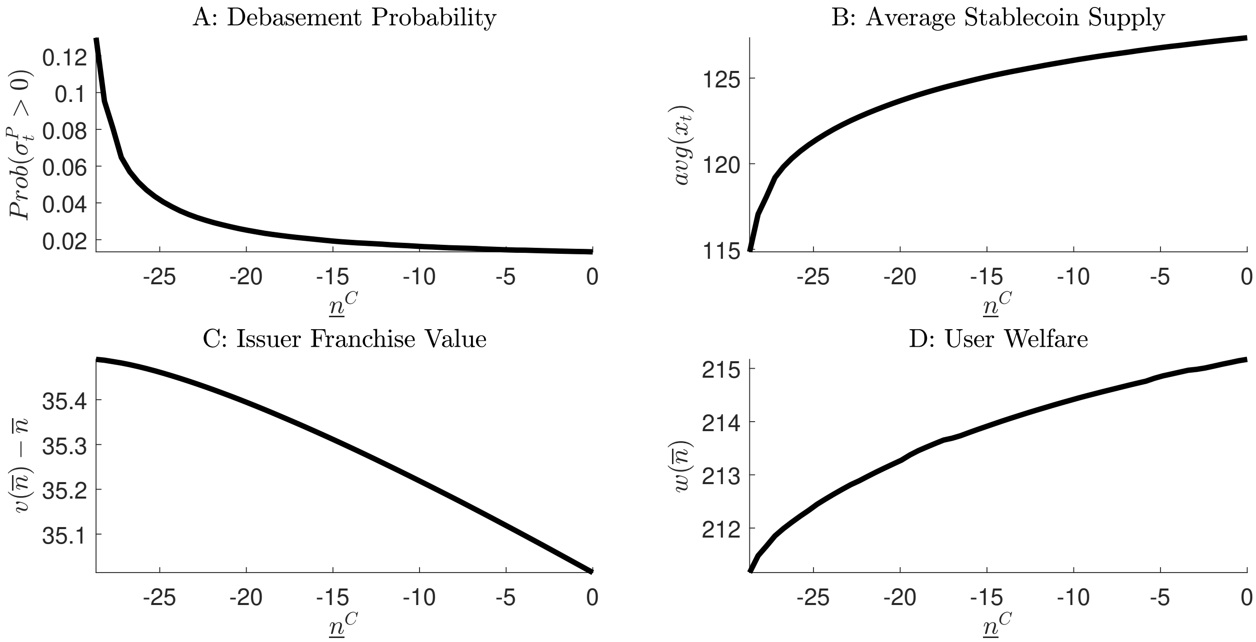


Figure 5: **The Impact of Capital Requirement.**

in Crypto-Assets Regulation (MiCA), the Amendments to Japan's Payment Services Act, and the stablecoin regulatory framework in Singapore's Payment Services Act.

We consider a regulatory lower bound on the issuer's net worth: $n_t \geq \underline{n}^C$ where $\underline{n}^C$ is greater than $\underline{n}$ in the laissez-faire case (see Proposition 4). Breaching the regulation forces the issuer into liquidation.[15] Note that $n_t \geq \underline{n}^C$ can also be self-imposed, as in practice, stablecoin issuers may advertise their commitment to over-collateralization, in which case $\underline{n}^C = 0$. We will show that such restriction reduces the issuer's payoff. The solution method is similar to the baseline case as detailed in Appendix F.1. In addition, since we evaluate users' welfare, Appendix F.2 shows how to derive ($K$-scaled) users' welfare as a function of $n$—that is, users' welfare is given by $w(n)K$, where $w(n)$ is the scaled welfare and we apply a discount rate of 5% to calculate user welfare.

In Figure 5, as the regulator tightens the capital requirement (i.e., increasing $\underline{n}^C$ starting from $\underline{n}^C = \underline{n}$), the probability of depegging is significantly reduced (see Panel A), and tighter regulation in fact increases stablecoin supply (see Panel B). While the users experience

[15] This implies that the issuer is infinitely risk-averse as $n_t$ approaches $\underline{n}^C$ (i.e., $\gamma(n)$ approaches $\infty$). The issuer meets the regulatory requirement by debasing stablecoin liabilities and offloading risk to the users, reducing $\sigma_n(n)$ and maintaining $\mu_n(n) > 0$, as we show in the appendix. The bound, $\underline{n}^C$, is never reached.

a welfare gain, the issuer's payoff decline (see Panel C and D), suggesting that capital requirement effectively transfers value from the issuer to users.[16]

We explore the mechanism by revisiting the law of motion of the issuer's net worth (11):

$$dn_t = \underbrace{(n_t + x_t)\mu dt + x_t\zeta_t dt - x_t r dt - x_t \kappa dt}_{\mu_n(n_t)dt} + \underbrace{[(n_t + x_t)\sigma - x_t\sigma_t^p]dZ_t}_{\sigma_n(n_t)dZ_t} - dy_t, \qquad (21)$$

where $x_t\zeta_t$ is the seigniorage revenue (see definition of $\zeta_t$ given by (12)):

$$x_t\zeta_t = x_t^{\xi} - \eta|\sigma_t^p|x_t. \qquad (22)$$

In the drift, $\mu_n(n_t)dt$, the issuer has two sources of revenue, the expected gain from reserve assets (funded by both stablecoin issuance and net worth), $(n_t + x_t)\mu dt$, and the seigniorage earned from issuing stablecoins, $x_t\zeta_t dt$ where either $x_t$ (the quantity margin) or $\zeta_t$ (the quality margin) declines when the issuer loses its net worth following negative shocks and thus has to reduce risk exposure.[17] These two sources of revenues differ in cyclicality: while the first source accrues at a fixed expected return $\mu$, the second comoves with the issuer's net worth.

The capital requirement mitigates this instability trap by forcing the issuer to derive a sufficiently high share of revenues from the first revenue source rather than the second source of procyclical seigniorage. A capital requirement, $n_t \geq \underline{n}^C$, implies that, given any level of stablecoin supply $x_t$, the issuer's first source of revenues (reserve assets' expected returns) are bounded below, i.e., $(n_t + x_t)\mu dt \geq (\underline{n}^C + x_t)\mu dt$. Note that forcing the issuer to hold a high level of reserve assets also adds reserve-asset risk: given any level of stablecoin supply, $x_t$, the first term in the diffusion, $\sigma_n(n_t)dZ_t$, is also bounded below, i.e., $(n_t + x_t)\sigma dt \geq (\underline{n}^C + x_t)\sigma dt$. As the issuer's net worth declines, the issuer deleverages (i.e., reduces $x_t$) or depegs the stablecoin to avoid hitting the capital requirement. This effect reduces $x_t\zeta_t$, further shrinking the revenue share of seigniorage relative to the expected reserve-asset return.

[16]We compute the issuer's valuation as $v(\overline{n}) - \overline{n}$. This is the payoff that an issuer earns, starting with an initial wealth of zero and raising equity $\overline{n}$ at $t = 0$ to start the business. $\overline{n}$ is the targeted level of equity once issuance is allowed, as we have shown in Section 3.3. For user welfare.

[17]The procyclicality of seigniorage revenues is shown in Proposition 5 and 9. In the parameter region where the condition (18) holds, $\zeta_t$ is a constant and $x_t$ is increasing in $n_t$. In the parameter region where the condition (18) does not hold, for $n_t \geq \widetilde{n}$, $\zeta_t$ is a constant and $x_t$ is increasing in $n_t$, and, for $n_t < \widetilde{n}$, $\sigma^p(n_t)$ is increasing in $n_t$, which implies $\zeta(n_t)$ is increasing in $n_t$ (procyclical), and $x(n_t)$ is a constant.

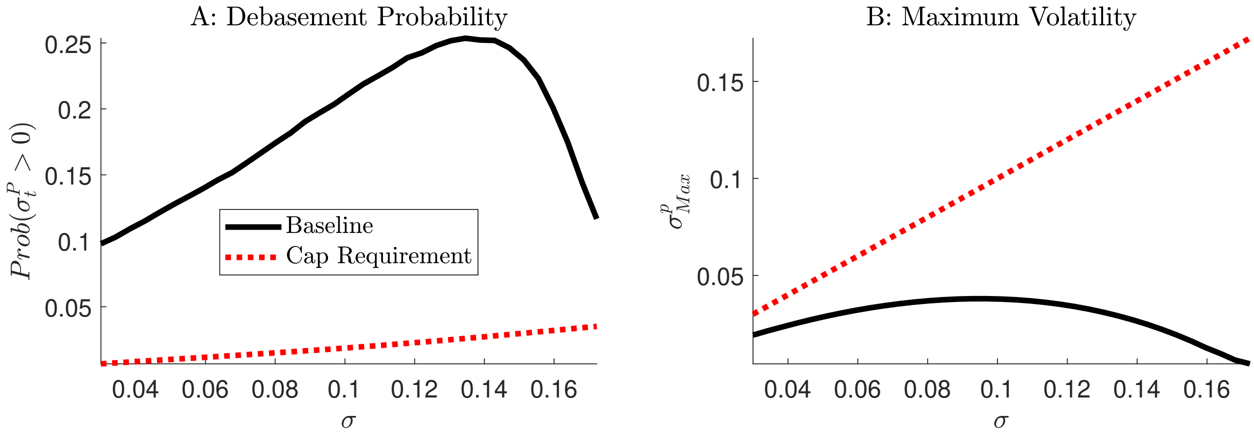


Figure 6: **Capital Requirement and Reserve-Asset Riskiness.**

In summary, what capital requirement does is to change the composition of the issuer's expected revenues, increasing the share from reserve assets and decreasing the share from procyclical seigniorage that is responsible for the instability trap. As a result, capital requirement improves stability (see Panel A of Figure 5). When the force of instability trap becomes weaker, the system would no longer be stuck in the low-$n_t$ region as in the laissez-faire case and thus spends more time in the high-$n_t$ region where the supply of stablecoins, $x_t$, is high. This explains why as capital requirement tightens (i.e., $\underline{n}^C$ increases), the average stablecoin supply, $x_t = x(n_t)$, computed using the stationary distribution of $n_t$, increases.

Admittedly, forcing the issuer to hold more reserve assets also means adding risk on the asset side of its balance sheet. When the issuer is undercapitalized, it needs to offload such risk on the liability side to the users. Therefore, capital requirement reduces the probability of depegging, but once depegging happens, instability can be worse than the laissez-faire case. This result is displayed in Figure 6. In Panel A, across different levels of reserve-asset riskiness, the depegging probability is significantly lower under capital requirement. In contrast, Panel B shows that the maximum of price volatility (i.e., the highest level of risk offloaded to the users) under capital requirement is greater than that in the laissez-faire case.

Figure 6 shows that capital requirement eliminates the risk paradox: the depegging probability and maximum price volatility are decreasing when $\sigma$ is sufficiently high. As discussed in Section 2.2, the inverse relationship between reserve-asset risk and stablecoin volatility in Proposition 7 is due to the issuer's precaution against the instability trap, which restrains the issuer from offloading risk to the users via depegging, once $\sigma$ is sufficiently high. Under capital requirement, the force of instability trap is weakened, and such precaution is no longer needed; in other words, capital requirement already mandates precaution against relying too much on the procyclical seigniorage as a profit source, and by weakening the force of instability trap, it substitutes the precaution against the instability trap.

**Discussion: the role of capital requirement.** The issuer has two sources of revenues, the returns on reserve assets that are funded by both net worth and stablecoin issuances and the seigniorage earned by issuing stablecoins. While the former depends on the broader financial markets, the latter depends on the issuer's stablecoin-management strategy and is endogenously procyclical—that is, following negative shocks that reduce the issuer's net worth, the issuer depegs the stablecoin to offload risk to the users, causing the seigniorage to decline. Our model shows that capital requirement effectively forces the stablecoin issuer to rely less on the procyclical seigniorage as a revenue source and more on the returns on reserve assets, thus improving stability. This role of capital requirement is unique to the stablecoin setting where the issuer's liability is subject to depegging by its own choice. In contrast, the traditional views on capital requirement emphasizes avoidance of runs, preventing systemic insolvency (externality and contagion), and incentive alignment between equity and debt investors, all motivated by studies on traditional financial institutions (e.g., banks).

# 5 The Value of Flexible Risk-Taking

## 5.1 Narrow banking

In addition to capital requirement, the U.S. GENIUS Act restricts stablecoin issuers to hold relatively safe reserve assets (i.e., assets with a low $\sigma$).[18] Therefore, the U.S. GENIUS Act is akin to a narrow-banking framework that entails both limits on reserve-asset riskiness and

[18] The permitted reserve assets include U.S. coin and currency, bank deposits, U.S. Treasury securities with a maturity of 93 days or less, repurchase agreements with a maturity of 7 days or less that are backed by Treasury bills with a maturity of 90 days or less, reverse repurchase agreements with a maturity of 7 days or less that are collateralized by that are collateralized by Treasury securities an overnight basis subject to overcollateralization, and reserves at the central bank. The assets above can be held via money market funds, provided that the funds do not invest in other assets.

a requirement that issuers maintain non-negative net worth.

Corollary 1 and Proposition 7 on the risk paradox show that reducing the riskiness of reserve assets enlarges the parameter region where depegging happens and can increase endogenous volatility of the stablecoin price. These findings suggest caution in limiting the riskiness of reserve assets. However, if reserve assets are perfectly safe, i.e., $\sigma = 0$, we know that the stablecoin price will be pegged at one, because without risk in reserve assets, the issuer has no reason to share risk with the stablecoin users via depegging. While the "quality" of stablecoins is maintained, the key question concerns the "quantity".

Next, we show that the narrow-banking requirements ($n_t \geq 0$ and $\sigma = 0$) significantly reduce the supply of stablecoins relative to the laissez-faire case. The following proposition summarizes the narrow-banking solution. We focus on the parameter region where depegging happens in the laissez-faire environment (i.e., the condition (18) does not hold).

**Proposition 10** (**Narrow banking**). *Under $n_t \geq 0$ and $\sigma = 0$, the stablecoin price is pegged at one, i.e., $p_t = 1$ and $\sigma_t^p = 0$. The value of stablecoins issued is $x^* = \left(\frac{\xi}{r+\kappa-\mu}\right)^{\frac{1}{1-\xi}}$. The issuer maintains a zero net worth, i.e., $n_t = 0$, and consumes all profits, i.e., $dy = x^*[\mu - (r - \zeta^* + \kappa)]dt$, where the marginal seigniorage, $\zeta^* = (x^*)^{\xi-1}$, is defined in (12).*

The value of stablecoins issued, $x^*$, is decreasing in $r+\kappa$, the users' discount rate (required return) and the issuer's operating cost, both of which have to be covered by the issuer, and $x^*$ is increasing in $\mu$, the issuer's return on reserve assets. The issuer earns a spread between reserve assets and stablecoin liabilities, including the users' utility from stablecoin holdings, $\zeta^*$ (the seigniorage). The issuer consumes all the profits: for every dollar of stablecoins issued and proceeds invested in the reserve asset, the issuer consumes the full spread, $\mu-(r-\zeta^*+\kappa)$.

In Panel A of Figure 7, we compare the value of stablecoins issued in the laissez-faire baseline model (the solid line) and that under narrow banking (the dashed line). Imposing the narrow-banking restrictions leads to a significantly lower amount of stablecoins that in turn translates into a significant decline of the users' welfare (see Panel B).[19]

One may argue that stablecoin price decline in the laissez-faire environment is harmful, and the destruction of stablecoin value is detrimental to welfare. Indeed, price volatility

[19]The risk-free rate which the users use as a discount rate is set to 5% (the same as the return on risk-free reserve assets.

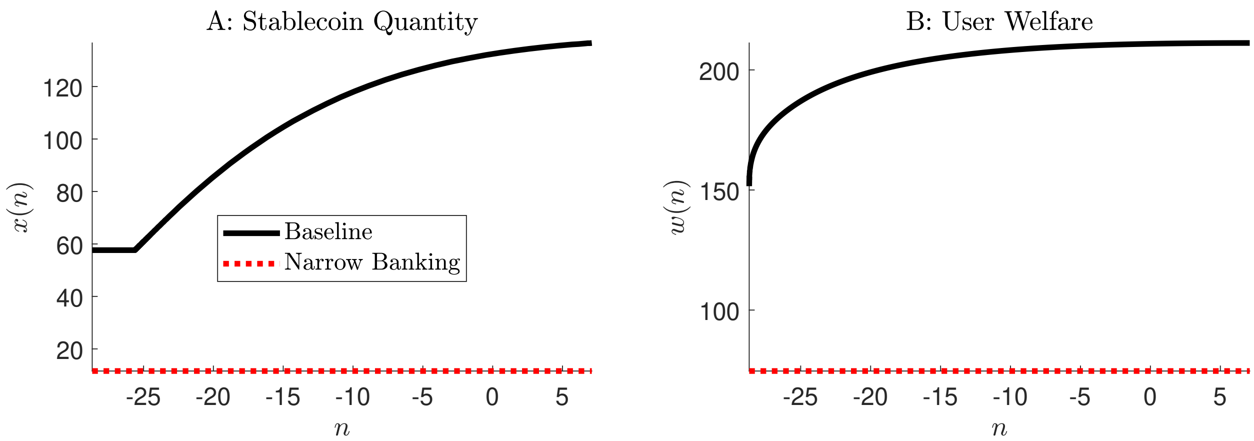


Figure 7: **Narrow Banking.**

reduces the users' utility from holding stablecoins, and as we have discussed in Section 2.2, an instability trap emerges once depegging happens. However, behind such phenomena is the active risk sharing between the stablecoin issuer and users. When the issuer's net worth is extremely low (i.e., below the threshold $\widetilde{n}$ given by Proposition 5), its sensitivity to risk is greater than the stablecoin users', so on a relative basis, the users have capacity to bear stablecoin-price fluctuations and provide insurance to the issuer.

Restricting the issuer to hold perfectly safe assets abandons the risk-sharing mechanism and forces the issuer to accept a lower expected return on reserve assets, which discourages intermediation (i.e., issuing stablecoins and investing in reserve assets). In the laissez-faire scenario with risk-taking, we have the calibrated value $\mu = 13.4\%$. In Figure 7, we have $\mu = 5\%$, which is a realistic value for returns on safe assets. As shown in Figure 7, the value of stablecoins issued and the users' welfare are both lower under the narrow-banking restrictions than the laissez-faire values even in the low net-worth region where depegging happens. In the high net-worth region, the laissez-faire values are even higher.

Our model solution also shows that imposing the narrow-banking requirements reduces the issuer's valuation by 33% relative to the laissez-faire case.[20] Overall, our analysis suggests caution against the narrow-banking framework, as it reduces both the users' welfare and the issuer's valuation. Forcing the issuer to avoid any risk-taking benefits the quality of

[20]We compute the issuer's valuation at the consumption boundary, $\overline{n}$, where we map variables in our model to Tether's values (see Section 3.1).

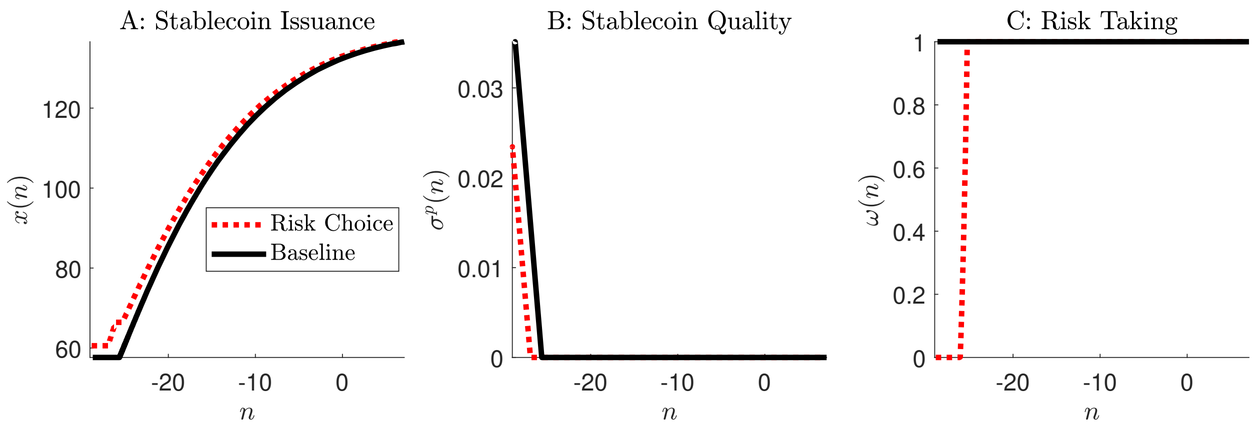


Figure 8: **Dynamic Choice of Reserve-Asset Riskiness.**

stablecoins (i.e., depegging never happens) but significantly reduces the quantity. A certain level of risk-taking can be Pareto-improving relative to narrow banking. Next, we show that more flexibility is preferred: allowing the issuer to not only take risk but dynamically adjust reserve-asset risk improves both the stablecoin quality and quantity.

## 5.2 Dynamic risk choice

In our baseline model, the issuer faces a fixed expected return, $\mu$, and volatility, $\sigma$, of the reserve assets. In the following, we allow the issuer to choose $\omega_t \in [w_L, w_H]$ and thereby change the expected return to $\mu_0 + \omega_t \alpha$ ($\alpha > 0$) and volatility to $\sigma_0 + \omega_t \sigma_\alpha$ ($\sigma_\alpha > 0$)—that is, the issuer faces a risk-return trade-off in its choice of $\omega_t$. The law of motion of the issuer's net worth, previously given by (11), is now extended to the following:

$$dn_t = \underbrace{[(\mu_0 + \omega_t\alpha)(n_t + x_t) + x_t\zeta_t - x_t r - x_t\kappa]}_{\mu_n(n_t)}dt + \underbrace{[(\sigma_0 + \omega_t\sigma_\alpha)(n_t + x_t) - x_t\sigma_t^p]}_{\sigma_n(n_t)}dZ_t - dy_t, \tag{23}$$

where $\zeta_t$ is the stablecoin user's marginal utility from holding stablecoins (i.e., the seigniorage) defined in (12). The issuer's problem has an additional control variable, $\omega_t$, but can be solved similarly as our baseline model is. We provide the solution details in Appendix F.4

In Figure 8, we compare our baseline model (solid line) and the extended model (dotted line) in the value of stablecoins issued (Panel A), price volatility or stablecoin "quality"

(Panel B), and the issuer's choice of $\omega(n)$ as a function of net worth, $n$. We set $\mu_0 = 12.9\%$, $\sigma_0 = 4.5\%$, $\alpha = 0.5\%$, and $\sigma_\alpha = 2.5\%$ and normalize $w_H$ to 1 so that the highest level of risk-taking, i.e., $\omega = 1$, brings the same expected return and volatility of reserve assets as in our baseline model. Moreover, we impose and $w_L = 0$ so that at the lowest level of risk-taking, the expected return and volatility of reserve assets are $\mu_0 = 12.9\%$ and $\sigma_0 = 4.5\%$. As shown in Panel C, $\omega$ decreases as the issuer loses net worth.

De-risking on the asset side of its balance sheet allows the issuer to refrain from risk-sharing via the liability side (i.e., offloading risk to the users through depegging), so Panel B of Figure 8 shows a smaller stablecoin-price volatility under the choice of $\omega_t$. Moreover, as shown in Panel A, the issuer supplies a higher value of stablecoins than the baseline case, especially in the low-$n$ region, because when it undercapitalized, the issuer can supply stablecoins but invest the proceeds more conservatively. Without the choice of risk-taking, the issuer has to either offload risk to the users (reducing the quality of stablecoins) or deleverage (reducing the quantity of stablecoins). Allowing asset-risk adjustment alleviates the tension between quality and quantity for an undercapitalized issuer.

# 6 Stablecoin Demand Dynamics

## 6.1 The general setup

We extend the model by introducing shocks to the stablecoin demand scaler, $K_t = K$:

$$\frac{dK_t}{K_t} = \mu_K dt + \sigma_K dZ_t^K, \tag{24}$$

where $\mu_K$ is a constant, $\sigma_K > 0$, and $Z_t^K$ is a standard Brownian motion. The correlation between the demand shock, $dZ_t^K$, and the shock to the issuer's reserve assets, $dZ_t$, is $\phi dt$.

The users take as given the price process, which now also loads on $dZ_t^K$:

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t + \sigma_{K,t}^p dZ_t^K, \tag{25}$$

where the shock loadings $\sigma_t^p$ and $\sigma_{K,t}^p$ are endogenously determined. We define

$$\Sigma_t^p := \sqrt{(\sigma_t^p)^2 + (\sigma_{K,t}^p)^2 + 2\phi\sigma_t^p\sigma_{K,t}^p}, \tag{26}$$

the instantaneous volatility of $dp_t/p_t$. In analogy to (2), the users' utility from holding stablecoins is given by $K_t u(x_t)$, where $u(x_t)$ is defined as follows

$$u(x_t) = \left(\frac{x_t^\xi}{\xi} - x_t\eta|\Sigma_t^p|\right), \tag{27}$$

where, as in the baseline model, $x_t = X_t/K_t$, and the users' optimal demand for stablecoin is scaled with $K_t$, i.e., $X_t = K_t x_t$ with

$$x_t = \left(\frac{1}{r - \mu_t^p + \eta|\Sigma_t^p| + f_t}\right)^{\frac{1}{1-\xi}}, \tag{28}$$

which is analogous to the stablecoin demand (5) in the baseline model.

Given the state variables, $N_t$ (net worth) and $K_t$ (stablecoin demand scaler), the issuer chooses the $K_t$-scaled stablecoin supply, $x_t$, consumption, $dY_t$, and as in the baseline model, the controls the volatilities $(\sigma_t^p, \sigma_{K,t}^p)$. The issuer implements these choices through fee policies and supply changes, i.e., $(f_t, S_t)_{t\geq 0}$, as we show. In Appendix F.5, we solve the issuer's problem with both reserve and demand shocks. We present an overview of the solution in Proposition F.1, while illustrating the results here with numerical analysis.

## 6.2 Stablecoin demand shocks and capital requirement

In this subsection, we focus on the role of demand shocks, setting $\mu_K = 0$. In practice, the correlation between the demand shock and reserve-asset shock, $\phi$, is likely to be positive, because the issuer (e.g., Tether) may hold cryptocurrencies in its reserve-asset portfolio, and cryptocurrencies' value is correlated with the adoption of stablecoins.

Such positive correlation exacerbates the instability trap, driven by the procyclicality of

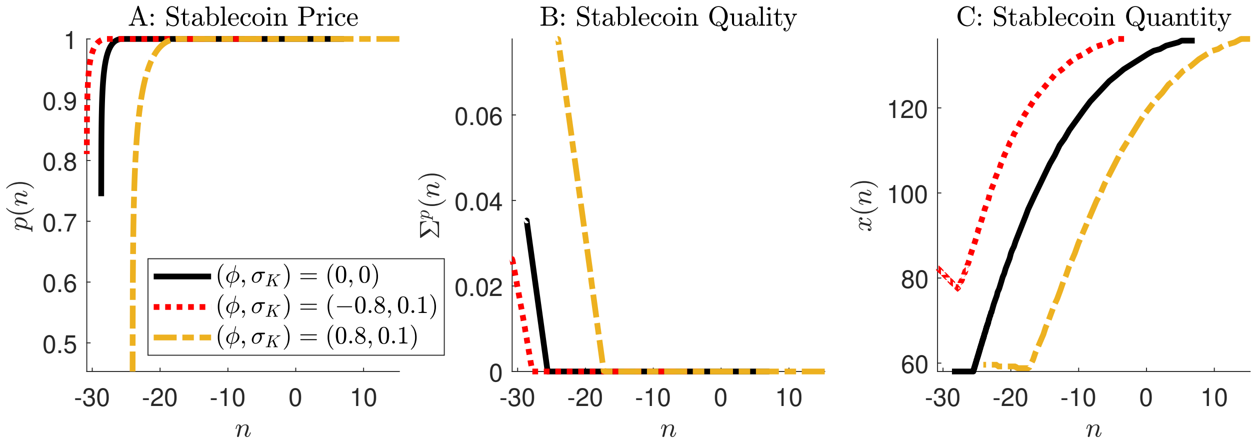


Figure 9: **Stablecoin Price and Quantity Dynamics under Demand Shocks.**

seigniorage revenues. The stablecoin issuer's total seigniorage revenues are given by

$$X_t \zeta_t = K_t \left( x_t^{\xi} - \eta |\Sigma_t^p| x_t \right), \tag{29}$$

When negative shocks to reserve assets significantly reduce the issuer's net worth, the issuer offloads risk to the users through depegging (i.e., $\Sigma_t^p$ increases) which reduces the seigniorage for any choice of stablecoin supply, $x_t$. Under $\phi > 0$, the negative reserve-asset shocks are likely to coincide with negative shocks to $K_t$, the demand scaler, which further reduces the seigniorage revenues—that is, $\phi > 0$ amplifies the procyclicality of seigniorage revenues. In Section 2.2, we have explained that such procyclicality leads to the instability trap.

In Figure 9, we plot the stablecoin price, price volatility, and value of stablecoins issued against the state variable, $n_t$ in Panel A, B, and C, respectively. The plots start from $\underline{n}$, the lower bound of $n_t$ in analogy to $\underline{n}$ in Proposition 4, and ends at $\overline{n}$, the upper bound where the issuer consumes. Note that both boundaries are endogenous and vary with the parameter values. Our numerical solution is based on parameter values from Section 3.1 and $\sigma_K = 10\%$ (i.e., the annualized volatility of the growth rate of demand scaler is 10%).

The solid black lines in Figure 9 represent the baseline model with a constant $K$, i.e., $\phi = 0$ and $\sigma_K = 0$. The yellow dashed lines represent the case of $\phi > 0$, i.e., the demand and reserve-asset shocks are positively correlated, while the red dotted lines represent the case of $\phi < 0$. As $\phi$ increases from -0.8 to zero and then to 0.8, depegging in the low-$n$

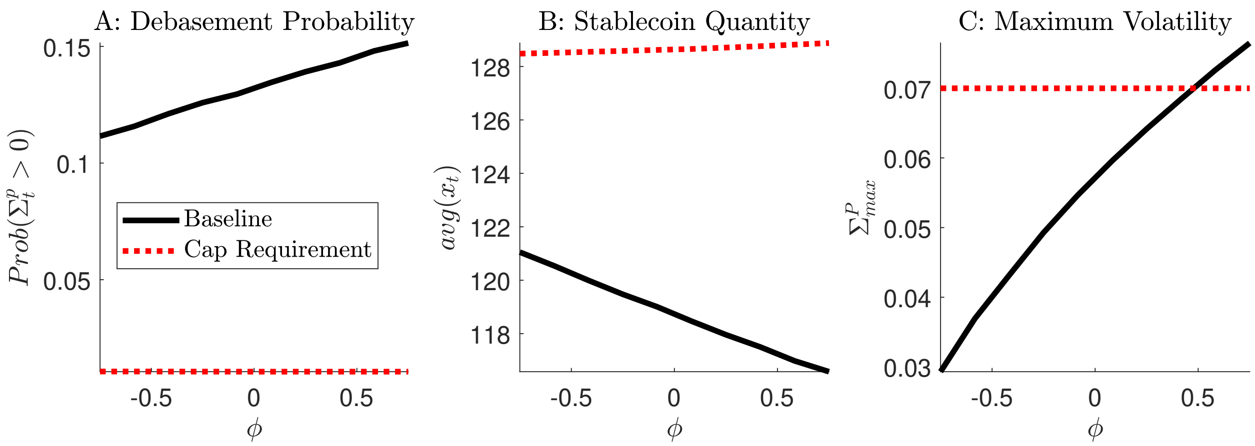


Figure 10: **Capital Requirement and Demand Shocks.**

region becomes more severe (see Panel A), and once depegging happens, the price fluctuates more aggressively (see Panel B). In addition, under positively correlated shocks, the issuer supplies less stablecoins (see Panel C). Figure 9 shows that when the demand shock and reserve-asset shock become positively correlated, both the quality and quantity of stablecoins suffer because the positive correlation amplifies the procyclicality of seigniorage revenues.

In Section 4, we have explained that introducing capital requirement reduces the issuer's reliance on the procyclical seigniorage as a revenue source. Therefore, it can make the issuer less sensitive to the correlation between the demand shock and reserve-asset shock, because $\phi > 0$ exacerbates instability by amplifying the procyclicality of seigniorage revenues. Figure 10 shows that this is indeed the case. We consider a capital requirement that requires the issuer to maintain non-negative net worth, i.e., $n_t \geq 0$. In Panel A, we show that the depegging probability is significantly reduced under capital requirement and is no longer sensitive to $\phi$. In Panel B, we compute the average value of stablecoin supply, $x_t$, based on the stationary probability distribution of state variables under different values of $\phi$. By reducing the issuer's reliance on procyclical seigniorage, capital requirement mitigates the force of instability trap and thereby increases the average supply of stablecoins (as discussed in Section 4), and importantly, it makes the supply less sensitive to $\phi$.

A caveat regarding capital requirement is that it increases the worst-case volatility of stablecoin price as we have discussed in Section 4. This is reflected in Panel C of Figure 10. When $\phi$ is high, capital requirement reduces the average volatility in the depegging region

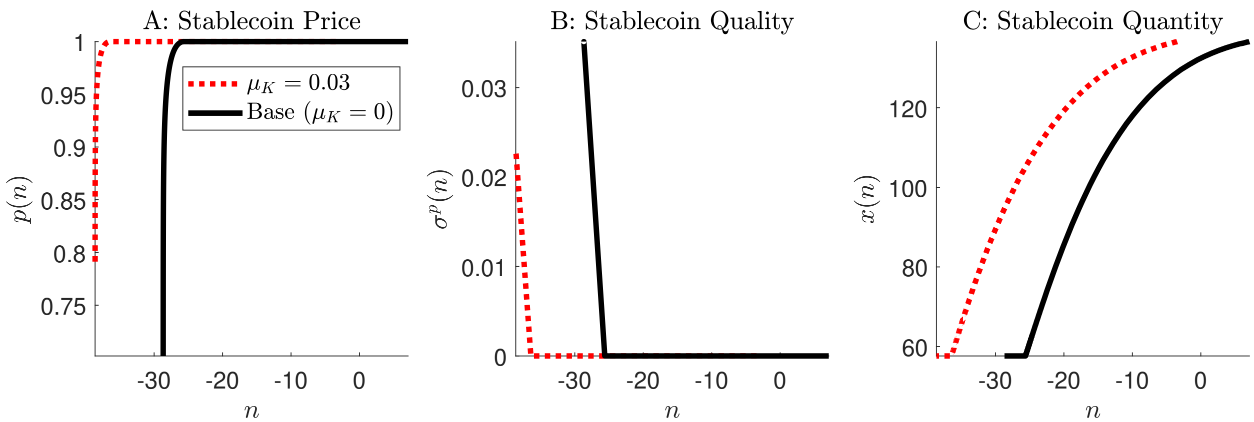


Figure 11: **Demand Growth Fosters Stability.**

(i.e., conditional on $\Sigma_t^p > 0$) relative to the laissez-faire case. However, when $\phi$ is sufficiently negative—that is, when the procyclicality of seigniorage revenues is already mitigated by the negative $\phi$, rendering the benefit of capital requirement less important—capital requirement makes the depegging region more unstable relative to the laissez-faire case.

In summary, our conclusion is that, while positively correlated reserve and demand shocks exacerbate instability, capital requirement makes the issuer less sensitive to this correlation, thereby reducing the probability of depegging. However, once depegging happens, capital requirement mitigates instability when the shock correlation is high but exacerbates instability relative to the laissez-faire case when the shock correlation is sufficiently negative.

## 6.3 Stablecoin demand growth and issuer valuation

The previous subsection focuses on the role of demand shocks, where we consider $\sigma_K > 0$ and $\mu_K = 0$. Next, we examine the impact of demand growth on the price, quantity, and quality dynamics of the stablecoin and issuer's equity valuation under $\sigma_K = 0$ and $\mu_K > 0$.

In Figure 11, we plot the stablecoin price (Panel A), stablecoin quality, i.e., the instantaneous price volatility (Panel B), and $K$-scaled stablecoin quantity (Panel C) against $K$-scaled net worth, $n$, under our baseline parameters (solid line; $\mu_K = 0$) and with demand growth at rate $\mu_K = 3\%$. Notably, Panel A and B show, respectively, that under greater demand growth, the stablecoin experiences less severe depegging in the low-$n$ region and fluctuates less. Intuitively, demand growth has a stabilizing effect: the trend in seigniorage revenues

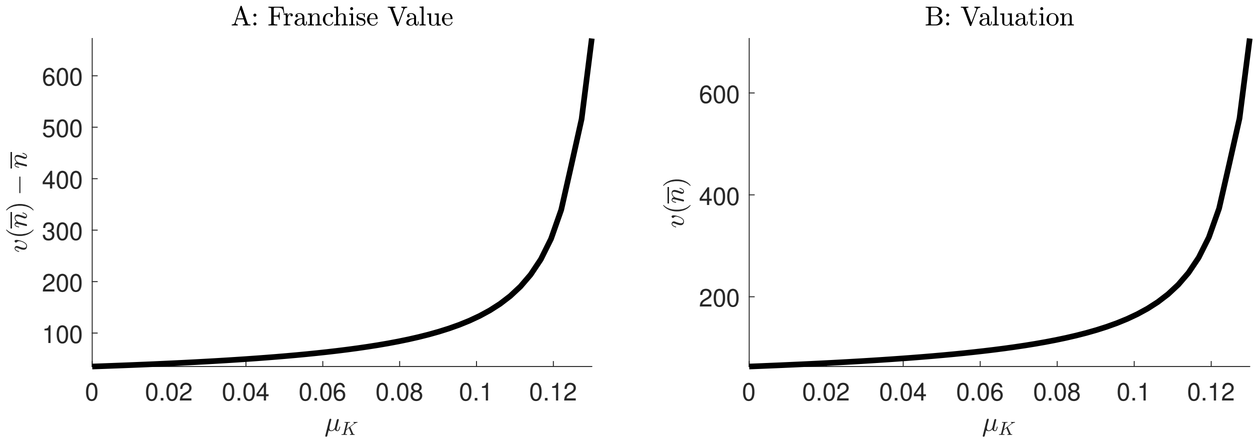


Figure 12: **Stablecoin Issuer Valuation.**

counterbalances the procyclicality of seigniorage revenues that is the source of instability.

Interestingly, Panel C of Figure 11 shows that the $K$-scaled value of stablecoins issued does not significantly differ under $\mu_K = 0$ versus $\mu_K = 3\%$. What a higher demand growth does is essentially shifting $x(n)$ to the left—that is, for any given level of stablecoin liabilities, the issuer maintains a lower net worth. Intuitively, as demand growth fosters stability, the issuer finds it less necessary to maintain net worth as a risk buffer.

Finally, we highlight the implications of our model on the issuer's equity valuation under demand growth rates. With the exception of Circle (the issuer of USDC), the majority of stablecoin issuers are private companies whose valuations are not publicly available. Our model provides a quantitative framework for valuing stablecoin issuers. The valuations may serve as a reference point for private investments and public offerings.

As reported by Cointelegraph.com, a recent market analysis estimated the valuation of Tether (the issuer of USDT, the largest stablecoin by market capitalization) at \$515 billion, which would make it the 19th most valuable company globally, ahead of firms like Costco and Coca-Cola. However, Tether CEO Paolo Ardoino has stated that the \$515 billion estimate may be "a bit bearish". Our model informs the valuation debate.

As discussed in Section 3.1, our model is calibrated to data on Tether, the issuer of USDT, at $n = \underline{n}$ (the issuer's consumption boundary), in line with Tether's significant amount of payout in 2024. In addition, when solving our model under different levels of demand growth, we impose a capital requirement, $n_t \geq 0$, in line with Tether's claim of maintaining over-collateralization. Next, using our model, we answer the following question: how strong the growth of USDT demand must be to support a valuation of Tether above \$500 billion?

In Panel A of Figure 12, we plot the value added, i.e., the difference between the equity valuation, $v(\overline{n})$, and book equity, $\overline{n}$, that reflects the franchise value, and in Panel B, we plot the equity valuation, $v(\overline{n})$. As a reminder, $v(\overline{n})$ is the $K_t$-scaled valuation where we consider $K_t$ equal to \$1 billion. From Panel B, a valuation above \$500 billion requires a demand growth rate, $\mu_K$, of at least 12%. This growth rate is roughly in line with the overall sector growth and rising competition within the sector. The World Economic Forum reported an average annualized growth rate of 28% as of Q1 2025. However, competition has risen. According to Coindesk.com (as cited by the BIS in their June 2025 bulletin), the number of active stablecoins has nearly doubled from the start of 2024 to June 2025.

# 7 Conclusion

This paper develops a dynamic model of stablecoin issuers, who represent a novel category of financial intermediaries because they retain discretion over the value of their liabilities. By analyzing the issuer's incentives to depeg, we show how this discretion introduces a new instability channel that is absent in models of traditional financial institutions. A key insight is the existence of an "instability trap," in which depegging reduces the issuer's seigniorage profits, slowing down its net worth rebuild after negative shocks and thereby making depegging more persistent. Our analysis also uncovers a risk paradox: higher reserve-asset risk can, counterintuitively, may reduce the probability and severity of depegging.

The model delivers important implications for regulation. The mainstream proposals that require stablecoins to be fully backed by assets with low risk may not resolve the underlying instability, as the risk paradox highlights. Forcing stablecoin issuers to hold perfectly safe assets with low returns hurts their profits and can significantly reduce stablecoin supply and users' welfare. In our setting, capital requirements play a new role: by reducing reliance on procyclical seigniorage from issuance, they improve stability. This finding sheds light on how capital regulation works in the stablecoin context, suggesting that the underlying mechanism differs significantly from those imposed on banks and other traditional intermediaries.

Finally, our model shows the value of granting stablecoin issuers' access to equity financing. Allowing issuers to share risk with external equity investors improves their resilience and the robustness of the peg. In addition, we provide a quantitative framework for valuing the issuers' equity. Taken together, these results clarify the economic forces that shape stablecoin supply, the fragility of their peg, and the regulatory levers available to address them. More broadly, the paper positions stablecoin issuers as a distinct class of intermediaries whose unique liability structure calls for tailored regulatory approaches, and provides a quantitative framework for evaluating the consequences of such policies.

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# Appendix

## A Preliminaries and Regularity Conditions

We impose the following regularity conditions and key parameter assumptions

1. We assume $\rho > \mu$. This assumption is standard in liquidity management models (Bolton et al., 2011), necessary for a well-behaved solution, and ensures that the issuer does not infinitely delay consumption.

2. We assume $r + \kappa > \mu$, i.e.,
$$\lambda = r + \kappa - \mu > 0.$$
The role of this assumption, as also discussed in the main text, is to introduce a cost of stablecoin issuance.

3. We assume $x_t \leq \bar{x}$, where $\bar{x}$ is a constant. A motivation of this assumption runs as follows. Stablecoins are issued on blockchains that have limited capacity for processing and recording transactions (e.g., Abadi and Brunnermeier (2019), Hinzen, John, and Saleh (2022)). In addition, regulatory restrictions or portfolio allocation considerations may lead to limits on stablecoin holdings. This assumption serves as a regularity condition, effectively ruling out a Ponzi schemes. We set $\bar{x}$ sufficiently high, in that the constraint never binds in optimum.

4. We assume $A_t = N_t + S_t P_t \geq 0$. In addition, $dY_t \geq 0$.

Finally, we emphasize that the Appendix follows a different structure and sequence than the main text. The main text presents results in line with the exposition and flow. The Appendix proves the findings according to the formal flow — potentially different from the expositional flow. The headers of the Sections will indicate where different Propositions and Corollaries from the main text are proven.

## B User Problem: Proof of Proposition 1

This Appendix solves the user problem and proves Proposition 1.

The representative user has wealth $N_t^u$, evolving according to (4). We conjecture (and verify) that the user's value function takes the form $V_t^u = N_t^u + v_t^u$, where $v_t^u$ does not depend on $N_t^u$ and only depends on aggregate states. The representative user takes price $p_t$, and price dynamics $dp_t$, and the dynamics $dv_t^u$ as given, when choosing $X_t$.

**User Optimization.** By the dynamic programming principle, a representative user solves the HJB equation:

$$rV_t^u dt = r(N_t^u + v_t^u)dt = \max_{X_t \geq 0, dY_t^u} \Big[dY_t^u + U(X_t)dt + \mathbb{E}_t^u[dN_t^u + dv_t^u]\Big], \tag{B.1}$$

where $\mathbb{E}_t^u$ is the user's time-$t$ expectation and $dV_t^u = d(v_t^u + N_t^u)$. It follows immediately from $dY_t \gtrless 0$ that the marginal utility from net worth equals 1.

Next, we use the budget constraint (4), that is,

$$dN_t^u = rN_t^u dt + X_t \left(-f_t dt + \frac{dp_t}{p_t} - rdt\right) - dY_t^u,$$

$X_t = x_t K$, as well as $U(X_t) = Ku(x_t)$ to rewrite the HJB equation as

$$rv_t^u dt = \max_{x_t \geq 0} \left\{ K \left[u(x_t)dt - rx_t dt - x_t f_t dt + x_t \mathbb{E}_t^u \left[\frac{dp_t}{p_t}\right]\right] + \mathbb{E}_t^u[dv_t^u] \right\}.$$

This also verifies our conjecture of the functional form of the value function.

As the user takes the dynamics of $p_t$ and $v_t^u$ as given, the choice of $x_t$ is determined according to the static optimization

$$\max_{x_t \geq 0} \left\{ u(x_t)dt - rx_t dt - x_t f_t dt + x_t \mathbb{E}_t^u \left[\frac{dp_t}{p_t}\right] \right\}, \tag{B.2}$$

Using that $u(x_t) = \frac{x_t^\xi}{\xi} - \eta x_t |\sigma_t^p|$, we obtain the first order condition with respect to $x_t$:

$$x_t^{\xi-1} dt - f_t dt + \mathbb{E}_t^u \left[\frac{dp_t}{p_t}\right] - rdt - \eta|\sigma_t^p|dt = 0. \tag{B.3}$$

We can solve (B.3) for:

$$x_t = \left( \frac{1}{r + f_t + \eta|\sigma_t^p| - \mathbb{E}_t^u \left[\frac{dp_t}{p_t dt}\right]} \right)^{\frac{1}{1-\xi}}. \tag{B.4}$$

Note that $\mathbb{E}_t^u \left[\frac{dp_t}{p_t}\right] = \mu_t^p dt$, so this expression for $x_t$ simplifies to (5), as desired.

# C Proof of Lemma 1 and State Variable Lower Bound

This Appendix provides a detailed derivation of the dynamics of net worth in (8), as well as presents the proof of Lemma 1.

Importantly, we also present the auxiliary Lemma 2, which will be used in solving the issuer's problem.

## C.1 Derivation of (8)

Here, we derive the law of motion by starting out from the issuer's reserve assets (an accounting identity) to provide details for the main text. Specifically, based on the model elements, the issuer's reserve assets evolve according to:

$$dA_t = A_t(\mu dt + \sigma dZ_t) + f_t X_t dt + dS_t(p_t + dp_t) - \kappa X_t dt - dY_t.$$

Note that issuance of stablecoins occurs at price $P_{t+dt} \simeq P_t + dP_t$. By Ito's Lemma:

$$dX_t = d(S_t p_t) = S_t dp_t + p_t dS_t + dS_t dp_t.$$

Thus:

$$\begin{aligned} dN_t = dA_t - dX_t &= A_t(\mu dt + \sigma dZ_t) + f_t X_t dt - S_t dp_t - \kappa X_t dt - dY_t \\ &= (N_t + p_t S_t)(\mu dt + \sigma dZ_t) - p_t S_t(\mu_t^p dt + \sigma_t^p dZ_t) + p_t S_t(f_t - \kappa)dt - dY_t. \end{aligned}$$

where we used that $dp_t = p_t(\mu_t^p dt + \sigma_t^p dZ_t)$. This expression coincides with (8), as desired.

## C.2 Proof of Lemma 1

First, we solve (5) for:

$$f_t = x_t^{\xi-1} - r + \mu_t^p - \eta|\sigma_t^p| = \zeta_t - r + \mu_t^p, \tag{C.1}$$

where $\zeta_t = x_t^{\xi-1} - \eta|\sigma_t^p|$.

Second, divide both sides of (8) by $K$ and use $S_t p_t = X_t = x_t K$ to obtain:

$$\frac{dN_t}{K} = (n_t + x_t)(\mu dt + \sigma dZ_t) - x_t(\mu_t^p dt + \sigma_t^p dZ_t) + x_t f_t dt - x_t \kappa dt - dy_t.$$

Inserting (C.1), we get

$$dn_t = [\mu(n_t + x_t) - x_t(r - \zeta_t) - x_t\kappa]dt + [\sigma(n_t + x_t) - x_t\sigma_t^p]dZ_t,$$

as desired.

We also rewrite the law of motion for $n_t$ for convenience. Inserting $\zeta_t = x_t^{\xi-1} - \eta|\sigma_t^p|$ and using $\lambda = r + \kappa - \mu$, we obtain:

$$dn_t = \left[\mu n_t + x_t^{\xi} - \lambda x_t - \eta x_t|\sigma_t^p|\right]dt + [\sigma(n_t + x_t) - x_t\sigma_t^p]dZ_t - dy_t. \tag{C.2}$$

Thus, $dn_t = \mu_n(n_t)dt + \sigma_n(n_t)dZ_t - dy_t$ with

$$\mu_n(n_t) = \mu n_t + x_t^{\xi} - \lambda x_t - \eta x_t|\sigma_t^p| \;\; \text{and } \sigma_n(n_t) = \sigma(n_t + x_t) - x_t\sigma_t^p.$$

## C.3 Auxiliary Results and Lower Bound

We state an auxiliary Lemma that we prove in this Appendix. This Lemma will be used for solving the issuer's problem.

**Lemma 2.** *Consider a strategy $\mathcal{S} := (x_t, \sigma_t^p, dy_t)_{t\geq 0}$. Let $\overline{\mathcal{S}}$ the set of strategies $\mathcal{S}$, satisfying $A_t = N_t + S_t p_t \geq 0$, $dy_t \geq 0$, $x_t \leq \overline{x}$, and $A_t \geq 0$ for all $t \geq 0$. Consider the net worth $n_t$ following the law of motion (11) (or, equivalently, (C.2)), governed by a strategy $\mathcal{S} \in \overline{\mathcal{S}}$. The following holds:*

1. *There exists a lower bound $\underline{n}'$ such that $n_t \geq \underline{n}$ with probability one. At the lower bound, we have $\mu_n(\underline{n}') \geq 0$, $\sigma_n(\underline{n}') = 0$, and $dy = dy(n) \leq n - \underline{n}'$.*

2. *Define the minimum feasible lower bound of $n_t$ via:*

$$\underline{n} := \inf\left\{\underline{n}' : n_t \geq \underline{n}' \text{ for some } \mathcal{S} \in \overline{\mathcal{S}}\right\} \tag{C.3}$$

*Then:*

$$\underline{n} = \begin{cases} -\left(\frac{\underline{x}^{\xi} - (r+\kappa-\mu)\underline{x} - \eta\sigma\underline{x}}{\mu - \eta\sigma}\right) & \text{if} \quad \mu - (1-\xi)(r+\kappa) - \eta\sigma > 0, \\ -\left(\frac{1}{r+\kappa}\right)^{\frac{1}{1-\xi}} & \text{if} \quad \mu - (1-\xi)(r+\kappa) - \eta\sigma \leq 0, \end{cases} \tag{C.4}$$

*where*

$$\underline{x} = \left(\frac{\xi}{r + \kappa - \mu + \eta\sigma}\right)^{\frac{1}{1-\xi}}.$$

*It follows for all $\mathcal{S} \in \overline{\mathcal{S}}$ that $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0$, as well as $dy \leq n - \underline{n}$.*

Importantly, Lemma 2 that net worth — under the issuer's optimized controls — will always satisfy $n_t \geq \underline{n}$ (with probability one). Put differently, $n_t < \underline{n}$ would imply a violation of $A_t \geq 0$, $x_t \leq \overline{x}$, or $dy_t \geq 0$ and thus cannot occur.

### C.3.1 Proof of Lemma 2 — Part I

Due to $x_t \leq \overline{x}$ (by assumption) and $A_t = K_t(x_t + n_t) \geq 0$ (by assumption), there exists $\underline{x}' \geq -\overline{x}$ such that $n_t \geq \underline{n}'$ (with probability one). In particular, $n_t \geq -\overline{x}$ with probability one. Again, note that $x_t \leq \overline{x}$ is a *regularity condition*; we can choose $\overline{x}$ arbitrarily large so that the constraint on $x_t$ never binds in equilibrium.

Due to $dy_t \geq 0$ and the dynamics of $n_t$ in (11), any lower boundary $\underline{n}'$ must satisfy $dy(\underline{n}') = 0$ (i.e., $dy_t \leq n_t - \underline{n}'$). In particular, no payouts occur at $n_t = \underline{n}'$ (i.e., $dy_t = 0$), as otherwise $n_t$ would fall below $\underline{n}'$. Likewise, in any other state $n_t$, payout cannot be so large that they push $n_t$ below $\underline{n}'$.

Moreover, at the lower boundary $\underline{n}'$, the drift of $n_t$ must be positive $\mu_n(\underline{n}') \geq 0$, as well as the volatility must be zero, i.e., $\sigma_n(\underline{n}') = 0$ — in order to prevent $n_t$ from falling below $\underline{n}'$.

### C.3.2 Proof of Lemma 2 — Part II

Since there exists a lower bound such $n_t \geq \underline{n}'$ with probability one by Part I, the set from (C.3) is non-empty and the infimum is well-defined. It follows that

$$\underline{n} := \inf\left\{n \in \mathbb{R} : \max_{x\in[0,\overline{x}],\sigma^p} \mu_n(n_t) \geq 0 \quad \text{s.t.} \quad \sigma_n(n_t) = 0\right\}$$

exists and satisfies $\underline{n} = (-\infty, +\infty)$.

By continuity, it follows that

$$\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0.$$

In what follows, we solve for the lower boundary $\underline{n}$ in closed form.

Doing so, we distinguish between two cases: (1) $\sigma^p(\underline{n}) > 0$ and (2) $\sigma^p(\underline{n}) = 0$. We omit time subscripts unless needed:

**Case (1): $\sigma^p(\underline{n}) > 0$.** First, consider $\sigma^p(\underline{n}) > 0$, and denote $x(\underline{n}) = \underline{x}$. Due to $\sigma_n(\underline{n}) = 0$, we have $\underline{x}\sigma^p(\underline{n}) = (\underline{x} + \underline{n})\sigma$. Therefore, the drift of net worth becomes

$$\mu_n(\underline{n}) = \max_{x\leq\overline{x}} \left[\mu\underline{n} - \lambda\underline{x} + \underline{x}^{\xi} - \eta\sigma(\underline{x} + \underline{n})\right].$$

Optimizing the drift over $\underline{x}$, we obtain

$$\underline{x} = \left(\frac{\xi}{\lambda + \eta\sigma}\right)^{\frac{1}{1-\xi}}.$$

Assuming $\mu - \eta\sigma > 0$, we can solve $\mu_n(\underline{n}) = 0$ for

$$\underline{n} = -\left(\frac{\underline{x}^\xi - \lambda\underline{x} - \eta\sigma\underline{x}}{\mu - \eta\sigma}\right) = -\left(\frac{\underline{x}^\xi - (r + \kappa - \mu)\underline{x} - \eta\sigma\underline{x}}{\mu - \eta\sigma}\right),$$

as stated.

Next, we determine the volatility $\sigma^p(\underline{n})$. To do so, calculate

$$\underline{n} + \underline{x} = \frac{(\mu + \lambda)\underline{x} - \underline{x}^\xi}{\mu - \eta\sigma} = \frac{\underline{x}\left[\xi\mu - (1-\xi)\lambda - \eta\sigma\right]}{\xi(\mu - \eta\sigma)}.$$

so that

$$\sigma^p(\underline{n}) = \sigma \max\left\{0, \frac{\xi\mu - (1-\xi)\lambda - \eta\sigma}{\xi(\mu - \eta\sigma)}\right\} \tag{C.5}$$

Thus, a necessary condition for $\sigma^p(\underline{n}) > 0$ is that $\xi\mu - (1-\xi)\lambda - \eta\sigma > 0$. This condition implies $\mu - \eta\sigma > 0$, due to $\xi \in (0, 1)$.

Further, we can rewrite

$$\xi\mu - (1-\xi)\lambda - \eta\sigma = \mu - (1-\xi)(r + \kappa) - \eta\sigma.$$

where $\lambda = r + \kappa - \mu$.

**Case (2): $\sigma^p(\underline{n}) = 0$.** Note that $\sigma^p(\underline{n}) = (x + n)\sigma - x\sigma^p = (x + n)\sigma = 0$. Thus, $x = x(\underline{n}) = -\underline{n}$. We can insert $x = -\underline{n}$ into the drift $\mu_n(n)$ from (C.2) to obtain

$$\mu_n(\underline{n}) = (\mu + \lambda)\underline{n} + (-\underline{n})^\xi.$$

We then solve $\mu_n(\underline{n}) = 0$ for

$$\underline{n} = -\left(\frac{1}{\mu + \lambda}\right)^{\frac{1}{1-\xi}}.$$

$$\xi\mu - (1-\xi)\lambda - \eta\sigma < 0; \lambda = r + \kappa - \mu$$

**Case Distinction.** We show that case (1) prevails and $\sigma^p(\underline{n}) > 0$ if and only if $\xi\mu - (1 -$

$\xi)\lambda - \eta\sigma > 0$.

First, we start with the "only if" implication. Note that $\sigma^p(\underline{n}) > 0$, i.e., case (1), requires that $\xi\mu - (1-\xi)\lambda - \eta\sigma > 0$. Thus, when $\xi\mu - (1-\xi)\lambda - \eta\sigma \leq 0$, we necessarily have $\sigma^p(\underline{n}) = 0$.

Second, we show that if $\xi\mu - (1-\xi)\lambda - \eta\sigma > 0$, then $\sigma^p(\underline{n}) > 0$. Suppose to the contrary that $\xi\mu - (1-\xi)\lambda - \eta\sigma > 0$ and $\sigma^p(\underline{n}) = 0$, leading to $\underline{n} = -\left(\frac{1}{\mu+\lambda}\right)^{\frac{1}{1-\xi}}$ and $x(\underline{n}) = -\underline{n}$ as shown above. Consider that the issuer selects in state $\underline{n} = -\left(\frac{1}{\mu+\lambda}\right)^{\frac{1}{1-\xi}}$ a different level of $x$, namely $x = -\underline{n}+\varepsilon$, while setting $x\sigma^p = \varepsilon\sigma$ for small $\varepsilon > 0$. Thus, $\sigma_n(\underline{n}) = (n+x)\sigma - x\sigma^p = 0$ under this alternative strategy. The alternative strategy implies drift of $n$ at $\underline{n}$ of

$$\begin{aligned}
\mu_n^\varepsilon(\underline{n}) &= \mu\underline{n} - \lambda(-\underline{n}+\varepsilon) + (-\underline{n}+\varepsilon)^\xi - \varepsilon\eta\sigma \\
&= \mu_n(\underline{n}) - \lambda\varepsilon - \eta\sigma\varepsilon + \xi(-\underline{n})^{\xi-1}\varepsilon + o(\varepsilon^2) \\
&= \varepsilon\big[\xi\mu - (1-\xi)\lambda - \eta\sigma\big] + o(\varepsilon^2),
\end{aligned}$$

where conducted a Taylor expansion around $\varepsilon = 0$ and used $\mu_n(\underline{n}) = 0$. As such, given $\xi\mu - (1-\xi)\lambda - \eta\sigma > 0$, there exists $\varepsilon > 0$ such that $\mu_n(\underline{n}) > 0$ and $\sigma_n(\underline{n})$ under $x = \underline{n} + \varepsilon$. By continuity, there exists $\underline{n}' < \underline{n}$ at which $\mu_n(\underline{n}') \geq 0$ and $\sigma_n(\underline{n}') = 0$, a contradiction. Likewise, we have shown that $x = \underline{n}$ and $\sigma^p = 0$ do not maximize $\mu_n(\underline{n})$, similarly yielding a contradiction.

Combining, we obtain (C.4), which was to show.

## C.4 Proof of Corollary 3

Corollary 3 directly follows from the closed-form expression (C.4) for the lower boundary presented in Lemma 2. This lower bound, as well will show next, will satisfy $v(\underline{n})$. Clearly, $\underline{n} < 0$.

Suppose that (18) holds, so that $\underline{n} = -\left(\frac{1}{r+\kappa}\right)^{\frac{1}{1-\xi}}$, which increases in $\kappa$ and $r$ and does not depend on $\sigma$ or $\eta$.

Suppose that (18) does not hold. Then, we can rewrite $\underline{n}$ as follows:

$$\underline{n} = -\frac{1-\xi}{\mu - \eta\sigma}\left(\frac{\xi}{r+\kappa-\mu+\eta\sigma}\right)^{\frac{\xi}{1-\xi}}.$$

Thus:

$$\frac{\partial \underline{n}}{\partial \sigma} = \frac{\eta}{(\mu-\eta\sigma)^2}\left(\frac{\xi}{r+\kappa-\mu+\eta\sigma}\right)^{\frac{\xi}{1-\xi}} \frac{\mu - \eta\sigma \;-\; (1-\xi)(r+\kappa)}{r+\kappa-\mu+\eta\sigma} > 0.$$

Finally, calculate

$$\frac{\partial \underline{n}}{\partial r} = \frac{\xi}{\mu - \eta\sigma}\left(\frac{\xi}{r+\kappa-\mu+\eta\sigma}\right)^{\frac{\xi}{1-\xi}} \frac{1}{r+\kappa-\mu+\eta\sigma} > 0.$$

# D Issuer Optimization

We now solve the issuer's problem, thereby proving various Propositions from the main text. The issuer's problem can be written as:

$$V_0 := K \max_{\mathcal{G}\in\overline{\mathcal{G}}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dy_t\right], \tag{D.1}$$

where the strategy $\mathcal{G} := (S_t,\ f_t,\ Y_t)_{t\geq 0}$ governs the state variable $N_t$ from (8). Here, $\overline{\mathcal{G}}$ denotes the set of strategies that satisfy the constraints $dY_t \geq 0$ and $A_t = N_t + S_t p_t \geq 0$. The issuer commits at time $t = 0$ to a strategy $\mathcal{G}$. We solve for the issuer's optimal strategy. In doing so, we conjecture that, under the optimal strategy, the stablecoin price $p_t$ is continuous and follows (1).

In Appendix G.2, we verify that the issuer indeed finds it optimal to implement a continuous price path, following (1). This is a consequence of the issuer's value function's concavity — established in Appendix D.4. In Appendix G.1, we show that full commitment could be relaxed, i.e., short-term commitment over an instant $[t, t+dt)$ plus a constraint on consumption is sufficient to implement the issuer's strategy.

## D.1 Preliminary Steps and Outline

The following argument then proceeds as follows. First, we solve the auxiliary problem, in which the issuer can directly choose $\mathcal{S} = (x_t, \sigma_t^p, dy_t)$ subject to $dy_t \geq 0$ and $A_t = N_t + S_t p_t \geq 0$. That is, we consider:

$$V_0^{Aux} := K \max_{\mathcal{G}\in\overline{\mathcal{G}}, \mathcal{S}\in\overline{\mathcal{S}}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dy_t\right] \tag{D.2}$$

subject to the dynamics of net worth in (8) or, equivalently, in (C.2). Here, $\overline{\mathcal{S}}$ is the set of transformed strategies $\mathcal{S} = (x_t, \sigma_t^p, dy_t)$ $dy_t \geq 0$ and $A_t = N_t + S_t p_t \geq 0$. The (optimized) issuer payoff under this auxiliary problem is, by construction, higher than that under (D.1), since the issuer has a wider range of options to optimize over.

Crucially, because payoffs are a function of the issuer's net worth and the dynamics of net worth depends on $(S_t, f_t)$ only via $(x_t, \sigma_t^p)$ — see (8) — it follows that

$$V_0^{Aux} = K \max_{\mathcal{S} \in \overline{\mathcal{S}}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dy_t\right].$$

More intuitively, when the issuer can choose $\mathcal{S}$, the strategy $\mathcal{G}$ is not payoff-relevant — in particular, $(S_t, f_t)$ affect issuer payoff only via $(x_t, \sigma_t^p)$, so it suffices to choose the latter two. The key arguments behind the solution to the issuer's (auxiliary) problem are presented in Parts I, II, and III of the proof.

Second, we map the issuer's optimal choice of $\mathcal{S} = (x_t, \sigma_t^p, dy_t)$ to a strategy $(S_t, f_t, dy_t)$. We show that for the optimal choice of $\mathcal{S}$, there exists $(S_t, f_t, dy_t)$ implementing this strategy $\mathcal{S}$, while satisfying all the imposed constraints. This implies

$$V_0 = K \max_{\mathcal{G} \in \overline{\mathcal{G}}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dy_t\right] = K \max_{\mathcal{S} \in \overline{\mathcal{S}}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dy_t\right],$$

as desired. These arguments are presented in Part IV of the proof (see Appendix D.5).

Third, in Appendix G.2, we verify that the issuer indeed finds it optimal to implement a continuous price path, following (1).

Finally, note that this Appendix — which is organized in several parts that build on each other — proves several results from the main text. Part I of the argument in Appendix D.2 proves Propositions 2 and (3). Part II in Appendix D.3 provides a proof for Propositions 3, 5, and 9, as well as Proposition 8. Part III in Appendix D.4 provides a proof for Propositions 5 and 7. Corollary 1 directly follows from these results. Part IV in Appendix D.5 demonstrates Corollary 2.

## D.2 Part I: HJB Equation and Boundaries — Proof of Propositions 2 and (3)

Let the issuer's scaled value function be denoted by $v(n_t)$, i.e., $V_t = Kv(n_t)$, for a given net worth $n_t$, that is:

$$v(n) = \mathbb{E}\left[\int_t^\infty e^{-\rho s} dy_s \Big| n_t = n\right] \tag{D.3}$$

We omit time subscript, as all quantities and payoffs will be functions of $n_t = n$ in optimum.

**HJB Equation.** We invoke the dynamic programming principle. By the integral representation in (D.2) (or in (D.3)), the issuer's value function solves the HJB equation (in the state space):

$$\rho v(n)dt = \max_{dy\geq 0,x,\sigma^p} \left\{ dy + v'(n)\big[\mu_n(n)dt - dy\big] + \frac{v''(n)\sigma_n(n)^2 dt}{2} \right\}. \tag{D.4}$$

where drift $\mu_n(n)$ and volatility $\sigma_n(n)$ of $n$ are defined (implicitly) in (8) or (C.2). Note that $dy$ satisfies $dy \in [n - \underline{n}]$ by Lemma 2. Also recall that $n \geq \underline{n}$ by the same Lemma.

**Upper Boundary.** We can optimize the right-hand-side over $dy \geq 0$ and obtain that consumption is optimal only if $v'(n) \leq 1$. As is standard, consumption occurs at a payout boundary $\overline{n}$, and follows a barrier strategy, that is, consumption causes $n$ to reflect at $\overline{n}$. As such, we have $v(n) = v(\overline{n}) + n - \overline{n}$ and $v'(n) = 1$ for $n > \overline{n}$. The location of the payout boundary is determined by smooth pasting and super contact conditions, that is,

$$v'(\overline{n}) = 1 \quad \text{and} \quad v''(\overline{n}) = 0. \tag{D.5}$$

Due to the (downward) reflection of $n$ at $\overline{n}$, the (endogenous) state space can be written as an interval $[\underline{n}, \overline{n}]$. Further, note that the value function satisfies $v'(n) \geq 1$ on the state space.

In the interior of the state space, i.e., for $n \in (\underline{n}, \overline{n})$, we have $dy = 0$. Inserting this relationship into (D.4) as well as plugging in the respective values for $\mu_n(n)$ and $\sigma_n(n)$, the HJB equation simplifies to:

$$\rho v(n) = \max_{\sigma^p,x} \left\{ v'(n)\Big[\mu n + x^{\xi} - \lambda x - \eta x|\sigma^p|\Big] + \frac{v''(n)}{2}\Big[\sigma(x+n) - x\sigma^p\Big]^2 \right\}. \tag{D.6}$$

Substituting $\lambda = r + \kappa - \mu$, equation (D.6) becomes (15) from the baseline.

**Lower Boundary.** Under the issuer's strategy, we have $n_t \geq \underline{n}$ with probability one, by Lemma 2 — where $\underline{n}$ is given in (C.4). When $n = \underline{n}$, then $\mathcal{S} \in \overline{\mathcal{S}}$ implies by means of Lemma 2 that $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0$, as well as $dy = 0$ — this means that the lower boundary is absorbing. Moreover, by means of (D.4), it follows that $v(\underline{n}) = 0$.

For the sake of exposition, the proof of the lower boundary being inaccessible (i.e., the lower boundary is never reached) will be provided in the Proof of Proposition 6 — see Appendix E.

Thus, the issuer's value function is the solution to (D.6) subject to

$$v(\underline{n}) = v'(\overline{n}) - 1 = v''(\overline{n}) = 0.$$

$v'(n) > 1$ for $n < \overline{n}$.

Overall, we have proven Proposition 2 and 4.

## D.3 Part II: Concavity of the Value Function and Optimal Controls — Proof of Propositions 3, 5, 9 and 8

We characterize the optimal controls. We start by proving the concavity of the

### D.3.1 Optimal Controls

We conjecture and verify that $v''(n) \in (-\infty, 0)$ on $n \in (\underline{n}, \overline{n})$, with $v''(\overline{n}) = 0$. Consider $n \in (\underline{n}, \overline{n})$. Note that $x + n \geq 0$ (which is equivalent to the constraint $A = N + Sp \geq 0$). Thus, the HJB equation (D.6) immediately implies that, in optimum, $\sigma^p \geq 0$. Con If interior, i.e., $\sigma^p > 0$ or $x \in (0, \overline{x})$, the optimal controls solve the first-order conditions, which, as easily can be verified, are sufficient for a maximum.

For $\sigma^p > 0$, the derivative in (D.6) with respect to $\sigma^p$ implies that

$$\frac{\partial v(\theta)}{\partial \sigma^p} \propto -\eta x + v''(n)\big(\sigma(x+n) - x\sigma^p\big)x.$$

When $\sigma^p > 0$, it solves the first-order condition $\frac{\partial v(\theta)}{\partial \sigma^p} = 0$. Thus, for $\gamma(n) = -\frac{v''(n)}{v'(n)}$:

$$x\sigma^p = \max\left\{0, \sigma(x+n) - \frac{\eta}{\gamma(n)}\right\}.$$

Suppose that $x\sigma^p > 0$ and insert the expression for $x\sigma^p$ into the HJB equation (15) to obtain:

$$\rho v(n) = \max_{x \geq 0}\left\{v'(n)\Big[\mu n - x\lambda + x^{\xi} - \eta(n+x)\sigma\Big] + \frac{v'(n)\eta^2}{2\gamma(n)}\right\}. \tag{D.7}$$

Thus, when $\sigma^p > 0$, the first-order condition with respect to $x$ becomes

$$v'(n)\big[-\lambda - \eta\sigma + \xi x^{\xi-1}\big] = 0,$$

which, using $v'(n) \geq 1$, we can solve for $x = x(n) = \underline{x}$

$$\underline{x} = \left(\frac{\xi}{\lambda + \eta\sigma}\right)^{\frac{1}{1-\xi}}. \tag{D.8}$$

Thus, $\underline{x}\sigma^p = \max\left\{0, \sigma(\underline{x}+n) - \frac{\eta}{\gamma(n)}\right\}$ and therefore

$$\sigma^p = \sigma^p(n) = \max\left\{0, \frac{\sigma(\underline{x}+n)}{\underline{x}} - \frac{\eta}{\gamma(n)\underline{x}}\right\}. \tag{D.9}$$

It follows that $\sigma^p(\overline{n}) = 0$, due to $v''(\overline{n}) = 0$ and by continuity.

Now, suppose that $\sigma^p = 0$. Then, the first order condition with respect to $x$ in the HJB equation (D.6) becomes

$$v'(n)\big[-\lambda + \xi x^{\xi-1}\big] + v''(n)\sigma^2(x+n) = 0 \quad \Longleftrightarrow \quad \xi x^{\xi-1} - \lambda = \gamma(n)\sigma^2(x+n). \tag{D.10}$$

Optimal $x = x(n)$ therefore is a function of $n$. As long as $v''(n) > -\infty$, we have $x + n > 0$. For $n = \overline{n}$, we have

$$x(\overline{n}) = x^* = \left(\frac{\xi}{\lambda}\right)^{\frac{1}{1-\xi}}. \tag{D.11}$$

Finally, when $\sigma^p = \sigma^p(n) > 0$, then $\sigma_n(n) = \sigma(\underline{x} + n) - \underline{x}\sigma^p = \frac{\eta}{\gamma(n)} > 0$. When $\sigma^p = 0$, then $\sigma_n(n) = (x+n)\sigma > 0$. Either way, $\sigma_n(n) > 0$ for all $n \in (\underline{n}, \overline{n}]$.

### D.3.2 Concavity of Value Function

Without loss of generality, we can prove the claim for $v''(n) > -\infty$. Part I implies that $\sigma_n(n) > 0$ if $v''(n) < 0$. Thus, we conjecture that $\sigma_n(n) > 0$ and then prove $v''(n) < 0$, thereby verifying the conjecture.

Rewrite the HJB equation (D.6) in the interior of the state space as

$$\rho v(n) = \max_{x,\sigma^p}\left\{\mu_n(n)v'(n) + \frac{v''(n)\sigma_n(n)^2}{2}\right\}$$

Assume that $v''(n)$ is differentiable — the set on which $v''(n)$ is not differentiable is not dense. In this case, we can invoke the envelope theorem to differentiate the HJB equation (15) under the optimal controls with respect to $n$:

$$v'''(n) = \frac{2}{\sigma_n(n)^2}\Big((\rho-\mu)v'(n) - v''(n)[\mu_n(n) + \sigma\sigma_n(n)]\Big) \tag{D.12}$$

In doing so, we have used that, when $\sigma^p = 0$, then $x$ solves the first order condition (D.10), so that $\frac{\partial v(n)}{\partial x} = 0$ and $\frac{\partial \sigma^p(n)}{\partial n} = 0$. On other hand, if $\sigma^p > 0$, then $x(n) = \underline{x}$,s so that $\frac{\partial x(n)}{\partial n} = 0$, and $\sigma^p$ solves the first order condition $\frac{\partial v(n)}{\partial \sigma^p} = 0$.

Evaluating above expression for $v'''(n)$ at $n = \overline{n}$, we obtain $\lim_{n\uparrow\overline{n}} v'''(n) > 0$. As such, $v''(n) < 0$ in a left-neighbourhood of $\overline{n}$.

Suppose now to the contrary there exists $n < \overline{n}$ such that $v''(n) \geq 0$, and define $n' = \sup\{n < \overline{n} : v''(n) \geq 0\}$. Note that, because $v(n)$ is strictly concave in a left-neighborhood of $\overline{n}$, we have $n' < \overline{n}$ as well as $v''(n) < 0$ on $(n', \overline{n})$ and $v'(n') > 1$. By continuity of $v''$, it follows that $v''(n') = 0$. As such, $v'''(n') > 0$. However, this implies that there exists $n'' > n'$

with $v''(n'') > 0$, contradicting the definition of $n'$. As such, the claim follows, and $v''(n) < 0$ for $n < \overline{n}$, which implies $v'(n) > 1$ for $n < \overline{n}$.

Overall, we have shown $v''(n) < 0$ on $(\underline{n}, \overline{n})$.

### D.3.3 Dynamics of Controls

We now show that (i) $\gamma(n)$ decreases with $n$, (ii) $\sigma^p(n)$ decreases with $n$, and (iii) $x(n)$ increases with $n$. In order to do so, we distinguish between $\sigma^p(n) > 0$ and $\sigma^p(n) = 0$.

**Case 1: $\sigma^p > 0$.** Suppose $\sigma^p > 0$. Then, we have in optimum $x = x(n) = \underline{x}$ and the HJB equation (D.7) holds. We then obtain

$$\rho\left(\frac{v(n)}{v'(n)}\right) = \left[\mu n - \underline{x}\lambda + \underline{x}^{\xi} - \eta(n + \underline{x})\sigma\right] + \frac{\eta^2}{2\gamma(n)}$$

Thus,

$$\frac{d}{dn}\left(\rho\left(\frac{v(n)}{v'(n)}\right) - \left[\mu n - \underline{x}\lambda + \underline{x}^{\xi} - \eta(n + \underline{x})\sigma\right]\right) = \frac{d}{dn}\left(\frac{\eta^2}{2\gamma(n)}\right).$$

Note that

$$\mathcal{A}(n) := \frac{d}{dn}\left(\rho\left(\frac{v(n)}{v'(n)}\right) - \left[\mu n - \underline{x}\lambda + \underline{x}^{\xi} - \eta(n + \underline{x})\sigma\right]\right) = \rho\left(1 - \frac{v''(n)v(n)}{v'(n)^2}\right) - \mu + \eta\sigma,$$

while $\frac{d}{dn}\left(\frac{\eta^2}{2\gamma(n)}\right) = -\frac{\eta^2\gamma'(n)}{2\gamma(n)^2}$. Due to $v''(n) \leq 0$, $\rho \geq \mu$, and $\lambda > 0$, it follows that $\mathcal{A}(n) > 0$. Due to $\mathcal{A}(n) = \frac{d}{dn}\left(\frac{\eta^2}{2\gamma(n)}\right) = -\frac{\eta^2\gamma'(n)}{2\gamma(n)^2}$, it follows $\gamma(n)$ decreases with $n$, i.e., $\gamma'(n) < 0$.

Furthermore, calculate

$$\frac{d}{dn}\left(\frac{1}{\gamma(n)}\right) = -\frac{\gamma'(n)}{\gamma(n)^2} = \frac{2\mathcal{A}(n)}{\eta^2} \geq \frac{2(\rho - \mu + \eta\sigma)}{\eta^2}.$$

Hence, when $\sigma^p(n) > 0$ — that is, $\underline{x}\sigma^p(n) = \sigma(\underline{x} + n) - \frac{\eta}{\gamma(n)} > 0$ — we have

$$\frac{d}{dn}\left(\sigma^p(n)\underline{x}\right) = \sigma - \eta\frac{d}{dn}\left(\frac{1}{\gamma(n)}\right) \leq \sigma - \frac{2(\rho - \mu + \eta\sigma)}{\eta} = -2\left(\frac{\rho - \mu}{\eta}\right) < 0,$$

so that $\sigma^p(n)$ decreases with $n$.

In turn, there exists unique point $\tilde{n} \in (\underline{n}, \overline{n})$ above which $\sigma^p(n) = 0$ and below which $\sigma^p > 0$. i.e., $\sigma^p(n) > 0$ for all $n < \tilde{n}$ and $\sigma^p(n) = 0$ for all $n \geq \tilde{n}$.

**Case 2: $\sigma^p = 0$** Suppose $\sigma^p = 0$, so $x$ solves the first order condition (D.10). Then, the HJB equation implies

$$\rho\left(\frac{v(n)}{v'(n)}\right)=\max_{x\geq 0}\left(\mu n-x\lambda+x^{\xi}-\frac{\sigma^2\gamma(n)(x+n)^2}{2}\right).$$

Using the envelope theorem, we can differentiate both sides with respect to $n$ to obtain (after rearranging):

$$\rho\left(1-\frac{v''(n)v(n)}{v'(n)^2}\right)-\mu=-\sigma^2(x+n)\left(\gamma(n)+\frac{\gamma'(n)(x+n)}{2}\right)$$

Due to $\rho>\mu$ and $v''(n)<0$, we obtain

$$-\sigma^2(x+n)\left(\gamma(n)+\frac{\gamma'(n)(x+n)}{2}\right)>0, \tag{D.13}$$

which implies $\gamma'(n)<\frac{-2\gamma(n)}{x+n}<0$.

Next, differentiate both sides of the first-order condition for $x$ — that is, (D.10) and $\xi x(n)^{\xi-1}-\lambda=\gamma(n)\sigma^2(x(n)+n)$ — with respect to $n$ to get:

$$\xi(\xi-1)x(n)^{\xi-2}x'(n)=\sigma^2\big[\gamma(n)\left(x'(n)+1\right)+\gamma'(n)\left(x(n)+n\right)\big].$$

Suppose that $x'(n)\leq 0$. Then, the left-hand-side is positive. The right-hand-side satisfies:

$$\sigma^2\big[\gamma(n)(x'(n)+1)+\gamma'(n)(x(n)+n))\big]\leq\sigma^2\big[\gamma(n)+\gamma'(n)(x(n)+n))\big]<0,$$

where we used (D.13). A contradiction. Thus, $x'(n)>0$. A

Overall, we have proven Proposition 3.

### D.3.4 Solving the Price — Proof of Proposition 8

Suppose that price is a function of $n_t=n$ only, in that $p_t=p(n_t)$. Then, by Ito's Lemma, the volatility of $p(n)$ is $p'(n)\sigma_n(n)$. At the same time, the price volatility equals $\sigma^p(n)p(n)$ by (1). As a result, we obtain

$$p'(n)\big[(x(n)+n)\sigma-x(n)\sigma^p(n)\big]=p'(n)\sigma_n(n)=p(n)\sigma^p(n).$$

For $n>\tilde{n}$, we have $\sigma^p(n)=0$, so $p'(n)=0$ and $p(n)$ is constant. We normalize $p(\overline{n})=p(\tilde{n})=1$.

Next, consider $n < \tilde{n}$, so that $x(n) = \underline{x}$ and $\sigma^p(n) = \frac{\sigma(\underline{x}+n)}{\underline{x}} - \frac{\eta}{\gamma(N)\underline{x}} > 0$. Thus,

$$\frac{d \ln p(n)}{dn} = \frac{p'(n)}{p(n)} = \frac{\sigma^p(n)}{\sigma_n(n)}.$$

Hence,

$$p(n) = \exp\left(-\int_n^{\tilde{n}} \frac{\sigma^p(\nu)}{\sigma_n(\nu)} d\nu\right) = \exp\left(-\int_n^{\tilde{n}} \frac{\gamma(\nu)\sigma(x(\nu)+\nu) - \eta}{\eta x(\nu)} d\nu\right), \tag{D.14}$$

which is equivalent to (20). This proves Proposition 8.

## D.4 Part III — Lower Boundary — Proof of Proposition 5 and 7

The previous part has shown that $\sigma^p(n)$ and $x(n)$ decrease in $n$. Note that $\sigma^p(\overline{n}) = 0$. Thus, there exists unique

$$\tilde{n} := \sup\{n \in [\underline{n}, \overline{n}] : \sigma^p(n) > 0\}$$

We now characterize the behavior at the lower boundary $\underline{n}$ given in (C.4). We also verify that under the optimal controls, the constraint $A \geq 0$ never binds in the interior of the state space.

**Proving Proposition 9 and 5.** Note that when (18) holds, then $\sigma^p(\underline{n}) = 0$ and $\sigma^p(n) = 0$ for all $n \in (\underline{n}, \overline{n})$. Then, the instability region is empty, i.e., $\tilde{n} = \underline{n}$. At $n = \underline{n}$, we have $\underline{n} + x(\underline{n}) = 0$, i.e., the constraint $A \geq 0$ binds at $n = \underline{n}$. Since $x(n) + n$ increases in $n$, we have $A > 0$ for all $n \in (\underline{n}, \overline{n})$. The expression for the price $p(n)$ from (20) implies $p(n) = 1$ for all $n \in (\underline{n}, \overline{n})$

In contrast, when (18) does not hold — that is, $\mu - (1-\xi)(r+\kappa) > \eta\sigma$ — then (D.15) holds, determining:

$$\sigma^p(\underline{n}) = \frac{\xi\mu - (1-\xi)\lambda - \eta\sigma}{\xi(\mu - \eta\sigma)} > 0. \tag{D.15}$$

In particular, because $\sigma^p(\underline{n}) > 0 = \sigma^p(\overline{n})$, with $\sigma^p(n)$ decreasing in $n$, we have $\tilde{n} \in (\underline{n}, \overline{n})$. Further, $\sigma_n(\underline{n}) = 0$ then implies $x(n) + n > 0$ for $n = \underline{n}$. Further, $x(n) + n$ increases in $n$, which implies that $A > 0$ in the entire state space. Importantly, the expression for the price in (20) implies $p(n) < 1$ for $n < \tilde{n}$, and $p(n) = 1$ for $n \geq \tilde{n}$.

This proves Proposition 5. Also note that for $n \leq \tilde{n}$, we have $x(n) = \underline{x}$, while, otherwise, $x'(n) > 0$ — see Part II. This proves the claim of Proposition 9

**Proving Proposition 7.** Next, combining the two cases, we obtain

$$\sigma^p(n) \leq \sup_n\{\sigma^p(n)\} = \sigma^p(\underline{n}) = \begin{cases} \sigma\left(\frac{\mu-(1-\xi)(r+\kappa)-\eta\sigma}{\xi(\mu-\eta\sigma)}\right) & \text{if} \quad \mu-(1-\xi)(r+\kappa) > \eta\sigma \\ 0 & \text{if} \quad \mu-(1-\xi)(r+\kappa) \leq \eta\sigma \end{cases}$$

which implies (19), as desired. Suppose that (18) does not hold, that is, $\sigma \in \left[0, \frac{\mu-(1-\xi)(r+\kappa)}{\eta}\right]$ Then, clearly, $\sigma^p(\underline{}) = 0$ for $\sigma = 0$ or $\sigma = \frac{\mu-(1-\xi)(r+\kappa)}{\eta}$. In the interior of this interval, we can calculate

$$\frac{\partial^2\sigma^p(\underline{n})}{\partial\sigma^2} = -\frac{2\mu\eta(1-\xi)(r+\kappa)}{\xi(\mu-\eta\sigma)^3} < 0.$$

Thus, $\sigma^p(\underline{n})$ is inverted U-shaped (or hump-shaped) and thus first increases, and then decreases on $\left[0, \frac{\mu-(1-\xi)(r+\kappa)}{\eta}\right]$. This proves Proposition 7.

## D.5 Part IV: Mapping optimal controls $(x, \sigma^p)$ to fee and issuance $(dS, f)$ — Proof of Corollary 2

We have solved for the optimal controls $(x, \sigma^p)$ as functions of $n$, as well as for the price $p(n)$ characterized in (20). We also have show that the constraint $A_t \geq 0$ is met under this choice of controls. Let $s = s(n) = S/K$ the issuer's scaled issuance strategy. We solve for the issuance strategy $ds = ds(n)$ and the fee $f = f(n)$ which implement the optimal levels of $x(n)$ and $\sigma^p(n)$. We show that $ds$ and $f$ are Markovian, i.e., they are functions of $n$ only.

**Issuance Strategy.** Having characterized the price in (D.14) as a function of $n$, notice that by Ito's Lemma:

$$\mu^p(n)p(n) = p'(n)\mu_n(n) + \frac{p''(n)\sigma_n(n)^2}{2}. \tag{D.16}$$

Thus, $p(n)$, $\sigma^p(n)$, and $\mu^p(n)$ are functions of $n$ only. It then follows by market clearing that $s(n) = x(n)/p(n)$ is a function of $n$ only. For $n > \tilde{n}$, we have $p(n) = 1$ and $x'(n) > 0$, so $x(n) = s(n)$. Then, $x'(n) > 0$ and $s(n)$ increases with $n$. For $n < \tilde{n}$, we have $x(n) = \underline{x}$ (see (D.8)) and $p'(n) > 0$, so $s'(n) < 0$ and $s(n)$ decreases with $n$.

This proves Corollary 2.

**Issuance Dynamics.** We characterize the issuance dynamics, i.e., $ds = ds(n)$. For $n \in (\tilde{n}, \overline{n})$, we have $\sigma^p(n) = 0$ and $p(n) = 1$. Thus, $ds(n) = dx(n)$. Using Ito's Lemma:

$$ds(n) = \left[x'(n)\mu_n(n) + \frac{x''(n)\sigma_n(n)^2}{2}\right] dt + x'(n)\sigma_n(n)dZ.$$

Because of $x'(n) > 0$, supply $s(n)$ expands (decreases) upon a positive (negative) shock

$dZ > 0$ ($dZ < 0$).

For $n \in (\underline{n}, \tilde{n})$, we have $\sigma^p(n) > 0$ and $x(n) = s(n)p(n) = \underline{x}$. Thus, $d(s(n)p(n)) = d(sp) = 0$. Stipulate

$$ds = \mu^s dt + \sigma^s dZ$$

and calculate

$$d(sp) = sdp + pds + dsdp = s(n)p(n)\big[\mu^p(n)dt + \sigma^p(n)dZ\big] + p(n)\big[\mu^s dt + \sigma^s dZ\big] + p(n)\sigma^s\sigma^p(n)dt = 0.$$

Dividing by $p(n)$, we obtain

$$s(n)\big[\mu^p(n)dt + \sigma^p(n)dZ\big] + \big[\mu^s dt + \sigma^s dZ\big] + \sigma^s\sigma^p(n)dt = 0.$$

Thus, we get $\sigma^s = \sigma^s(n) = -s(n)\sigma^p(n) < 0$.

In addition, we solve $s(n)\mu^p(n) + \mu^s(n) + \sigma^s\sigma^p(n) = 0$ for

$$\mu^s(n) = -s(n)\mu^p(n) + s(n)(\sigma^p(n))^2$$

**Fee.** Next, inverting (5), we obtain

$$\mu^p(n) = r + f(n) - x(n)^{\xi-1} + \eta\sigma^p(n) \quad \Longleftrightarrow \quad f(n) = x(n)^{\xi-1} - \eta\sigma^p(n) - r + \mu^p(n).$$

This shows that $f(n)$ is a function of $n$ — it is uniquely determined given $x(n)$ and $\sigma^p(n)$.

Overall, we have verified that the issuance strategy is Markovian, i.e., a function of $n$. We have also verified that there exist fee structure and issuance policies that implement the desired levels of $x = x(n)$ and $\sigma^p = \sigma^p(n)$. This validates our approach to consider $(x, \sigma^p)$ instead of $(ds, f)$ as control variables.

# E Proof of Proposition 6

We prove in Parts I and II that a stationary density exists — which requires proving that $\underline{n}$ is never reached (i.e., is inaccessible). We also show in Part II that the stationary density decreases in $n$ over $(\underline{n}, \tilde{n})$.

We recall that Proposition 6 is proven under (18) not being met, that is, $\mu - (1-\xi)(r + \kappa) - \eta\sigma > 0$. We can rewrite this condition to $\xi\mu - (1-\xi)\lambda > \sigma$ — which implies $\sigma^p(\underline{n}) > 0$ and $\tilde{n} \in (\underline{n}, \overline{n})$.

## E.1 Part I — Derivation of Feller Condition and KFE

We show that a stationary density exists and is non-degenerate, which boils down to showing that the lower boundary is not attainable. For this sake, we conjecture that the lower boundary is indeed not attainable, and verify this claim.

Given our conjecture, a stationary density exists. In the interior of the state space for $n \in (\underline{n}, \overline{n})$ when $\sigma_n(n)$ is twice differentiable, the stationary density $g(n)$ satisfies the Kolmogorov forward (Fokker Planck) equation:

$$0 = -\frac{\partial}{\partial n}\big[\mu_n(n)g(n)\big] + \frac{1}{2}\frac{\partial^2}{\partial n^2}\big[\sigma_n(n)^2 g(n)\big]. \tag{E.1}$$

Define

$$\hat{G}(n) := -\mu_n(n)g(n) + \frac{1}{2}\frac{\partial}{\partial n}\big[\sigma_n(n)^2 g(n)\big], \tag{E.2}$$

Due to $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0$, we have $\hat{G}(\underline{n}) = 0$.

Next, we can integrate (E.1) from $\underline{n}$ to $n$ to obtain

$$0 = \hat{G}(n) - \hat{G}(\underline{n}) = \hat{G}(n).$$

This yields:

$$\mu_n(n)g(n) = \frac{1}{2}\frac{\partial}{\partial n}\big[\sigma_n(n)^2 g(n)\big]. \tag{E.3}$$

The ODE (E.3) satisfies the normalization condition $\int_{\underline{n}}^{\overline{n}} g(n)dn = 1$.

Define the scaled stationary density $\hat{g}(n) = \sigma_n(n)^2 g(n)$, so that

$$\hat{g}'(n) = 2\mu_n(n)g(n) = 2\hat{g}(n)\left(\frac{\mu_n(n)}{\sigma_n(n)^2}\right).$$

That is,

$$\frac{d\ln\hat{g}(n)}{dn} = \frac{\hat{g}'(n)}{\hat{g}(n)} = 2\left(\frac{\mu_n(n)}{\sigma_n(n)^2}\right).$$

The boundary $\underline{n}$ is absorbing, since $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0$ — according to Lemma 2.

A non-degenerate stationary density, with the absorbing boundary at $\underline{n}$, exists if the boundary condition $\hat{g}(\underline{n}) = 0$ can be satisfied together with $\hat{g}(\hat{n}) > 0$ for $\hat{n} > \underline{n}$; in this case, the boundary $\underline{n}$ is never reached or inaccessible. For this to happen, we need that

$$\ln\hat{g}(n') = \ln\hat{g}(\hat{n}) - 2\int_{n'}^{\hat{n}} \frac{\mu_n(n)}{\sigma_n(n)^2}dn$$

tends to $-\infty$, as $n' \downarrow \underline{n}$ for some $\hat{n} \in (\underline{n}, \overline{n})$; see Brunnermeier and Sannikov (2014) for an analogous argument in a similar context. A sufficient condition is

$$\lim_{n' \downarrow \underline{n}} \int_{n'}^{\hat{n}} \frac{\mu_n(n)}{\sigma_n(n)^2} dn = +\infty. \tag{E.4}$$

Thus, to indeed show and verify that a stationary density consists, we need to prove (E.4).

In the following two parts, we show that (E.4) is met, which then implies that $\underline{n}$ is never reached and a stationary distribution of states exists.

## E.2 Part II — Proof of (E.4)

Consider $n \in (\underline{n}, \hat{n})$ for some $\hat{n} > \underline{n}$. Without loss of generality, pick $\hat{n} < \tilde{n}$, i.e., $n < \tilde{n}$, so that

$$\sigma_n(n) = \frac{\eta}{\gamma(n)}$$

and

$$\sigma^p(n) = \frac{(n+x)\sigma}{x} - \frac{\eta}{\gamma(n)}.$$

### E.2.1 Auxiliary Result

To begin with, we rewrite the drift of $n$ for $n \in (\underline{n}, \tilde{n})$, $\mu_n(n)$, as follows:

$$\begin{aligned}
\mu_n(n) &= \mu n - \lambda \underline{x} + \underline{x}^{\xi} - \eta \underline{x} \sigma^p(n) \\
&= (\mu - \eta\sigma)(n - \underline{n}) + \mu \underline{n} - \lambda \underline{x} + \underline{x}^{\xi} - \eta\sigma(\underline{x} + \underline{n}) + \frac{\eta^2 \underline{x}}{\gamma(n)} \\
&= (\mu - \eta\sigma)(n - \underline{n}) + \mu_n(\underline{n}) + \frac{\eta^2 \underline{x}}{\gamma(n)} \geq (\mu - \eta\sigma)(n - \underline{n}),
\end{aligned} \tag{E.5}$$

where $\mu_n(\underline{n}) = 0$. As $(\mu - \eta\sigma) > 0$, to prove (E.4) it suffices to show that

$$\int_{n'}^{\hat{n}} \frac{n - \underline{n}}{\sigma_n(n)^2} dn \propto \int_{n'}^{\hat{n}} \left[(n - \underline{n})\gamma(n)^2\right] dn = \int_{n'}^{\hat{n}} \left[\frac{n - \underline{n}}{(1/\gamma(n))^2}\right] dn \tag{E.6}$$

tends to $\infty$, as $n' \to \underline{n}$. That is, for $\hat{n} \in (\underline{n}, \tilde{n})$, we show $\lim_{n' \to \underline{n}} \int_{n'}^{\hat{n}} \left[\frac{n - \underline{n}}{(1/\gamma(n))^2}\right] dn = +\infty$.

Next, we show that there exists constant $\mathcal{K} > 0$ such that

$$\frac{1}{\gamma(n)} < \mathcal{K}(n - \underline{n}) \tag{E.7}$$

for $n$ close to $\underline{n}$. Note that (E.7) implies $\gamma(n) > \frac{1}{\mathcal{K}(n - \underline{n})}$ for all $n \in (\underline{n}, \hat{n})$ for $\hat{n}$ sufficiently close to $\underline{n}$.

Given this, we obtain for $\hat{n}$ sufficiently close to $\underline{n}$ and $n' \in (\underline{n}, \hat{n})$:

$$\mathcal{B}(n') := \int_{n'}^{\hat{n}} \left[ \frac{n - \underline{n}}{(1/\gamma(n))^2} \right] dn \geq \int_{n'}^{\hat{n}} \frac{1}{\mathcal{K}^2 (n - \underline{n})} = \frac{1}{\mathcal{K}^2} \Big[ \ln(\hat{n} - \underline{n})] - \ln(n' - \underline{n}) \Big].$$

Note that $\lim_{n' \downarrow \underline{n}} \Big[ \ln(\hat{n} - \underline{n})] - \ln(n' - \underline{n}) \Big] = +\infty$.

Thus, when (E.7) holds, then $\lim_{n' \to \underline{n}} \mathcal{B}(n') = +\infty$, which implies (E.4). Thus, once we have proven (E.7) — which we do in the next part — the proof is complete.

### E.2.2 Proof of (E.7)

First, we conduct a Taylor expansion of $1/\gamma(n)$ around $n'$:

$$\frac{1}{\gamma(n)} = \frac{1}{\gamma(n')} - \frac{\gamma'(n')}{\gamma(n')^2} \cdot (n - n') + O((n - n')^2). \tag{E.8}$$

We then take the limit $n' \downarrow \underline{n}$, where $\lim_{n' \downarrow \underline{n}} \frac{1}{\gamma(n')} = 0$, due to $\sigma_n(\underline{n}) = \frac{\eta}{\gamma(\underline{n})} = 0$. Doing so, we obtain:

$$\frac{1}{\gamma(n)} = - \lim_{n' \downarrow \underline{n}} \left( \frac{\gamma'(n')}{\gamma(n')^2} \right) \cdot (n - n') + O((n - \underline{n})^2). \tag{E.9}$$

Thus, in order to establish (E.7), we need to show $\frac{\gamma'(n)}{\gamma(n)}$ remains bounded in a right-neighbourhood of $\underline{n}$.

Calculate

$$\gamma'(n) = \frac{-v'''(n) v'(n) + (v''(n))^2}{(v'(n))^2} = \gamma(n)^2 - \frac{v'''(n)}{v'(n)}. \tag{E.10}$$

Notice from (D.12) — which follows from differentiating both sides of the HJB equation (D.6) with respect to $n$ (assuming differentiability) — that

$$\begin{aligned} v'''(n) &= \frac{2}{\sigma_n(n)^2} \Big( (\rho - \mu) v'(n) - v''(n) [\mu_n(n) + \sigma \sigma_n(n)] \Big) \\ &= \frac{2\gamma(n)^2}{\eta^2} \left\{ (\rho - \mu) v'(n) - v''(n) \left[ \left( (\mu - \eta\sigma)(n - \underline{n}) + \frac{\eta^2}{\gamma(n)} \right) + \frac{\sigma\eta}{\gamma(n)} \right] \right\}. \end{aligned}$$

Hence,

$$\frac{v'''(n)}{v'(n)} = \frac{2\gamma(n)^2}{\eta^2} \Big[ \rho - \mu + \eta\sigma + \eta^2 + \gamma(n) \left[ (\mu - \eta\sigma)(n - \underline{n}) \right] \Big]. \tag{E.11}$$

Using (E.10), we obtain

$$-\frac{\gamma'(n)}{\gamma(n)^2} = -1 + \frac{v'''(n)}{v'(n)(\gamma(n))^2} \tag{E.12}$$
$$= \frac{2}{\eta^2}\Big[\rho - \mu + \eta\sigma + \gamma(n)\left[(\mu - \eta\sigma)(n - \underline{n})\right]\Big] + 1.$$

Without loss of generality, we can consider that $\gamma(n) \le \frac{1}{\hat{K}(n-\underline{n})}$ for $\hat{K} > 0$; otherwise, there exists $K'$ such that $\frac{1}{\gamma(n)} < K'(n - \underline{n})$ and the proof is complete.

Under this assumption:

$$-\frac{\gamma'(n)}{\gamma(n)^2} \le \frac{2}{\eta^2}\left[\rho - \mu + \eta\sigma + \frac{\mu - \eta\sigma}{\hat{K}}\right] + 1$$

Thus, the Taylor expansion (E.9) then implies that there exists constant $\mathcal{K} > 0$ such that

$$0 \le \frac{1}{\gamma(n)} < \mathcal{K}(n - \underline{n}) + O((n - \underline{n})^2).$$

Thus, for $n$ sufficiently close to $\underline{n}$, we have $\frac{1}{\gamma(n)} < \mathcal{K}(n - \underline{n})$, i.e., (E.7) holds and, by means of the previous findings, we obtain (E.4).

## E.3 Part III: Stationary Density is Decreasing on $(\underline{n}, \tilde{n})$

We show that the stationary density is decreasing on $(\underline{n}, \tilde{n})$. To start with, we rewrite (E.3) as follows:

$$g(n)\left[\mu_n(n) - \sigma_n(n)\sigma_n'(n)\right] = \frac{\sigma_n(n)^2 g'(n)}{2}.$$

Thus, $\mu_n(n) - \sigma_n(n)\sigma_n'(n)$ determines the sign of $g'(n)$. We have to show that $\mu_n(n) - \sigma_n(n)\sigma_n'(n) < 0$ for $n \in (\underline{n}, \tilde{n})$.

Note that on $(\underline{n}, \tilde{n})$, we have $\sigma_n(n) = \frac{\eta}{\gamma(n)}$ so that $\sigma_n'(n) = -\frac{\eta\gamma'(n)}{\gamma(n)^2}$ and

$$\sigma_n'(n)\sigma_n(n) = -\frac{\eta^2\gamma'(n)}{\gamma(n)^3}.$$

Moreover, using (E.5), we obtain for all $n \in (\underline{n}, \tilde{n})$:

$$\mu_n(n) = (\mu - \eta\sigma)(n - \underline{n}) + \frac{\eta^2}{\gamma(n)}.$$

Using (E.10), we calculate

$$\sigma_n(n)\sigma_n'(n) = -\frac{\eta^2}{\gamma(n)} + \frac{\eta^2 v'''(n)}{v'(n)\gamma(n)^3}$$

Using (E.11), we can calculate:

$$\begin{aligned}\sigma_n(n)\sigma_n'(n) &= -\frac{\eta^2}{\gamma(n)} + \frac{2}{\gamma(n)}\Big[\rho - \mu + \eta\sigma + \eta^2 + \gamma(n)\left[(\mu - \eta\sigma)(n - \underline{n})\right]\Big] \\ &= \frac{\left(2(\rho - \mu + \eta\sigma) + \eta^2\right)}{\gamma(n)} + 2(\mu - \eta\sigma)(n - \underline{n}).\end{aligned}$$

Thus,

$$\mu_n(n) - \sigma_n'(n)\sigma_n(n) = -(\mu - \eta\sigma)(n - \underline{n}) - \frac{2(\rho - \mu + \eta\sigma)}{\gamma(n)} < 0.$$

Note that because (18) does not hold, we have $\mu > \eta\sigma$. In addition, we have $\rho > \mu$. Thus, $g'(n) < 0$ for $n \in (\underline{n}, \tilde{n})$.

# F Extensions, Model Variants, and Other Results

## F.1 Capital Requirement and Narrow Banking: Proof of Proposition 10

### F.1.1 Capital Requirement

The model variant with capital requirement imposes that $n_t \geq \underline{n}^C$ — without loss of generality, we assume $\underline{n}^C > \underline{n}$ (a capital requirement $\underline{n}^C < \underline{n}$ would yield our baseline and the capital requirement is irrelevant).

The issuer chooses a strategy such that the capital requirement is met at all times. Since our model does not have jump shocks, this is feasible, as we argue below. One could microfound that the issuer always respects the capital requirement, for instance, by assuming that, upon violation, the regulator liquidates the issuer (leading to zero payoff for the issuer). Since imposing the capital requirement limits the strategy space, it is immediate that the capital requirement reduces the issuer's ex-ante payoff.

**Solution.** The solution is as in the baseline, in that the HJB equation (15) or, equivalently, (D.6) applies. The only change relative to the baseline lies in the boundary conditions. With capital requirement $n_t \geq \underline{n}^C$, the boundary conditions become the standard smooth pasting and super contact conditions for the upper boundary — that is, $v'(\overline{n}) - 1 = v''(\bar{}) = 0$ — plus the boundary condition

$$\lim_{n \downarrow \underline{n}^C} \gamma(n) = +\infty, \tag{F.1}$$

where, as we recall, $\gamma(n) = \frac{-v''(n)}{v'(n)}$ is the issuer's effective risk-aversion. As we show, the boundary condition (F.1) ensures that net worth $n_t$ never falls below $\underline{n}^C$ and violates the capital requirement. Intuitively, it reflects that the issuer becomes prohibitively risk-averse versus violating the capital requirement, for reasons argued above.

The optimal controls are characterized in Appendix D.3 and Appendix D. We now prove that the boundary $\underline{n}^C$ is never reached in the interesting cases that the capital requirement stipulates over-collateralization (i.e., $\underline{n}^C$) or that (18) does not hold — i.e., the baseline features instability and debasement.

**Proof that $\underline{n}^C$ is never reached.** We show that $\underline{n}^C > \underline{n}$ is never reached and $n_t > \underline{n}^C$ for all times $t$ when at least one of the two conditions holds: (1) (18) does *not* hold or (2) $\underline{n}^C$ is close to zero.

In doing so, we assume $\sigma > 0$ and that the capital requirement exceeds the lower boundary from (C.4).

Conjecture $\sigma^p(n) > 0$ in a neighbourhood of $\underline{n}^C$. It follows from (D.9) and (F.1) that

$$\sigma^p(\underline{n}^C) := \lim_{n \downarrow \underline{n}^C} \sigma^p(n) = \sigma + \frac{\sigma \underline{n}^C}{\underline{x}},$$

where $\underline{x}$ is from (D.8).

As $\underline{n}^C > \underline{n}$, it follows that $\sigma^p(\underline{n}^C) > 0$, verifying above conjecture. Also, note that the volatility of $dn$ vanishes as $n$ approaches $\underline{n}^C$, in that $\lim_{n \downarrow \underline{n}^C} \sigma_n(n) = \lim_{n \downarrow \underline{n}^C} \frac{\eta}{\gamma(n)} = 0$.

The claim follows from showing $\mu_n(\underline{n}^C) = \lim_{n \downarrow \underline{n}^C} \mu_n(n) > 0$. Analogously to (E.5), we calculate for $\underline{n}^C > \underline{n}$:

$$\mu_n(\underline{n}^C) = \mu \underline{n}^C - \lambda \underline{x} + \underline{x}^\xi - \eta \underline{x} \sigma^p(\underline{n}^C) = (\mu - \eta\sigma)\underline{n}^C + \underline{x}^\xi - (\lambda + \eta\sigma)\underline{x}.$$

Clearly, $\underline{x}^\xi - (\lambda + \eta\sigma)\underline{x} > 0$. Thus, when $\underline{n}^C = 0$, then $\mu_n(\underline{n}^C) > 0$.

When (18) does not hold, then $\mu > \eta\sigma$, and we can, analogously to (E.5), rewrite $\mu_n(\underline{n}^C) = (\mu - \eta\sigma)(\underline{n}^C - \underline{n}) > 0$.

### F.1.2 Narrow banking

Our model nests the special case of narrow banking upon setting $\underline{n}^C = 0$ — that is, the issuer has positive net worth — and $\sigma = 0$ — that is, reserve assets are risk-free. Then, clearly, $\sigma^p(n) = 0$ for all $n \geq 0$ and $p(n) = 1$.

**Solution.** The HJB equation (15) then reduces to

$$\rho v(n) = \max_{x \geq -n} v'(n)\big(\mu n - \lambda x + x^{\xi}\big)$$

for all $n \geq 0$ where $v'(n) \geq 1$. The optimization with respect to $x$ yields

$$x(n) = x^* = \left(\frac{\xi}{\lambda}\right)^{\frac{1}{1-\xi}}$$

Without risk, there is no need to hold any buffer, so that Similarly to above, it can be shown that $\overline{n} = \underline{n}$.

We now prove that $\overline{n} = \underline{n} = 0$. Suppose to the contrary $\overline{n} > 0$. Then, $x(\overline{n}) = x^*$ and $v'(\overline{n}) = 1$ and

$$v(\overline{n}) = \overline{n} + \frac{-\lambda x(\overline{n}) + x(\overline{n})^{\xi} - (\rho - \mu)\overline{n}}{\rho}.$$

Due to $\rho > \mu$, it then follows that there exists $\varepsilon > 0$ such that $\overline{n} - \varepsilon > \underline{n}$, $x(\overline{n} - \varepsilon) = x^*$, and

$$v(\overline{n} - \varepsilon) + \varepsilon = \varepsilon + \frac{v'(\overline{n} - \varepsilon)}{\rho}\big(\mu(\overline{n} - \varepsilon) - \lambda x + x^{\xi}\big) \geq \frac{(\rho - \mu)\varepsilon}{\rho} + v(\overline{n}),$$

where we used the HJB equation at $\overline{n} - \varepsilon$ to substitute for $v(\overline{n} - \varepsilon)$ and that $v'(\overline{n} - \varepsilon) \geq 1$. Therefore, $v(\overline{n} - \varepsilon) + \varepsilon > v(\overline{n})$. Thus, the issuer can achieve strictly higher payout by paying out $\varepsilon > 0$, contradicting the hypothesis that $\overline{n}$ is the payout boundary.

Thus, whenever $n > 0$, the issuer immediately consumes $n$ dollars and continues with zero at net worth. At $n = 0$, the issuer's value function under narrow banking then reads:

$$v^{Narrow} = \frac{1}{\rho}\left\{\left(\frac{\xi}{\lambda}\right)^{\frac{\xi}{1-\xi}} - \lambda\left(\frac{\xi}{\lambda}\right)^{\frac{1}{1-\xi}}\right\} = \frac{(1-\xi)}{\rho}\left(\frac{\xi}{\lambda}\right)^{\frac{\xi}{1-\xi}}.$$

For $n \geq 0$, the value function then becomes $v^{Narrow} + n$. The volatility of net worth is zero and the drift equals $\mu_n(0) = \rho v^{Narrow} > 0$, i.e., under narrow banking, the issuer continuous consumes according to $dy = \mu_n(0)dt$.

Finally, scaled user welfare under narrow banking becomes (see next Section for details)

$$w^{Narrow} = \frac{(1-\xi)}{\xi \hat{r}}(x^*)^{\xi} = \frac{(1-\xi)}{\xi \hat{r}}\left(\frac{\xi}{\lambda}\right)^{\frac{\xi}{1-\xi}},$$

where $\hat{r}$ is the discount rate applied to calculate user welfare.

## F.2 Calculating Welfare

Given the optimal controls $(x(n), \sigma^p(n))$, we can calculate user welfare. User welfare — like issuer valuation — scales with $K$, in that $W_t = Kw(n)$ for $n_t = n$. Scaled user welfare can be written as

$$w(n) = \mathbb{E}\left[\int_t^\infty e^{-\hat{r}(s-t)}\Big(u(x_s) - x_s f_s + (\mu_s^p - r)x_s\Big)ds\Big)\right] \tag{F.2}$$

where we discount at rate $\hat{r}$ — which is a flexible choice. It can be equal to $r$, but does not necessarily have to be. The rate $\hat{r}$ reflects a "social planner's" time preference, as opposed to necessarily the risk-free rate.

Thus, user welfare is expected discounted value of (i) convenience utility $u(x_s)$ and (ii) capital gains (excess returns) $(\mu_p^s - r)x_s$ from stablecoins, net of (iii) fees levied, $x_s f_s$.

Next, using (5), we get $x_t^{\xi-1} = r + f_t - \eta|\sigma_t^p| + \mu_t^p$. Inserting $u(x_t) = \frac{x_t^\xi}{\xi} - x_t\eta|\sigma_t^p|$, we calculate for the flow utility term in round brackets:

$$u(x_t) - x_t f_t + (\mu_t^p - r)x_t = \frac{x_t^\xi}{\xi} - \eta|\sigma_t^p| - x_t x_t^{\xi-1} = \frac{(1-\xi)x_t^\xi}{\xi}. \tag{F.3}$$

Having solved for $x(n), \sigma^p(n)$, we obtain $\mu_n(n)$ and $\sigma_n(n)$. Then, the integral expression (F.2) together with (F.3) implies that welfare $w(n)$ satisfies the ODE:

$$\hat{r}w(n) = \frac{(1-\xi)x(n)^\xi}{\xi} + w'(n)\mu_n(n) + \frac{w''(n)\sigma_n(n)^2}{2}. \tag{F.4}$$

This ODE is solved subject to $w'(\overline{n}) = 0$, since $n$ reflects a the upper boundary $\overline{n}$. Moreover, at the lower boundary $\underline{n}$, the drift and volatility of $n$ vanish so that $w(\underline{n}) = \frac{(1-\xi)\underline{x}^\xi}{\xi r}$, where $\underline{x}$ is from (D.8).

With capital requirement, i.e., $n^C > \underline{n}$, the ODE remains unchanged, but the boundary condition at the lower boundary changes to:

$$\lim_{n \downarrow n^C} \hat{r}w(n) = \frac{(1-\xi)\underline{x}^\xi}{\xi} + \lim_{n \downarrow n^C} w'(n)\mu_n(n)$$

## F.3 Equity Issuance

We sketch the solution with costly equity issuance.

Since raising equity entails only a fixed cost $F$ (though a variable cost component could easily be introduced) and new equity investors are risk-neutral and competitive (i.e., they require shares worth of one dollar for each dollar contributed), it is optimal for the issuer to raise enough equity to restore net worth to the target level. Indeed, when the issuer raises equity and incurs the fixed cost, each additional dollar raised requires giving up one dollar of ownership. However, doing so increases the issuer's valuation by $v'(n) \geq 1$ dollars, with equality only when net worth reaches the target level, $n = \overline{n}$. As is standard, the issuer raises equity when $n$ reaches a lower threshold $\underline{n}^E$.

The argument to determine this lower boundary follows Bolton, Wang, and Yang (2025). We consider the scaled fixed cost of equity issuance as $f = \frac{F}{K}$. At the equity issuance boundary $\underline{n}_E$, the issuer's continuation payoff becomes

$$v(\overline{n}) - (\overline{n} - \underline{n}_E) - f,$$

as it continues with valuation $v(\overline{n})$ post-issuance, yet must raise $(\overline{n} - \underline{n}_E) + F$ dollars of equity The issuer optimally delays as much as possible to avoid incurring the fixed cost. Thus, at the equity issuance boundary, the issuer will be indifferent between issuing equity and continuing, and liquidating, in that:

$$v(\underline{n}^E) = v(\overline{n}) - f - (\overline{n} - \underline{n}^E) = 0.$$

Using the HJB equation (15) and $v'(\overline{n}) - 1 = v''(\overline{n}) = 0$, we get

$$v(\overline{n}) - \overline{n} = \frac{(x^*)^\xi - \lambda x^* - (\rho - \mu)\overline{n}}{\rho},$$

for $x(\overline{n}) = x^*$ from (D.11). Thus, we can solve for the equity issuance boundary $\underline{n}^E$ through:

$$\underline{n}^E = -\left(\frac{(x^*)^\xi - \lambda x^* - (\rho - \mu)\overline{n}}{\rho} - f\right).$$

We note now that the issuer can either proceed with raising equity at the lower boundary — in which case the lower boundary coincides with $\underline{n}^E$ — or can adopt the baseline strategy — in which case there is no equity issuance and the lower boundary equals $\underline{n}$.

**Solution.** The issuer optimally chooses the option that gives itself more "financial leeway." In particular, the lower boundary in the state space then becomes

$$\underline{n}' = \min\{\underline{n}, \underline{n}^E\},$$

where $\underline{n}$ is from (C.4). The analysis goes through as before with this modified lower boundary. Specifically, the value function satisfies the HJB equation (15). This HJB equation is then solved subject the boundary conditions

$$v'(\overline{n}) - 1 = v''(\overline{n}) = v(\underline{n}') = 0.$$

The controls $(x, \sigma^p)$ are determined according to the HJB equation analogous to the baseline — see Appendix D.

**Price with Equity Issuance.** Since equity issuance is lumpy and induces a discrete jump in net worth, implementing the stablecoin price requires some care. We now specify the price function $p(n)$ satisfying equation (20), given the controls $(x, \sigma^p)$. As in the baseline case, we focus on a price function that satisfies $p(\overline{n}) = 1$—that is, the peg is maintained at the target net worth level. However, when $\underline{n}' = \underline{n}^E$ and the solution involves equity issuance, we may have $\lim_{n \downarrow \underline{n}^E} \sigma^p(n) > 0$ and $p(\underline{n}^E) = \lim_{n \downarrow \underline{n}^E} p(n) < 1$. In this case, as $n$ approaches $\underline{n}^E$, a discontinuous jump in price arises. Absent further assumptions, this discontinuity creates an arbitrage opportunity as $n \to \underline{n}^E$. To rule out such arbitrage, we assume that users must surrender a portion of their stablecoin holdings—that is, their holdings $S$, or equivalently their scaled holdings $s$, are reduced. This is analogous to debt forgiveness in bankruptcy, where creditors relinquish part of their claims to enable the firm to continue operating. Loosely speaking, in our context, users similarly forgo part of their claims to facilitate the recapitalization of the issuer and allow it to continue operations, thereby avoiding liquidation and the complete loss of stablecoin value.

To eliminate arbitrage, we assume that, per unit of stablecoin held, a user retains only a fraction $p(\underline{n}^E)$. This ensures that the dollar value of one unit of stablecoin remains unchanged across the equity issuance event, effectively accounting for the "loss." This adjustment prevents arbitrage and supports the pricing rule in equation (20). We omit further implementation details for brevity.

## F.4 Dynamic Risk Taking: Section 5.2

We now provide the heuristic solution to the model variant with dynamic risk taking from Section 5.2

**Dynamics of $dn$.** We can rewrite the dynamics of $n_t$ according to:

$$dn_t = \left[(\mu + \omega_t \alpha) n_t + x_t^{\xi} - \lambda x_t - \eta x_t |\sigma_t^p|\right] dt + \left[(\sigma + \omega_t \sigma_\alpha)(n_t + x_t) - x_t \sigma_t^p\right] dZ_t - dy_t. \quad \text{(F.5)}$$

where $\omega_t \in [w_L, w_H]$ for constants $w_L \leq w_H$. Above law of motion defines drift and volatility of $dn$, i.e., $\mu_n(n)$ and $\sigma_n(n)$.

**Lower Boundary $\underline{n}$.** The same logic as in Lemma 2 applies, albeit with an additional control $\omega_t$. Analogously to Lemma 2 from the baseline, the lower boundary in this model variant is defined as:

$$\underline{n} := \inf\left\{n \in \mathbb{R} : \max_{x\in[0,\overline{x}],\omega\in[w_L,w_H],\sigma^p\geq 0} \mu_n(n_t) \geq 0 \quad \text{s.t.} \quad \sigma_n(n_t) = 0\right\}. \tag{F.6}$$

Consider $\sigma^p(\underline{n}) > 0$. Then, $\sigma_n(\underline{n}) = 0$ implies $x\sigma^p = (\sigma + \omega\sigma_\alpha)(x + n)$ and

$$\mu_n(\underline{n}) = \mu\underline{n} - \lambda\underline{x} + \underline{x}^\xi + \omega(\underline{x} + \underline{n})\alpha - \eta(\sigma + \omega\sigma_\alpha)(\underline{x} + \underline{n})$$

for $x(\underline{n}) =: \underline{x}$.

Optimizing $\mu_n(\underline{n})$ over $\omega$, we get

$$\omega(\underline{n}) = \underline{\omega} := w_L + (w_H - w_L)\mathbb{I}\{\alpha \geq \eta\sigma_\alpha\}.$$

Next, we can optimize over $\underline{x}$ to obtain:

$$\underline{x} = \left(\frac{\xi}{\lambda + \eta\sigma - \underline{\omega}(\alpha - \eta\sigma_\alpha)}\right)^{\frac{1}{1-\xi}}$$

Finally, we can solve for $\underline{n}$ by solving $\mu_n(\underline{n})$ under $\omega = \underline{\omega}$ and $x = \underline{x}$:

$$\underline{n} = \left(\frac{\underline{x}^\xi - \hat{\lambda}\underline{x} - \eta\sigma\underline{x}}{\hat{\mu} - \eta\sigma}\right)$$

where

$$\hat{\mu} := \mu + \underline{\omega}(\alpha - \eta\sigma_\alpha) \quad \text{and} \quad \hat{\lambda} := \lambda - \underline{\omega}(\alpha - \eta\sigma_\alpha).$$

As in the baseline, we also consider the second case where $\underline{n} + x(\underline{n}) = 0$.

Overall, we obtain:

$$\underline{n} = \begin{cases} -\left(\frac{\underline{x}^\xi - \hat{\lambda}\underline{x} - \eta\sigma\underline{x}}{\hat{\mu} - \eta\sigma}\right) & \text{if} \quad \hat{\mu} - (1-\xi)(r+\kappa) > \eta\sigma \\ -\left(\frac{1}{\hat{\mu} + \hat{\lambda}}\right)^{\frac{1}{1-\xi}} & \text{if} \quad \hat{\mu} - (1-\xi)(r+\kappa) \leq \eta\sigma \end{cases} \tag{F.7}$$

Expression F.7 illustrates that the lower boundary is determined analogously to the baseline and risk-taking results in a transformation of the parameters $\mu$ and $\lambda$ (toward $\hat{\mu}$ and $\hat{\lambda}$, respectively). In addition, the level of $\underline{x}$ is determined differently than in the baseline.

**HJB Equation.** The issuer's value function satisfies $V_t = Kv(n_t)$. Given the law of motion of net worth in (F.5), the scaled value function $v(n)$ solves on the endogenous state space $(\underline{n}, \overline{n})$ with $dD = 0$:

$$\rho v(n) = \max_{\sigma^p, x, \omega \in [w_L, w_H]} \Big\{ v'(n)\Big(\mu n - \lambda x + x^\xi - \eta x|\sigma^p| + \omega(x+n)\alpha\Big) + \frac{v''(n)}{2}\Big((\sigma + \omega\sigma_\alpha)(n+x) - x\sigma^p\Big)^2 \Big\}. \tag{F.8}$$

The usual smooth pasting and super contact conditions apply at the upper boundary $\overline{n}$, that is, $v'(\overline{n}) - 1 = v''(\overline{n}) = 0$. One can show that the value function is strictly concave. In addition, we have $v(\underline{n}) = 0$.

We now determine the optimal controls in the interior of the state space. Also recall the definition of the issuer's effective risk aversion, i.e., $\gamma(n) = \frac{-v''(n)}{v'(n)}$.

**Optimization with respect to $\sigma^p(n)$.** As in the baseline, we can solve for optimal $\sigma^p(n)$ via

$$\sigma^p(n) = \max\left\{0, \frac{(\sigma + \omega\sigma_\alpha)(x+n)}{x} - \frac{\eta}{\gamma(n)x}\right\}.$$

One can show that when $\sigma^p(n) > 0$, we have $\omega = \underline{\omega}$ and $x = \underline{x}$.

**Optimization with respect to $\omega$.** The optimization with respect to $\omega$ yields:

$$\omega = \begin{cases} w_L & \text{if} \quad \alpha < \sigma_\alpha\sigma_n(n)\gamma(n) \\ \hat{\omega} \in [w_L, w_H] & \text{if} \quad \alpha = \sigma_\alpha\sigma_n(n)\gamma(n) \\ w_H & \text{if} \quad \alpha > \sigma_\alpha\sigma_n(n)\gamma(n). \end{cases}$$

The choice of $\omega$ or, equivalentl, $\Omega = \omega(x+n)$ simplifies as follows.

Note that when $\alpha \geq \eta\sigma_\alpha$, setting $\omega = w_H$ is optimal in any state $n$, in that $\underline{\omega} = w_H$. To see this, observe that the issuer could always raise $\omega(x+n)$ by one marginal unit and raise $x\sigma^p$ by $\sigma_\alpha$ units. This change leaves, by construction, the volatility $\sigma_n(n)$ unchanged, and raises the drift $\mu_n(n)$ by $\alpha - \eta\sigma_\alpha$. It is therefore optimal when $\alpha \geq \eta\sigma_\alpha$.

Otherwise, when $\alpha < \eta\sigma_\alpha$, then $\omega = w_L = 0$ whenever $\sigma^p(n) > 0$, i.e., for $n < \tilde{n}$. To see this, suppose to the contrary that $\omega(n) > 0$ and $\sigma^p(n) > 0$. Then, the issuer could lower $\omega(n)(x+n)$ by one (marginal) unit and $x\sigma^p(N)$ by $\sigma_\alpha$ units, whilst leaving volatility $\sigma_n(n)$ unchanged. However, this change raises the drift by $\eta\sigma_\alpha - \alpha > 0$, and thus is profitable.

**Optimization with respect to $x$.** When $\sigma^p(n) > 0$, then $x = \underline{x}$. When $\sigma^p = 0$, the first order condition with respect to $x$ becomes

$$-\lambda x + \xi x^{\xi-1} = (\sigma + \omega\sigma_\alpha)\Big((\sigma + \omega\sigma_\alpha)(x+n)\Big)\gamma(n).$$

**Price.** As in the baseline, the price satisfies $p(n) = \exp\left(-\int_n^{\tilde{n}} \frac{\sigma^p(n)}{\sigma_n(n)} dn\right)$.

## F.5 Demand Shocks — Solution and Proof of Proposition F.1

We extend the model by introducing shocks to the stablecoin demand scaler, $K_t = K$:

$$\frac{dK_t}{K_t} = \mu_K dt + \sigma_K dZ_t^K, \tag{F.9}$$

where $\mu_K$ is a constant, $\sigma_K \geq 0$, and $Z_t^K$ is a standard Brownian motion. The correlation between the demand shock, $dZ_t^K$, and the shock to the issuer's reserve assets, $dZ_t$, is $\phi dt$ — that is, $dZ_t \cdot dZ_t^K = \phi dt$.

The users take as given the price process:

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t + \sigma_{K,t}^p dZ_t^K, \tag{F.10}$$

where drift $\mu_t^p$, and the shock loadings $\sigma_t^p$ and $\sigma_{K,t}^p$ are endogenously determined. We define

$$\Sigma_t^p := \sqrt{(\sigma_t^p)^2 + (\sigma_{K,t}^p)^2 + 2\phi\sigma_t^p\sigma_{K,t}^p} \tag{F.11}$$

the instantaneous volatility of $dp_t/p_t$. Define the scaled stablecoin value as $x_t = \frac{X_t}{K_t}$, analogously to the baseline.

In analogy to (2), the users' utility from holding stablecoins is given by $K_t u(x_t)$, where $u(x_t)$ is defined as follows

$$u(x_t) = \left(\frac{x_t^\xi}{\xi} - x_t\eta|\Sigma_t^p|\right). \tag{F.12}$$

Going through the same steps as in Appendix B (and replacing $\sigma_t^p$ by $\Sigma_t^p$), the users' optimal demand for stablecoin is scaled with $K_t$, i.e., $X_t = K_t x_t$ with

$$x_t = \left(\frac{1}{r - \mu_t^p + \eta|\Sigma_t^p| + f_t}\right)^{\frac{1}{1-\xi}}. \tag{F.13}$$

This expression is analogous to the stablecoin demand (5) in the baseline model. Also, recall that $X_t = S_t p_t$ by market clearing.

### F.5.1 Dynamics of $n$ and $X_t$

Next, we invert $X_t = K_t \left(\frac{1}{r+f_t-\mu_t^p+\eta|\Sigma_t^p|}\right)^{\frac{1}{1-\xi}}$ to obtain

$$\mu_t^p = \left(-K_t^{1-\xi}X_t^{\xi-1} + f_t + r + \eta|\Sigma_t^p|\right). \tag{F.14}$$

That is,

$$dp_t = (r+f_t)p_t dt - p_t\Big(X_t^{\xi-1}K_t^{1-\xi} - \eta|\Sigma_t^p|\Big)dt + p_t\sigma_t^p dZ_t + p_t\sigma_{K,t}^p dZ_t^K. \tag{F.15}$$

Next, multiply both sides of (F.15) by $S_t$ and use $X_t = S_t p_t$ (market clearing) to obtain

$$S_t dp_t = rX_t dt - \Big(K_t^{1-\xi}X_t^{\xi} - f_t X_t - \eta X_t|\Sigma_t^p|\Big)dt + X_t\sigma_t^p dZ_t + X_t\sigma_{K,t}^p dZ_t^K.$$

Ito's product rule implies $d(S_t p_t) = dS_t p_t + p_t dS_t + dS_t dp_t$. Therefore, we calculate

$$dX_t = (r+f_t)X_t dt - \Big(K_t^{1-\xi}X_t^{\xi} - \eta X_t|\Sigma_t^p|\Big)dt + X_t\sigma_t^p dZ_t + X_t\sigma_{K,t}^p dZ_t^K + dS_t(p_t+dp_t). \tag{F.16}$$

Using the dynamics of reserve assets

$$dA_t = A_t(\mu dt + \sigma dZ_t) + f_t X_t dt + dS_t(p_t + dp_t) - \kappa X_t dt - dY_t,$$

we calculate the dynamics of net worth via $dN_t = dA_t - dX_t$:

$$\begin{aligned} dN_t = &\Big[\mu N_t - \lambda X_t + X_t^{\xi}K_t^{1-\xi} - \eta X_t|\Sigma_t^p|\Big]dt \\ &+ \big[\sigma(N_t+X_t) - X_t\sigma_t^p\big]dZ_t - X_t\sigma_{K,t}^p dZ_t^K - dY_t. \end{aligned} \tag{F.17}$$

Note that one could also write down the law of motion of net worth more directly as accounting identity. Also, recall that $\lambda = \mu - (r+\kappa)$.

We omit time subscripts in what follows. Next, we calculate the dynamics of scaled liquid net worth $n$ by combining aforementioned law of motion of $N$ with the law of motion of $K$ in (F.9). For this sake, calculate using Ito's Lemma

$$dn = \frac{dN}{K} - \frac{n}{K}dK - \frac{1}{K^2} < dN, dK > + \frac{2N}{2K^3} < dK, dK >$$

As such, using (F.17) and (F.9), as well as $dy = \frac{dY}{K}$, we calculate

$$dn = \Big[(\mu - \mu_K)n - x\lambda + x^{\xi} - \eta x \Sigma^p\Big]dt + \sigma_K\big[n\sigma_K + x\sigma_K^p - \phi\big(\sigma(x+n) - x\sigma^p\big)\big]dt$$
$$- \big[n\sigma_K + x\sigma_K^p\big]dZ^K + \big[\sigma(n+x) - x\sigma^p\big]dZ - dy. \tag{F.18}$$

We now denote the volatility on $dZ$ by $\sigma_n(n) = \sigma(x+n) - x\sigma^p$ and the volatility on $dZ^K$ by $\sigma_n^K(n) = -(n\sigma_K + x\sigma_K^p)$. We denote the drift of $dn$ by $\mu_n(n)$.

### F.5.2 Issuer Problem and Solution

The issuer chooses its issuance $(dS_t)_{t\geq 0}$, the fee policy $(f_t)_{t\geq 0}$, and $(dY_t)_{t\geq 0}$ maximizes

$$V_0 = \mathbb{E}\left[\int_0^{\infty} e^{-\rho t} dY_t\right],$$

subject to (F.17), (F.9), $A_t \geq 0$, and $dY_t \geq 0$. As in the baseline, we consider scaled quantities and the payoff-relevant state (besides scaling) is $n = n_t$. Since $(dS_t, f_t)$ affects $dn_t$ from (F.18) only via $(x_t, \sigma_t^p, \sigma_{K,t}^p)$, we can solve the issuer's problem by considering a relaxed problem, where the issuer chooses $(x_t, \sigma_t^p, \sigma_{K,t}^p, dy_t)_{t\geq 0}$. One can then show that this choice can be implemented, analogously to the arguments from Appendix D.5. See Appendix F.5.6.

In what follows, we restrict attention to strategies that implement $\sigma_t^p, \sigma_{K,t}^p \geq 0$. This assumption resembles a standard monotonicity assumption in security design and optimal contracting: The stablecoin price must load positively on the shocks, in that it can never decrease following positive demand or reserve shocks.[21]

The following Proposition summarizes the key findings of this model variant.

**Proposition F.1** (Solution with Demand Shocks)**.** *With demand shocks, the issuer's value function solves* (F.20) *subject to* $v(\underline{n}) = v'(\overline{n}) - 1 = v''(\overline{n}) = 0$. *The following holds:*

1. *For* $n > 0$, *we have* $\sigma_K^p = 0$ *and* $\sigma^p = \sigma^p(n) = \max\left\{0, \frac{\sigma(x+n) - \phi n \sigma_K}{x} - \frac{\eta}{x\gamma(n)}\right\}$. *Otherwise, for* $n < 0$, *price volatility terms satisfy:*

$$\sigma^p = \max\left\{0, \frac{\sigma(n+x)}{x}\left(1 - \frac{\eta}{\gamma(n)\pi(n)}\right)\right\}$$
$$\sigma_K^p = \max\left\{0, -\frac{n\sigma_K}{x}\left(1 - \frac{\eta}{\gamma(n)\pi(n)}\right)\right\},$$

[21] In principle, negative values of $\sigma_K^p$ could be optimal when $n > 0$ and $\eta$ is sufficiently small. Although such cases cannot be ruled out analytically, they appear to be corner or edge cases. Accordingly, we do not consider them further. Moreover, a negative $\sigma_K^p$ would imply that prices decline in response to a positive demand shock—a counterfactual scenario we do not highlight.

*where* $\gamma(n) = \frac{-v''(n)}{v'(n)}$ *and* $\pi(n)$ *is defined in* (F.23).

2. *The stablecoin price satisfies*

$$p(n) = \exp\left( - \int_{n}^{\tilde{n}} \frac{\gamma(\nu)\pi(\nu) - \eta}{\eta x(\nu)} d\nu \right),$$

*where* $\tilde{n} = \inf\{n' \in (\underline{n}, \overline{n}) : \sigma^p(n) = \sigma^p_K(n) = 0$ *for all* $n \geq n'\}$.

### F.5.3 HJB Equation and Solution Details

We now solve the dynamic optimization and derive the HJB equation. Doing so, we conjecture and verify that $V(N,K) = v(n)K$. To solve for the issuer's value function, we use shorthand notation $V = V(N,K)$ and denote partial derivatives by subscripts. We conjecture that in the interior of the endogenous state space, there is no consumption, i.e., $dD = 0$. Then, by dynamic programming, the value function $V$ solves in the interior of the state space the following HJB equation:

$$\begin{aligned}\rho V = \max_{X,\sigma^p,\sigma^p_K} \Bigg\{ & V_K K \mu_K + \frac{V_{KK}(\sigma_K K)^2}{2} + V_N \Big[\mu N - X\lambda + K^{1-\xi} X^{\xi} - \eta X |\Sigma^p|\Big] \\ + \frac{V_{NN}}{2} & \Big( \big[\sigma(N+X) - X\sigma^p\big]^2 + (X\sigma^p_K)^2 - 2\phi\sigma^p_K X \big[\sigma(N+X) - X\sigma^p\big] \Big) \\ & + V_{NK}\sigma_K K \Big( - X\sigma^p_K + \phi\big[\sigma(N+X) - X\sigma^p\big]\Big) \Bigg\}. \end{aligned} \tag{F.19}$$

Next, calculate for $V = V(N,K) = Kv(n)$ the derivatives $V_K = v(n) - v'(n)n$, $V_{NN} = v''(n)/K$, $V_{KK} = v''(n)n^2/K$, $V_{KN} = -v''(n)n/K$. Inserting these relations back in to (F.19) and simplifying yields in the interior of the state space:

$$\begin{aligned}(\rho - \mu_K)v(n) = \max_{x,\sigma^p \geq 0, \sigma^p_K \geq 0} \Bigg\{ & v'(n)\big[(\mu - \mu_K)n - \lambda x + x^{\xi} - \eta x |\Sigma^p|\big] \\ & + v''(n) \left[ \frac{(n\sigma_K + x\sigma^p_K)^2}{2} + \frac{(\sigma(n+x) - x\sigma^p)^2}{2} \right] \\ & - v''(n)\phi(n\sigma_K + x\sigma^p_K)(\sigma(n+x) - x\sigma^p) \Bigg\}. \end{aligned} \tag{F.20}$$

The state space is characterized by an interval $(\underline{n}, \overline{n})$ with endogenous upper and lower boundaries. The upper boundary $\overline{n}$ is a payout boundary and (scaled) consumption/dividends $dD$ causes $n$ to reflect at $\overline{n}$. The upper boundary satisfies the standard smooth pasting and super contact conditions, i.e.,

$$v'(\overline{n}) - 1 = v''(\overline{n}) = 0.$$

The lower boundary is determined analogously to the baseline as follows:

$$\underline{n} := \inf \left\{ n \in \mathbb{R} : \max_{x \in [0,\overline{x}], \sigma^p \geq 0} \mu_n(n_t) \geq 0 \quad \text{s.t.} \quad \sigma_n(n_t) = \sigma_n^K(n_t) = 0 \right\},$$

where $\mu_n(n)$, $\sigma_n(n)$, and $\sigma_n^K(n)$ are defined in (F.18). We have $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = \sigma_n^K(\underline{n}) = 0$, and thus $v(\underline{n}) = 0$.

We now turn to analyze the optimal controls and characterize them as functions of $n$ only.

### F.5.4 Controls

We can restate the optimization in (F.20) by dividing both sides by $v'(n)$ and using $\gamma(n) = \frac{-v''(n)}{v'(n)}$. The optimization in (F.20) becomes then equivalent to:

$$\begin{aligned} &\max_{x, \sigma^p \geq 0, \sigma_K^p \geq 0} \Bigg\{ (\mu - \mu_K) n - x\lambda + x^\xi - x\eta |\Sigma^p| \Bigg] \\ &- \gamma(n) \left[ \frac{(n\sigma_K + x\sigma_K^p)^2}{2} + \frac{(\sigma(n+x) - x\sigma^p)^2}{2} - \phi(n\sigma_K + x\sigma_K^p)(\sigma(n+x) - x\sigma^p) \right] \Bigg\} \end{aligned}$$

where $\Sigma^p$ is a function of $\sigma^p$ and $\sigma_K^p$.

Suppose that $\sigma^p > 0$ and $\sigma_K^p > 0$. Then, they solve the first-order conditions:

$$\begin{aligned} x\sigma^p &= \sigma(n+x) - \frac{\eta}{\gamma(n)} \left( \frac{\sigma^p + \phi\sigma_K^p}{\Sigma^p} \right) - \phi(n\sigma_K + x\sigma_K^p), \\ x\sigma_K^p &= -n\sigma_K - \frac{\eta}{\gamma(n)} \left( \frac{\sigma_K^p + \phi\sigma^p}{\Sigma^p} \right) + \phi(\sigma(n+x) - x\sigma^p). \end{aligned}$$

We can rearrange to obtain:

$$\begin{aligned} \left( x + \frac{\eta}{\gamma(n)\Sigma^p} \right) \sigma^p &= \sigma(n+x) - \eta \left( \frac{\phi\sigma_K^p}{\gamma(n)\Sigma^p} \right) - \phi(n\sigma_K + x\sigma_K^p) \\ \left( x + \frac{\eta}{\gamma(n)\Sigma^p} \right) \sigma_K^p &= -n\sigma_K - \eta \left( \frac{\phi\sigma^p}{\gamma(n)\Sigma^p} \right) + \phi(\sigma(n+x) - x\sigma^p) \end{aligned}$$

Thus, we can solve:[22]

$$\sigma^p = \frac{\Sigma^p\, \sigma(n+x)}{\Sigma^p\, x + \frac{\eta}{\gamma(n)}}, \qquad \sigma_K^p = -\frac{\Sigma^p\, n\sigma_K}{\Sigma^p\, x + \frac{\eta}{\gamma(n)}} \tag{F.21}$$

It follows that $\sigma_K^p = 0$ whenever $n \geq 0$. Under these circumstances, we get $\Sigma^p = \sigma^p$ and

$$\sigma^p(n) = \max\left\{0, \frac{\sigma(x+n) - \phi n \sigma_K}{x} - \frac{\eta}{\gamma(n)x}\right\} \tag{F.22}$$

Next, we define

$$\pi(n) := \sqrt{\sigma^2(n+x)^2 + n^2(\sigma_K)^2 - 2\phi\, n\, \sigma(n+x)\sigma_K}. \tag{F.23}$$

Inserting (F.11) in (F.21) and using our definition of $\pi(n)$, as well as after accounting for $\sigma^p, \sigma_K^p \geq 0$, we obtain for $n < 0$:

$$\sigma^p = \sigma^p(n) = \max\left\{0, \frac{\sigma(n+x)}{x}\left(1 - \frac{\eta}{\gamma(n)\pi(n)}\right)\right\} \tag{F.24}$$

$$\sigma_K^p = \sigma_K^p(n) = \max\left\{0, \frac{\max\{0, -n\sigma_K\}}{x}\left(1 - \frac{\eta}{\gamma(n)\pi(n)}\right)\right\}. \tag{F.25}$$

We note that when $n < 0$, then $\sigma^p(n) > 0 \iff \sigma_K^p(n) > 0$, i.e., both volatilities are positive if and only if $\gamma(n)\pi(n) > \eta$. Also note that $\sigma^p(n) = 0$ implies $\sigma_K^p(n) = 0$.

Accordingly, for $n < 0$, the standard deviation of price becomes

$$\Sigma^p = \max\left\{0, \frac{1}{x}\left(\pi(n) - \frac{\eta}{\gamma(n)}\right)\right\}, \tag{F.26}$$

while, for $n \geq 0$, we have $\Sigma^p = \sigma^p$.

Finally, the first-order condition with respect to $x$ becomes

$$-\lambda + \xi x^{\xi-1} - \left[\sigma^2(n+x) - \phi n \sigma_K \sigma\right] \min\left\{\gamma(n), \frac{\eta}{\pi(n)}\right\}$$

where $\gamma(n) > \frac{\eta}{\pi(n)}$ if and only if $\sigma^p > 0$.

[22] Note that (F.21) suggests that a negative $\sigma_K^p$ may be optimal in certain edge cases—particularly when $n > 0$ and $\eta$ is small. However, in this region, the issuer is generally less risk-averse (compared to the $n < 0$ case), and it is typically optimal to set $\sigma_K^p = 0$, even absent the constraint $\sigma_K^p \geq 0$. Overall, instances where $\sigma_K^p < 0$ is optimal are rare and confined to corners of the parameter space. By contrast, choosing a negative $\sigma^p$ is never optimal, so the constraint $\sigma^p \geq 0$ is effectively non-binding—that is, if $\sigma^p = 0$ is chosen, it would also be optimal without the constraint.

As in the baseline, we again define:

$$\tilde{n} = \inf\{n' \in (\underline{n}, \overline{n}) : \sigma^p(n) = \sigma^p_K(n) = 0 \;\text{ for all } n \geq n'\}$$

We focus on parameters such that $\tilde{n} \in (\underline{n}, \overline{n})$, i.e., there exist values of $n \in (\underline{n}, \overline{n})$ such that $\sigma^p(n) > 0$ or $\sigma^p_K(n) > 0$.

### F.5.5 Price

We determine the token price as a function of $n$, i.e., $p_t = p(n_t)$. When $\sigma^p(n), \sigma^p_K(n)$, the price $p(n)$ solves the following ODEs:

$$\sigma^p p(n) = p'(n)\big[\sigma(x+n) - x\sigma^p\big] = p'(n)\left(\frac{\eta\sigma(x+n)}{\gamma(n)\pi(n)}\right) \tag{F.27}$$

$$\sigma^p_K p(n) = p'(n)\big[-n\sigma_K - x\sigma^p_K\big] = -p'(n)\left(\frac{\eta n\sigma_K}{\gamma(n)\pi(n)}\right). \tag{F.28}$$

When $\sigma^p(n) > 0 = \sigma^p_K(n)$ Note that

$$\sigma^p_K = \frac{\sigma(x+n)}{-n\sigma_K},$$

so these two equations are equivalent.

Suppose $\sigma^p(n) > 0$, so that $\sigma^p(n) = \frac{\sigma(n+x)}{x}\left(1 - \frac{\eta}{\gamma(n)\pi(n)}\right)$, as well as

$$\sigma_n(n) = (n+x)\sigma - x\sigma^p(n) = \frac{\eta\sigma(n+x)}{\gamma(n)\pi(n)}.$$

We then can solve (F.27) subject to $p(\tilde{n}) = 1$ to obtain:

$$p(n) = \exp\left(-\int_n^{\tilde{n}} \frac{\sigma^p(\nu)}{\sigma_n(\nu)} d\nu\right) = \exp\left(-\int_n^{\tilde{n}} \frac{\gamma(\nu)\pi(\nu) - \eta}{\eta x(\nu)} d\nu\right), \tag{F.29}$$

as desired.

### F.5.6 Implementing $(x, \sigma^p, \sigma^p_K)$ via $(dS, f)$

Having solved for $(x, \sigma^p, \sigma^p_K)$, as well as the price $p(n)$ as functions for $n$ (see (F.29)), we determine the optimal (scaled) supply process $ds$ and fee policies $f$ to implement the optimal controls $(x, \sigma^p, \sigma^p_K)$. The optimal controls pin down $\mu_n(n)$, $\sigma_n(n)$ and $\sigma^K_n(n)$ by means of the law of motion (F.17), that $dn = \mu_n(n)dt + \sigma_n(n)dZ + \sigma^K_n(n)dZ^K$.

First, note that, given $x(n), p(n)$, the supply process $s(n)$ is uniquely determined via $s(n) = \frac{x(n)}{p(n)}$. Similar to the arguments from Appendix D.5, one could further characterize the dynamics of $ds$ which follow $ds(n) = \mu^s(n)dt + \sigma^s(n)dZ + \sigma^s_K(n)dZ^K$. One could determine the drift and volatility terms — omitted here, since not essential.

Next, calculate by Ito's Lemma:

$$\mu^p(n) = p'(n)\mu_n(n) + \frac{p''(n)}{2}\left[(\sigma_n^K(n))^2 + (\sigma_n(n))^2 + 2\phi\sigma_n^K(n)\sigma_n(n)\right].$$

Using (F.13), we obtain

$$f(n) = x(n)^{\xi-1} - \eta\Sigma^p(n) - r + \mu^p(n),$$

which pins downs the fee process as a function of $n$.

# G Commitment and Continuous Price Process

## G.1 Relaxing Long-Term Commitment

While we assume that the issuer commits at time $t = 0$ to a long-term issuance plan, we argue that it is sufficient to instead have short-term commitment paired with a restriction on dividend payouts. As shown in Appendix D—and in particular, Appendix D.5—the optimal controls $(x_t, \sigma_t^p)_{t\geq 0}$ can be implemented through an appropriate choice of fees, and issuance $(f_t, ds_t)_{t\geq 0}$.

In essence, to implement the optimal controls $(x_t, \sigma_t^p)_{t\geq 0}$, it suffices for the issuer to commit, at each time $t$, to strategies over the infinitesimal interval $[t, t+dt]$. Specifically, this involves: (i) setting issuance as $ds_t = \mu_s dt + \sigma dZ$, (ii) choosing a fee $f$, and (iii) ensuring consumption satisfies $dy \leq n - \underline{n}$, where $\underline{n}$ is defined in (C.4). This local commitment implements the optimal strategy. In fact, it is equivalent to allowing the issuer to select the auxiliary controls $x(n)$ and $\sigma^p(n)$ for the next instant.

By the dynamic programming underlying the determination of the optimal controls pinned down by the HJB equation (15), this short-term commitment implements the optimal strategy.

In the optimum, all quantities and controls are functions of the state variable $n$. However, this differs from the standard notion of a Markov Perfect Equilibrium. In our setting, at time $t$, the issuer's control — specifically, the issuance $ds$ over $[t, t+dt]$ — depends not only on the current state $n_t = n$, but also on the realization of the shock $dZ_t$, which effectively determines the next state $n_{t+dt}$. A similar logic applies to the choice of $\sigma^p(n)$: the issuer effectively commits to an issuance strategy that governs the price change over the next instant. Intuitively, the issuer's controls are contingent not only on the current state $n_t = n$, but also on the future state $n_{t+dt}$ — a feature that, as shown in Jermann and Xiang (2024), is central to enabling commitment.

In a Markov Perfect Equilibrium, the controls—specifically, issuance—must depend only on the current state $n$, and not on the realization of the shock, and hence not on the next-period state.

## G.2 Generalized Price Process and Issuance Strategy: Verifying the Optimality of a Continuous Price Path

In the baseline solution, the issuer's strategy $S_t$ — or $s_t/K$ and the price process $p_t$ are continuous diffusion processes. To solve for the optimal strategy, we focused on a continuous price process — which only loads on fundamental shocks $dZ_t$ — and then determined the issuance. One might be concerned that the focus on continuous price and issuance processes, as well as the focus on fundamental uncertainty $dZ_t$, are restrictive. Here, we show that the issuer optimally implements continuous price and issuance, whereby price and issuance only load on $dZ_t$.

We now allow for a generalized price process, which is akin to allowing for generalized issuance:

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t + d\ell_t.$$

Here, $d\ell_t$ is a general process, which can be deterministic and stochastic. However, the increments of $\ell_t$ and $Z_t$ are (without loss of generality) orthogonal — that is, $d\ell_t \cdot dZ_t = 0$.

Importantly, if stochastic, $d\ell_t$ captures the stochastic price/issuance unrelated to $dZ_t$, in that $d\ell_t$ is orthogonal to $dZ_t$. It satisfies standard regularity conditions (i.e., it is almost surely finite and square integrable). Since the price process is controlled by issuance, allowing for general $d\ell_t$ is akin to allowing for generalized issuance. All other elements remain unchanged.

We solve for the dynamics of the state variable $n$ under these generalized assumptions, and write down the HJB equation. Notably, we assume that price fluctuations related to $d\ell_t$ do not affect the users' convenience utility — i.e., they are not "priced." We then show that, even under this favorable treatment of price adjustments $d\ell_t$, stipulating $d\ell_t = 0$ is optimal. This implies that price process $p_t$ and issuance strategy $s_t$ are continuous diffusion processes in optimum.

**User Optimization and Dynamics of $x$.** Analogously to the baseline, (B.4) holds, in that:

$$x_t = \left( \frac{dt}{rdt + f_t dt - \eta|\sigma_t^p|dt - \mathbb{E}_t^u\left[\frac{dp_t}{p_t}\right]} \right)^{\frac{1}{1-\xi}}.$$

Next, rewrite this relationship to obtain

$$\mathbb{E}_t^u\left[\frac{dp_t}{p_t}\right] = \mu_t^p dt + \mathbb{E}_t^u[d\ell_t] = \left(-x_t^{\xi-1} + f_t + r + \eta|\sigma_t^p|\right) dt. \tag{G.1}$$

We note that $d\ell_t$ might represent a lumpy price change that occurs with an atom of probability, so $\mathbb{E}_t^u[d\ell_t]$ is not of order $dt$. We also allow the fee process to not be lumpy (to potentially offset $d\ell_t$), but, instead of introducing additional notation, we would capture this by $f_t \in \{-\infty, \infty\}$ where $f_t = +\infty$ captures a lump-sum fee and $f_t = -\infty$ a lump-sum rebate.

Next, calculate

$$dp_t = p_t\mu_t^p dt + p_t\sigma_t^p dZ_t + p_t d\ell_t = (r + f_t)p_t dt - p_t\left(x_t^{\xi-1} - \eta|\sigma_t^p|\right)dt + p_t\sigma_t^p dZ_t + p_t(d\ell_t - \mathbb{E}_t^u[d\ell_t]).$$

Next, multiply both sides of above by $s_t$ and use $x_t = s_t p_t$ (market clearing) to obtain

$$s_t dp_t = (r + f_t)x_t dt - \left(x_t^{\xi} - \eta x_t|\sigma_t^p|\right)dt + x_t\sigma_t^p dZ_t + x_t(d\ell_t - \mathbb{E}_t^u[d\ell_t]).$$

Ito's product rule implies $d(s_t p_t) = ds_t p_t + p_t ds_t + ds_t dp_t$. Then, we calculate for $x_t = s_t p_t$:

$$dx_t = (r + f_t)x_t dt - \left(x_t^{\xi} - \eta x_t|\sigma_t^p|\right) dt + x_t\sigma_t^p dZ_t + x_t\left(d\ell_t - \mathbb{E}_t^u[d\ell_t]\right) + ds_t(p_t + dp_t). \tag{G.2}$$

**Dynamics of $n$.** The reserves follow

$$dA_t = \mu A_t dt - \kappa X_t dt + \sigma A_t dZ_t - f_t X_t dt + dS_t(p_t + dp_t) - dY_t,$$

We can calculate for $n_t = N_t/K = A_t/K - x_t$:

$$dn_t = \left[\mu(n_t + x_t) - \lambda x_t + x_t^{\xi} - \eta x_t|\sigma_t^p|\right]dt + \left[\sigma(n_t + x_t) - x_t\sigma_t^p\right]dZ_t - dD_t - x_t\left(d\ell_t - \mathbb{E}_t^u[d\ell_t]\right). \tag{G.3}$$

The issuer's commitment to a strategy implies $\mathbb{E}_t^u = \mathbb{E}_t$, i.e., the issuer's and the user's expectation operators coincide at any point in time. (The net worth could be also derived more directly, but we provide details here, since we are dealing with a different price process.)

**HJB Equation.** We note $d\ell_t$ enters the issuer's payoff only via the state variable $n_t$.

When $d\ell_t$ is deterministic, i.e., $\mathbb{E}_t[d\ell_t] = d\ell_t$, then the issuer's commitment immediately implies $d\ell_t = \mathbb{E}_t^u[d\ell_t]$. In this case, $d\ell_t$ drops out and the law of motion of $n$ is as in the baseline. It follows that $d\ell_t$ is payoff-irrelevant, since $n_t$ is the only state variable and $d\ell_t$ enters the issuer's payoff only via $n_t$. Then, $d\ell_t$ is payoff-irrelevant and can be set to zero.

Since $d\ell_t$ affects the state variable only via its deviation from the mean $d\ell_t - \mathbb{E}_t^u[d\ell_t]$ and deterministic $d\ell_t$ is payoff irrelevant, we can without loss of generality focus on a stochastic process $d\ell_t$ that satisfies $\mathbb{E}_t[d\ell_t] = 0$ and $d\ell_t \cdot dZ_t = 0$. Thus, $d\ell_t$ captures price movements from randomization by the issuer. We will show that such randomization is sub-optimal, notably, even if we do not assume additional risk-aversion for the user regarding price fluctuations stemming from such randomization. We focus on $d\ell_t$ following a Levy process.

By the Levy-Ito decomposition, we can write the process $d\ell = d\ell_t$ as mean-zero process according to:

$$d\ell_t = \sigma_\ell dZ_t^\ell + \Delta_t^\ell\big(dN_t^\ell - \mathbb{E}_t[dN_t^\ell]\big) \tag{G.4}$$

where $dN_t^\ell \in \{0,1\}$ is a jump process with $\mathbb{E}_t[dN_t^\ell] = \Lambda_t^\ell dt$ — the intensity $\Lambda_t^\ell$ can potentially be infinite, capturing lumpy price adjustements with an atom of probability. In addition, $dZ_t^\ell$ is a Brownian motion orthogonal to fundamental shocks $dZ_t$, i.e., $dZ_t^\ell \cdot dZ_t = 0$. Without loss of generality, there is only one jump component.

The HJB equation then satisfies:

$$\begin{aligned}\rho v(n) = \max_{\sigma^p, x \geq 0} &\left\{ v'(n)\Big(\mu n - \lambda x + x^\xi - \eta x|\sigma^p|\Big) + \frac{v''(n)}{2}\Big[\sigma(x+n) - x\sigma^p\Big]^2 \right\} \\ &\max_{\sigma_\ell} \frac{v''(n)x^2\sigma_\ell^2}{2} + \max_{\Delta^\ell, \Lambda^\ell \geq 0} \Lambda^\ell \Big[v(n - x\Delta^\ell) - v(n) + v'(n)x\Delta^\ell\Big].\end{aligned} \tag{G.5}$$

with upper boundary $\overline{v}$, satisfying $v'(\overline{n}) - 1 = v''(\overline{n}) = 1$. In addition, there is some lower boundary $\underline{n}$ with $v(\underline{n}) = 0$.

**Proving $d\ell_t = 0$.** We conjecture that the value function is strictly concave, i.e., $v''(n) < 0$ for $n \in (\underline{n}, \overline{n})$. Given this conjecture, we will show that setting $d\ell_t = 0$ is optimal so that the above HJB equation collapses to (D.6) or, equivalently, (15). Thus, the value function coincides with the one of the baseline, which, as we have shown, is strictly concave, verifying our conjecture.

First, note that due to concavity, $v''(n) \leq 0$, we have $\sigma_\ell = 0$ for all $n \in (\underline{n}, \overline{n})$.

Suppose $\Lambda^\ell > 0$. The term $\Big[v(n - x\Delta^\ell) - v(n) + v'(n)x\Delta^\ell\Big]$ is zero for $\Delta^\ell = 0$, while it has derivative with respect to $x\Delta^\ell$ of $\mathcal{U}(x\Delta^\ell) = -v'(n - x\Delta^\ell) + v'(n)$. Due to concavity, it follows that $\max_{\Delta^\ell, \Lambda^\ell \geq 0} \Lambda^\ell \Big[v(n - x\Delta^\ell) - v(n) + v'(n)x\Delta^\ell\Big] = 0$. Overall, $d\ell_t = 0$.

# H Calibration Details

In the appendix, we supplement details regarding our parameter calibration.

**Calibrating $\mu$ and $\sigma$.** For the parameters $\mu$ and $\sigma$, we calibrate them based on the disclosed information about Tether's reserve asset portfolio. Tether is the largest stablecoin issuer. Our data is from Tether's latest financial report as of December 31, 2024.

Tether's reserves are allocated as follows: 82.3% in cash and cash equivalents (primarily T-bills), 3.7% in precious metals (primarily gold), 5.5% in Bitcoin, 5.7% secured loans, and 2.77% other investments — where we exclude 0.01% corporate bonds and round to the first decimal. To choose $\mu$ and $\sigma$, we obtain the annualized returns of the individual components over the past two years (relative to the Tether report date on 12/31/2024) and compute a weighted average of annualized returns and return volatility. However, we do not account for correlation among individual components, which makes our estimate of $\sigma$ a conservative lower bound since the correlation is likely positive.

For Bitcoin, annualized returns in 2023 and 2024 were 155%, and 121%, respectively. Based on these numbers, we assume a conservative return of 100%, which is below the annualized return in the past two years. We make this assumption since 2024 was an extraordinary year for Bitcoin, also due to the regulatory approval of Bitcoin ETF and the election of a crypto-friendly administration in the U.S. We directly calculate the annualized return volatility of Bitcoin over the same time period, which equals about 49%.[23] This is lower than the annualized return volatility in previous years; in comparison, the annualized return volatility was about 64% for the calendar year 2022 and even higher in Bitcoin's earlier days.[24] For precious metals, approximated by gold, we calculate an annualized return of 18.60% based on a 13% return in 2023, and 27% in 2024. As for Bitcoin, we calculate the annualized return volatility of Gold—using the gold ETF (ticker: GLD)—from 01/01/2023 to 12/31/2024. We obtain an annualized return volatility of 14.3%.

For cash and cash equivalents, we take the T-bill ETF (with ticker BIL). For 2023-2024, we calculate an annualized return of about 5%, and an annualized volatility of 0.27%. For secured loans, we assume that they are similar in terms of risk and return to relatively short-term investment-grade corporate bonds, as those contained in the ETF with ticker SLQD.

[23] We calculate annualized returns of an asset as follows. First, we calculate daily percentage returns based on adjusted closing prices from 01/01/2023 to 12/31/2024. Second, we calculate the standard deviation of daily returns over the same time period. Third, multiply this number by $\sqrt{\#\text{Trading Days}}$, where we assume 365 trading days for cryptocurrency and 252 trading days for more standard assets. The results are similar using log returns or percentage returns.

[24] For instance, industry reports by Fidelity and Blackrock suggest that Bitcoin exhibited higher volatility in earlier years, with annualized return volatility often being close to or exceeding 100%.

For SLQD and over 2023-2024, we calculate an annualized return of about 5.4% and an annualized return volatility of 2.4%. This estimate is in line with the volatility for secured loans reported in an industry report by Invesco.

For other investments, we assume that these comprise other cryptocurrencies and altcoins riskier than Bitcoin currently or comparable to Bitcoin in its early days. We set the return of other investment to the same level as that for Bitcoin, i.e., 100%, but assume they are twice as risky and set the volatility to 100% — as we verify, this number coincides with the annualized return volatility of Solana from 2023 to 2024 which we calculate as 99%.

Thus, we calculate:

$$\mu = \underbrace{0.823 \cdot 5\%}_{\text{T-Bills}} + \underbrace{0.055 \cdot 100\%}_{\text{Bitcoin}} + \underbrace{0.037 \cdot 18.60\%}_{\text{Gold}} + \underbrace{0.057 \cdot 5.4\%}_{\text{Secured Loans}} + \underbrace{0.0277 \cdot 100\%}_{\text{Other}} \approx 13.39\%,$$

$$\sigma = \underbrace{0.823 \cdot 0.27\%}_{\text{T-Bills}} + \underbrace{0.055 \cdot 49\%}_{\text{Bitcoin}} + \underbrace{0.037 \cdot 14.23\%}_{\text{Gold}} + \underbrace{0.057 \cdot 2.4\%}_{\text{Secured Loans}} + \underbrace{0.0277 \cdot 100\%}_{\text{Other}} \approx 6.35\%.$$

Consequently, we set $\mu = 0.134$. For volatility, we round up and set $\sigma = 0.07$ as ignoring correlations (which are likely positive on a net basis) makes our calculation a lower bound.

**USDT convenience yield.** As explained in the main text, the convenience yield of USDT is used to calibrate $\xi$ that governs the users' demand elasticity (see (5)). Following Ma et al. (2023), we match the marginal convenience to the lending rate offered on DeFi lending protocols, specifically Aave (one of the largest protocols). We use Aavescan to retrieve historical data on USDT lending rates in 2024, which are available from the time of download (in our case, 02/09/2025) roughly one year back to 02/16/2024. We calculate the average lending rate (after winsorizing at the 98% levels) in year 2024 (based on the available datapoints) and obtain lending rate of about 6.4%. Solving $(136.6)^{\xi-1} = 0.064$ yields $\xi = 44.1\%$.[25]

[25]The rationale is that in equilibrium, stablecoin users must be indifferent at the margin between holding the stablecoin for the conveneince yield or lending it out on DeFi platforms (e.g., Aave).