Financial Intermediation with Fragile Seigniorage: A Dynamic Model of Stablecoin Issuers*

Ye Li† Simon Mayer‡

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Abstract

We develop a unified model of stablecoins that demonstrates novel mechanisms of fragility and quantitative implications for the issuer's stablecoin issuance, buyback, and redemption strategies, payout and financing policies, equity valuation, and responses to regulations. The issuer earns seigniorage from supplying stablecoins, but negative shocks to reserve assets can cause depegging, reducing seigniorage precisely when needed the most by the issuer to recover its balance-sheet capacity and restore the peg. This fragility of seigniorage creates an instability trap. In addition, a risk paradox emerges: restricting the riskiness of reserve assets can make depegging more likely, challenging the conventional views. Finally, we show that for stablecoin issuers, capital requirements serve a distinct role that is absent for traditional financial intermediaries: by limiting the issuer's reliance on the fragile seigniorage as a source of profits, they enhance stability and eliminate the risk paradox.

Keywords: Stablecoin, depegging, regulations, financial intermediation, instability, dynamic corporate finance, intermediary asset pricing, production-based asset pricing

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†University of Washington. E-mail: liye@uw.edu

‡Carnegie Mellon University. E-mail: simonmay@andrew.cmu.edu.

1 Introduction

The rapid growth of stablecoins in market capitalization and the regulatory momentum in major economies have drawn enormous attention. Governments view stablecoins as instruments to extend the international reach of their currencies, while major financial institutions are closely examining the competitive threats posed by stablecoin issuers. Stablecoins are blockchain-based assets pegged to fiat currencies, typically backed by the issuers' reserve assets. The risk profile of reserve assets is central to ongoing debates, with restrictions on risk-taking being a primary regulatory focus, as exemplified by the GENIUS Act in the U.S.

How does reserve-asset risk transmit to stablecoin users through depegging? What trade-offs shape an issuer's decision to depeg? How to evaluate various regulatory proposals? We develop a dynamic model of stablecoin issuers to address these questions.

Unlike contemporaneous studies that focus on specific aspects of stablecoins, we jointly analyze the issuer's key decisions, including depegging, stablecoin issuance, redemption and buyback policies, fees or interest payments for stablecoin users, external financing, and payout policies. This unified model uncovers novel mechanisms of instability. Our continuous-time model is tractable for an analytical characterization of the equilibrium, yet rich enough to be calibrated to data for quantitative implications on key issues—such as the severity and persistence of depegging, the benefits of granting issuers access to equity financing (as illustrated by Circle's IPO), the valuation of issuers' equity, and the distinct effects of capital requirements and reserve-asset risk limits that are the two pillars of regulatory frameworks.

At the core of our model is an endogenously fragile seigniorage. By intermediating between reserve assets and stablecoins, the issuer earns a return spread. This spread constitutes a form of seigniorage: because stablecoin users value the transactional utility of the stablecoin, they are willing to accept a lower return, which reduces the issuer's funding cost. After negative shocks hit the reserve assets, the issuer's net worth declines, which may induce the issuer to devalue its stablecoin liabilities. Depegging lowers the stablecoin's transactional utility and thus reduces seigniorage—precisely when it is most needed by the issuer to rebuild net worth and to restore the peg. As a result, the stablecoin enters an instability trap.

Our baseline model considers a laissez-faire environment where the issuer faces no regulations and is not bound to maintain the peg, having discretion over its liability value.[1] This benchmark is essential for evaluating the effects of regulations. Stablecoin issuers differ from banks that commit to redeem deposits at par on a first-come, first-served basis, provided they have liquidity—a commitment underpinning the classic run mechanism based on banks' liq-

[1] According to the contracts that the major issuers offer to stablecoin users, a stablecoin is neither a definitive claim on the issuers or their assets that presumably back the stablecoin. An issuer can change the redemption terms "without prior notice" (see, for example, Tether's Terms of Service at https://tether.to/en/legal).

uidity mismatch and coordination failures among depositors (Diamond and Dybvig, 1983).[2] Our model features run-like dynamics but follows another strand of literature on financial instability: instead of liquidity transformation, it emphasizes risk transformation and the role of net worth in determining intermediation capacity (e.g., He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014). In the literature review, we compare the two approaches and highlight how our analysis complements studies based on liquidity transformation.

Next, we summarize the model of the stablecoin issuer and its main results. As in Biais, Bisière, Bouvard, Casamatta, and Menkveld (2023), stablecoin users derive transactional benefits (or monetary utility) from holding the stablecoin. Such utility reduces the users' required return, thereby creating *seigniorage revenues* for the issuer in the form of reducing funding costs. On the asset side of the issuer's balance sheet are reserve assets, while the liability side consists of outstanding stablecoins and the issuer's net worth. Stablecoin users observe the issuer's net worth, which serves as the state variable driving the equilibrium dynamics. Therefore, financial instability in our model does not arise from a lack of balance-sheet disclosure or from accounting misconduct by the stablecoin issuer. Instead, we emphasize that financial instability emerges from more fundamental economic forces that persist even when disclosure problems are fully resolved in the stablecoin space.

The issuer maintains net worth above an endogenous threshold, $\underline{n}$. Falling to this threshold implies that the revenues from reserve assets are insufficient to cover the costs of stablecoin liabilities, leading to a permanent loss of profitability—an absorbing state where the value function is zero. This induces concavity in the value function, rendering the issuer effectively risk-averse. In an extension, the issuer may raise equity financing to restore net worth, but effective risk aversion still exists due to the costs of external financing.[3]

The users' monetary utility decreases when depegging happens—that is, when the stablecoin price falls below one and fluctuates. What triggers depegging are the negative shocks to the issuer's reserve assets.[4] In effect, the issuer dynamically transforms one type of assets (its reserve assets) into another type of assets (stablecoins), using its net worth as a risk buffer, much like in securitization (e.g., Malamud, Rui, and Whinston, 2013). Below, we describe the shock transmission mechanism that gives rise to an *instability trap*.

[2]Related, it is worth noting that stablecoin issuers are not money market mutual funds (MMMF) that operate under NAV-based redemption rules. They may be regulated into MMMFs depending on the evolving legal landscape, but in a laissez-faire environment—the benchmark for evaluating regulations—they are not.

[3]The issuer's objective is to maximize the present value of consumption flows (payout) with a risk-neutral preference but it becomes effective risk-averse under financial frictions as in the literature on dynamic corporate finance (e.g., Bolton, Chen, and Wang, 2011; Décamps, Mariotti, Rochet, and Villeneuve, 2011; Hugonnier, Malamud, and Morellec, 2015; Abel and Panageas, 2023) .

[4]In reality, some stablecoin issuers hold safer assets than others, but it is impossible to completely avoid risk. For quantitative implications, we calibrate our model parameters, including the risk-return profile of reserve assets, to data on Tether, the issuer of USDT (the largest stablecoin by market capitalization).

When negative shocks reduce the issuer's net worth, its effective risk aversion rises endogenously. To reduce risk exposure and preserve net worth against subsequent shocks, the issuer faces two options, deleveraging and depegging. When deleveraging, the issuer reduces the outstanding amount of stablecoin liabilities, selling reserve assets and using the proceeds to repurchase stablecoins out of circulation (*quantity adjustment*). This is the preferred option if the issuer's net worth is above a threshold, $\widetilde{n}$; otherwise, the issuer does not maintain the redemption at par, and depegging occurs (*quality adjustment*). Note that in our model, the market price of the stablecoin is always equal to the redemption value, as we abstract from market structure frictions that could otherwise generate a wedge between the redemption value (or primary-market price) and the secondary-market price.[5]

Below $\widetilde{n}$, as depegging lowers the stablecoin users' utility, an inward shift of their demand curve takes place, representing redemption requests and resembling a run. The further the issuer's net worth falls, the more severe depegging becomes, as in the process, the issuer supplies more stablecoins albeit at increasingly lower prices, desperately to raise revenues. Once the system enters this region of low net worth, it gets trapped here for a long time. The stationary density increases sharply as net worth approaches $\underline{n}$.

What lies at the heart of this instability trap is the procyclicality and endogenous fragility of seigniorage. Seigniorage is given by the stablecoin quantity multiplied by the users' monetary utility per unit of stablecoin. In response to negative reserve-asset shocks, seigniorage declines because the issuer either reduces the stablecoin quantity (deleverages) or quality (depegs the stablecoin) to preserve net worth. A lower seigniorage in turn slows down the rebuild of net worth, resulting in persistently high effective risk aversion and low seigniorage.

Depegging is disciplined by the users. When the users' monetary utility has a sufficiently high sensitivity to stablecoin price fluctuation, the issuer refrains from depegging at all values of net worth and focuses on quantity adjustment (deleveraging) as the only risk-management tool. Therefore, one contribution of this paper is to characterize the parameter condition under which, even in a laissez-faire environment where reserve assets are risky, a stablecoin is perfectly stable in price (though its quantity varies significantly).

Holding users' sensitivity to price fluctuations in their monetary utility fixed, the parameter region in which depegging never occurs expands as the riskiness of reserve assets increases. This *risk paradox* is due to the issuer's precaution against the instability trap. When the reserve-asset risk is higher, offloading risk to users via depegging causes more severe demand contraction. As a result, the issuer optimally chooses a smaller scale of intermediation and risk transformation—that is, instead of quality reduction (depegging), the

[5]Thus, our model does not address the tiny deviations from the \$1 peg (sometimes even above \$1) observed in stablecoin prices, which are driven by market structure frictions.

issuer prefers quantity adjustment (deleveraging) for managing its risk exposure.

In addition, we show that in the parameter region where depegging actually happens, the highest level of stablecoin volatility has an inverse-U shaped relationship with reserve-asset risk. If reserve assets are risk-free, depegging does not occur as there are no shocks. As reserve-asset risk rises, stablecoin volatility rises. However, once reserve-asset risk is above a threshold, the issuer's precaution against the instability trap kicks in forcefully, causing stablecoin volatility to be decreasing in reserve-asset risk—another form of risk paradox.

Since the problem lies in reserve-asset risk, a natural question is: what if risk-taking is regulated? The GENIUS Act restricts the types of assets that stablecoin issuers can hold. Our results on the risk paradox suggests caution: the relationship between reserve-asset risk and stablecoin volatility can be inverse, as long as some risk—even at a low level—exists in the issuer's reserves. What if issuers were instead required to hold only perfectly safe assets? In addition, the GENIUS Act requires issuers to maintain positive net worth (i.e., a capital requirement), effectively restricting the stablecoin issuer to be a narrow bank. Our calibrated model shows that, relative to the laissez-faire benchmark, this narrow-banking framework lowers the supply of stablecoins by 73–88% and users' welfare by 60–70%, depending on the issuer's net worth, and cuts the issuer's equity valuation by 33%.[6]

Allowing the issuer to take some risk—and thereby earn higher expected returns on reserve assets—can be beneficial. It strengthens the incentive to intermediate, and when well capitalized, the issuer optimally passes through a portion of reserve-asset returns to stablecoin users in the form of interest payments (i.e., negative user fees), resembling the "rewards" that stablecoin issuers offer to users in practice. By contrast, when restricted to narrow banking, the issuer compensates for the limited returns on reserves by raising user fees, which ultimately reduces stablecoin demand and lowers overall user welfare.

We also show that more flexibility in the issuer's asset choice is beneficial in general. In an extension, we allow the issuer to dynamically adjust the riskiness of reserve assets. The issuer takes more risk to earn a higher expected return on assets when it is well-capitalized, and in response to negative shocks, it de-risks the asset portfolio as a preferred way to reduce risk exposure before engaging in stablecoin depegging (offloading risk to the stablecoin users).

As previously discussed, there are two pillars of regulatory frameworks, risk-taking restriction and capital requirement. We find that capital requirement alone already significantly improves stability and user welfare. Adding an additional requirement of risk-free reserves only leads to marginal improvement. Next, we discuss capital requirements and their unique role in regulating stablecoin issuers. This role is absent for traditional financial institutions.

[6]A direct implication of our analysis is that imposing regulations that constrain stablecoin issuers to operate like money market mutual funds holding only safe assets, or as narrow banks, may be suboptimal.

Reserve assets are funded by the issuer's net worth (equity) and stablecoin issuances. Increasing equity reduces the issuer's reliance on seigniorage—the liability side of balance sheet—as a source of profits and increases the share of profits from the equity-funded reserve assets. Because reserve-asset return is determined in the broader financial markets and thus exogenous to the depegging dynamics, it does not suffer from the feedback loop behind the procyclicality of seigniorage (i.e., comovement between seigniorage and net worth) that leads to the instability trap. Therefore, capital requirement improves stability by regulating the issuer's profit composition between the return on reserve assets and procyclical seigniorage.

Capital requirement also eliminates the risk paradox in the parameter region where depegging happens: as reserve-asset risk increases, stablecoin volatility increases (i.e., the inverse relationship no longer exists). In the laissez-faire environment, risk paradox emerges from the issuer's precaution against the instability trap. Under capital requirement, such precaution is no longer needed because the force of instability trap is already weak.

Finally, we extend our model to account for stochastically growing stablecoin demand. In practice, demand varies with technology changes, competition, and growth or contraction of the community that transacts on blockchains. Because some stablecoin issuers hold cryptocurrencies in their asset portfolio, the value of reserve assets and stablecoin demand tend to be correlated. We find that such correlation amplifies the procyclicality of seigniorage and exacerbates the instability trap. Capital requirement dampens this harmful impact by reducing the issuer's reliance on procyclical seigniorage. This result reveals a distinctive role of capital requirement in the presence of randomly evolving stablecoin demand.

Extending our model to incorporate the growth of stablecoin demand brings important insights into our model mechanism. Demand growth improves stability by imputing a trend in the issuer's seigniorage profits that counterbalances the procyclical fluctuation along the trend. As a result, for any given level of stablecoin liabilities, the issuer maintains a lower net worth when the growth rate of demand is higher. Intuitively, as demand growth fosters stability, the issuer finds it less necessary to maintain net worth to buffer shocks.

Currently, the majority of stablecoin issuers are private companies. The recent IPO of Circle, the issuer of stablecoin USDC, has brought enormous attention to stablecoin issuers' access to equity-market financing and their equity valuation. We show that being able to raise external equity significantly improves the stability of stablecoin because equity issuance essentially allows the stablecoin issuer to share risk with external equity investors, adding a risk-management tool beyond deleveraging and depegging. A major hurdle to public listing is the ambiguity in the valuation of the issuer's equity and, in particular, its connection with the growing trend of stablecoin adoption. By incorporating a rich set of modeling ingredients—such as, the issuer's profits from both reserve-asset returns and stablecoin seigniorage, its dynamic decision to deleverage and depeg, its payout (consumption) policy, and dynamic fees charged on stablecoin users—our model provides a valuation framework that goes beyond the discounted cash-flow analysis. We calibrate the model to data on Tether. Under different projections of demand growth, we map out the implied valuation of Tether's equity.

Related Literature. There is a growing literature on cryptocurrencies ("tokens").[7] The endogenous interactions among depegging, tokens' transactional benefits, and the issuer's seigniorage in our model generate novel theoretical results—including the instability trap and risk paradox—that are distinctive to stablecoins and do not arise for other types of transactional tokens (e.g., Biais, Bisière, Bouvard, Casamatta, and Menkveld, 2023). As in Gryglewicz, Mayer, and Morellec (2021), Cong, Li, and Wang (2022), Jermann (2023, 2024) and Jermann and Xiang (2025b), we develop a dynamic model that provides a unified account of various aspects of tokenomics, including token issuances, fees/remuneration for token holders, the issuer's payout and financing, and valuation of the issuer's equity. In addition, our calibrated model delivers quantitative implications.

Stablecoin issuers form a new category of financial intermediaries.[8] There are two main approaches to modeling financial intermediaries and financial instability: one centered on liquidity transformation and the other on risk transformation. The existing stablecoin literature largely adopts the former, with the classic run mechanism of Diamond and Dybvig (1983) at the core. In contrast, our paper takes the latter approach, following He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), that facilitates an analysis of the transmission of the issuer's reserve-asset risk to stablecoin users via depegging and highlights the role of the issuer's net worth. Next, we compare the two approaches.

The liquidity transformation approach requires two ingredients. First, the stablecoin is liquid in the sense that the issuer commits to first-come, first-serve redemption at par until it exhausts resources. Second, the issuer's reserve assets are illiquid—that is, it incurs costs when liquidating assets to meet redemption requests. Together they lead to coordination failure among stablecoin holders that is key to the instability mechanism in those models.[9]

[7]Prior theoretical studies focus on various aspects from the blockchain technology and the formation of decentralized consensus to tokens as platform currencies and financing instruments, including, but not limited to, Biais, Bisière, Bouvard, and Casamatta (2019); Saleh (2020); Sockin and Xiong (2020, 2023); Cong, Li, and Wang (2021); Chod and Lyandres (2021); Gryglewicz, Mayer, and Morellec (2021); Hinzen, John, and Saleh (2022); Biais et al. (2023); Brunnermeier and Payne (2023); Goldstein, Gupta, and Sverchkov (2024); John, Rivera, and Saleh (2025). For surveys, see Brunnermeier, James, and Landau (2019); John, O'Hara, and Saleh (2022); Makarov and Schoar (2022); John, Kogan, and Saleh (2023).

[8]We study stablecoin issuers that intermediate between the reserve assets and stablecoin liabilities. We do not study algorithmic stablecoins like TerraUSD (Liu, Makarov, and Schoar, 2023).

[9]In a run, depositors front-run one another, causing a coordination failure, because early withdrawal, if met at par value, imposes asset liquidation costs on those that withdraw later. Many stablecoin models adopt this feature (e.g., Gorton and Zhang, 2021; Routledge and Zetlin-Jones, 2022; Uhlig, 2022; Gorton,

Neither ingredient appears in our model. Instead, the issuer may depeg the stablecoin following negative shocks to reserve assets or negative shocks to stablecoin demand (i.e., redemption requests), in line with the current practice.[10] The issuer holds risky yet liquid reserve assets, which is also in line with the reality.[11] When issuing stablecoins backed by risky reserves, the issuer engages in a risk transformation. Its intermediation capacity depends on its net worth as in models of intermediary asset pricing (e.g., He and Krishnamurthy, 2012, 2013; Brunnermeier and Sannikov, 2014; Drechsler, Savov, and Schnabl, 2018; Kondor and Vayanos, 2019; Malamud, Schrimpf, and Zhang, 2025). This risk-transformation approach allows us to uncover how shocks to reserve assets are transmitted to stablecoin users through depegging. This is central to evaluating regulatory proposals that restrict issuers' risk-taking and to clarifying the problems such restrictions aim to solve.

The instability mechanism faced by stablecoin issuers still differs from that of financial intermediaries in intermediary-based asset pricing models. When negative shocks reduce balance-sheet capacity (tied to intermediaries' net worth), stablecoin issuers deleverage as other intermediaries but have another lever to pull—depegging, albeit facing discipline from the stablecoin users. We emphasize that the procyclicality of stablecoin issuers' seigniorage contrasts sharply with the countercyclicality of risk premia that intermediaries earn in intermediary asset pricing models. Those intermediaries in bad times face falling asset prices and rising risk premia, which lead to stronger expected net-worth growth and accelerate recovery (e.g., He and Krishnamurthy, 2013). By contrast, in our setting the stablecoin issuer experiences a decline in seigniorage after negative shocks, which causes the instability trap.

The stablecoin issuer's intermediation capacity depends on its effective risk aversion, which is inversely related to net worth. Restoring net worth after negative shocks requires either internal earnings accumulation or external equity issuance. The former is slow due to the procyclicality of seigniorage, while the latter incurs financing costs. Hence, our modeling approach is closely related to dynamic agency problem and corporate finance models, where firms' effective risk aversion arises from financial constraints (e.g., DeMarzo and Sannikov, 2006; Biais, Mariotti, Plantin, and Rochet, 2007; Biais, Mariotti, Rochet, and Villeneuve, 2010; Décamps, Mariotti, Rochet, and Villeneuve, 2011; Bolton, Chen, and Wang, 2011; Hugonnier, Malamud, and Morellec, 2015; Hartman-Glaser, Mayer, and Milbradt, 2025).

Capital requirements are a cornerstone of financial regulation and feature prominently in

Klee, Ross, Ross, and Vardoulakis, 2022; Ma, Zeng, and Zhang, 2023; Bertsch, 2023; Ahmed, Aldasoro, and Duley, 2024; Goel, Lewrick, and Agarwal, 2025). d'Avernas, Maurin, and Vandeweyer (2022) do not have the classic runs while focusing on adoption risk, without reserve asset risk and the associated risk transformation.

[10] For example, USDT holders possess an unsecured contractual right to redeem at par, but Tether's Terms of Service can be amended "without prior notice." (source: https://tether.to/en/legal).

[11] In Appendix A, we provide an overview of Tether's reserve portfolio as part of calibration exercises.

proposed frameworks for stablecoins—often in the form of over-collateralization mandates, such as those in the U.S. GENIUS Act. Our paper uncovers a novel role for capital requirements: they reduce stablecoin issuers' dependence on procyclical seigniorage. This role has no counterpart in models of capital regulation for traditional financial institutions (e.g., Van den Heuvel, 2008; Begenau, 2020; Jermann and Xiang, 2025a; Rivera, 2025).

Our model offers distinctive asset-pricing perspectives on stablecoins and their issuers. Stablecoin price depends on the expected future paths of reserve-asset risk offloaded by the issuer to users via depegging. Unlike intermediary-based asset-pricing models that emphasize intermediaries as asset buyers, we model stablecoin issuers, following the focus on asset issuers in the production-based asset-pricing literature (see, e.g., Cochrane (1991); Jermann (1998); Gomes, Kogan, and Zhang (2003); see Kogan and Papanikolaou (2012) for a review). In addition, our model provides a framework for valuing stablecoin issuers' equity shares, which takes into account the issuer's various state-contingent decisions (e.g., depegging).

2 Model

Consider an infinite-horizon economy in continuous time. An issuer supplies stablecoins to users who derive utility from their stablecoin holdings. This utility, modeled in line with the literature on monetary assets, generates a demand curve that evolves dynamically driven by the endogenous risk of depegging and fees charged by the issuer. The issuer's proceeds from stablecoin issuance and fees are invested in reserve assets, which the issuer uses as sources of funds for managing the stablecoin supply (e.g., repurchasing stablecoins out of circulation). The issuer also decides on its consumption (i.e., pays itself a dividend). In the following, we first introduce the demand for stablecoins and then set up the issuer's problem.

2.1 Stablecoin demand

We model a unit mass of representative stablecoin users. The generic consumption goods ("dollars") are the numeraire in this economy. Let p_t denote the price of one unit of stablecoin in dollars. When pegged to the numeraire, $p_t = 1$; otherwise, $p_t < 1$ represents depegging.

A unit mass of atomic users take as given the equilibrium process of stablecoin price,

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t \,, \tag{1}$$

where μ_t^p and σ_t^p in the drift and diffusion terms, respectively, are determined after we solve the issuer's optimal strategy, and dZ_t is a Brownian shock to the issuer's reserve assets, which is the only risk in our baseline model.[12] In an extended model, we consider shocks to K_t that scales the users' demand and allow such demand shocks to be correlated with Z_t. Note that we do not distinguish the redemption value (primary-market price) from the secondary market price, as we abstract from market structure frictions that may otherwise generate a wedge. When reducing their holdings, stablecoin users may either redeem the stablecoins (i.e., sell them back to the issuer) or sell them in the market; through market clearing, these sales in the market are ultimately absorbed (bought back) by the issuer.

The representative user derives the following monetary utility (transactional benefits) from holding the stablecoin:

$$U(X_t) := \frac{1}{\xi} K_t^{1-\xi} X_t^{\xi} - X_t \eta |\sigma_t^p| = \underbrace{\left(\frac{1}{\xi} x_t^{\xi} - x_t \eta |\sigma_t^p|\right)}_{u(x_t)} K_t, \tag{2}$$

where the parameter ξ is a constant in $(0, 1)$, K_t is the demand scaler, X_t is the numeraire value of stablecoin holdings, and $x_t = X_t/K_t$.[13] Quantity variables are homogeneous of degree one in K_t (i.e., the model is "scale-invariant"). In our baseline model, we consider a constant demand scaler, $K_t = K$. When calibrating the model for quantitative analysis, we set K to one billion so that the quantities variables match their empirical counterparts. In Section 7, we extend our model to allow K_t to evolve stochastically over time. The variation of K_t will reflect changes in the stablecoin's user base and may capture potential competition from other stablecoins or payment instruments in general.

The first component of $u(x_t) = \frac{1}{\xi} x_t^{\xi} - x_t \eta |\sigma_t^p|$ is increasing in the numeraire value of stablecoin holdings in line with the modeling approach of real balance-in-utility in monetary economics (e.g., Baumol, 1952; Tobin, 1956; Feenstra, 1986; Freeman and Kydland, 2000).[14] The second component is decreasing in the stablecoin price's shock sensitivity, $|\sigma_t^p|$. The user's safety preference is captured by η (> 0). Such safety preference can be motivated, for example, by the role of stablecoins as means of payment. As a transaction medium, an asset must be information-insensitive and thereby deters private information acquisition, preventing asymmetric information on the payment instrument between trade counterparties (e.g., Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999). Safety preference is defined on the absolute value of σ_t^p as any loading on the issuer's asset shock (the source of "information" in the baseline model), whether positive or negative, generates information sensitivity.

[12]Internet Appendix F shows that implementing a continuous price path is in fact optimal for the issuer.

[13]There always exists a trivial equilibrium where p_t is a constant and equal to zero and $X_t = 0$ (implying $u(X_t) = 0$). We focus on the equilibrium where $p_t > 0$.

[14]This approach of modeling money demand functions has received empirical support (e.g., Poterba and Rotemberg, 1986; Lucas and Nicolini, 2015; Sunderam, 2015; Nagel, 2016; Krishnamurthy and Li, 2022).

The user has a quasi-linear instantaneous utility over dt that constitutes the utility from stablecoin holdings and her consumption, $Ku(x_t)dt + dY_t^u$, where Y_t^u is the cumulative (undiscounted) consumption process with superscript "u" for users. The user is risk-neutral in the consumption streams and discount them at the risk-free rate r (> 0). The user chooses x_t at any $t \in [0, \infty)$ to maximize the life-time utility

$$\max_{\{x_t\}_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-rt}\left[dY_t^u + Ku(x_t)dt\right]\right]. \tag{3}$$

Let N_t^u denote a representative user's wealth. The user faces the budget constraint:

$$dN_t^u = rN_t^u dt + Kx_t(\mu_t^p dt + \sigma_t^p dZ_t - f_t dt - rdt) - dY_t^u \tag{4}$$

and the transversality condition, $\lim_{s\to\infty} e^{-r(s-t)}\mathbb{E}_t[N_s^u] = 0$. The user allocates wealth between a risk-free asset and stablecoins. The stablecoin holdings, $X_t = Kx_t$, earns an excess return $dp_t/p_t - f_t dt - rdt$ where $dp_t/p_t = \mu_t^p dt + \sigma_t^p dZ_t$ and f_t is the fees charged by the issuer.

Proposition 1 (**Stablecoin demand**). *The users' demand is given by $X_t = Kx_t$, where*

$$x_t = \left(\frac{1}{r - \mu_t^p + \eta|\sigma_t^p| + f_t}\right)^{\frac{1}{1-\xi}}. \tag{5}$$

The user's demand has several intuitive properties. It is increasing in the expected return from price change, μ_t^p, decreasing in the fees, f_t, and the riskiness of the stablecoin, σ_t^p, and the user's discount rate r, which is the prevailing risk-free rate in the economy. If at time t the stablecoin is pegged to one dollar, then its price does not fluctuation, that is $\mu_t^p = 0$ and $\sigma_t^p = 0$, which implies a downward-sloping demand curve: $X_t = K\left(\frac{1}{r+f_t}\right)^{\frac{1}{1-\xi}}$, where $r + f_t$ is essentially the carry cost (the forgone interest-rate difference) for holding stablecoins. As will be made clear, we have $r - \mu_t^p + \eta|\sigma_t^p| + f_t > 0$ in equilibrium, i.e., x_t in (5) is well-defined.

In equilibrium, the stablecoin market clears:

$$X_t = S_t p_t, \tag{6}$$

where S_t is the total outstanding units of stablecoin (i.e., the aggregate nominal supply). Next, we set up the issuer' s problem with supply as part of the issuer's optimal strategy.

2.2 The stablecoin issuer

The issuer chooses a strategy, $(S_t, f_t, dY_t)_{t\geq 0}$, that involves the processes of stablecoin supply S_t, fees f_t, and consumption, Y_t (or equivalently, consumption, dY_t). We assume $dY_t \geq 0$ to capture restricted access to external equity financing, as, for example, in Bolton, Chen, and Wang (2011).[15] In Section 4.3, we allow the issuer to raise equity at a cost. At $t = 0$, the issuer is endowed with net worth, N_0 (i.e., the initial equity position).

At time t, the liability side of the issuer's balance sheet includes its net worth, N_t, and outstanding stablecoins, $p_t S_t$, so on the asset side, its total reserve assets are $A_t = N_t + p_t S_t$. We assume $A_t \geq 0$—that is, the issuer does not take short position in the reserve assets. The issuer's assets generate return $\mu dt + \sigma dZ_t$ over dt, where dZ_t is a standard Brownian shock, the only source of risk in our baseline model. Later in Section 7, we extend our model to incorporate shocks to users' demand. In our quantitative analysis, we calibrate μ and σ based on the composition and risk-return profiles of major stablecoin issuers' reserve assets.

In Internet Appendix C, we derive the law of motion of N_t, the issuer's net worth:

$$dN_t = (N_t + p_t S_t)(\mu dt + \sigma dZ_t) - p_t S_t(\mu_t^p dt + \sigma_t^p dZ_t) + p_t S_t f_t dt - p_t S_t \kappa dt - dY_t. \tag{7}$$

The first term is the return on its reserve assets. The second term is the price appreciation of stablecoin liabilities ($dp_t/p_t = \mu_t^p dt + \sigma_t^p dZ_t$). The third term represents the fee revenues. The operating cost, κdt per numeraire value of stablecoins, broadly reflects the issuer's expenses for sustaining the stablecoin's utility or function for its users, for example, by supporting and promoting it as a means of payment. And, the last term is the issuer's consumption. Note that when issuing more stablecoins and investing the proceeds in reserve assets, the issuer simultaneously expands liabilities and assets, which does not change the net worth, so dN_t only depends on the relative appreciation or depreciation of *existing* assets and liabilities (i.e., the first two terms), fee revenues, operating costs, and consumption.

In the law of motion (7), the right side contains the current state, N_t, the stablecoins supply, S_t, price, p_t (and its rate of change over dt, i.e., dp_t/p_t), fees, f_t, and issuer's consumption (dividend payouts), dY_t. Note that once the supply process is given, the price process is determined by the market-clearing condition under the stablecoin demand characterized in Proposition 1. Therefore, dN_t essentially depends on N_t and the strategy $(S_t, f_t, dY_t)_{t\geq 0}$.

We introduce a parameter condition:

$$\lambda := r + \kappa - \mu > 0\,. \tag{8}$$

[15]As in Brunnermeier and Sannikov (2014), negative consumption is equivalent to equity issuance.

Consider the hypothetical scenario where users' utility from stablecoin holdings is absent, i.e., $u(x_t) = 0$. Then the users requires a return of r to hold stablecoins, and on top of that, the issuer also covers the operational cost, κ. Under $r + \kappa - \mu > 0$, such pure financial intermediation—that is, the issuer raises funds via stablecoin issuances and invests in the reserve assets without providing any utility to users—is not profitable. Therefore, the condition (8) states that the users' utility, $u(x_t)$, from stablecoin holdings is the source of *seigniorage* for the issuer and ultimately justifies a positive amount of stablecoin supply.

The issuer chooses a strategy $(S_t, f_t, dY_t)_{t\geq 0}$ to maximize the present value of its lifetime consumption (i.e., the expected discounted value of dividends):

$$V_0 := \max_{(S_t, f_t, dY_t)_{t\geq 0}} \mathbb{E}\left[\int_0^{\infty} e^{-\rho t} dY_t\right]. \tag{9}$$

We assume $\rho > \mu$, which is standard in dynamic corporate finance and macro-finance models with financial constraints, e.g., Décamps et al. (2011), Bolton, Chen, and Wang (2011), Brunnermeier and Sannikov (2014) among others. Impatience induces consumption; otherwise, the issuer never consumes and always accumulates financial slack (net worth) so that eventually the external financing constraint (i.e., $dY_t \geq 0$) no longer matters. As ρ determines the issuer's willingness to grow net worth (rather than consume it), it is calibrated by matching data on stablecoin issuers' net worth. Appendix A provides calibration details.

3 Model Solution

We characterize the solution to the issuer's problem and provide analytical results describing the model dynamics. The formal treatment of the issuer's optimization—and the proofs for the Propositions—are provided in Internet Appendices C and D. The next section focuses on quantitative analysis. Specifically, we analyze the issuer's optimal strategy, including its decisions on stablecoin depegging, issuances, fees, and consumption. We also characterize the issuer's franchise value through the value function and to what extent the stablecoin can be under-collateralized (i.e., the issuer's net worth can turn negative). The model reveals several distinct phenomena about stablecoins, such as the instability trap and risk paradox.

3.1 State variable, strategy space, and value function

In the following, we start our analysis by simplifying the issuer's strategy space. As previously discussed, we divide all quantity variables by K, the stablecoin-demand scaler, and use n_t, s_t, and dy_t to denote, respectively, the K-scaled net worth (N_t/K), stablecoin supply (S_t/K), and consumption (dY_t/K). We simply refer to n_t, which is the state variable in our model, as the issuer's net worth, s_t as supply, and dy_t as consumption. The next lemma summarizes the law of motion of n_t. It shows that the strategy space of three stochastic processes, i.e., $(S_t)_{t\geq 0}, (f_t)_{t\geq 0}$, and $(dY_t)_{t\geq 0}$, can be transformed into a more tractable form.

Lemma 1 (State variable law of motion). *The issuer's net worth, n_t, evolves as follows:*

$$dn_t = \underbrace{[\mu(n_t + x_t) - x_t(r - \zeta_t) - x_t\kappa]}_{\mu_n(n_t)}dt + \underbrace{[\sigma(n_t + x_t) - x_t\sigma_t^p]}_{\sigma_n(n_t)}dZ_t - dy_t, \tag{10}$$

where ζ_t is the stablecoin user's marginal utility from holding stablecoins:

$$\zeta_t = u'(x_t) = x_t^{\xi-1} - \eta|\sigma_t^p|. \tag{11}$$

As shown in Internet Appendix C, the issuer's value function (i.e., continuation value) can be expressed as a function of the state variable $n_t = n$ only, satisfying $V(n_t) = Kv(n_t)$, where $v(n_t)$, the K-scaled value function, is simply refer to as value function below.

Three variables determine the law of motion of n_t: 1) the value or quantity of stablecoins, x_t; 2) the instantaneous volatility of stablecoin (price) returns, σ_t^p; 3) the issuer's consumption, dy_t. Once they are determined as functions of n_t, the dynamics given by (10) yield an autonomous law of motion. Note that $(S_t, f_t, dY_t)_{t\geq 0}$ affects the dynamics of n_t only through $(x_t, \sigma_t^p, dy_t)_{t\geq 0}$. Therefore, instead of characterizing optimal processes for stablecoin supply, fees, and the issuer's consumption, we solve the auxiliary problem of optimization over $(x_t, \sigma_t^p, dy_t)_{t\geq 0}$. Solving the optimal $(x_t, \sigma_t^p, dy_t)_{t\geq 0}$ also leads to clear intuitions on the economic mechanisms. We then show that this auxiliary problem indeed solves the problem given by (9), in that the optimal choice of $(x_t, \sigma_t^p, dy_t)_{t\geq 0}$ can be implemented via a strategy $(S_t, f_t, dY_t)_{t\geq 0}$ (see Internet Appendix C for details on the solution).

The law of motion of n_t given by (10) has several intuitive properties. In the drift, $\mu_n(n_t)$, the first term represents the expected return on reserve assets funded by the issuer's net worth and stablecoin issuances. The second term is the cost of issuing stablecoins: the issuer must compensate the users their required rate of return, r, minus the user's marginal utility from holding stablecoins, ζ_t, which is the marginal *seigniorage* earned by the issuer. The third term is the operating cost. The diffusion, $\sigma_n(n_t)$, includes the risk in the reserve-asset return, and the second part captures the risk borne by stablecoin holders. Under $\sigma_t^p > 0$, the issuer effectively offloads risk to users. Under these circumstances, the stablecoin price and value of the issuer's stablecoin liabilities decline following a negative shock to reserve assets. This effect reduces the risk exposure of the issuer's net worth. We will show that this risk sharing mechanism is key for understanding the stablecoin issuer's incentives to maintain the peg.

Next, we characterize the issuer's optimal strategy and how it affects the law of motion of n_t, starting with the issuer's consumption choice. The next proposition shows that the issuer consumes, $dy_t > 0$, only when n_t reaches $\overline{n}$, an endogenous upper boundary of n_t.

Proposition 2 (**The issuer's consumption**). *There exists $\overline{n}$, a reflecting upper bound of n_t with the following properties: 1) the issuer consumes variations of n_t that move n_t beyond $\overline{n}$: when n_t reaches $\overline{n}$, $dy_t = dn_t$ if $dn_t > 0$, and $dy_t = 0$ if $dn_t \leq 0$; 2) consumption optimality implies that the value function satisfies the following two boundary conditions:*

$$v'(\overline{n}) - 1 = 0, \text{ and, } v''(\overline{n}) = 0. \tag{12}$$

The issuer does not consume (i.e., $dy_t = 0$ in the law of motion (10)) if $n_t < \overline{n}$. Thus, for $n < \overline{n}$, the value function, $v(n)$, satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho v(n) = \max_{\sigma^p, x} \left\{ v'(n) \Big[\mu(n+x) - rx + x^{\xi} - \eta x |\sigma^p| - \kappa x \Big] + \frac{v''(n)}{2} \Big[\sigma(x+n) - x\sigma^p \Big]^2 \right\}. \tag{13}$$

We suppress the time subscripts to simplify the notations. The following Proposition summarizes the properties of value function and introduces the issuer's effective risk aversion.

Proposition 3 (**Value function**). *The value function, $v(n)$, is strictly increasing and concave in n for $n < \overline{n}$, i.e., $v'(n) > 0$ and $v''(n) < 0$. The issuer's effective risk aversion based on the value function is defined as*

$$\gamma(n) = -\frac{v''(n)}{v'(n)} > 0, \tag{14}$$

for $n < \overline{n}$. $\gamma(n)$ is strictly decreasing in n, i.e., $\gamma'(n) < 0$. At $n = \overline{n}$, $\gamma(n) = 0$.

The issuer's effective risk aversion arises from its financial constraint—that is, it cannot raise external funds and must rely its own net worth for managing the stablecoin. The wedge between $v'(n)$, the marginal value of net worth ("retained earnings" accumulated up to time t), and 1, the marginal value of consumption ("payout"), reflects how tight the financial constraint is. At the consumption (upper) boundary of n_t, the issuer is effectively unconstrained with $v'(\overline{n}) = 1$ (see the boundary condition (12)) and not risk-averse, i.e., $\gamma(\overline{n}) = 0$. At any $n < \overline{n}$, we have $v'(n) > 1$ (implied by the concavity of $v(n)$ and $v'(\overline{n}) = 1$), and $v'(n)$ is higher when n falls further below $\overline{n}$, indicating that the issuer is more financially constrained and thus more risk-averse (i.e., $\gamma'(n) < 0$). In Section 4.3 where we allow the issuer to raise external financing at a cost, this pattern remains. In the next subsection, we show that $\gamma(n)$ is key for understanding the issuer's decision to depeg the stablecoin.

Finally, to complete the characterization of the state space and value function, we characterize the lower bound of n_t in the following proposition, denoted by $\underline{n}$. In the Internet Appendix, (C.7) provides the closed-form expression for $\underline{n}$.

Proposition 4 (State variable lower bound). *There exists $\underline{n}$ ($< \overline{n}$), a lower bound of n_t (i.e., $n_t \geq \underline{n}$) that is absorbing. The value function is zero at $\underline{n}$:*

$$v(\underline{n}) = 0\,. \tag{15}$$

The intuition behind $\underline{n}$ as an absorbing lower bound is as follows. The condition $n_t = a_t - x_t \geq \underline{n}$ states that the issuer needs to maintain adequate reserves, $a_t = \frac{A_t}{K}$, relative to its stablecoin liabilities, x_t. If its reserves fall short, the issuer lacks the revenues from reserve assets to cover the costs of its stablecoin liabilities and thus cannot generate profits to grow net worth.[16] Once n falls to $\underline{n}$, the issuer has no prospect of recovery: both the drift and diffusion of n_t fall to zero. This permanent lack of profitability translates to a zero continuation value, which the issuer seeks to avoid. In order to maintain n_t above $\underline{n}$, the issuer may have to depeg the stablecoin, which we discuss in the next subsection.

In summary, the issuer's value function solves (13), a differential equation for $v(n)$, subject to the boundary conditions (12) and (15), and its net worth evolves according to (10) in $[\underline{n}, \overline{n}]$. We also show that for all $n > \underline{n}$, the issuer maintains strictly positive reserve assets (does not short sell assets) and the constraint $A_t \geq 0$ never binds, in that $n + x > 0$; accordingly, we omit it going forward and do not explicitly account for it in the HJB equation optimization.

3.2 Depegging, risk paradox, and instability trap

The next proposition lays out when and how depegging happens in equilibrium.

Proposition 5 (Stablecoin quality and optimal depegging). *Under the condition,*

$$\mu - (1-\xi)(r+\kappa) - \eta\sigma \leq 0\,, \tag{16}$$

the issuer optimally sets $\sigma^p(n) = 0$ and the stablecoin price is always pegged to one, i.e., $p(n) = 1$ for all $n \in (\underline{n}, \overline{n}]$. If the condition (16) does not hold, there exists a unique $\widetilde{n} \in (\underline{n}, \overline{n})$ that separates two regions:

- *For $n \geq \widetilde{n}$, the peg holds, i.e., $p(n) = 1$ and $\sigma^p(n) = 0$;*
- *For $n < \widetilde{n}$, depegging happens, i.e., $p(n) < 1$, and the stablecoin price comoves with the issuer's reserve-asset shocks, i.e., $p'(n) > 0$ and $\sigma^p(n) > 0$, with $\sigma^p(n)$ decreasing in n.*

[16] As previously discussed, the issuer must compensate the users their required return, r, minus their marginal utility from holding stablecoins, ζ_t, and incurs the operating cost, κ.

As previously discussed and specified in (2), the quality or "moneyness" of stablecoin is inversely related to $\sigma_t^p = \sigma^p(n_t)$. The proposition above shows that the issuer always maintains the peg if the condition (16) holds; otherwise, depegging happens once the issuer's net worth falls below a threshold. In the following, we first explain the intuition behind the condition (16) and then discuss depegging when the condition does not hold.

According to Proposition 3, the issuer's effective risk aversion, $\gamma(n)$, is highest when n approaches $\underline{n}$, where the issuer's risk-bearing capacity is exhausted. To reduce risk exposure, the issuer can either reduce stablecoin issuance, i.e., deleverage (the quantity margin), or offload asset risk to the users through depegging and allowing the stablecoin price to fluctuate with asset shocks, i.e., $\sigma^p(n) > 0$ (the quality margin). The latter option—that is, issuing a marginal unit of stablecoin, investing the proceeds in reserve assets, and offloading the reserve-asset risk to the stablecoin users—generates a marginal profit of $\mu-(1-\xi)(r+\kappa)-\eta\sigma$. Here, $\mu-(1-\xi)(r+\kappa)$ is akin to a net interest margin: the marginal asset funded by stablecoin issuance earns μ, but the issuer must compensate the users' required return, r, and cover the operating cost, κ, facing an overall cost of $(1-\xi)(r+\kappa)$, where $(1-\xi)$ is tied to the users' demand elasticity (see (5)). The last term, $\eta\sigma$, is the reduction of users' marginal utility from holding stablecoins (i.e., the issuer's seigniorage given by (11)). Therefore, if the condition (16) holds, i.e., $\mu - (1-\xi)(r+\kappa) - \eta\sigma < 0$, compromising quality is not optimal, and the issuer would prefer adjusting quantity to reduce risk exposure. Since $\gamma(n)$ is the highest and the issuer's incentive to depeg strongest as n approaches $\underline{n}$, if adjusting the quality margin is not profitable near $\underline{n}$, the issuer would not depeg when n is higher. Therefore, if the condition (16) holds, depegging does not happen in the whole range of n.

If the condition (16) does not hold, the issuer depegs the stablecoin and offloads risk to the users by setting $\sigma^p(n) > 0$, once its net worth falls below the threshold $\widetilde{n}$ and it becomes sufficiently risk-averse. As shown in Proposition 3, $\gamma(n)$ rises as n falls. Therefore, the issuer shares more risk with the users by increasing $\sigma^p(n)$ when n falls further below $\widetilde{n}$. Under $p'(n) > 0$ and $\sigma^p(n) > 0$, when the issuer's reserve assets are hit by negative shocks, its liabilities—the value of stablecoins—decline as well, mitigating the impact on its net worth.

An increase in η—the parameter for users' stability preference—enlarges the parameter region where depegging does not happen. Under a higher η, the reduction of seigniorage triggered by the issuer's depegging is larger, thus making the quality adjustment less attractive as a way to control risk exposure (the issuer focuses on the quantity adjustment).

Interestingly, increasing σ (reserve-asset riskiness) while holding other parameters fixed makes the condition (16) more likely to hold, which implies a greater parameter region where a perfectly safe stablecoin can be sustained. In contrast, reducing σ—that is, the issuer's reserve assets are safer—enlarges the parameter region where depegging happens.

The following proposition summarizes this result of risk paradox.

Corollary 1 (Risk paradox: parameter region). *Higher reserve-asset risk, σ, shrink the parameter region where the condition (16) does not hold and depegging happens.*

When reserve assets are riskier, the parameter region where the condition (16) fails and depegging happens actually shrinks, and the region without depegging expands. This seemingly counterintuitive result is due to the users' discipline on the issuer's depegging decision. As shown in (5), offloading risk to users dampens demand. When σ is higher, reducing stablecoin quality as a way to control risk exposure (i.e., sharing risk with the users) causes a larger reduction of seigniorage. As a result, the issuer would prefer quantity adjustment (i.e., reducing stablecoin issuance or deleveraging) when its net worth is low. As shown below by our results on the "instability trap", once depegging happens, it causes persistent demand destruction. Under a higher σ, such demand destruction is more significant.

In the parameter region where depegging can happen (i.e., the condition (16) does not hold), we observe the following self-reinforcing dynamics, which we call the "instability trap." In this parameter region, the depegging happens and the price fluctuates, when the issuer's net worth, n, falls below the critical threshold (i.e., $p(n) < 1$ and $\sigma^P(n) > 0$ for $n < \widetilde{n}$ in Proposition 5). Depegging reduces the seigniorage revenues.[17] The further n declines, the lower the issuer's profits from the seigniorage is as depegging becomes more severe and $\sigma^P(n)$ increases. Profit reduction slows net-worth rebuild, trapping the system in the low-n region where depegging becomes persistent. Put differently, seigniorage revenues are *procyclical*, comoving with the issuer's net worth. This procyclicality underlies the instability trap.

The next proposition characterizes the stationary distribution of state variable that describes the amount of time the system spends at each level of the issuer's net worth, n. The instability trap manifests itself into the rising probability density of the stationary distribution as n falls, which reflects that the lower boundary $\underline{n}$ is absorbing.

Proposition 6 (Instability trap). *If the condition (16) does not hold and thus depegging happens at $n < \widetilde{n}$, where $\widetilde{n}$ is defined in Proposition 5, the stationary density of state variable n, denoted by $g(n)$, exists, and is strictly decreasing in n, i.e., $g'(n) < 0$, for $n < \widetilde{n}$.*

Next, we introduce a new aspect of risk paradox in our model. The next proposition describes what happens within the parameter region where the condition (16) does not hold and depegging happens once the issuer's net worth, n, falls below $\widetilde{n}$ given by Proposition 5. In this parameter region, the highest level of volatility is *inverted U-shaped* in σ.

[17] The seigniorage revenues are given by $x_t\zeta_t$. Specifically, when n_t falls below $\widetilde{n}$, a further decline raises $\sigma^p(n_t)$, so the seigniorage per dollar value of stablecoin, $\zeta_t = \zeta(n_t)$ in Lemma 1), decreases. Moreover, we will show in Proposition 9, $x_t = x(n_t)$ is a constant for $n_t < \widetilde{n}$. Therefore, $x_t\zeta_t$ is decreasing in n_t.

Proposition 7 (**Risk paradox: endogenous volatility**). *When the condition (16) does not hold (depegging happens when $n < \widetilde{n}$ in Proposition 5), stablecoin volatility satisfies*

$$\sigma^p(n) \leq \sup_{n \in (\underline{n}, \overline{n})} \{\sigma^p(n)\} = \sigma \left(\frac{\mu - (1-\xi)(r+\kappa) - \eta\sigma}{\xi(\mu - \eta\sigma)} \right), \tag{17}$$

where the supremum is first increasing in σ and, when σ is sufficiently high, decreasing in σ.

Intuitively, volatility starts at zero when $\sigma = 0$, i.e., $\sigma^P(n) = 0$ for any n under $\sigma = 0$. As σ increases, the issuer starts sharing risk with the users in the low-n region, so the highest level of $\sigma^p(n)$ rises. This is the scaling effect of σ on the right side of (17).

The risk paradox emerges once σ passes a threshold—that is, a higher σ leads to a reduction in the maximum level of stablecoin-price volatility. The scaling effect is dominated by the issuer's precaution against the instability trap triggered by quality adjustment (i.e., risk-sharing with the users). The instability trap is more potent a force under a higher σ. Instead of quality adjustment, the issuer relies more on the quantity margin (reduces stablecoin issuance) to control risk exposure when σ is higher.

Having summarized the dynamics of stablecoin price, we introduce the next proposition that provides an intuitive representation of stablecoin price.

Proposition 8 (**Stablecoin price and risk-sharing**). *Stablecoin price can be written as*

$$p(n) = \exp\left(-\int_n^{\widetilde{n}} \frac{\sigma^p(\nu)}{\sigma_n(\nu)} d\nu \right), \tag{18}$$

in the parameter region where depegging happens (i.e., when the condition (16) does not hold), where the threshold $\widetilde{n}$ is given by Proposition 5.

As shown in Proposition 5, $p(n) = 1$ for any n if the condition (16) holds; otherwise, the proposition above shows that the price of stablecoin reflects the extent to which the issuer shares reserve-asset risk with the users. The ratio $\frac{\sigma^p(\nu)}{\sigma_n(\nu)}$ measures the fraction of risk exposure in the issuer's net worth that has been offloaded to the users. Since only when $n < \widetilde{n}$, the issuer shares risk (sets $\sigma^p(n) > 0$), the upper limit of the integral is $\widetilde{n}$. Intuitively, the more risk the issuer offloads to the users, the lower the stablecoin price is.

3.3 Stablecoin supply dynamics

So far, our analysis focuses on stablecoin price or "quality". The next proposition summarizes the quantity dynamics, i.e., how the value of stablecoins outstanding evolves.

Proposition 9 (**Issuance dynamics**). *If condition (16) holds, the issuer supplies stablecoins worth $x(n)$ that is strictly increasing in its net worth n, i.e., $x'(n) > 0$. If condition (16) does not hold, $x'(n) > 0$ if $n > \widetilde{n}$, and for $n < \widetilde{n}$, $x(n) = \underline{x} = \left(\frac{\xi}{\lambda+\eta\sigma}\right)^{\frac{1}{1-\xi}}$, a constant.*

A key message from the proposition above is that the issuer creates more stablecoins when it accumulates net worth. Supplying stablecoins backed by the reserve assets requires risk-taking capacity that is inversely tied to the issuer's effective risk aversion, $\gamma(n)$, and from Proposition 3, $\gamma'(n) < 0$. Increasing $x(n)$, the value of stablecoins issued, allows the issuer to earn the spread between the reserve assets and stablecoins as sources of funds. The stablecoin funding cost is $r - \zeta_t$ as shown in n_t's law of motion (10). In Lemma 1, we show that ζ_t is the users' marginal utility from holding stablecoins, which reduces the issuer's funding cost and is essentially a form of *seigniorage* for the money supplier. However, a higher $x(n)$ also means the issuer bears more risk, as shown by the diffusion term in (10), unless the issuer allows the stablecoin price to fluctuate and thereby share risk with the users. Sharing risk, i.e., increasing $\sigma^p(n)$, reduces the seigniorage per dollar of stablecoins issued, as shown in (11). Therefore, the issuer faces a trade-off between seigniorage profits and risk exposure when determining the value of stablecoins issued, $x(n)$.

The next corollary derives the dynamics of $s(n) = x(n)/p(n)$, the nominal supply or units of stablecoins outstanding from the results in Proposition 5 and 9.

Corollary 2 (**Stablecoin supply**). *If the condition (16) holds, we have $s'(n) > 0$; otherwise, $s'(n) > 0$ for $n > \widetilde{n}$, where $\widetilde{n}$ is defined in Proposition 5, and $s'(n) < 0$ for $n < \widetilde{n}$.*

If the condition (16) holds, the stablecoin is always pegged (see Proposition 5), so under $p(n) = 1$, $s(n) = x(n)/p(n) = x(n)$. Therefore, $s(n) = x(n)$ is strictly increasing in n (see Proposition 9). If the condition (16) does not hold, we have $s(n) = x(n)$ under $p(n) = 1$ for $n \geq \widetilde{n}$ (see Proposition 9) with $s'(n) = x'(n) > 0$ (see Proposition 5) and, for $n < \widetilde{n}$, we have $p'(n) > 0$ (see Proposition 9) and $x(n)$ being a constant (see Proposition 5), so $s(n) = x(n)/p(n)$ is decreasing in n. The units of stablecoins supplied, $s(n)$, is *U-shaped* in n. The issuer follows a pecking-order strategy when negative shocks erode its net worth: first, when n is still above the critical threshold $\widetilde{n}$, it deleverages by reducing the value and units of stablecoins supplied, i.e., $x'(n) = s'(n) > 0$, while maintaining the peg; second, once n falls below $\widetilde{n}$, the dollar value of stablecoins is held constant at $\underline{x}$, and the issuer offloads risk to the users through depegging, reducing the quality of stablecoins as shown in Proposition 5. In addition, the further its net worth falls, the more units of stablecoins it issues albeit at lower prices as depegging intensifies.

In summary, our approach to solve the issuer's optimization was as follows. Proposition 2 shows the issuer's optimal consumption, dy_t. In Lemma 1, we show that the law of motion of the state variable, n_t, depends on (S_t, f_t) only via (x_t, σ_t^p), whose optimal choices are given by Propositions 5 and 9. Finally, we characterize the supply, s_t, in Corollary 2 and, in Internet Appendix C.6, the fees, f_t that implement optimal (x_t, σ_t^p).

3.4 Under-collateralization

Finally, we turn to a widely debated question about stablecoins: can under-collateralization be sustained? The next proposition confirms that this is indeed the case: the lower bound for n_t, the issuer's net worth—the difference between the value of reserve assets and that of outstanding stablecoins—is negative, which gives the maximum level of under-collateralization.

Corollary 3 (Under-collateralization). *The lower bound $\underline{n}$ in Proposition 4 has a closed-form expression given by* (C.7) *in the Internet Appendix; it is negative and satisfies $\partial \underline{n}/\partial r > 0$. Further, in the parameter region where depegging occurs (i.e., when the condition* (16) *does not hold), $\underline{n}$ satisfies $\partial \underline{n}/\partial \sigma > 0$ and is never reached (i.e., it is inaccessible).*

The issuer maintains its net worth above the absorbing bound $\underline{n}$ where the franchise value falls to zero. The bound is negative, which implies that the stablecoins can be under-collateralized. When users' required return increases, $\underline{n}$ becomes less negative, implying a smaller room for under-collateralization. The issuer has to deliver a higher return to the users, so its seigniorage and franchise value decline, and its net worth, n, cannot fall too far below zero before the franchise value (or value function) reaches zero.

If the condition (16) holds, the issuer always maintains the peg, as shown in Proposition 5, and as a result, the riskiness of its reserve asset, σ, does not affect the lower bound of its net worth. However, when the condition (16) does not hold and depegging happens in the low-net worth region, the users absorb the issuer's asset risk. Therefore, the riskier the issuer's asset is, the more risk the users absorb, which either dampens their stablecoin demand or, to sustain demand, leads to less fees to the issuer (i.e., fee reduction as users' risk compensation). Both forces reduce the issuer's franchise value, and the second force requires the issuer to hold more reserve assets. These mechanisms raise the lower bound, $\underline{n}$.

One may argue that under-collateralization is infeasible because over-collateralization is necessary to meet the users' withdrawal. Such argument ignores the fact that any withdrawal can be met if the issuer is willing to depeg the stablecoin. As shown in Proposition 5, maintaining the peg or not is the issuer's choice, which is different from the traditional corporate finance settings where debt repayments are legally binding and failing to repay leads to bankruptcy. In other words, the value of the issuer's liabilities—stablecoins—is chosen by the issuer itself.[18] Such choice is priced in through the users' expectation of

[18] We also want to highlight that the issuer maintains a positive continuation value—that is, $n_t > \underline{n}$ and

price volatility as shown in (see (5)). Expectation of depegging weakens the users' demand and reduces the issuer's seigniorage—that is, the stablecoin issuer faces a trade-off between preserving net worth by offloading asset risk to the users (i.e., maintaining $n_t > \underline{n}$) and sustaining the users' demand and seigniorage (i.e., ζ_t in Lemma 1).

A "run-like" phenomenon can emerge in our model if the condition (16) does not hold. Following negative shocks to the issuer's assets, its net worth declines. As shown in Proposition 5, the price of stablecoin falls and volatility rises, which dampens the users' demand (see Proposition 1). Such phenomenon of depegging and withdrawal (i.e., a decrease of users' demand) resembles a run but is not due to coordination failure that is seen in Diamond and Dybvig (1983) among others; instead, it is due to the issuer's optimal decision to dynamically share reserve-asset risk with the users. As previously discussed and formalized in Proposition 6, such run-like behavior is self-reinforcing, resulting in the instability trap.[19]

4 Quantitative Analysis

We calibrate the model, conduct quantitative analysis of equilibrium dynamics, and evaluate several regulatory proposals. Our numerical solution is obtained by solving the differential equation for $v(n)$ (the HJB equation (13)) with the boundary conditions (12) and (15).

4.1 Model calibration

We calibrate our model to Tether, the largest stablecoin issuer. Its U.S. dollar stablecoin is USDT. Tether's report as of December 31, 2024 states that Tether holds reserve assets of \$143.7 billion, with liabilities (stablecoins outstanding) of \$136.6 billion, resulting in equity (net worth) of \$7.1 billion (Tether Transparency Report). In our model, all quantity variables are scaled by K, which we set to 1 billion. We map the balance-sheet status of Tether to $n_t = \overline{n}$, i.e., the level (upper bound) of issuer's net worth that triggers payout, which is in line with the fact that Tether made a sizable payout in 2024. Therefore, we have $x(\overline{n}) = 136.6$ and $\overline{n} = 7.1$. We adjust the composite parameter λ (defined in (8)) and the issuer's discount rate ρ to match these two numbers. As a result, obtain $\lambda = 0.0282$ and $\rho = 0.1371$.

We calibrate ξ that governs the users' demand elasticity (see (5)). As shown in Proposition 5, the stablecoin is pegged at $p(\overline{n}) = 1$ at this level of the issuer's net worth, which is in line with the robust peg of USDT around 2024 year end. At $\overline{n}$, $\sigma^p(\overline{n}) = 0$, so the users' marginal utility from stablecoin holdings is $\zeta(\overline{n}) = x(\overline{n})^{\xi-1} = 136.6^{\xi-1}$ (see (11)), which we

$v(n_t) > 0$. Thus, it does not voluntarily shut down the operation. Therefore, the users' withdrawal cannot force the issuer into bankrupt, and the issuer is not willing to declare bankruptcy on its own.

[19] In Proposition 6, the absorbing lower bound $\underline{n}$ is never reached so the stationary distribution exists.

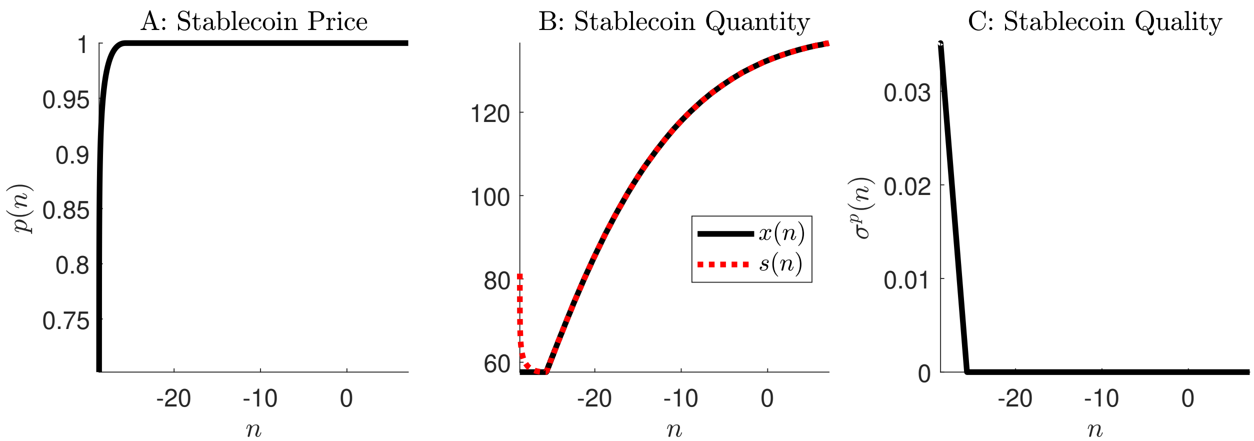

Figure 1: **Stablecoin Price and Quantity Dynamics.**

calibrate to the marginal convenience yield of USDT to obtain $\xi = 0.441$. In Appendix A, we provide details on how to measure the convenience yield of USDT.

We set μ and σ to 13.4% and 7%, respectively, based on Tether's disclosure of reserve assets. In Appendix A, we provide details on the decomposition of reserve assets into different asset classes and how we compute the returns and volatilities of each asset class. Note that in our model, r and κ appear in equilibrium conditions together with μ as part of the composite parameter, $\lambda = r + \kappa - \mu$. Once λ and μ are set, we no longer need to pin down r and κ.

Finally, for η that represents the users' risk sensitivity, we consider a value that is sufficiently low so the condition (16) for perfect stability does not hold and depegging is possible. Given the parameter values above, the upper bound for η is 0.61. For the lack a direct empirical counterpart, we set η to 0.25 and report in Figure 2 results of comparative statics across different values of η, such as $\eta = 0.1$ and 0.4; our results are robust to the choice of η.

Our calibrated model sheds light on Tether's payouts. According to Proposition 2, the issuer pays out $dn_t > 0$ at $n_t = \overline{n}$, where dn_t has the drift and diffusion components. We have a drift $\mu_n(\overline{n}) = 5.84$, which is \$5.84 billion as quantity variables are scaled by $K = \$1$ billion. The diffusion component is random, loading on the Brownian shocks. Tether reported a \$10 billion payout for 2024. Through the lens of our model, it includes the \$5.84 billion expected component and \$4.16 billion due to unexpected gains from reserve-asset shocks. In Section 7, we extend our model to allow the demand-scaler K to evolve stochastically over time and discuss how our model can be used to value stablecoin issuers' equity.

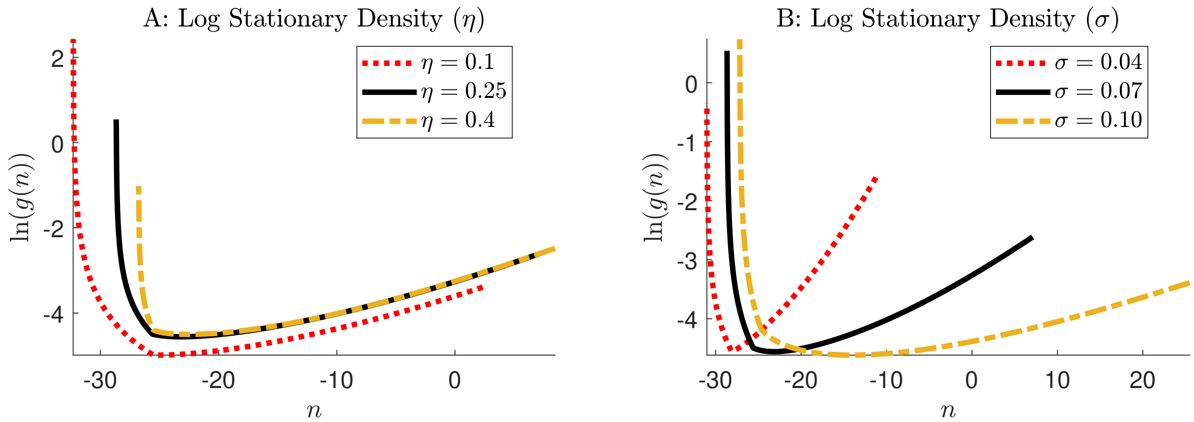

Figure 2: **The Stationary Probability Density and Instability Trap.**

4.2 Model performances

Price and issuance dynamics. Figure 1 illustrates the model dynamics under the calibrated parameters. In each panel, we plot the endogenous variable against the issuer's net worth, n. In Panel A, we plot the stablecoin price, $p(n)$. The peg holds until the issuer's net worth falls below $\widetilde{n}$—the critical threshold in Proposition 5. This threshold is significantly below zero. Near $\underline{n}$, the maximum deviation from the peg is about 30%. In Panel B, we plot the instantaneous volatility of dp/p (see (1)). As depegging takes place in the low-n region, the (annualized) volatility ranges from 0% to above 3.5%.

In Panel B of Figure 1, we plot the value of stablecoins supplied by the issuer (solid line). It is strictly increasing in its net worth for $n > \widetilde{n}$, reaching \$136.6 billion at $\overline{n}$, which is one of our calibration targets. The value of stablecoins supplied drops by almost 60% once n falls below $\widetilde{n}$. As shown in Proposition 9, the value of stablecoin supplied is a constant if $n < \widetilde{n}$. Thus, as the price declines when n falls further below $\widetilde{n}$, more units of stablecoins are issued. In Panel C, we plot the units of stablecoin or the nominal supply in the dotted line. It shows a sharp upturn as n approaches $\underline{n}$, which, in practice, is often viewed as the stablecoin issuer desperately trying to raise revenues by expanding supply in spite of a rapidly falling price.

Panel C of Figure 1 illustrates the risk-sharing mechanism behind depegging. As discussed in the previous section, depegging happens because the issuer wants to offload risk to the users, which manifests into a rising $\sigma^p(n)$ as n falls below $\widetilde{n}$. As shown in (10), a higher $\sigma^p(n)$ reduces the issuer's risk exposure—that is, for $n < \widetilde{n}$, when shocks hit the issuer's assets and n varies, the value of its stablecoin liabilities adjusts as well.

Instability trap. Once the issuer's net worth falls below $\widetilde{n}$, it faces a difficult trade-off: risk-sharing through depegging is necessary for net worth preservation but the resultant

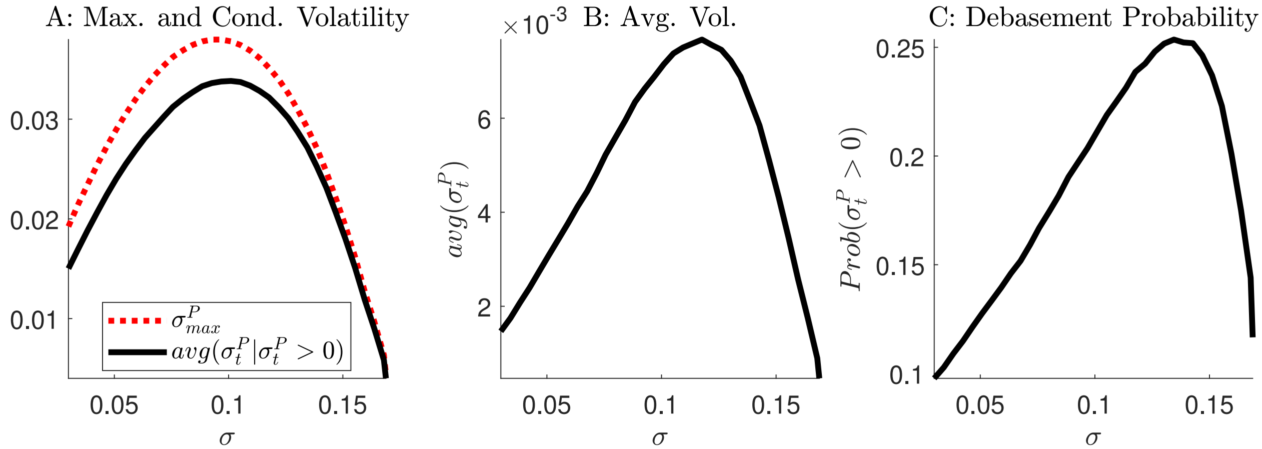

Figure 3: **Risk Paradox.**

demand destruction and decline of profits (i.e., the reduction of seigniorage ζ_t given by (11)) slows down the rebuild of net worth. The endogenous procyclicality of seigniorage revenues leads to the instability trap in Proposition 6. In Figure 2, we plot the logarithm of stationary probability density of n. The stationary density shows the amount of time the system spends at different levels of n. In Panel A, we plot it for different values of the stablecoin users' risk sensitivity, η, and in Panel B, we plot the stationary density for different levels of reserve-asset riskiness, σ. Across different values of η and σ, the pattern is consistent: the instability trap emerges and is represented by the rising density as n falls below $\widetilde{n}$.

Panel A of Figure 2 shows that instability trap emerges across different values of η. It also delivers an interesting message: as η increases, the system spends less time in the region near $\underline{n}$—that is, the force of instability trap weakens. When the users become more averse to fluctuation in the stablecoin price, the issuer is more "disciplined" in its decision to share risk with the users through depegging, so the downward spiral of depegging, demand and profit destruction, and persistently low net worth is less likely to be triggered.

Panel B of Figure 2 plots the stationary probability density under different values of σ. The instability trap intensifies as σ increases, as shown by sharper upturns in the density function in the low-n region. This relationship may eventually reserves as σ increases further: the issuer' precaution against the instability trap, which can be rather destructive under high σ, makes it refrain from depegging too often, which makes the stablecoin price more stable.

Risk paradox. In Figure 3, we demonstrate the risk paradox. Note that there are two aspects of risk paradox. The first aspect is about the parameter region where a perfect peg is maintained (Proposition 1). This result is straightforward as one can simply inspect how σ affects the condition (16). Figure 3 is not about this aspect of risk paradox. The figure is about the second aspect, i.e., the results in Proposition 7: in the parameter region where depegging happens, an inverted-U shaped relationship exists between the highest level of stablecoin-price volatility and the riskiness of the issuer's reserve assets, which is illustrated by the dotted line in Panel A. We also plot the average volatility against n, where the average is computed from the stationary probability density of n conditional on depegging (i.e., $p(n) < 1$, or equivalently, $\sigma^p(n) > 0$). A similar inverted-U shape emerges.

As previously discussed in Section 3.2, the inverted-U shape results from two forces. First, under $\sigma = 0$, the issuer does not need to share any risk with the users, so the stablecoin price does not fluctuate. As σ increases, the need for risk sharing arises, which leads to depegging and stablecoin-price fluctuation. The second force is about the issuer's endogenous precaution against the instability trap, and it becomes the dominant force once σ surpasses a certain level. In Panel B of Figure 3, we show the inverted U-shaped relationship between the average volatility and σ. Different from the solid line in Panel A, the average volatility here is computed over the full state space (not conditional on $n < \widetilde{n}$, i.e., being in the depegging region). In Panel C, we plot the probability of $n < \widetilde{n}$ (depegging) in the long run using the stationary distribution of n and demonstrate a similar inverted-U shape.

Overall, our results on the risk paradox is about the transmission of exogenous risk in the issuer's reserve assets to the endogenous fluctuation of stablecoin price. One may argue that we can legally force stablecoin issuers to hold reserves in perfectly safe assets. However, as we will show in Section 6.1, such restriction significantly reduces stablecoin supply and the users' welfare. Moreover, in reality, perfectly safe assets are rare. Even for the U.S. Treasuries, once the maturity goes beyond one year, significant risk emerges due to changes in inflation, the stand of monetary policy, and currency exchange rates for foreign investors.

4.3 Equity issuance

We extend our model by allowing the issuer to raise equity. In practice, equity financing may come from traditional sources, such as public offerings and venture capital investments, or the issuance of "governance tokens" that resemble equity.[20] We introduce issuance costs following dynamic corporate finance models (e.g., Riddick and Whited, 2009; Décamps, Mariotti, Rochet, and Villeneuve, 2011; Bolton, Chen, and Wang, 2011). Specifically, the issuer faces a fixed cost F and, as in the literature, raises equity to a targeted level, denoted by n^E.

Since the equity investors are competitive (i.e., they require shares worth of one dollar for each dollar contributed), it is optimal for the issuer to raise equity as long as the marginal value, $v'(n)$, is greater than one, once the issuance cost is incurred. As shown in Proposition

[20] One example of governance token is MKR, the governance token of the MakerDAO protocol that issues DAI, the fourth largest stablecoin by market capitalization.

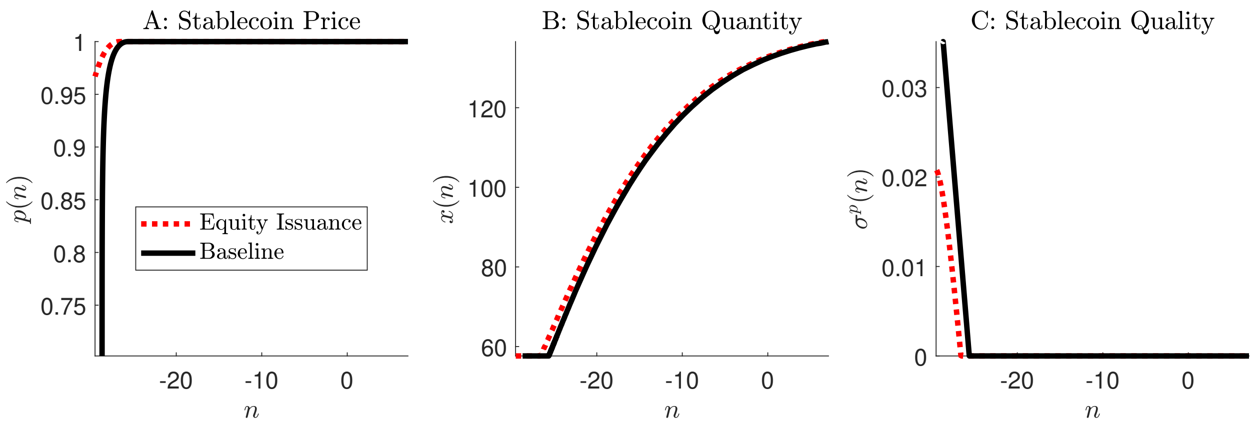

Figure 4: **Equity Issuance.**

3, the value function is strictly concave, so the marginal value of equity, $v'(n)$, is greater than one for any $n < \overline{n}$, and equal to one at $\overline{n}$. Therefore, we have $n^E = \overline{n}$. Internet Appendix E.3 provides details on the solution. We set $F = 6$ in our numerical solution.[21]

In Figure 4, we plot the stablecoin price (Panel A), the value of stablecoin issued (Panel B), and the price volatility or "stablecoin quality" (Panel C) against the issuer's net worth n, where we use solid lines for the baseline model and dotted lines for the extended model with equity issuance. The ability to raise equity allows the issuer to share risk with equity investors, albeit at a cost. Therefore, the issuer relies less on the stablecoin users for risk sharing. In particular, the ability to raise equity financing allows the issuer to limit depegging. Overall, the stablecoin becomes more stable, and the issuer supplies a higher value of stablecoins. Our findings indicate that granting stablecoin issuers access to equity markets—as exemplified by the recent IPO of Circle that issues USDC, the second-largest stablecoin by market capitalization–enhances both the stability and supply of stablecoins.

5 Capital Requirement

The U.S. GENIUS Act (Guiding and Establishing National Innovation for U.S. Stablecoins Act), signed into law on July 18, 2025, requires stablecoin issuers to hold reserve assets backing stablecoins on an at least 1-to-1 basis, (i.e., the issuer's net worth cannot be negative, $n_t \geq 0$). Similar regulations have been adopted in other regions, such as the EU's Markets in Crypto-Assets Regulation (MiCA), the Amendments to Japan's Payment Services Act, and the stablecoin regulatory framework in Singapore's Payment Services Act.

[21] Given that the amount of equity raised at the lower boundary is close to 40, this implies an issuance cost of 15% of money raised, which is conservative relative to the underpricing of Circle (the issuer of stablecoin USDC) implied by the immediate market response after the initial public offering.

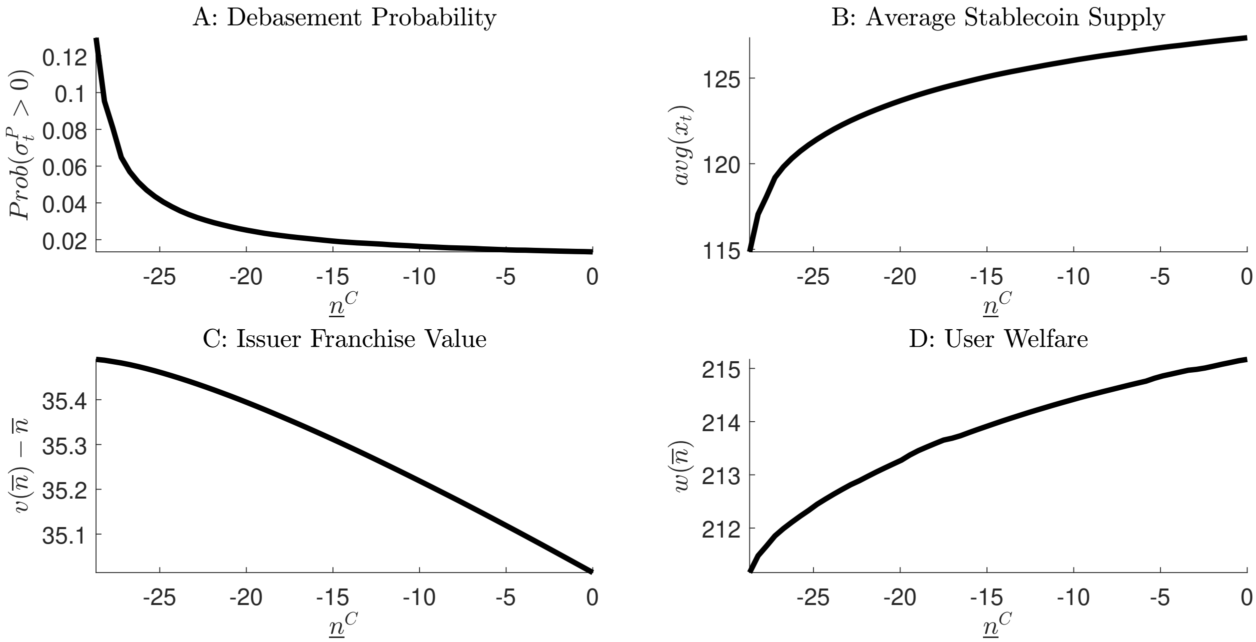

Figure 5: **The Impact of Capital Requirement.**

We consider a regulatory lower bound on the issuer's net worth: $n_t \geq \underline{n}^C$ where $\underline{n}^C$ is greater than $\underline{n}$ in the laissez-faire case (see Proposition 4). Breaching the regulation forces the issuer into liquidation.[22] Note that $n_t \geq \underline{n}^C$ can also be self-imposed, as in practice, stablecoin issuers may advertise their commitment to over-collateralization, in which case $\underline{n}^C = 0$. We will show that such restriction reduces the issuer's payoff. The solution method is similar to the baseline case as detailed in Internet Appendix E.1. In addition, since we evaluate users' welfare, Internet Appendix E.2 shows how to derive (K-scaled) users' welfare as a function of n—that is, users' welfare is given by $w(n)K$, where $w(n)$ is the scaled welfare and we apply a discount rate of 5% to calculate user welfare.

In Figure 5, as the regulator tightens the capital requirement (i.e., increasing $\underline{n}^C$ starting from $\underline{n}^C = \underline{n}$), the probability of depegging is significantly reduced (see Panel A), and tighter regulation in fact increases stablecoin supply (see Panel B). While the users experience a welfare gain, the issuer's payoff decline (see Panel C and D), suggesting that capital requirement effectively transfers value from the issuer to users.[23]

[22] This implies that the issuer is infinitely risk-averse as n_t approaches $\underline{n}^C$ (i.e., $\gamma(n)$ approaches ∞). The issuer meets the regulation by debasing stablecoin liabilities and offloading risk to the users, reducing $\sigma_n(n)$ and maintaining $\mu_n(n) > 0$, as we show in the Internet Appendix. The bound, $\underline{n}^C$, is never reached.

[23] We compute the issuer's valuation as $v(\overline{n}) - \overline{n}$. This is the payoff that an issuer earns, starting with an initial wealth of zero and raising equity $\overline{n}$ at $t = 0$ to start the business. $\overline{n}$ is the targeted level of equity once issuance is allowed, as we have shown in Section 4.3.

We explore the mechanism by revisiting the law of motion of the issuer's net worth (10):

$$dn_t = \underbrace{(n_t + x_t)\mu dt + x_t\zeta_t dt - x_t r dt - x_t\kappa dt}_{\mu_n(n_t)dt} + \underbrace{[(n_t + x_t)\sigma - x_t\sigma_t^p]dZ_t}_{\sigma_n(n_t)dZ_t} - dy_t,$$

where $x_t\zeta_t$ is the seigniorage revenue (see definition of ζ_t given by (11)):

$$x_t\zeta_t = x_t^\xi - \eta|\sigma_t^p|x_t. \tag{19}$$

In the drift, $\mu_n(n_t)dt$, the issuer has two sources of revenue, the expected gain from reserve assets (funded by both stablecoin issuance and net worth), $(n_t + x_t)\mu dt$, and the seigniorage earned from issuing stablecoins, $x_t\zeta_t dt$ where either x_t (the quantity margin) or ζ_t (the quality margin) declines when the issuer loses its net worth following negative shocks and thus has to reduce risk exposure.[24] These two sources of revenues differ in cyclicality: while the first source accrues at a fixed expected return μ, the second—through both x_t and ξ_t—comoves with the issuer's net worth, rendering seigniorage revenues *procyclical*.

The capital requirement mitigates the instability trap by forcing the issuer to derive a sufficiently high share of revenues from the first revenue source rather than the second source of procyclical seigniorage. A capital requirement, $n_t \geq \underline{n}^C$, implies that, given any level of stablecoin supply x_t, the issuer's first source of revenues (reserve assets' expected returns) are bounded below, i.e., $(n_t + x_t)\mu dt \geq (\underline{n}^C + x_t)\mu dt$. Note that forcing the issuer to hold a high level of reserve assets also adds reserve-asset risk: given any level of stablecoin supply, x_t, the first term in the diffusion, $\sigma_n(n_t)dZ_t$, is also bounded below, i.e., $(n_t + x_t)\sigma dt \geq (\underline{n}^C + x_t)\sigma dt$. As the issuer's net worth declines, the issuer deleverages (i.e., reduces x_t) or depegs the stablecoin to avoid hitting the capital requirement. This effect reduces $x_t\zeta_t$, further shrinking the revenue share of seigniorage relative to the expected reserve-asset return.

In summary, what capital requirement does is to change the composition of the issuer's expected revenues, increasing the share from reserve assets and decreasing the share from procyclical seigniorage that is responsible for the instability trap. As a result, capital requirement improves stability (see Panel A of Figure 5). When the force of instability trap becomes weaker, the system would no longer be stuck in the low-n_t region as in the laissez-faire case and thus spends more time in the high-n_t region where the supply of stablecoins, x_t, is high. This explains why, as capital requirement tightens (i.e., $\underline{n}^C$ increases), the average stablecoin supply, $x_t = x(n_t)$, computed using the stationary distribution of n_t, increases.

[24] The procyclicality of seigniorage revenues is shown in Proposition 5 and 9. In the parameter region where the condition (16) holds, ζ_t is a constant and x_t is increasing in n_t. In the parameter region where the condition (16) does not hold, for $n_t \geq \widetilde{n}$, ζ_t is a constant and x_t is increasing in n_t, and, for $n_t < \widetilde{n}$, $\sigma^p(n_t)$ is increasing in n_t, which implies $\zeta(n_t)$ is increasing in n_t (procyclical), and $x(n_t)$ is a constant.

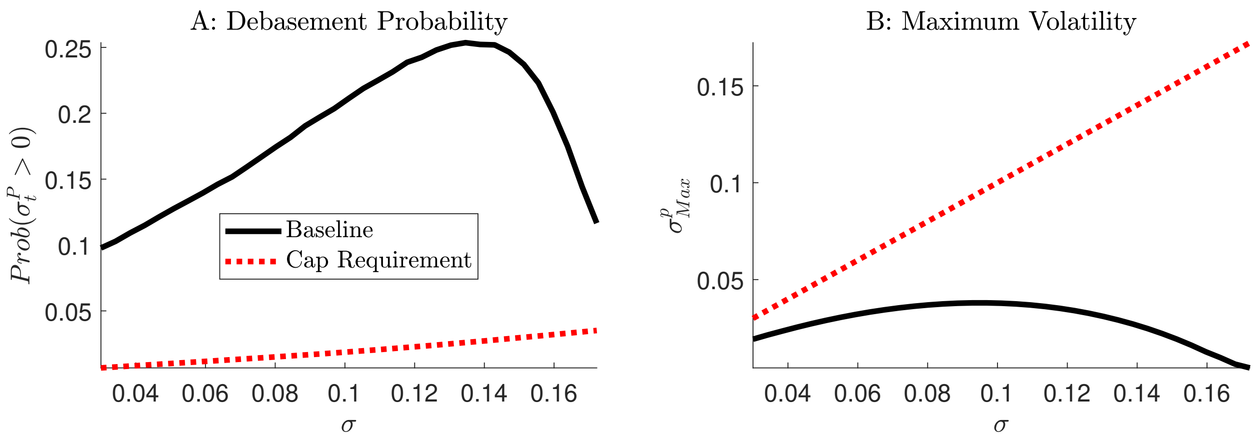

Figure 6: **Capital Requirement and Reserve-Asset Riskiness.**

Admittedly, forcing the issuer to hold more reserve assets also means adding risk on the asset side of its balance sheet. When the issuer has low net worth, it needs to offload such risk on the liability side to the users. Therefore, capital requirement reduces the probability of depegging, but once depegging happens, instability can be worse than the laissez-faire case. This result is displayed in Figure 6. In Panel A, across different levels of reserve-asset riskiness, the depegging probability is significantly lower under capital requirement. In contrast, Panel B shows that the maximum of price volatility (i.e., the highest level of risk offloaded to the users) under capital requirement is greater than that in the laissez-faire case.

Figure 6 shows that capital requirement eliminates the risk paradox: the depegging probability and maximum price volatility are no longer decreasing in σ when σ is high. As discussed in Section 3.2, the inverse relationship between reserve-asset risk and stablecoin volatility in Proposition 7 is due to the issuer's precaution against the instability trap, which restrains the issuer from offloading risk to the users via depegging once σ is sufficiently high. Under capital requirement, the force of instability trap is weakened, and such precaution is no longer needed; in other words, capital requirement already mandates precaution against relying too much on the procyclical seigniorage as a profit source, and by weakening the force of instability trap, it substitutes the voluntary precaution against the instability trap.

Discussion: the role of capital requirement. The issuer has two sources of revenues, the returns on reserve assets that are funded by both net worth and stablecoin issuances and the seigniorage earned by issuing stablecoins. While the former depends on the broader financial markets, the latter depends on the issuer's stablecoin-management strategy and is endogenously procyclical—that is, following negative shocks that reduce the issuer's net worth, the issuer depegs the stablecoin to offload risk to the users, causing the seigniorage to decline. Our model shows that capital requirement effectively forces the stablecoin issuer to rely less on the procyclical seigniorage as a revenue source and more on the returns on reserve assets, thus improving stability. This role of capital requirement is unique to the stablecoin setting where the issuer's liability is subject to depegging by its own choice. In contrast, the traditional views on capital requirement emphasizes avoidance of runs, preventing systemic insolvency (externality and contagion), and incentive alignment between equity and debt investors, all motivated by studies on traditional financial institutions (e.g., banks).

6 The Value of Flexible Risk-Taking

6.1 Narrow banking

In addition to capital requirement, the U.S. GENIUS Act restricts stablecoin issuers to hold relatively safe reserve assets (i.e., assets with a low σ).[25] Therefore, the U.S. GENIUS Act is akin to a narrow-banking framework that entails both limits on reserve-asset riskiness and a requirement that issuers maintain non-negative net worth.

Corollary 1 and Proposition 7 on the risk paradox show that reducing the riskiness of reserve assets enlarges the parameter region where depegging happens and can increase endogenous volatility of the stablecoin price. These findings suggest caution in limiting the riskiness of reserve assets. However, if reserve assets are perfectly safe, i.e., $\sigma = 0$, we know that the stablecoin price will be pegged at one, because without risk in reserve assets, the issuer has no reason to offload risk to stablecoin users via depegging. While the "quality" of stablecoins is maintained, the key question concerns the "quantity".

Next, we show that the narrow-banking requirements ($n_t \geq 0$ and $\sigma = 0$) significantly reduce the supply of stablecoins relative to the laissez-faire case. The following proposition summarizes the narrow-banking solution.

Proposition 10 (Narrow banking). *Under $n_t \geq 0$ and $\sigma = 0$, the stablecoin price is pegged at one, i.e., $p_t = 1$ and $\sigma_t^p = 0$. The value of stablecoins issued is $x^* = \left(\frac{\xi}{r+\kappa-\mu}\right)^{\frac{1}{1-\xi}}$. The issuer maintains a zero net worth, i.e., $n_t = 0$, and consumes all profits, i.e., $dy = x^*[\mu - (r - \zeta^* + \kappa)]dt$, where the marginal seigniorage, $\zeta^* = (x^*)^{\xi-1}$, is defined in (11).*

[25] The permitted reserve assets include U.S. coin and currency, bank deposits, U.S. Treasury securities with a maturity of 93 days or less, repurchase agreements with a maturity of 7 days or less that are backed by Treasury bills with a maturity of 90 days or less, reverse repurchase agreements with a maturity of 7 days or less that are collateralized by that are collateralized by Treasury securities an overnight basis subject to over-collateralization, and reserves at the central bank. The assets above can be held via money market funds, provided that the funds do not invest in other assets.

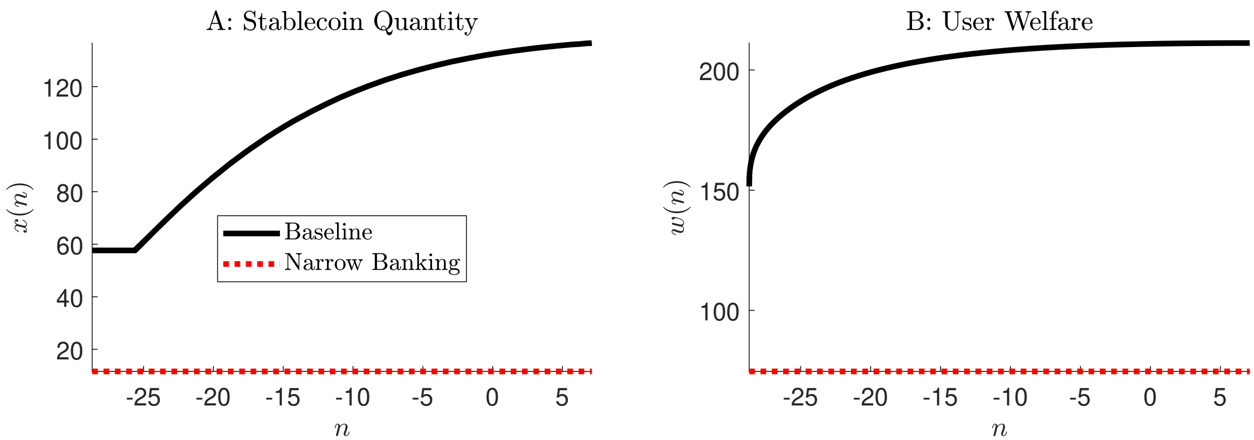

Figure 7: **Narrow Banking.**

The value of stablecoins issued, x^*, is decreasing in $r+\kappa$, the users' discount rate (required return) and the issuer's operating cost, both of which have to be covered by the issuer, and x^* is increasing in μ, the issuer's return on reserve assets. The issuer earns a spread between reserve assets and stablecoin liabilities, including the users' utility from stablecoin holdings, ζ^* (the seigniorage). The issuer consumes all the profits: for every dollar of stablecoins issued and proceeds invested in the reserve asset, the issuer consumes the full spread, $\mu-(r-\zeta^*+\kappa)$.

In Panel A of Figure 7, we compare the value of stablecoins issued in the laissez-faire baseline model (the solid line) and that under narrow banking (the dashed line). Imposing the narrow-banking restrictions leads to a significantly lower amount of stablecoins that in turn translates into a significant decline of the users' welfare (see Panel B).[26]

In the laissez-faire scenario with risk-taking, we have the calibrated value $\mu = 13.4\%$. In Figure 7, we have $\mu = 5\%$, a realistic value for returns on safe assets. As shown in Figure 7, the value of stablecoins issued and users' welfare are both lower under the narrow-banking restrictions than the laissez-faire values even in the low net-worth region where depegging happens. In the high net-worth region, the laissez-faire values are even higher.

The reduction in stablecoin supply and user welfare arises because restricting issuers to hold only perfectly safe assets lowers the expected return on reserves, thereby discouraging intermediation (i.e., the issuance of stablecoins and investment in reserve assets). To offset the decline in reserve returns, the issuer raises user fees, which in turn dampens stablecoin demand and reduces welfare. Consequently, whether narrow banking enhances user welfare depends on whether the gains from improved stability outweigh the foregone returns that necessitate higher fees, as the issuer seeks to maintain profitability.

Our model solution also shows that imposing the narrow-banking requirements reduces

[26]The users' risk-neutral discount rate is set to 5%, same as the return on risk-free reserve assets.

the issuer's valuation by 33% relative to the laissez-faire case.[27] Overall, our analysis suggests caution against the narrow-banking framework, as it reduces both the users' welfare and the issuer's valuation. Forcing the issuer to avoid risk-taking at all costs can improve the quality of stablecoins (i.e., depegging never happens) but significantly reduces the demand of stablecoins (due to higher fees) and equilibrium quantity. A certain level of risk-taking can be Pareto-improving relative to narrow banking. Next, in Section 6.2, we show that more flexibility is preferred: allowing the issuer to not only take risk but dynamically adjust reserve-asset risk improves both the stablecoin quality and quantity.

In Section 5, our analysis shows that capital requirement alone already substantially reduces instability and improves welfare for stablecoin users relative to the laissez-faire benchmark and thus even more so relative to narrow banking (as we have shown that the laissez-faire benchmark dominates narrow banking). Therefore, imposing an additional requirement of risk-free reserves on top of the capital requirement can be counterproductive: it yields a marginal improvement in making the stablecoin price perfectly pegged, eliminating the low-probability event of depegging all together, but causes a large decline in user welfare.

6.2 Flexible risk choice

In our baseline model, the issuer faces a fixed expected return, μ, and volatility, σ, of the reserve assets. In the following, we allow the issuer to choose $\omega_t \in [w_L, w_H]$ and thereby change the expected return to $\mu_0 + \omega_t \alpha$ ($\alpha > 0$) and volatility to $\sigma_0 + \omega_t \sigma_\alpha$ ($\sigma_\alpha > 0$)—that is, the issuer faces a risk-return trade-off in its choice of ω_t. The law of motion of the issuer's net worth, previously given by (10), is now extended to the following:

$$dn_t = \underbrace{[(\mu_0 + \omega_t \alpha)(n_t + x_t) + x_t \zeta_t - x_t r - x_t \kappa]}_{\mu_n(n_t)} dt + \underbrace{[(\sigma_0 + \omega_t \sigma_\alpha)(n_t + x_t) - x_t \sigma_t^p]}_{\sigma_n(n_t)} dZ_t - dy_t,$$

where ζ_t is the stablecoin user's marginal utility from holding stablecoins (i.e., the seigniorage) defined in (11). The issuer's problem has an additional control variable, ω_t, but is solved similarly as our baseline model is. We provide the solution details in Internet Appendix E.4.

In Figure 8, we compare our baseline model (solid line) and the extended model (dotted line) in the value of stablecoins issued (Panel A), price volatility or stablecoin "quality" (Panel B), and the issuer's choice of $\omega(n)$ as a function of net worth, n. We set $\mu_0 = 12.9\%$, $\sigma_0 = 4.5\%$, $\alpha = 0.5\%$, and $\sigma_\alpha = 2.5\%$ and normalize w_H to 1 so that the highest level of risk-taking, i.e., $\omega = 1$, brings the same expected return and volatility of reserve assets as

[27]We compute the issuer's valuation at the consumption boundary, $\overline{n}$, where we map variables in our model to Tether's values (see Section 4.1).

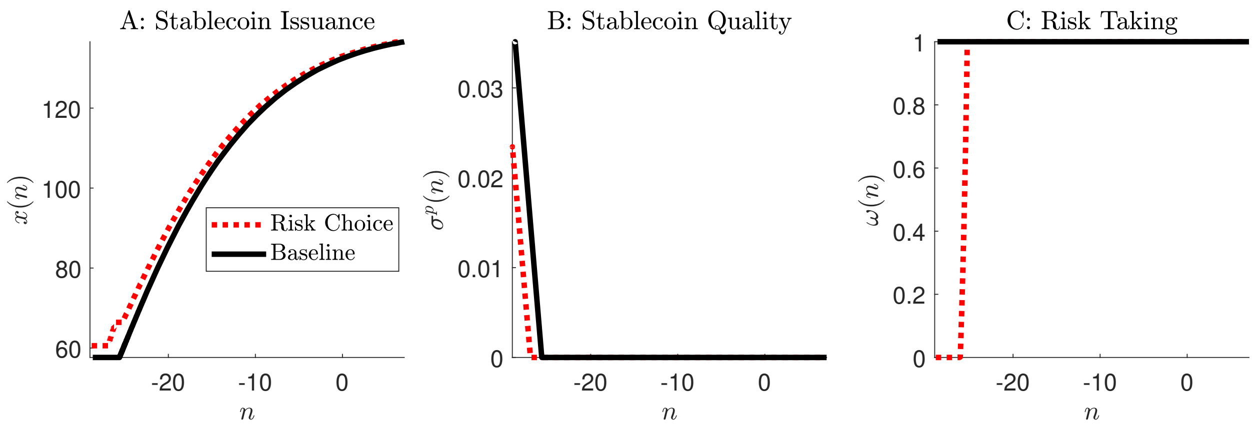

Figure 8: **Dynamic Choice of Reserve-Asset Riskiness.**

in our baseline model. Moreover, we impose and $w_L = 0$ so that at the lowest level of risk-taking, the expected return and volatility of reserve assets are $\mu_0 = 12.9\%$ and $\sigma_0 = 4.5\%$. As shown in Panel C, ω decreases as the issuer loses net worth.

De-risking on the asset side of its balance sheet allows the issuer to refrain from risk-sharing via the liability side (i.e., offloading risk to the users through depegging), so Panel B of Figure 8 shows a smaller stablecoin-price volatility under the flexible choice of ω_t. Moreover, as shown in Panel A, the issuer supplies a higher value of stablecoins than the baseline case, especially in the low-n region, because when it undercapitalized, the issuer can supply stablecoins but invest the proceeds more conservatively. Without the choice of risk-taking, the issuer has to either offload risk to the users (reducing the quality of stablecoins) or deleverage (reducing the quantity of stablecoins). Allowing asset-risk adjustment alleviates the tension between quality and quantity for an undercapitalized issuer.

Overall, our analysis in this section points towards the benefits of allowing the stablecoin issuer to flexibly manage its reserve risk. We demonstrate quantitatively the cost of forbidding risk-taking completely in Section 6.1 and, in Section 6.2, we show how the issuer behaves when allowed to adjust risk exposure in a state-contingent fashion.

7 Stablecoin Demand Dynamics

7.1 The general setup

We extend the model by introducing shocks to the stablecoin demand scaler, $K_t = K$:

$$\frac{dK_t}{K_t} = \mu_K dt + \sigma_K dZ_t^K, \tag{20}$$

where μ_K is a constant, $\sigma_K \geq 0$, and Z_t^K is a standard Brownian motion. The correlation between the demand shock, dZ_t^K, and the shock to the issuer's reserve assets, dZ_t, is ϕdt.

The users take as given the price process, which now also loads on dZ_t^K:

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t + \sigma_{K,t}^p dZ_t^K, \tag{21}$$

where the shock loadings σ_t^p and $\sigma_{K,t}^p$ are endogenously determined. We define

$$\Sigma_t^p := \sqrt{(\sigma_t^p)^2 + (\sigma_{K,t}^p)^2 + 2\phi\sigma_t^p\sigma_{K,t}^p}, \tag{22}$$

the instantaneous volatility of dp_t/p_t. In analogy to (2), the users' utility from holding stablecoins is given by $K_t u(x_t)$, where $u(x_t)$ is defined as follows

$$u(x_t) = \left(\frac{x_t^\xi}{\xi} - x_t\eta|\Sigma_t^p|\right). \tag{23}$$

As in the baseline model, $x_t = X_t/K_t$, and the users' optimal demand for stablecoin is scaled with K_t, i.e., $X_t = K_t x_t$ with

$$x_t = \left(\frac{1}{r - \mu_t^p + \eta|\Sigma_t^p| + f_t}\right)^{\frac{1}{1-\xi}}. \tag{24}$$

This expression is analogous to the stablecoin demand (5) in the baseline model.

Given the state variables, N_t (net worth) and K_t (stablecoin demand scaler), the issuer chooses the K_t-scaled stablecoin supply, x_t, consumption, dY_t, and as in the baseline model, the controls the volatilities $(\sigma_t^p, \sigma_{K,t}^p)$. The issuer implements these choices through fee policies and supply changes, i.e., $(f_t, S_t)_{t\geq 0}$, as we show. In Internet Appendix E.5, we solve the issuer's problem with both reserve and demand shocks. We present an overview of the solution in Proposition E.1, while illustrating the results here with numerical analysis.

7.2 Stablecoin demand shocks and capital requirement

In this subsection, we focus on the role of demand shocks, setting $\mu_K = 0$. In practice, the correlation between the demand shock and reserve-asset shock, ϕ, is likely to be positive, because the issuer (e.g., Tether) may hold cryptocurrencies in its reserve-asset portfolio, and cryptocurrencies' value is correlated with the adoption of stablecoins.

Such positive correlation exacerbates the instability trap, driven by the procyclicality of

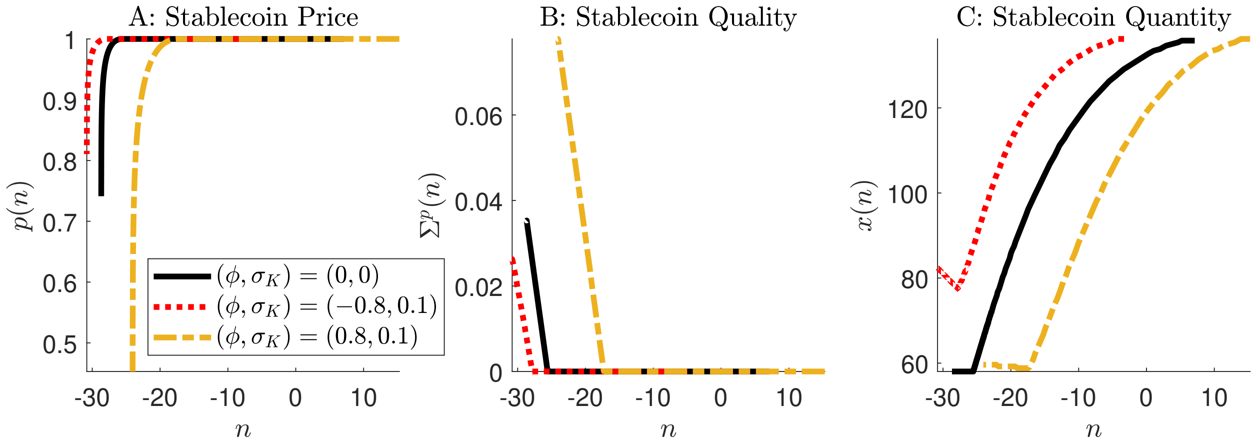

Figure 9: **Stablecoin Price and Quantity Dynamics under Demand Shocks.**

seigniorage revenues. The stablecoin issuer's total seigniorage revenues are given by

$$X_t \zeta_t = K_t \left(x_t^{\xi} - \eta |\Sigma_t^p| x_t \right), \tag{25}$$

When negative shocks to reserve assets significantly reduce the issuer's net worth, the issuer offloads risk to the users through depegging (i.e., Σ_t^p increases) which reduces the seigniorage for any choice of stablecoin supply, x_t. Under $\phi > 0$, the negative reserve-asset shocks are likely to coincide with negative shocks to K_t, the demand scaler, which further reduces the seigniorage revenues—that is, $\phi > 0$ amplifies the procyclicality of seigniorage revenues. In Section 3.2, we have explained that such procyclicality leads to the instability trap.

In Figure 9, we plot the stablecoin price, price volatility, and value of stablecoins issued against the state variable, n_t in Panel A, B, and C, respectively. The plots start from $\underline{n}$, the lower bound of n_t in analogy to $\underline{n}$ in Proposition 4, and end at $\overline{n}$, the upper bound where the issuer consumes. Note that both boundaries are endogenous and vary with the parameter values. Our numerical solution is based on parameter values from Section 4.1 and $\sigma_K = 10\%$ (i.e., the annualized volatility of the growth rate of demand scaler is 10%).

The solid black lines in Figure 9 represent the baseline model with a constant K, i.e., $\phi = 0$ and $\sigma_K = 0$. The yellow dashed lines represent the case of $\phi > 0$, i.e., the demand and reserve-asset shocks are positively correlated, while the red dotted lines represent the case of $\phi < 0$. As ϕ increases from -0.8 to zero and then to 0.8, depegging in the low-n region becomes more severe (see Panel A), and once depegging happens, the price fluctuates more aggressively (see Panel B). In addition, under positively correlated shocks, the issuer supplies less stablecoins (see Panel C). Figure 9 shows that when the demand shock and reserve-asset shock become positively correlated, both the quality and quantity of stablecoins

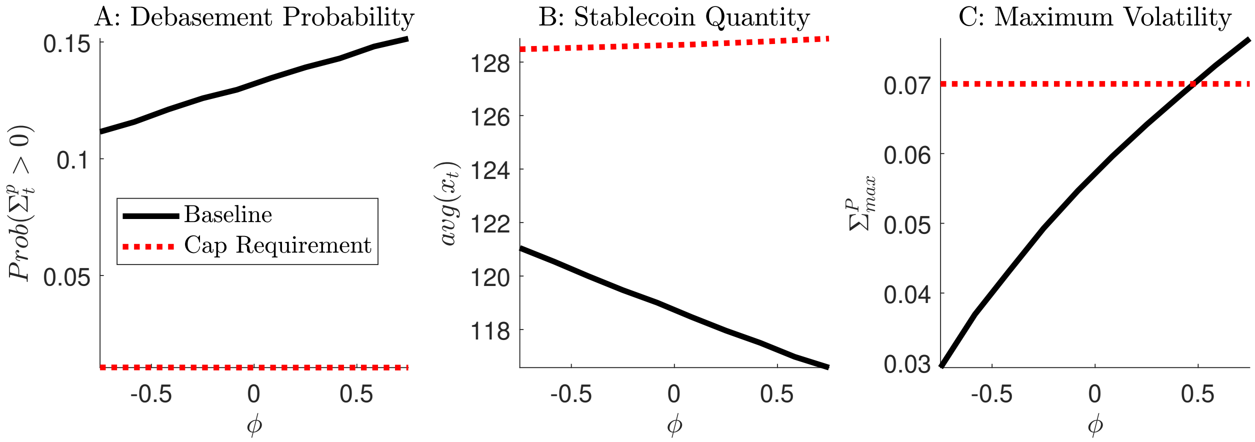

Figure 10: **Capital Requirement and Demand Shocks.**

suffer because the positive correlation amplifies the procyclicality of seigniorage revenues.

In Section 5, we have explained that introducing capital requirement reduces the issuer's reliance on the procyclical seigniorage as a revenue source. Therefore, it can make the issuer less sensitive to the correlation between the demand shock and reserve-asset shock, because $\phi > 0$ exacerbates instability by amplifying the procyclicality of seigniorage revenues. Figure 10 shows that this is indeed the case. We consider a capital requirement that requires the issuer to maintain non-negative net worth, i.e., $n_t \geq 0$. In Panel A, we show that the depegging probability is significantly reduced under capital requirement and, basically, is no longer sensitive to ϕ. In Panel B, we compute the average value of stablecoin supply, x_t, based on the stationary probability distribution of state variables under different values of ϕ. By reducing the issuer's reliance on procyclical seigniorage, capital requirement mitigates the force of instability trap and thereby increases the average supply of stablecoins (as discussed in Section 5), and importantly, it makes the supply less sensitive to ϕ.

A caveat regarding capital requirement is that it increases the worst-case volatility of stablecoin price as we have discussed in Section 5. This is reflected in Panel C of Figure 10. When ϕ is high, capital requirement reduces the average volatility in the depegging region (i.e., conditional on $\Sigma_t^p > 0$) relative to the laissez-faire case. However, when ϕ is sufficiently negative—that is, when the procyclicality of seigniorage revenues is already mitigated by the negative ϕ, rendering the benefit of capital requirement less important—capital requirement makes the depegging region more unstable relative to the laissez-faire case.

In summary, our conclusion is that, while positively correlated reserve and demand shocks exacerbate instability, capital requirement makes the issuer less sensitive to this correlation, thereby reducing the probability of depegging. However, once depegging happens, capital requirement mitigates instability when the shock correlation is high but exacerbates instabil-

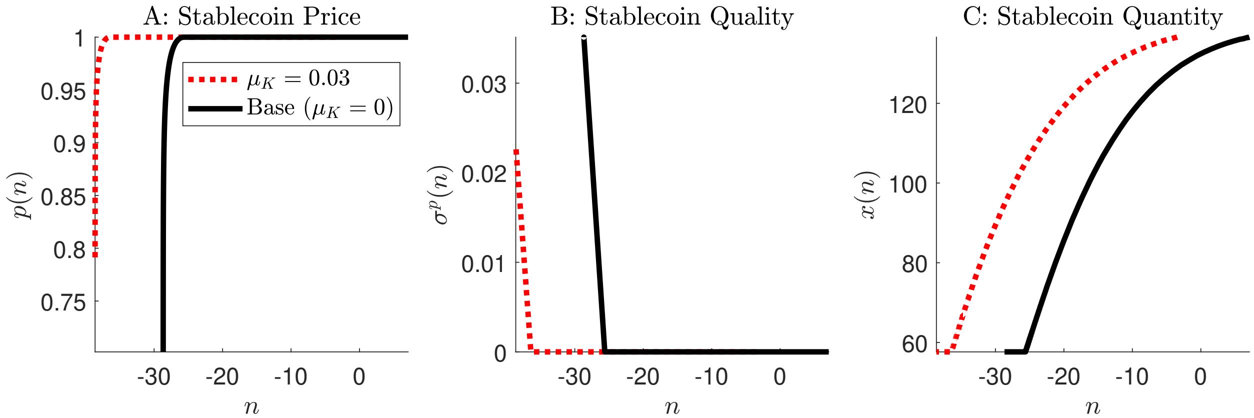

Figure 11: **Demand Growth Fosters Stability.**

ity relative to the laissez-faire case when the shock correlation is sufficiently negative. Thus, capital requirement is most valuable in the presence of positively demand and reserve risk.

7.3 Stablecoin demand growth and issuer valuation

The previous subsection focused on the role of demand shocks, where we considered $\sigma_K > 0$ and $\mu_K = 0$. Next, we examine the impact of demand growth on the price, quantity, and quality dynamics of the stablecoin and issuer's equity valuation under $\sigma_K = 0$ and $\mu_K > 0$.

In Figure 11, we plot the stablecoin price (Panel A), stablecoin quality, i.e., the instantaneous price volatility (Panel B), and K-scaled stablecoin quantity (Panel C) against K-scaled net worth, n, under our baseline parameters (solid line; $\mu_K = 0$) and with demand growth at rate $\mu_K = 3\%$. Notably, Panels A and B show, respectively, that under greater demand growth, the stablecoin experiences less severe depegging in the low-n region and fluctuates less. Intuitively, demand growth has a stabilizing effect: the trend in seigniorage revenues counterbalances the procyclicality of seigniorage revenues that is the source of instability.

Panel C of Figure 11 shows that introducing demand growth essentially shifts the K-scaled value of stablecoins $x(n)$ to the left—that is, for any given level of stablecoin liabilities, the issuer maintains a lower net worth. Intuitively, as demand growth fosters stability, the issuer finds it less necessary to maintain net worth as a risk buffer.

Finally, we highlight the implications of our model on the issuer's equity valuation under demand growth rates. With the exception of Circle (the issuer of USDC), the majority of stablecoin issuers are private companies whose valuations are not publicly available. Our model provides a quantitative framework for valuing stablecoin issuers. The valuations may serve as a reference point for private investments and public offerings.

As reported by Cointelegraph.com, a recent market analysis estimated the valuation of

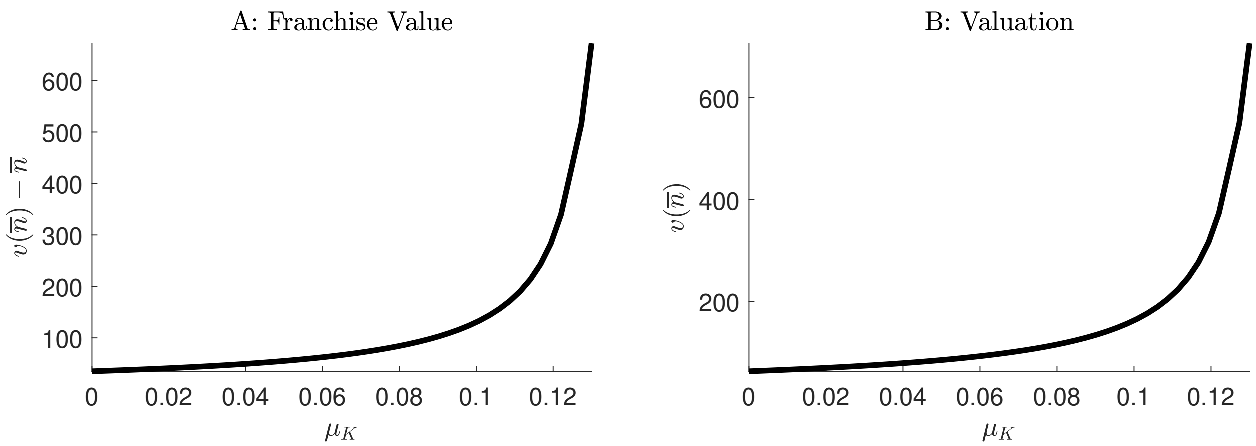

Figure 12: **Stablecoin Issuer Valuation.**

Tether (the issuer of USDT, the largest stablecoin by market capitalization) at \$515 billion, which would make it the 19th most valuable company globally, ahead of firms like Costco and Coca-Cola. However, Tether CEO Paolo Ardoino has stated that the \$515 billion estimate may be "a bit bearish". Our model informs the valuation debate.

As discussed in Section 4.1, our model is calibrated to data on Tether, the issuer of USDT, at $n = \underline{n}$ (the issuer's consumption boundary), in line with Tether's significant amount of payout in 2024. In addition, when solving our model under different levels of demand growth, we impose a capital requirement, $n_t \geq 0$, in line with Tether's claim of maintaining over-collateralization. Next, using our model, we answer the following question: how strong the growth of USDT demand must be to support a valuation of Tether above \$500 billion?

In Panel A of Figure 12, we plot the value added, i.e., the difference between the equity valuation, $v(\overline{n})$, and book equity, $\overline{n}$, that reflects the franchise value, and in Panel B, we plot the equity valuation, $v(\overline{n})$. As a reminder, $v(\overline{n})$ is the K_t-scaled valuation where we consider K_t equal to \$1 billion. From Panel B, a valuation above \$500 billion requires a demand growth rate, μ_K, of at least 12%. This growth rate is roughly in line with the overall sector growth and rising competition within the sector. The World Economic Forum reported an average annualized growth rate of 28% as of Q1 2025. However, competition has risen. According to Coindesk.com (as cited by the BIS in their June 2025 bulletin), the number of active stablecoins has nearly doubled from the start of 2024 to June 2025.

8 Conclusion

This paper develops a dynamic model of stablecoin issuers, who represent a novel category of financial intermediaries that earn procyclical and fragile seigniorage. By analyzing the issuer's incentives to depeg, we show how this decision introduces a new instability channel that is absent in models of traditional financial institutions. A key insight is the existence of an "instability trap," in which depegging reduces the issuer's seigniorage profits, slowing down its net worth rebuild after negative shocks, and thereby making depegging more persistent. Our analysis also uncovers a risk paradox: higher reserve-asset risk can, counterintuitively, reduce the probability and severity of depegging.

The model delivers important implications for regulation. The mainstream proposals that require stablecoins to be backed by assets with low risk may not resolve the underlying instability, as the risk paradox highlights. Forcing stablecoin issuers to hold perfectly risk-free assets with low returns hurts their profits and can significantly reduce users' welfare, as the issuer compensates for the lower reserve returns by raising user fees.

Capital requirements instead are effective tools to improve stability and welfare. They play a new role: by reducing the issuer's reliance on procyclical seigniorage from issuance, they improve stability. This finding sheds light on how capital regulation works in the stablecoin context, suggesting that the underlying mechanism differs significantly from those imposed on banks and other traditional financial intermediaries.

Our model shows the value of granting stablecoin issuers' access to equity financing. Allowing issuers to share risk with external equity investors improves their resilience and the robustness of the peg. In addition, we provide a framework for valuing the issuers' equity.

Taken together, these results clarify the economic forces that shape stablecoins' supply, the fragility of their peg, and the regulatory levers available to improve stability. More broadly, our paper positions stablecoin issuers as a distinct class of intermediaries whose unique liability structure calls for tailored regulatory approaches, and we provide a quantitative framework for evaluating the consequences of such policies.

Finally, our model is both flexible and tractable, allowing for an analytical characterization of the equilibrium. At the same time, it is rich enough to be calibrated to data on real-world stablecoin issuers. We emphasize that the flexible structure could be extended in various directions—such as, incorporating a richer treatment of demand risk (including the classic run mechanism) and analyzing competition by embedding our single-issuer framework into one with multiple issuers. We view these as promising avenues for future research.

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Appendix in the Main Text

A Calibration Details

We provide additional details regarding our parameter calibration in Section 4.1.

Calibrating μ and σ. For the parameters μ and σ, we calibrate them based on the disclosed information about Tether's reserve asset portfolio. Tether is the largest stablecoin issuer. Our data is from Tether's latest financial report as of December 31, 2024.

Tether's reserves are allocated as follows: 82.3% in cash and cash equivalents (primarily T-bills), 3.7% in precious metals (primarily gold), 5.5% in Bitcoin, 5.7% secured loans, and 2.77% other investments — where we exclude 0.01% corporate bonds and round to the first decimal. To choose μ and σ, we obtain the annualized returns of the individual components over the past two years (relative to the Tether report date on 12/31/2024) and compute a weighted average of annualized returns and return volatility. We do not model cross-asset correlations explicitly. Instead, we aggregate volatilities as a simple weighted sum.[28]

For Bitcoin, annualized returns in 2023 and 2024 were 155%, and 121%, respectively. Based on these numbers, we assume a conservative return of 100%, which is below the annualized return in the past two years. We make this assumption since 2024 was an extraordinary year for Bitcoin, also due to the regulatory approval of Bitcoin ETF and the election of a crypto-friendly administration in the U.S. We directly calculate the annualized return volatility of Bitcoin over the same time period, which equals about 49%.[29] This is lower than the annualized return volatility in previous years; in comparison, the annualized return volatility was about 64% for the calendar year 2022 and even higher in Bitcoin's earlier days.[30]

For precious metals, approximated by gold, we calculate an average annual return of about 19.7% based on the arithmetic average of a 12.69% return in 2023, and 26.66% in 2024, using the gold ETF (ticker: GLD) and data from Yahoo Finance. As for Bitcoin, we calculate the annualized return volatility of Gold—using the gold ETF (ticker: GLD)—from 01/01/2023 to 12/31/2024. We obtain an annualized return volatility of 14.3%.

For cash and cash equivalents, we take the T-bill ETF (with ticker BIL). For 2023-2024, we calculate an annualized return of about 5%, and an annualized volatility of 0.27%. For secured loans, we assume that they are similar in terms of risk and return to relatively short-term investment-grade corporate bonds, as those contained in the ETF with ticker SLQD.

[28] This is akin to assuming perfectly positive correlations and therefore provides a conservative upper bound on portfolio volatility—a stance justified by tail risks and other risks not reflected in realized volatility over the past two years. As the contribution breakdown below shows, Bitcoin and the Other investments bucket dominate the portfolio's volatility, with the remaining assets playing a minor role. Since Other investments likely contain crypto-assets, their correlation with Bitcoin is plausibly high and positive.

[29] For an asset, we calculate annualized returns and their standard deviation as follows. First, we calculate daily percentage returns based on adjusted closing prices from 01/01/2023 to 12/31/2024. Second, we calculate the standard deviation of daily returns over the same time period. Third, multiply this number by $\sqrt{\#\text{Trading Days}}$, where we assume 365 trading days for cryptocurrency and 252 trading days for more standard assets. The results are similar using log returns or percentage returns.

[30] For instance, industry reports by Fidelity and Blackrock suggest that Bitcoin exhibited higher volatility in earlier years, with annualized return volatility often being close to or exceeding 100%.

For SLQD and over 2023-2024, we calculate an annualized return of about 5.4% and an annualized return volatility of 2.4%. This estimate is in line with the volatility for secured loans reported in an industry report by Invesco.

For other investments, we assume that these comprise other cryptocurrencies and altcoins riskier than Bitcoin currently or comparable to Bitcoin in its early days. We set the return of other investment to the same level as that for Bitcoin, i.e., 100%, but assume they are twice as risky and set the volatility to 100% — as we verify, this number coincides with the annualized return volatility of Solana from 2023 to 2024 which we calculate as 99%. Thus, we calculate:

$$\mu = \underbrace{0.823 \cdot 5\%}_{\text{T-Bills}} + \underbrace{0.055 \cdot 100\%}_{\text{Bitcoin}} + \underbrace{0.037 \cdot 19.7\%}_{\text{Gold}} + \underbrace{0.057 \cdot 5.4\%}_{\text{Secured Loans}} + \underbrace{0.0277 \cdot 100\%}_{\text{Other}} \approx 13.4\%,$$
$$\sigma = \underbrace{0.823 \cdot 0.27\%}_{\text{T-Bills}} + \underbrace{0.055 \cdot 49\%}_{\text{Bitcoin}} + \underbrace{0.037 \cdot 14.3\%}_{\text{Gold}} + \underbrace{0.057 \cdot 2.4\%}_{\text{Secured Loans}} + \underbrace{0.0277 \cdot 100\%}_{\text{Other}} \approx 6.35\%.$$

Consequently, we set $\mu = 0.134$. For volatility, we round up and set $\sigma = 0.07$ accounting for potential tail risks other risks, which realized volatility over the last few years does not capture.

USDT convenience yield. As explained in main text, the convenience yield of USDT is used to calibrate ξ that governs the users' demand elasticity (see (5)). Following Ma et al. (2023), we match marginal convenience to the lending rate offered on DeFi lending protocols, specifically Aave (one of the largest protocols). We use Aavescan to retrieve historical data on USDT lending rates in 2024, which are available from the time of download (in our case, 02/09/2025) roughly one year back to 02/16/2024. We calculate the average lending rate (after winsorizing at 2% and 98% levels) in year 2024 (based on the available datapoints) and obtain lending rate of about 6.4%. Solving $(136.6)^{\xi-1} = 0.064$ yields $\xi = 44.1\%$.[31] Again, recall that 136.6 is the market cap of USDT in $ billions, as discussed in Section 4.1.

[31] The rationale is that in equilibrium, stablecoin users must be indifferent at the margin between holding the stablecoin for the convenience yield or lending it out on DeFi platforms (e.g., Aave).

Internet Appendix

A Preliminaries and Regularity Conditions

We impose the following regularity conditions and key parameter assumptions, some of which were introduced earlier but are repeated here for clarity and a better overview:

First, as discussed in the main text, we assume non-negative dividend payouts, $dY_t \geq 0$, and $\rho > \mu$. The latter assumption is standard in liquidity management models (Bolton et al., 2011), necessary for a well-behaved solution, and ensures that the issuer does not infinitely delay consumption. Second, we assume $r + \kappa > \mu$, i.e., $\lambda = r + \kappa - \mu > 0$. The role of this assumption, as also discussed in the main text, is to introduce a cost of stablecoin issuance.

Importantly, we impose the regularity condition that x_t is bounded from above. There exists a constant $\overline{x} > 0$ such that $x_t \leq \overline{x}$.[32] Throughout the paper, we set $\overline{x}$ sufficiently high, so that the constraint never binds in optimum—we thus do not explicitly account for $\overline{x}$ in our expressions. The condition is only invoked in the proofs and to ensure the issuer's dynamic problem in Appendix C is well-behaved.[33]

The Appendix follows a different structure and sequence than the exposition in the main text to present the derivation formally step-by-step. The section headers will indicate where different Propositions, Lemmata, and Corollaries from the main text are proven.

B User Optimization (Proof of Proposition 1)

The representative user has wealth N_t^u, evolving according to (4). It follows from $dY_t^u \gtrless 0$ (i.e., dY_t^u is not sign-restricted) that the marginal utility from net worth equals 1. We conjecture (and verify) that the user's value function takes the form $V_t^u = N_t^u + v_t^u$, where v_t^u does not depend on N_t^u and only depends on aggregate states. The representative user takes price p_t, and price dynamics dp_t, and the dynamics dv_t^u as given, when choosing X_t.

By the dynamic programming principle, a representative user solves the HJB equation:

$$rV_t^u dt = r(N_t^u + v_t^u)dt = \max_{X_t \geq 0, dY_t^u} \Big[dY_t^u + U(X_t)dt + \mathbb{E}_t^u[dN_t^u + dv_t^u] \Big], \tag{B.1}$$

where $\mathbb{E}_t^u$ is the user's time-t expectation and $dV_t^u = d(v_t^u + N_t^u)$.

Next, we use the budget constraint (4), that is, $dN_t^u = rN_t^u dt + X_t \left(-f_t dt + \frac{dp_t}{p_t} - rdt \right) - dY_t^u$, $X_t = x_t K$, as well as $U(X_t) = Ku(x_t)$ to rewrite the HJB equation as

[32] One motivation is that stablecoins are issued on blockchains that have limited capacity for processing and recording transactions (e.g., Abadi and Brunnermeier (2019), Hinzen, John, and Saleh (2022)). In addition, regulatory restrictions or users' portfolio allocation considerations may lead to limits on stablecoin holdings—for instance, users simply have limited capital. This assumption serves as a regularity condition, in particular ruling out Ponzi schemes; it is used in the proof of Lemma 2.

[33] As will become clear, we employ similar solution methods as those in the dynamic contracting and security design literature. The assumption $x_t \leq \overline{x}$ is in the same spirit as boundedness assumptions routinely imposed in this literature (see, e.g., Sannikov (2008)).

$$rv_t^u dt = \max_{x_t \geq 0} \left\{ K \left(u(x_t)dt - rx_t dt - x_t f_t dt + x_t \mathbb{E}_t^u \left[\frac{dp_t}{p_t} \right] \right) + \mathbb{E}_t^u [dv_t^u] \right\}.$$

This also verifies our conjecture of the functional form of the value function.

As the user takes the dynamics of p_t and v_t^u as given, the choice of x_t is determined according to the static optimization $\max_{x_t \geq 0} \left\{ u(x_t)dt - rx_t dt - x_t f_t dt + x_t \mathbb{E}_t^u \left[\frac{dp_t}{p_t} \right] \right\}$. Using that $u(x_t) = \frac{x_t^\xi}{\xi} - \eta x_t |\sigma_t^p|$, we obtain the first order condition with respect to x_t: $x_t^{\xi-1} dt - f_t dt + \mathbb{E}_t^u \left[\frac{dp_t}{p_t} \right] - rdt - \eta |\sigma_t^p| dt = 0$. We can solve for:

$$x_t = \left(\frac{1}{r + f_t + \eta |\sigma_t^p| - \mathbb{E}_t^u [dp_t]/(p_t dt)} \right)^{\frac{1}{1-\xi}}. \tag{B.2}$$

Note that $\mathbb{E}_t^u \left[\frac{dp_t}{p_t} \right] = \mu_t^p dt$, so (B.2) simplifies to (5), as desired. As will become clear, the denominator in (B.2) will be positive in equilibrium, so that (B.2) is well-defined.

C Issuer Optimization

Net Worth Dynamics. We derive the law of motion of the issuer's net worth from the issuer's reserve assets to supplement details for the main text. The assets evolve as

$$dA_t = \underbrace{A_t(\mu dt + \sigma dZ_t)}_{\text{Return on assets}} + \underbrace{f_t X_t dt}_{\text{Fee Revenues}} + \underbrace{dS_t(p_t + dp_t)}_{\text{Issuance proceeds}} - \underbrace{\kappa X_t dt}_{\text{Operating Cost}} - \underbrace{dY_t}_{\text{Payout}}. \tag{C.1}$$

Note that issuance of stablecoins occurs at price $p_{t+dt} \simeq p_t + dp_t$. By Ito's Lemma: $dX_t = d(S_t p_t) = S_t dp_t + p_t dS_t + dS_t dp_t$. Thus,

$$\begin{aligned} dN_t &= dA_t - dX_t = A_t(\mu dt + \sigma dZ_t) + f_t X_t dt - S_t dp_t - \kappa X_t dt - dY_t \\ &= (N_t + p_t S_t)(\mu dt + \sigma dZ_t) - p_t S_t (\mu_t^p dt + \sigma_t^p dZ_t) + p_t S_t (f_t - \kappa) dt - dY_t, \end{aligned}$$

where we used $dp_t = p_t(\mu_t^p dt + \sigma_t^p dZ_t)$. This expression coincides with (7), as desired.

Proof of Lemma 1. First, we solve (5) for: $f_t = x_t^{\xi-1} - r + \mu_t^p - \eta |\sigma_t^p| = \zeta_t - r + \mu_t^p$, where $\zeta_t = x_t^{\xi-1} - \eta |\sigma_t^p|$. Second, divide both sides of (7) by K and use $S_t p_t = X_t = x_t K$ to obtain: $\frac{dN_t}{K} = (n_t + x_t)(\mu dt + \sigma dZ_t) - x_t(\mu_t^p dt + \sigma_t^p dZ_t) + x_t f_t dt - x_t \kappa dt - dy_t$. Inserting above expression for f_t, we get $dn_t = [\mu(n_t + x_t) - x_t(r - \zeta_t) - x_t \kappa] dt + [\sigma(n_t + x_t) - x_t \sigma_t^p] dZ_t - dy_t$, as desired. Inserting $\zeta_t = x_t^{\xi-1} - \eta |\sigma_t^p|$ and using $\lambda = r + \kappa - \mu$ — defined in (8) — we obtain:

$$dn_t = \left[\mu n_t + x_t^\xi - \lambda x_t - \eta x_t |\sigma_t^p| \right] dt + [\sigma(n_t + x_t) - x_t \sigma_t^p] dZ_t - dy_t. \tag{C.2}$$

We have the drift and diffusion: $\mu_n(n_t) = \mu n_t + x_t^\xi - \lambda x_t - \eta x_t |\sigma_t^p|$, $\sigma_n(n_t) = \sigma(n_t + x_t) - x_t \sigma_t^p$.

Issuer Optimization Problem. We restate the full program of the issuer in K-scaled terms. Let $a_t = A_t/K$, $dy_t = dY_t/K$, and $s_t = S_t/K$, where K is a constant scaling parameter. The issuer's problem in K-scaled terms can be written as:

$$V_0 := v_0 K = K \max_{\mathcal{G} \in \overline{\mathcal{G}}} \mathbb{E} \left[\int_0^\infty e^{-\rho t} dy_t \right], \tag{C.3}$$

where the strategy $\mathcal{G} := (S_t,\, f_t,\, dY_t)_{t\geq 0}$ — or its K-scaled equivalent $(s_t,\, f_t,\, dy_t)_{t\geq 0}$ — governs n_t. We will work with K-scaled strategies and payoffs going forward. The issuer optimizes its objective (i.e., solves (C.3)) subject to:

$$
\begin{aligned}
x_t &= \left(\frac{1}{r-\mu_t^p+\eta|\sigma_t^p|+f_t}\right)^{\frac{1}{1-\xi}};\\
dp_t &= p_t[\mu_t^p dt+\sigma_t^p dZ_t]; \quad x_t = s_t p_t;\\
dn_t &= (n_t+p_t s_t)(\mu dt+\sigma dZ_t) - p_t s_t(\mu_t^p dt+\sigma_t^p dZ_t) + p_t s_t f_t dt - p_t s_t \kappa dt - dy_t,
\end{aligned}
\tag{C.4}
$$

as well as $dy_t \geq 0$, $a_t = n_t + s_t p_t \geq 0$, and $x_t \leq \overline{x}$. (As explained in Appendix A, the constraint $x_t \leq \overline{x}$ is a regularity condition; we take $\overline{x}$ arbitrarily large, so that this constraint does not bind in optimum.) Let $\overline{\mathcal{G}}$ denote the set of feasible strategies satisfying these constraints, that is, (C.4) in addition to $dy_t \geq 0$, $a_t \geq 0, x_t \in [0, \overline{x}]$.

By Lemma 1, proven above, we can rewrite dn_t according to (10) or, equivalently, (C.2). As explained below, our formulation solves the issuer's full-commitment problem (Ramsey problem). Our recursive formulation tracks the stablecoin market capitalization x_t (real balances) as a promised value (state variable), allowing us to characterize the optimum using methods from the dynamic contracting/security-design literature (e.g., Sannikov (2008)).

Remark: State rotation. In the dynamic optimization above, n_t and x_t are state variables. One could rotate the state space and consider (a_t, x_t) instead of (n_t, x_t) as state variables, since $a_t = n_t + x_t$, i.e., given x_t, a_t and n_t are simple linear transformations of each other. Thus, we could have set up the problem with the above constraints, except that the dynamics of dn_t, are replaced by $da_t = dA_t/K$, where dA_t is given in (C.1). In that case, we could have rotated the state space back to (n_t, x_t) to arrive at the same dynamic optimization that we have set out in this Appendix. Overall, the rotation of the state space from (a_t, x_t) to (n_t, x_t) and vice versa is without loss of generality, and does not change the results.

C.1 Outline of the Solution

The following argument and the solution to the issuer's optimization proceed as follows. (For convenience, we often omit time subscripts going forward.)

First, in Part I, in Appendix C.2, we derive the auxiliary Lemma 2 that follows from above constraints, and applies to all feasible strategies. This Lemma implies that net worth $n_t = n$ is bounded from below by $\underline{n}$ (available in closed-form in (C.7)), and puts some state constraints on the optimization. These constraints also restrict feasible dividend payouts to satisfy $dy \in [0, n - \underline{n}]$, as, in state n, a dividend payout $dy > n - \underline{n}$ would cause n to fall below $\underline{n}$. Lemma 2 establishes a necessary condition for the issuer to be able to deliver the "promise" x_t to the users in the recursive formulation of our problem.

Second, in Part II in subsection C.3, we solve the auxiliary problem, in which the issuer can directly choose $(\sigma_t^p)_{t\geq 0}$, in addition to $\mathcal{G} \in \overline{\mathcal{G}}$. That is, we consider:

$$
V_0^{Aux} := K v_0^{Aux} = K \max_{\mathcal{G}\in\overline{\mathcal{G}},(\sigma_t^p)_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dy_t\right], \tag{C.5}
$$

subject to the same constraints, i.e., (C.4), $dy_t \geq 0$, $a_t = n_t + s_t p_t \geq 0$, and $x_t \leq \overline{x}$. Issuer payoff under this auxiliary problem is, by construction, at least as high as that under (C.3), since the issuer has a wider range of options to optimize over. That is, $v_0^{Aux} \geq v_0$.

Part II in Appendix C.3 also shows that in the auxiliary problem, the dimensionality of the state space can be reduced. In particular, x_t can be adjusted via (ds_t, f_t) and thus drops out as state (i.e., becomes control variable) in the dynamic optimization, so n_t remains the sole state variable. We then show that the (scaled) value function will be a function $v(n)$ of n only, solving the HJB (13). Let $\mathcal{S} = (x_t, \sigma_t^p, dy_t)_{t\geq 0}$ denote the optimal policies.

Third, in Part V of the proof (see Appendix C.6), we map the issuer's optimal choice of $\mathcal{S} = (x_t, \sigma_t^p, dy_t)_{t\geq 0}$ to $(s_t, f_t, dy_t)_{t\geq 0}$. We show that for the optimal choice of $\mathcal{S}$, there exists $(s_t, f_t, dy_t)_{t\geq 0}$ — that is, $\mathcal{G} \in \overline{\mathcal{G}}$ — implementing this strategy $\mathcal{S}$, while satisfying all the imposed constraints. This implies $V_0 = V_0^{Aux}$ (i.e., $v_0 = v_0^{Aux}$), as well as

$$V_0 = Kv(n) = K \max_{(x_t, \sigma_t^p, dy_t)_{t\geq 0}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dy_t \mid n_0 = n\right],$$

subject to $(x_t, \sigma_t^p, dy_t)_{t\geq 0}$ being feasible (i.e., the corresponding strategy $\mathcal{G}$ is feasible). This validates that the approach from the main text, where (x, σ^p) are chosen according to the HJB (13), yields the solution to the full problem (9).

Taken together, we formulate the problem using dynamic programming with (n_t, x_t) as state variables, where x_t serves as the promised value, encoding the issuer's commitments to users. A key result is that the dimensionality can be reduced: n_t emerges as the sole state variable, while x_t effectively becomes a control variable (which can be adjusted instantaneously through issuance), alongside σ_t^p and dy_t.

Fourth, in Appendix F, we verify that the issuer indeed finds it optimal to implement a continuous price path, with p_t following (1) with endogenous $(\mu_t^p, \sigma_t^p)_{t\geq 0}$.

Importantly, we conjecture and verify that for all $n > \underline{n}$, the issuer maintains strictly positive reserve assets and the constraint $A_t \geq 0$ never binds, in that $n + x > 0$; accordingly, we omit this constraint going forward, solve the issuer's optimization without it, and verify in Appendix C.5 in Part IV that $n + x > 0$ for all $n > \underline{n}$.

Finally, this Appendix (organized in several parts that build on each other) proves several results from the main text, albeit in a different order than they appear in the main text.[34] Part I in Appendix C.2 proves an auxiliary Lemma, which also derives the lower boundary $\underline{n}$ in closed-form. Part II of the argument in Appendix C.3 proves Propositions 2 and 4. Part III in Appendix C.4 provides a proof for Propositions 3 and 8. Part IV in Appendix C.5 provides a proof for Propositions 5, 7, and 9. Corollary 1 directly follows from Proposition 5. Part V in Appendix C.6 demonstrates Corollary 2.

C.2 Part I: Auxiliary Lemma and Lower Bound

The following Lemma follows from the issuer's optimization constraints. Recall drift and volatility of n, μ_n and σ_n from (10), which depend on (n, x), and σ^p.

[34] Main text flow is organized for expositional purposes and to provide intuition. In contrast, the Appendix organization follows the theoretical and formal argument.

Lemma 2. *Consider the net worth $n_t = n$ following the law of motion* (10) *(or, equivalently,* (C.2)*), governed by a feasible strategy $\mathcal{G} \in \overline{\mathcal{G}}$, satisfying $n_t + x_t \geq 0$, $dy_t \geq 0$, $x_t \leq \overline{x}$ for all $t \geq 0$, and* (C.4)*. Then, the following holds:*

1. *There exists a lower bound $\underline{n}'$ such that $n_t \geq \underline{n}'$ with probability one. If this lower bound is reached, we have $\mu_n(\underline{n}') \geq 0$, $\sigma_n(\underline{n}') = 0$; payouts must respect the lower bound, implying $dy \leq n - \underline{n}'$.*

2. *Define the lowest viable boundary of $n_t = n$ via:*

$$\underline{n} := \inf\Big\{n \in \mathbb{R} : \max_{x\in[0,\overline{x}],\, \sigma^p\in\mathbb{R}} \mu_n(n) \geq 0 \quad s.t. \quad n + x \geq 0,\ \sigma_n(n) = 0\Big\}. \tag{C.6}$$

   where $\mu_n = \mu_n(n)$ and $\sigma_n = \sigma_n(n)$ are defined in (10)*, and depend on x and σ^p (we suppress this dependence). Then:*

$$\underline{n} = \begin{cases} -\left(\frac{\underline{x}^{\xi}-(r+\kappa-\mu)\underline{x}-\eta\sigma\underline{x}}{\mu-\eta\sigma}\right) & \text{if} \quad \mu-(1-\xi)(r+\kappa)-\eta\sigma > 0, \\ -\left(\frac{1}{r+\kappa}\right)^{\frac{1}{1-\xi}} & \text{if} \quad \mu-(1-\xi)(r+\kappa)-\eta\sigma \leq 0. \end{cases} \tag{C.7}$$

   Here:

$$\underline{x} = \left(\frac{\xi}{r+\kappa-\mu+\eta\sigma}\right)^{\frac{1}{1-\xi}} = \left(\frac{\xi}{\lambda+\eta\sigma}\right)^{\frac{1}{1-\xi}}; \quad \lambda = r+\kappa-\mu. \tag{C.8}$$

   At $n = \underline{n}$, the viability conditions bind and under the optimizing (x, σ^p), we have $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0$ (as well as $\underline{n} + x \geq 0$). Further, payouts satisfy $dy = dy(n) \leq n - \underline{n}$.

Importantly, Lemma 2 identifies a feasibility/viability lower boundary. Under any feasible strategy, net worth is bounded below by $-\overline{x}$. The level $\underline{n}$ in (C.6) is the lowest boundary that can be made viable by appropriate controls (i.e., diffusion can be shut down and drift made nonnegative). Under the optimal (and any feasible) strategy, net worth never falls below $\underline{n}$, and payouts respect the state constraint at the boundary. The

C.2.1 Proof of Lemma 2 — Part A

Due to $x_t \leq \overline{x}$ and $A_t = K(x_t + n_t) \geq 0$, there exists $\underline{n}' \geq -\overline{x}$ such that $n_t \geq \underline{n}'$ (with probability one). In particular, $n_t \geq -\overline{x}$ with probability one.[35] Due to $dy_t \geq 0$ and the dynamics of n_t in (10), payouts need to satisfy $dy_t \leq n_t - \underline{n}'$. In particular, if $\underline{n}'$ were reached, no payouts occur in $n_t = \underline{n}'$, as otherwise n_t would fall below $\underline{n}'$. Likewise, in any other state n_t, payouts cannot be so large that they push n_t below $\underline{n}'$. Moreover, if the lower boundary $\underline{n}'$ is reached, the drift of n_t must be positive $\mu_n(\underline{n}') \geq 0$, as well as the volatility must be zero, i.e., $\sigma_n(\underline{n}') = 0$ — in order to prevent n_t from falling below $\underline{n}'$.

C.2.2 Proof of Lemma 2 — Part B

Since, by Part A above, there exists a lower bound such that $n_t \geq \underline{n}'$ with probability one, the set from (C.6) is non-empty and the infimum over the set (i.e., $\underline{n}$) is well-defined. We

[35] Again, note that $x_t \leq \overline{x}$ is a *regularity condition*; we can choose $\overline{x}$ arbitrarily large so that the constraint on x_t never binds in equilibrium.

first solve a relaxed problem in (C.6) that drops the constraint $n + x \geq 0$, and obtain a candidate boundary $\underline{n}$. We then verify ex post that $\underline{n} + x(\underline{n}) \geq 0$, so the candidate indeed solves (C.6). In what follows, we solve for the lower boundary $\underline{n}$ in closed form.

Doing so, we distinguish between two cases: (1) $\sigma^p(\underline{n}) > 0$ and (2) $\sigma^p(\underline{n}) = 0$. We omit time subscripts unless needed:

Case (1): $\sigma^p(\underline{n}) > 0$. First, consider $\sigma^p(\underline{n}) > 0$, and denote $x(\underline{n}) = \underline{x}$. Due to $\sigma_n(\underline{n}) = 0$, we have $\underline{x}\sigma^p(\underline{n}) = (\underline{x} + \underline{n})\sigma$. Therefore, the drift of net worth becomes $\mu_n(\underline{n}) = \max_{x \leq \overline{x}} \Big[\mu\underline{n} - \lambda\underline{x} + \underline{x}^\xi - \eta\sigma(\underline{x} + \underline{n})\Big]$. Optimizing the drift over $\underline{x}$, we obtain (C.8), that is: $\underline{x} = \left(\frac{\xi}{\lambda+\eta\sigma}\right)^{\frac{1}{1-\xi}}$. (Again, we assume that $\overline{x}$ is sufficiently large so that $x \leq \overline{x}$ does not bind.) Considering $\mu - \eta\sigma > 0$, we can solve $\mu_n(\underline{n}) = 0$ for

$$\underline{n} = -\left(\frac{\underline{x}^\xi - \lambda\underline{x} - \eta\sigma\underline{x}}{\mu - \eta\sigma}\right) = -\left(\frac{\underline{x}^\xi - (r + \kappa - \mu)\underline{x} - \eta\sigma\underline{x}}{\mu - \eta\sigma}\right),$$

as stated. Next, we determine the volatility $\sigma^p(\underline{n})$. To do so, calculate

$$\underline{n} + \underline{x} = \frac{(\mu + \lambda)\underline{x} - \underline{x}^\xi}{\mu - \eta\sigma} = \frac{\underline{x}\,[\xi\mu - (1 - \xi)\lambda - \eta\sigma]}{\xi(\mu - \eta\sigma)} > 0, \tag{C.9}$$

which also implies that reserve assets are strictly positive at $\underline{n}$ (i.e., $A_t \geq 0$ or $n_t + x_t \geq 0$ does not bind). Moreover:

$$\sigma^p(\underline{n}) = \sigma \max\left\{0, \frac{\xi\mu - (1 - \xi)\lambda - \eta\sigma}{\xi(\mu - \eta\sigma)}\right\}. \tag{C.10}$$

Thus, a necessary condition for $\sigma^p(\underline{n}) > 0$ is that $\xi\mu - (1 - \xi)\lambda - \eta\sigma > 0$. This condition implies $\mu - \eta\sigma > 0$, due to $\xi \in (0, 1)$.

Further, for $\lambda = r + \kappa - \mu$, we can rewrite $\xi\mu - (1 - \xi)\lambda - \eta\sigma = \mu - (1 - \xi)(r + \kappa) - \eta\sigma$.

Case (2): $\sigma^p(\underline{n}) = 0$. Note that $\sigma_n(\underline{n}) = (x + n)\sigma - x\sigma^p = (x + n)\sigma = 0$, i.e., $a = n + x = 0$. Thus, $x = x(\underline{n}) = -\underline{n}$. We can insert $x = -\underline{n}$ into the drift $\mu_n(n)$ from (C.2) to obtain

$$\mu_n(\underline{n}) = (\mu + \lambda)\underline{n} + (-\underline{n})^\xi.$$

We then solve $\mu_n(\underline{n}) = 0$ for $\underline{n} = -\left(\frac{1}{\mu+\lambda}\right)^{\frac{1}{1-\xi}} = -\left(\frac{1}{r+\kappa}\right)^{\frac{1}{1-\xi}}$.

Case Distinction. We show that case (1) prevails and $\sigma^p(\underline{n}) > 0$ if and only if $\xi\mu - (1 - \xi)\lambda - \eta\sigma > 0$. First, we start with the "only if" implication. Note that $\sigma^p(\underline{n}) > 0$, i.e., case (1), requires that $\xi\mu - (1 - \xi)\lambda - \eta\sigma > 0$. Thus, when $\xi\mu - (1 - \xi)\lambda - \eta\sigma \leq 0$, we necessarily have $\sigma^p(\underline{n}) = 0$.

Second, we show that if $\xi\mu - (1 - \xi)\lambda - \eta\sigma > 0$, then $\sigma^p(\underline{n}) > 0$. Suppose to the contrary that $\xi\mu - (1 - \xi)\lambda - \eta\sigma > 0$ and $\sigma^p(\underline{n}) = 0$, leading to $\underline{n} = -\left(\frac{1}{\mu+\lambda}\right)^{\frac{1}{1-\xi}}$ and $x(\underline{n}) = -\underline{n}$ as shown above. Consider that the issuer selects in state $\underline{n} = -\left(\frac{1}{\mu+\lambda}\right)^{\frac{1}{1-\xi}}$ a different level of x, namely $x = -\underline{n} + \varepsilon$, while setting $x\sigma^p = \varepsilon\sigma$ for small $\varepsilon > 0$.

Thus, $\sigma_n(\underline{n}) = (n + x)\sigma - x\sigma^p = 0$ under this alternative strategy. The alternative strategy implies drift of n at $\underline{n}$ of

$$\begin{aligned}\mu_n^\varepsilon(\underline{n}) &= \mu\underline{n} - \lambda(-\underline{n} + \varepsilon) + (-\underline{n} + \varepsilon)^\xi - \varepsilon\eta\sigma \\ &= \mu_n(\underline{n}) - \lambda\varepsilon - \eta\sigma\varepsilon + \xi(-\underline{n})^{\xi-1}\varepsilon + O(\varepsilon^2) = \varepsilon\big[\xi\mu - (1-\xi)\lambda - \eta\sigma\big] + O(\varepsilon^2),\end{aligned}$$

where we conducted a Taylor expansion around $\varepsilon = 0$ and used $\mu_n(\underline{n}) = 0$.

Given $\xi\mu - (1-\xi)\lambda - \eta\sigma > 0$, there exists $\varepsilon > 0$ such that $\mu_n^\varepsilon(\underline{n}) > 0$ and $\sigma_n(\underline{n}) = 0$ under $x = -\underline{n} + \varepsilon$. By continuity, there exists $\underline{n}' < \underline{n}$ at which $\mu_n(\underline{n}') \geq 0$ and $\sigma_n(\underline{n}') = 0$, a contradiction. Likewise, we have shown that $x = -\underline{n}$ and $\sigma^p = 0$ do not maximize $\mu_n(\underline{n})$, similarly yielding a contradiction. Combining, we obtain (C.7).

C.3 Part II: HJB Equation (Proof of Propositions 2 and 4)

Consider the auxiliary problem whereby the issuer can choose $(\sigma_t^p)_{t\geq 0}$. Appendix C.6 shows that optimal σ_t^p can be implemented through an appropriate issuance and fee policy, i.e., the auxiliary problem solves the issuer's full problem.

It suffices to track real (dollar) balances of stablecoins, $x_t = s_t p_t$ (by market clearing), because users' utility, the issuer's balance-sheet constraint, $a_t = n_t + x_t \geq 0$, and the dynamics of net worth in (10) depend on p_t and s_t only through the stablecoin's total dollar value x_t. Given $x_t = s_t p_t$, the split between stablecoin quantity s_t and price p_t is a choice of units and is payoff-irrelevant. Furthermore, the issuer can choose σ_t^p, and the optimization depends on issuance and fee policies only via the transfer control dT_t defined in (C.12) below. In sum, x_t is a sufficient statistic for user demand and the issuer's constraints, so we can solve the optimization without tracking p_t and s_t separately: p_t and s_t are not additional state variable because they enter payoffs and constraints only through x_t.

We characterize the dynamics of x_t. Inverting (5), we obtain $\mu_t^p = r + f_t + \eta|\sigma_t^p| - x_t^{\xi-1}$. Thus, we can rewrite $dp_t = p_t\mu_t^p dt + p_t\sigma_t^p dZ_t = (r + f_t)p_t dt - p_t\Big(x_t^{\xi-1} - \eta|\sigma_t^p|\Big)dt + p_t\sigma_t^p dZ_t$. Then, multiply both sides of the above equation by s_t and use $x_t = s_t p_t$ (market clearing) to obtain $s_t dp_t = (r + f_t)x_t dt - \Big(x_t^\xi - \eta x_t|\sigma_t^p|\Big)dt + x_t\sigma_t^p dZ_t$. Ito's product rule implies $d(s_t p_t) = ds_t p_t + p_t ds_t + ds_t dp_t$. Then, we calculate for $x_t = s_t p_t$:

$$dx_t = (r + f_t)x_t dt - \Big(x_t^\xi - \eta x_t|\sigma_t^p|\Big)dt + x_t\sigma_t^p dZ_t + ds_t(p_t + dp_t). \tag{C.11}$$

Define

$$dT_t = -ds_t(p_t + dp_t) - f_t x_t dt. \tag{C.12}$$

which represents the net dollar transfer from issuer to users. Note that dT_t is a choice variable; it can be determined by the issuer through issuance strategy and fee policies. dT_t can be positive or negative.

Under a transversality condition, $\lim_{u\to\infty} e^{-r(u-t)}\,\mathbb{E}_t[x_u] = 0$, which is implied by $x_u \leq \overline{x}$, and square integrability of σ_t^p (a standard regularity condition), we can integrate:

$$x_t = \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \Big(x_s^\xi ds - \eta x_s |\sigma_s^p| ds + dT_s \Big) \right]. \tag{C.13}$$

Consequently, we can write the issuer's optimization as follows:

$$v_0 := \max_{(\sigma_t^p, dT_t, dy_t)_{t \geq 0}} \mathbb{E} \left[\int_0^\infty e^{-\rho t} dy_t \right], \tag{C.14}$$

subject to (10) and (C.11) (equivalently (C.13)) for all $t \geq 0$, as well as $n_t + x_t \geq 0$, $dy_t \geq 0$, and $x_t \leq \overline{x}$. Moreover, Lemma 2 implies the payout restriction $dy_t \in [0, n_t - \underline{n}]$.

Similar to Jermann and Xiang (2025b), we are solving the full commitment problem of the issuer (also known as Ramsey problem). The problem is mathematically analogous to a dynamic contracting or security-design problem, as, e.g., in Sannikov (2008). Equation (C.13) (or (C.11) in differential form) plays the role of a promise-keeping constraint: the outstanding real balances x_t restrict the set of feasible continuation policies, i.e., $(x_s, \sigma_s^p, dT_s, dy_s)_{s \geq t}$.

Accordingly, x_t is a *promised-payoff* or *promised-value* state variable in the issuer's recursive formulation, encoding the issuer's past commitment in terms of delivering stablecoin value. Crucially, enriching the state space to include $x_t = x$ and tracking states $(n_t, x_t) = (n, x)$ ensures that the dynamic programming representation captures the issuer's full-commitment optimum: at any time t, continuation policies are evaluated subject to delivering the inherited promise x_t. Thus, the dynamic programming principle applies here after enriching the state space.

Crucially, the lower boundary established in Lemma 2 is a necessary condition for the promise x_t to be feasible: Any violation of $n \geq \underline{n}$ or $dy > n - \underline{n}$ would imply the promise is not feasible, in that the promise-keeping constraint (C.13) (or (C.11)) cannot be satisfied.

Optimization and Generalized HJB. The issuer's optimization has two state variables, $(n_t, x_t) = (n, x)$. The continuation value under the optimal policy is given by $F = F(n, x) := \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} dy_s \mid (n_t, x_t) = (n, x) \right]$, solving the HJB equation:

$$\begin{aligned} \rho F dt = \max_{dy \in [0, n - \underline{n}], dT, \sigma^p} \Bigg\{ & dy + F_n \Big[\mu n + x^\xi - \eta x |\sigma^p| - (r + \kappa - \mu) x \Big] dt - F_n dy \\ & + F_x \Big[(rx + \eta x |\sigma^p| - x^\xi) dt - dT \Big] + \frac{F_{nn}}{2} (\sigma(n + x) - x\sigma^p)^2 dt + \frac{F_{xx} d[x, x] + 2F_{xn} d[x, n]}{2} \Bigg\}. \end{aligned} \tag{C.15}$$

Here, $d[x, x]$ is the quadratic variation of dx and $d[x, n]$ the quadratic covariation between dn and dx. Assume F is twice continuously differentiable in the interior of the state space. Note that dy is a singular payout control that reflects n from the edges of the state space back into the interior.[36] $dy > 0$ is optimal only if $F_n \leq 1$, whereas $F_n > 1$ implies $dy = 0$.

Optimal Transfer dT. Next, consider the optimization with respect to the transfer dT. Note that dT affects the objective only through the term $-F_x \, dT$, while dn does not directly

[36] Formally, the HJB equation associated with a singular control problem takes the form of a variational inequality; with some abuse of notation, we use the standard HJB notation with dy entering.

depend on dT; only dx does, with dT reducing x one-for-one. Therefore, the issuer can adjust x instantaneously via dT **without changing** n. In particular, issuing an additional dollar worth of stablecoins increases reserves, A, by one dollar, and liabilities (outstanding stablecoin value), X, by the same amount, leaving net worth N and thus n unchanged.

In the interior of the state space where $a > 0$, $x \in (0, \overline{x})$, both positive and negative adjustments of x via dT are feasible. Thus, the issuer can move from (n, x) to $(n, x + \varepsilon)$ for any $\varepsilon \gtrless 0$ by choosing $dT = -\varepsilon$, where $|\varepsilon|$ is sufficiently small. Since, in state (n, x), setting $dT = -\varepsilon$ is feasible but not necessarily optimal, we have $F(n, x) \geq F(n, x + \varepsilon)$. Likewise, in state $(n, x + \varepsilon)$, setting $dT = \varepsilon$ and moving toward (n, x) is always an option, so $F(n, x + \varepsilon) \geq F(n, x)$. Consequently, $F(n, x + \varepsilon) = F(n, x)$ for all sufficiently small $\varepsilon \gtrless 0$.

In the interior of the state space, the issuer can marginally increase (decrease) x via $dT < 0$ ($dT > 0$), while leaving n unchanged. It follows that $F_x \equiv 0$ in the interior of the state space and along the optimal path. Consequently, we have $F_{xx} = F_{xn} = 0$ there as well.

HJB equation (13) **and** x **as control.** As shown above, on the interior of the state space the value function is independent of x, so that n is the only relevant state variable, i.e., $F_x \equiv 0$. Indeed, the issuer can adjust x through dT, leaving n unchanged. Equivalently to considering the control dT, we may treat x as a control variable in the recursive problem: for each n, the issuer chooses $x \in [0, \overline{x}]$ (implemented via dT) together with the remaining controls. Hence, although x is the promised value in the recursive formulation, the issuer can adjust it through dT; i.e., x "drops out" as state. Formally, define the reduced value function $v(n) := \sup_{x \in [0, \overline{x}]} F(n, x)$, and solve the associated HJB in n with x chosen optimally.

Consider the interior of the state space, where $dy = 0$. Inserting $F_x = F_{xx} = F_{xn} = 0$ in (C.15) and canceling dt terms on both sides, the generalized HJB equation from above simplifies to (in the interior of the state space):

$$\rho v(n) = \max_{\sigma^p, x \in [0, \overline{x}]} \left\{ v'(n) \Big(\mu n - \lambda x + x^{\xi} - \eta x |\sigma^p| \Big) + \frac{v''(n) \big[\sigma(x + n) - x\sigma^p \big]^2}{2} \right\}, \tag{C.16}$$

where $\lambda = r + \kappa - \mu$. Note that (C.16) is equivalent to (13). Again, the transfer process dT is used to implement the desired path of x, subject to the feasibility restrictions $dT \leq a$ and $x \in (0, \overline{x})$; Appendix C.5 verifies that in the interior of the state space, $a > 0$ holds (so positive and negative adjustments, $dT \gtrless 0$, are indeed feasible along the optimal path).

Upper Boundary. As is standard, consumption occurs at a payout boundary $\overline{n}$, and follows a barrier strategy, that is, consumption causes n to reflect at $\overline{n}$. As such, we have $v(n) = v(\overline{n}) + n - \overline{n}$ and $v'(n) = 1$ for $n > \overline{n}$. The location of the payout boundary is determined by smooth pasting and super contact conditions, that is, $v'(\overline{n}) = 1$ and $v''(\overline{n}) = 0$. Due to the (downward) reflection of n at $\overline{n}$, the (endogenous) state space can be written as an interval $[\underline{n}, \overline{n}]$. Further, note that the value function satisfies $v'(n) \geq 1$ on $[\underline{n}, \overline{n}]$.

Lower Boundary. Under the issuer's (optimal) strategy, we have $n \geq \underline{n}$ with probability one, by Lemma 2, where $\underline{n}$ is given in (C.7). When $n = \underline{n}$, then, by means of Lemma 2, $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0$, as well as $dy = 0$ — this means that the lower boundary is absorbing. Moreover, by means of (13), it follows that $v(\underline{n}) = 0$.[37] Taken together, the issuer's value

[37]In the proof of Proposition 6 in Appendix D, we show that the lower boundary being inaccessible (i.e.,

function is the solution to (13) subject to $v(\underline{n}) = v'(\overline{n}) - 1 = v''(\overline{n}) = 0$. Overall, we have proven Propositions 2 and and 4.

C.4 Part III: Concavity of the Value Function and Optimal Controls (Proof of Propositions 3 and 8)

We first characterize the optimal controls, conjecturing concavity. Then, we prove the concavity of the value function, verifying this conjecture.

C.4.1 Optimal Controls

We conjecture and verify that $v''(n) \in (-\infty, 0)$ on $n \in (\underline{n}, \overline{n})$, with $v''(\overline{n}) = 0$. Consider $n \in (\underline{n}, \overline{n})$. Note that $x + n \geq 0$ (which is equivalent to the constraint $A = N + Sp \geq 0$). Thus, the HJB equation (C.16) immediately implies that, in optimum, $\sigma^p \geq 0$.

If interior, i.e., $\sigma^p > 0$ or $x \in (0, \overline{x})$, the optimal controls solve the first-order conditions, which, as easily can be verified, are sufficient for a maximum.

For $\sigma^p > 0$, the derivative in (C.16) with respect to σ^p implies that, after omitting positive constants: $\frac{\partial v(n)}{\partial \sigma^p} \propto -v'(n)\eta x - v''(n)\big(\sigma(x+n) - x\sigma^p\big)x$. When $\sigma^p > 0$, then $\sigma^p = \sigma^p(n)$ solves the first-order condition $\frac{\partial v(n)}{\partial \sigma^p} = 0$. Thus, we obtain:

$$x\sigma^p = \max\left\{0, \sigma(x+n) - \frac{\eta}{\gamma(n)}\right\}, \quad \text{for} \quad \gamma(n) := -\frac{v''(n)}{v'(n)}.$$

where we account for the fact that σ^p does not become negative in optimum.

Suppose $x\sigma^p > 0$ and insert the expression for $x\sigma^p$ into the HJB equation (C.16):

$$\rho v(n) = \max_{x \geq 0}\left\{v'(n)\Big[\mu n - x\lambda + x^{\xi} - \eta(n+x)\sigma\Big] + \frac{v'(n)\eta^2}{2\gamma(n)}\right\}. \tag{C.17}$$

Thus, when $\sigma^p > 0$, the first-order condition with respect to x becomes $v'(n)\big[-\lambda - \eta\sigma + \xi x^{\xi-1}\big] = 0$. Using $v'(n) \geq 1$, we can solve this FOC for $x = x(n) = \underline{x}$ where, as in (C.8): $\underline{x} = \left(\frac{\xi}{\lambda + \eta\sigma}\right)^{\frac{1}{1-\xi}}$. Thus, $\underline{x}\sigma^p = \max\left\{0, \sigma(\underline{x} + n) - \frac{\eta}{\gamma(n)}\right\}$ and therefore

$$\sigma^p = \sigma^p(n) = \max\left\{0, \frac{\sigma(\underline{x} + n)}{\underline{x}} - \frac{\eta}{\gamma(n)\underline{x}}\right\}. \tag{C.18}$$

It follows that $\sigma^p(\overline{n}) = 0$, due to $v''(\overline{n}) = 0$. Thus, when $\sigma^p(n) > 0$, then $\sigma_n(n) = \frac{\eta}{\gamma(n)}$, which is strictly positive for $n > \underline{n}$.

Now, suppose that $\sigma^p = 0$. Then, the first order condition with respect to x in the HJB equation (C.16) becomes

$$v'(n)\big[-\lambda + \xi x^{\xi-1}\big] + v''(n)\sigma^2(x+n) = 0 \quad \Longleftrightarrow \quad \xi x^{\xi-1} - \lambda = \gamma(n)\sigma^2(x+n). \tag{C.19}$$

the lower boundary is never reached) whenever (16) does not hold and depegging occurs.

Optimal $x = x(n)$ therefore is a function of n. As long as $v''(n) > -\infty$, we have $x + n > 0$. For $n = \overline{n}$, we have

$$x(\overline{n}) = x^* := \left(\frac{\xi}{\lambda}\right)^{\frac{1}{1-\xi}}. \tag{C.20}$$

Finally, when $\sigma^p = \sigma^p(n) > 0$, then $\sigma_n(n) = \sigma(\underline{x} + n) - \underline{x}\sigma^p = \frac{\eta}{\gamma(n)} > 0$. When $\sigma^p = 0$, then $\sigma_n(n) = (x + n)\sigma > 0$. Either way, $\sigma_n(n) > 0$ for all $n \in (\underline{n}, \overline{n}]$, where $x + n > 0$.

C.4.2 Concavity of Value Function

Without loss of generality, we can prove the claim for $v''(n) > -\infty$. Part I implies that $\sigma_n(n) > 0$ if $v''(n) < 0$. Thus, we conjecture that $\sigma_n(n) > 0$ and then prove $v''(n) < 0$, thereby verifying the conjecture. Rewrite the HJB equation (C.16) in the interior of the state space as $\rho v(n) = \max_{x,\sigma^p} \left\{ \mu_n(n)v'(n) + \frac{v''(n)\sigma_n(n)^2}{2} \right\}$. We proceed under the standard assumption that $v''(n)$ is differentiable almost everywhere (viscosity solution properties ensure the validity of this approach).

In this case, we can invoke the envelope theorem to differentiate the HJB equation (13) under the optimal controls with respect to n:

$$v'''(n) = \frac{2}{\sigma_n(n)^2}\Big((\rho - \mu)v'(n) - v''(n)\big[\mu_n(n) + \sigma\sigma_n(n)\big]\Big). \tag{C.21}$$

In doing so, we have used that, when $\sigma^p = 0$, then x solves the first order condition (C.19), so that $\frac{\partial v(n)}{\partial x} = 0$ and $\frac{\partial \sigma^p(n)}{\partial n} = 0$. On other hand, if $\sigma^p > 0$, then $x(n) = \underline{x}$, so that $\frac{\partial x(n)}{\partial n} = 0$, and σ^p solves the first order condition $\frac{\partial v(n)}{\partial \sigma^p} = 0$.

Evaluating above expression for $v'''(n)$ at $n = \overline{n}$, we obtain $\lim_{n \uparrow \overline{n}} v'''(n) > 0$. As such, $v''(n) < 0$ in a left-neighbourhood of $\overline{n}$. Suppose now to the contrary there exists $n < \overline{n}$ such that $v''(n) \geq 0$, and define $n' = \sup\{n < \overline{n} : v''(n) \geq 0\}$. Note that, because $v(n)$ is strictly concave in a left-neighborhood of $\overline{n}$, we have $n' < \overline{n}$ as well as $v''(n) < 0$ on $(n', \overline{n})$ and $v'(n') > 1$. By continuity of v'', it follows that $v''(n') = 0$. As such, $v'''(n') > 0$, due to $\rho > \mu$. However, this implies that there exists $n'' > n'$ with $v''(n'') > 0$, contradicting the definition of n'. As such, the claim follows, and $v''(n) < 0$ for $n < \overline{n}$, which implies $v'(n) > 1$ for $n < \overline{n}$. Overall, we have shown $v''(n) < 0$ on $(\underline{n}, \overline{n})$.

C.4.3 Dynamics of Controls

We now show that (i) $\gamma(n)$ decreases with n, (ii) $\sigma^p(n)$ decreases with n (strictly so when $\sigma^p(n) > 0$), and (iii) $x(n)$ increases with n (strictly so when $x(n) > \underline{x}$). In order to do so, we distinguish between $\sigma^p(n) > 0$ and $\sigma^p(n) = 0$.

Case 1: $\sigma^p = \sigma^p(n) > 0$. Suppose $\sigma^p > 0$. Then, we have in optimum $x = x(n) = \underline{x}$ and the HJB equation (C.17) holds. We then obtain

$$\rho\left(\frac{v(n)}{v'(n)}\right) = \Big[\mu n - \underline{x}\lambda + \underline{x}^{\xi} - \eta(n + \underline{x})\sigma\Big] + \frac{\eta^2}{2\gamma(n)}.$$

Thus, differentiating both sides with respect to n yields:

$$\frac{d}{dn}\left(\rho\left(\frac{v(n)}{v'(n)}\right)-\left[\mu n-\underline{x}\lambda+\underline{x}^{\xi}-\eta(n+\underline{x})\sigma\right]\right)=\frac{d}{dn}\left(\frac{\eta^2}{2\gamma(n)}\right).$$

Note that

$$\mathcal{A}(n):=\frac{d}{dn}\left(\rho\left(\frac{v(n)}{v'(n)}\right)-\left[\mu n-\underline{x}\lambda+\underline{x}^{\xi}-\eta(n+\underline{x})\sigma\right]\right)=\rho\left(1-\frac{v''(n)v(n)}{v'(n)^2}\right)-\mu+\eta\sigma,$$

while $\frac{d}{dn}\left(\frac{\eta^2}{2\gamma(n)}\right)=-\frac{\eta^2\gamma'(n)}{2\gamma(n)^2}$. Due to $v''(n)\leq 0$, $\rho\geq\mu$, it follows that $\mathcal{A}(n)>0$. Due to $\mathcal{A}(n)=\frac{d}{dn}\left(\frac{\eta^2}{2\gamma(n)}\right)=-\frac{\eta^2\gamma'(n)}{2\gamma(n)^2}$, it follows $\gamma(n)$ decreases with n, i.e., $\gamma'(n)<0$.

Furthermore, calculate

$$\frac{d}{dn}\left(\frac{1}{\gamma(n)}\right)=-\frac{\gamma'(n)}{\gamma(n)^2}=\frac{2\mathcal{A}(n)}{\eta^2}\geq\frac{2(\rho-\mu+\eta\sigma)}{\eta^2}.$$

Hence, when $\sigma^p(n)>0$ — that is, $\underline{x}\sigma^p(n)=\sigma(\underline{x}+n)-\frac{\eta}{\gamma(n)}>0$ — we have

$$\frac{d}{dn}\left(\sigma^p(n)\underline{x}\right)=\sigma-\eta\frac{d}{dn}\left(\frac{1}{\gamma(n)}\right)\leq\sigma-\frac{2(\rho-\mu+\eta\sigma)}{\eta}=-\sigma-2\left(\frac{\rho-\mu}{\eta}\right)<0,$$

so that $\sigma^p(n)$ decreases with n. Due to continuity, $\lim_{n\to\underline{n}}\sigma^p(n)>0$, and $\sigma^p(n)=0$ in a left-neighbourhood of $\overline{n}$, there exists unique point $\tilde{n}\in(\underline{n},\overline{n})$ above which $\sigma^p(n)=0$ and below which $\sigma^p>0$. i.e., $\sigma^p(n)>0$ for all $n<\tilde{n}$ and $\sigma^p(n)=0$ for all $n\geq\tilde{n}$.

Case 2: $\sigma^p=\sigma^p(n)=0$. Suppose $\sigma^p=0$, so x solves the first order condition (C.19), that is, $\frac{\partial v(n)}{\partial x}=0$. Then, the HJB equation implies under the optimal choice of x:

$$\rho\left(\frac{v(n)}{v'(n)}\right)=\left(\mu n-x\lambda+x^{\xi}-\frac{\sigma^2\gamma(n)(x+n)^2}{2}\right).$$

Using the envelope theorem, we can differentiate both sides with respect to n to obtain:

$$\rho\left(1-\frac{v''(n)v(n)}{v'(n)^2}\right)-\mu=-\sigma^2(x+n)\left(\gamma(n)+\frac{\gamma'(n)(x+n)}{2}\right).$$

Due to $\rho>\mu$ and $v''(n)<0$, we obtain

$$-\sigma^2(x+n)\left(\gamma(n)+\frac{\gamma'(n)(x+n)}{2}\right)>0, \tag{C.22}$$

which implies $\gamma'(n)<\frac{-2\gamma(n)}{x+n}<0$.

Next, differentiate both sides of the first-order condition for x — that is, (C.19) and $\xi x(n)^{\xi-1}-\lambda=\gamma(n)\sigma^2(x(n)+n)$ — with respect to n to get:

$$\xi(\xi-1)x(n)^{\xi-2}x'(n)=\sigma^2\left[\gamma(n)\left(x'(n)+1\right)+\gamma'(n)\left(x(n)+n\right)\right].$$

Suppose $x'(n) \leq 0$. Then, the left-hand-side is non-negative. The right-hand-side satisfies:

$$\sigma^2\big[\gamma(n)(x'(n)+1)+\gamma'(n)(x(n)+n))\big] \leq \sigma^2\big[\gamma(n)+\gamma'(n)(x(n)+n))\big] < 0,$$

where we used (C.22). A contradiction. Thus, $x'(n) > 0$.

Overall, we have proven Proposition 3.

C.4.4 Solving the Price: Proof of Proposition 8

Suppose that price is a function of $n_t = n$ only, in that $p_t = p(n_t)$. Take the optimal controls, $x = x(n)$ and $\sigma^p = \sigma^p(n)$, as well as the resulting volatility of n, i.e., $\sigma_n(n)$.

Then, by Ito's Lemma, the volatility of $p(n)$ is $p'(n)\sigma_n(n)$. At the same time, the price volatility equals $\sigma^p(n)p(n)$ by (1). As a result, we obtain

$$p'(n)\big[(x(n)+n)\sigma - x(n)\sigma^p(n)\big] = p'(n)\sigma_n(n) = p(n)\sigma^p(n).$$

For $n > \tilde{n}$, we have $\sigma^p(n) = 0$, so $p'(n) = 0$ and $p(n)$ is constant. We normalize $p(\overline{n}) = p(\tilde{n}) = 1$. Next, consider $n < \tilde{n}$, so that $x(n) = \underline{x}$ and $\sigma^p(n) = \frac{\sigma(\underline{x}+n)}{\underline{x}} - \frac{\eta}{\gamma(n)\underline{x}} > 0$. Thus, $\frac{d\ln p(n)}{dn} = \frac{p'(n)}{p(n)} = \frac{\sigma^p(n)}{\sigma_n(n)}$. Hence,

$$p(n) = \exp\left(-\int_n^{\tilde{n}} \frac{\sigma^p(\nu)}{\sigma_n(\nu)}d\nu\right) = \exp\left(-\int_n^{\tilde{n}} \frac{\gamma(\nu)\sigma(x(\nu)+\nu)-\eta}{\eta x(\nu)}d\nu\right), \tag{C.23}$$

which is equivalent to (18). This proves Proposition 8.

C.5 Part IV: Proof of Propositions 5, 7, 9 and Verifying $A \geq 0$

The previous part has shown that $\sigma^p(n)$ decreases and $x(n)$ increases in n. Note that $\sigma^p(\overline{n}) = 0$. Thus, there exists unique $\tilde{n} := \sup\{n \in [\underline{n}, \overline{n}] : \sigma^p(n) > 0\}$; moreover, $p(n) = 1$ for $n \geq \tilde{n}$, while $p(n) < 1$ for $n < \tilde{n}$. We now characterize the behavior at the lower boundary $\underline{n}$ given in (C.7).

Proving Propositions 9 and 5. Note that when (16) holds, then $\sigma^p(\underline{n}) = 0$ and $\sigma^p(n) = 0$ for all $n \in (\underline{n}, \overline{n})$. Then, the instability region is empty, i.e., $\tilde{n} = \underline{n}$. At $n = \underline{n}$, we have $\underline{n} + x(\underline{n}) = 0$, i.e., the constraint $A \geq 0$ binds at $n = \underline{n}$. Since $x(n) + n$ increases in n, we have $A > 0$, i.e., $n + x > 0$, for all $n \in (\underline{n}, \overline{n})$. The expression for the price $p(n)$ from (18) implies $p(n) = 1$ for all $n \in (\underline{n}, \overline{n})$. Further, by the previous Part III in Section C.4 ("Dynamics of Controls"), we have $x'(n) > 0$.

In contrast, when (16) does not hold — that is, $\mu - (1-\xi)(r+\kappa) > \eta\sigma$ — then (C.10) holds, determining:

$$\sigma^p(\underline{n}) = \sigma\left(\frac{\xi\mu - (1-\xi)\lambda - \eta\sigma}{\xi(\mu - \eta\sigma)}\right) = \sigma\left(\frac{\mu - (1-\xi)(r+\kappa) - \eta\sigma}{\xi(\mu-\eta\sigma)}\right) > 0. \tag{C.24}$$

In particular, because $\sigma^p(\underline{n}) > 0 = \sigma^p(\overline{n})$, with $\sigma^p(n)$ decreasing in n, we have $\tilde{n} \in (\underline{n}, \overline{n})$. Further, $\sigma_n(\underline{n}) = (x(\underline{n})+\underline{n})\sigma - x(\underline{n})\sigma^p(\underline{n}) = 0$ for $\sigma^p(\underline{n}), x(\underline{n}) = \underline{x} > 0$ then implies $x(n)+n > 0$ for $n = \underline{n}$. Moreover, $x(n) + n$ increases in n. This implies that $A > 0$ (i.e., $x + n > 0$) in the entire state space.

Importantly, the expression for the price in (18) implies $p(n) < 1$ for $n < \tilde{n}$, and $p(n) = 1$ for $n \geq \tilde{n}$. This proves Proposition 5. Also note that for $n \leq \tilde{n}$, we have $x(n) = \underline{x}$, while, otherwise, $x'(n) > 0$ — see Part III ("Dynamics of Controls"). This proves Proposition 9.

In addition, and importantly, we have also verified that $A > 0$ (i.e., $a = x + n > 0$) on $(\underline{n}, \overline{n}]$. By continuity, this implies $A \geq 0$.

Proving Proposition 7. Next, combining the two cases, we obtain

$$\sigma^p(n) \leq \sup_{n \in (\underline{n}, \overline{n})} \{\sigma^p(n)\} = \sigma^p(\underline{n}) = \begin{cases} \sigma\left(\frac{\mu-(1-\xi)(r+\kappa)-\eta\sigma}{\xi(\mu-\eta\sigma)}\right) & \text{if} \quad \mu-(1-\xi)(r+\kappa) > \eta\sigma \\ 0 & \text{if} \quad \mu-(1-\xi)(r+\kappa) \leq \eta\sigma, \end{cases}$$

which implies (17), as desired. Suppose that (16) does not hold, that is, $\sigma \in \left[0, \frac{\mu-(1-\xi)(r+\kappa)}{\eta}\right]$ Then, clearly, $\sigma^p(\underline{n}) = 0$ for $\sigma = 0$ or $\sigma = \frac{\mu-(1-\xi)(r+\kappa)}{\eta}$. In the interior of this interval, we can calculate $\frac{\partial^2 \sigma^p(\underline{n})}{\partial \sigma^2} = -\frac{2\mu\eta(1-\xi)(r+\kappa)}{\xi(\mu-\eta\sigma)^3} < 0$. Thus, $\sigma^p(\underline{n})$ is inverted U-shaped (or hump-shaped) and thus first increases, and then decreases on $\left[0, \frac{\mu-(1-\xi)(r+\kappa)}{\eta}\right]$. This proves Proposition 7.

C.6 Part V: Implementing (x, σ^p) (Proof of Corollary 2)

We have solved for the optimal controls (x, σ^p) as functions of n, as well as for the price $p(n)$ characterized in (18). We also have shown in Part IV in Section C.5 that the constraint $A \geq 0$ (i.e., $a = n + x \geq 0$) is met under this choice of controls. Let $s = s(n) = S/K$ the issuer's scaled issuance strategy. We solve for the issuance strategy $ds = ds(n)$ and the fee $f = f(n)$ which implement the optimal levels of $x(n)$ and $\sigma^p(n)$ (where $\sigma^p(n) \geq 0$). We show that ds and f are Markovian, i.e., they are functions of n only.

Issuance Strategy. Having characterized the price in (C.23) as a function of n, notice that by Ito's Lemma:

$$\mu^p(n)p(n) = p'(n)\mu_n(n) + \frac{p''(n)\sigma_n(n)^2}{2}. \tag{C.25}$$

Thus, $p(n)$, $\sigma^p(n)$, and $\mu^p(n)$ are functions of n only. It then follows by market clearing that $s(n) = x(n)/p(n)$ is a function of n only. For $n > \tilde{n}$, we have $p(n) = 1$ and $x'(n) > 0$, so $x(n) = s(n)$. Then, $x'(n) > 0$ and $s(n)$ increases with n. For $n < \tilde{n}$, we have $x(n) = \underline{x}$ (see (C.8)) and $p'(n) > 0$, so $s'(n) < 0$ and $s(n)$ decreases with n. This proves Corollary 2.

Issuance Dynamics. We characterize the issuance dynamics, i.e., $ds = ds(n)$. For $n \in (\tilde{n}, \overline{n})$, we have $\sigma^p(n) = 0$ and $p(n) = 1$. Thus, $ds(n) = dx(n)$. Using Ito's Lemma:

$$ds(n) = \left[x'(n)\mu_n(n) + \frac{x''(n)\sigma_n(n)^2}{2}\right]dt + x'(n)\sigma_n(n)dZ.$$

Because of $x'(n) > 0$, supply $s(n)$ expands (decreases) upon a positive (negative) shock $dZ > 0$ ($dZ < 0$). For $n \in (\underline{n}, \tilde{n})$, we have $\sigma^p(n) > 0$ and $x(n) = s(n)p(n) = \underline{x}$. Thus, $d(s(n)p(n)) = d(sp) = 0$. We stipulate $ds = \mu^s dt + \sigma^s dZ$, and solve for drift and diffusion terms. First, we calculate $d(sp) = sdp + pds + dsdp$. Thus:

$$d(sp) = s(n)p(n)\left[\mu^p(n)dt + \sigma^p(n)dZ\right] + p(n)\left[\mu^s dt + \sigma^s dZ\right] + p(n)\sigma^s\sigma^p(n)dt = 0.$$

Dividing by $p(n)$, we obtain

$$s(n)\big[\mu^p(n)dt + \sigma^p(n)dZ\big] + \big[\mu^s dt + \sigma^s dZ\big] + \sigma^s\sigma^p(n)dt = 0.$$

Thus, $\sigma^s = \sigma^s(n) = -s(n)\sigma^p(n) < 0$. Next, solve $s(n)\mu^p(n) + \mu^s(n) + \sigma^s\sigma^p(n) = 0$ for $\mu^s(n) = -s(n)\mu^p(n) + s(n)(\sigma^p(n))^2$.

Fee. Next, using $\sigma^p(n) \geq 0$ and inverting (5), we obtain

$$\mu^p(n) = r + f(n) - x(n)^{\xi-1} + \eta\sigma^p(n) \quad \Longleftrightarrow \quad f(n) = x(n)^{\xi-1} - \eta\sigma^p(n) - r + \mu^p(n).$$

This shows that $f(n)$ is a function of n — it is uniquely determined given $x(n)$ and $\sigma^p(n)$.

Overall, we have verified that the issuance strategy is Markovian, i.e., a function of n. We have also verified that there exist fee structure and issuance policies that implement the desired levels of $x = x(n)$ and $\sigma^p = \sigma^p(n)$. This validates our approach to consider (x, σ^p) instead of (ds, f) as control variables. Further, by the previous Part IV, these controls satisfy $n + x > 0$, i.e., $A \geq 0$ does not bind, on $(\underline{n}, \overline{n})$.

D Proof of Proposition 6 and Corollary 3

We start by proving Proposition 6. We prove in Parts I and II that a stationary density exists — which requires proving that $\underline{n}$ is never reached (i.e., is inaccessible). We also show in Part II that the stationary density decreases in n over $(\underline{n}, \tilde{n})$. We recall that Proposition 6 is proven under (16) not being met, that is, $\mu - (1-\xi)(r+\kappa) - \eta\sigma > 0$. We can rewrite this condition to $\xi\mu - (1-\xi)\lambda > \eta\sigma$ — which implies $\sigma^p(\underline{n}) > 0$ and $\tilde{n} \in (\underline{n}, \overline{n})$.

Corollary 3. Corollary 3 follows from the inaccessibility of the lower bound $\underline{n}$ (under (16) not being met), and its closed-form solution from (C.7) (Lemma 2). The comparative statics follow by direct calculation. Clearly, $\underline{n} < 0$. When (16) holds, so that $\underline{n} = -\left(\frac{1}{r+\kappa}\right)^{\frac{1}{1-\xi}}$, then $\underline{n}$ increases in κ and r and does not depend on σ or η. When (16) does not hold, we have: $\underline{n} = -\frac{1-\xi}{\mu-\eta\sigma}\left(\frac{\xi}{r+\kappa-\mu+\eta\sigma}\right)^{\frac{\xi}{1-\xi}}$. Thus: $\frac{\partial \underline{n}}{\partial \sigma} = \frac{\eta}{(\mu-\eta\sigma)^2}\left(\frac{\xi}{r+\kappa-\mu+\eta\sigma}\right)^{\frac{\xi}{1-\xi}} \frac{\mu-\eta\sigma\, -\, (1-\xi)(r+\kappa)}{r+\kappa-\mu+\eta\sigma} > 0$, and $\frac{\partial \underline{n}}{\partial r} = \frac{\xi}{\mu-\eta\sigma}\left(\frac{\xi}{r+\kappa-\mu+\eta\sigma}\right)^{\frac{\xi}{1-\xi}} \frac{1}{r+\kappa-\mu+\eta\sigma} > 0$.

D.1 Part I — Derivation of Feller Condition and KFE

We show that a stationary density exists and is non-degenerate, which boils down to showing that the lower boundary is not attainable. To this end, we conjecture that the lower boundary is indeed not attainable, and verify this claim.

Given our conjecture, a stationary density exists. In the interior of the state space for $n \in (\underline{n}, \overline{n})$ when $\sigma_n(n)$ is twice differentiable, the stationary density $g(n)$ satisfies the Kolmogorov forward (Fokker-Planck) equation:

$$0 = -\frac{\partial}{\partial n}\big[\mu_n(n)g(n)\big] + \frac{1}{2}\frac{\partial^2}{\partial n^2}\big[\sigma_n(n)^2 g(n)\big]. \tag{D.1}$$

Define $\hat{G}(n) := -\mu_n(n)g(n) + \frac{1}{2}\frac{\partial}{\partial n}\big[\sigma_n(n)^2 g(n)\big]$. Due to $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0$ and the conjecture that $\underline{n}$ is not attained (so there is zero probability mass in that state), we have $\hat{G}(\underline{n}) = 0$.

Next, we can integrate (D.1) from $\underline{n}$ to n to obtain $0 = \hat{G}(n) - \hat{G}(\underline{n}) = \hat{G}(n)$. This yields:

$$\mu_n(n)g(n) = \frac{1}{2}\frac{\partial}{\partial n}\left[\sigma_n(n)^2 g(n)\right]. \tag{D.2}$$

The ODE (D.2) satisfies the normalization condition $\int_{\underline{n}}^{\overline{n}} g(n)dn = 1$.

Define the scaled stationary density $\hat{g}(n) = \sigma_n(n)^2 g(n)$, so that

$$\hat{g}'(n) = 2\mu_n(n)g(n) = 2\hat{g}(n)\left(\frac{\mu_n(n)}{\sigma_n(n)^2}\right) \quad \text{and} \quad \frac{d\ln\hat{g}(n)}{dn} = \frac{\hat{g}'(n)}{\hat{g}(n)} = 2\left(\frac{\mu_n(n)}{\sigma_n(n)^2}\right).$$

The boundary $\underline{n}$ is absorbing (if it were reached), since $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = 0$, according to Lemma 2.

A non-degenerate stationary density, with the absorbing boundary at $\underline{n}$, exists if the boundary condition $\hat{g}(\underline{n}) = 0$ can be satisfied together with $\hat{g}(\hat{n}) > 0$ for $\hat{n} > \underline{n}$; in this case, the boundary $\underline{n}$ is never reached or inaccessible. For this to happen, we need that $\ln\hat{g}(n') = \ln\hat{g}(\hat{n}) - 2\int_{n'}^{\hat{n}} \frac{\mu_n(n)}{\sigma_n(n)^2}dn$ tends to $-\infty$, as $n' \downarrow \underline{n}$ for some $\hat{n} \in (\underline{n}, \overline{n})$; see Brunnermeier and Sannikov (2014) for an analogous argument in a similar context. A sufficient condition for $\underline{n}$ to be inaccessible and the stationary density to exist is:

$$\lim_{n' \downarrow \underline{n}} \int_{n'}^{\hat{n}} \frac{\mu_n(n)}{\sigma_n(n)^2}dn = +\infty. \tag{D.3}$$

Thus, to indeed show and verify that a stationary density exists, we need to prove (D.3).

In the following two parts, we show that (D.3) is met, which then implies that $\underline{n}$ is never reached and a stationary distribution of states exists.

D.2 Part II — Proof of (D.3)

Consider $n \in (\underline{n}, \hat{n})$ for some $\hat{n} > \underline{n}$. Without loss of generality, pick $\hat{n} < \tilde{n}$, i.e., $n < \tilde{n}$, so that $\sigma^p > 0$. Thus, by our previous results from Appendix C.4, we have $\sigma^p(n) = \frac{(n+x)\sigma}{x} - \frac{\eta}{\gamma(n)x}$ (see (C.18)), and $x = \underline{x}$ (see (C.8)). This implies $\sigma_n(n) = (n+x)\sigma - x\sigma^p = \frac{\eta}{\gamma(n)}$.

D.2.1 Auxiliary Result

To begin with, we rewrite the drift of n for $n \in (\underline{n}, \tilde{n})$, i.e., $\mu_n(n)$, as follows:

$$\begin{aligned}
\mu_n(n) = \mu n - \lambda\underline{x} + \underline{x}^\xi - \eta\underline{x}\sigma^p(n) &= (\mu - \eta\sigma)(n - \underline{n}) + \mu\underline{n} - \lambda\underline{x} + \underline{x}^\xi - \eta\sigma(\underline{x} + \underline{n}) + \frac{\eta^2}{\gamma(n)} \\
&= (\mu - \eta\sigma)(n - \underline{n}) + \mu_n(\underline{n}) + \frac{\eta^2}{\gamma(n)} \geq (\mu - \eta\sigma)(n - \underline{n}),
\end{aligned} \tag{D.4}$$

where $\mu_n(\underline{n}) = 0$. As $(\mu - \eta\sigma) > 0$, to prove (D.3) it suffices to show that

$$\int_{n'}^{\hat{n}} \frac{n - \underline{n}}{\sigma_n(n)^2}dn \propto \int_{n'}^{\hat{n}} \left[(n - \underline{n})\gamma(n)^2\right]dn = \int_{n'}^{\hat{n}} \left[\frac{n - \underline{n}}{(1/\gamma(n))^2}\right]dn \tag{D.5}$$

tends to ∞, as $n' \to \underline{n}$. That is, for $\hat{n} \in (\underline{n}, \tilde{n})$, we show $\lim_{n' \to \underline{n}} \int_{n'}^{\hat{n}} \left[\frac{n - \underline{n}}{(1/\gamma(n))^2}\right]dn = +\infty$.

Next, we show that there exists constant $\mathcal{K} > 0$ such that

$$\frac{1}{\gamma(n)} < \mathcal{K}(n - \underline{n}) \tag{D.6}$$

for n close to $\underline{n}$. Note that (D.6) implies $\gamma(n) > \frac{1}{\mathcal{K}(n-\underline{n})}$ for all $n \in (\underline{n}, \hat{n})$ when $\hat{n}$ is sufficiently close to $\underline{n}$. Given this, we obtain for $\hat{n}$ sufficiently close to $\underline{n}$ and $n' \in (\underline{n}, \hat{n})$:

$$\mathcal{B}(n') := \int_{n'}^{\hat{n}} \left[\frac{n - \underline{n}}{(1/\gamma(n))^2}\right] dn \geq \int_{n'}^{\hat{n}} \frac{1}{\mathcal{K}^2(n - \underline{n})} = \frac{1}{\mathcal{K}^2}\Big[\ln(\hat{n} - \underline{n})] - \ln(n' - \underline{n})\Big].$$

Note that $\lim_{n'\downarrow\underline{n}} \Big[\ln(\hat{n} - \underline{n})] - \ln(n' - \underline{n})\Big] = +\infty$.

Thus, when (D.6) holds, then $\lim_{n'\to\underline{n}} \mathcal{B}(n') = +\infty$, which implies (D.3). Thus, once we have proven (D.6) — which we do in the next part — the proof is complete, i.e., the stationary density exists and the lower boundary is not attained.

D.2.2 Proof of (D.6)

First, we conduct a Taylor expansion of $1/\gamma(n)$ around n':

$$\frac{1}{\gamma(n)} = \frac{1}{\gamma(n')} - \frac{\gamma'(n')}{\gamma(n')^2} \cdot (n - n') + O((n - n')^2). \tag{D.7}$$

We then take the limit $n' \downarrow \underline{n}$, where $\lim_{n'\downarrow\underline{n}} \frac{1}{\gamma(n')} = 0$, due to $\sigma_n(\underline{n}) = \frac{\eta}{\gamma(\underline{n})} = 0$. Thus:

$$\frac{1}{\gamma(n)} = -\lim_{n'\downarrow\underline{n}} \left(\frac{\gamma'(n')}{\gamma(n')^2}\right) \cdot (n - \underline{n}) + O((n - \underline{n})^2). \tag{D.8}$$

Thus, in order to establish (D.6), we need to show $\frac{\gamma'(n)}{\gamma(n)^2}$ remains bounded in a right-neighbourhood of $\underline{n}$. Also note that $\frac{\gamma'(n)}{\gamma(n)} < 0$, since $\gamma'(n) < 0$.

Calculate

$$\gamma'(n) = \frac{-v'''(n)v'(n) + (v''(n))^2}{(v'(n))^2} = \gamma(n)^2 - \frac{v'''(n)}{v'(n)}. \tag{D.9}$$

Notice from (C.21) — which follows from differentiating both sides of the HJB equation (C.16) with respect to n (assuming differentiability) — that

$$\begin{aligned} v'''(n) &= \frac{2}{\sigma_n(n)^2}\Big((\rho - \mu)v'(n) - v''(n)[\mu_n(n) + \sigma\sigma_n(n)]\Big) \\ &= \frac{2\gamma(n)^2}{\eta^2}\left\{(\rho - \mu)v'(n) - v''(n)\left[\left((\mu - \eta\sigma)(n - \underline{n}) + \frac{\eta^2}{\gamma(n)}\right) + \frac{\sigma\eta}{\gamma(n)}\right]\right\}, \end{aligned}$$

where we used $\mu_n(n) = (\mu - \eta\sigma)(n - \underline{n}) + \frac{\eta^2}{\gamma(n)}$, which follows from (D.4). Hence,

$$\frac{v'''(n)}{v'(n)} = \frac{2\gamma(n)^2}{\eta^2}\Big[\rho - \mu + \eta\sigma + \eta^2 + \gamma(n)(\mu - \eta\sigma)(n - \underline{n})\Big]. \tag{D.10}$$

Using (D.9), we obtain

$$-\frac{\gamma'(n)}{\gamma(n)^2} = -1 + \frac{v'''(n)}{v'(n)(\gamma(n))^2} = \frac{2}{\eta^2}\Big[\rho - \mu + \eta\sigma + \gamma(n)\left[(\mu - \eta\sigma)(n - \underline{n})\right]\Big] + 1.$$

Without loss of generality, we can consider that $\gamma(n) \leq \frac{1}{\hat{K}(n-\underline{n})}$ for $\hat{K} > 0$; otherwise, there exists K' such that $\frac{1}{\gamma(n)} < K'(n - \underline{n})$ and the proof is complete.

Under this assumption:

$$-\frac{\gamma'(n)}{\gamma(n)^2} \leq \frac{2}{\eta^2}\left[\rho - \mu + \eta\sigma + \frac{\mu - \eta\sigma}{\hat{K}}\right] + 1,$$

that is, $-\frac{\gamma'(n)}{\gamma(n)^2}$ (or its negative) remains bounded in a right-neighbourhood of $\underline{n}$.

Thus, the Taylor expansion (D.8) then implies that there exists constant $\mathcal{K} > 0$ such that

$$0 \leq \frac{1}{\gamma(n)} < \mathcal{K}(n - \underline{n}) + O((n - \underline{n})^2).$$

Thus, for n sufficiently close to $\underline{n}$, we have $\frac{1}{\gamma(n)} < \mathcal{K}(n - \underline{n})$, i.e., (D.6) holds and, by means of the previous findings, we obtain (D.3).

D.3 Part III: Stationary Density is Decreasing on $(\underline{n}, \tilde{n})$

To show that the stationary density is decreasing on $(\underline{n}, \tilde{n})$, we rewrite (D.2) as follows:

$$g(n)\big[\mu_n(n) - \sigma_n(n)\sigma_n'(n)\big] = \frac{\sigma_n(n)^2 g'(n)}{2}.$$

Thus, $\mu_n(n) - \sigma_n(n)\sigma_n'(n)$ determines the sign of $g'(n)$. We have to show that $\mu_n(n) - \sigma_n(n)\sigma_n'(n) < 0$ for $n \in (\underline{n}, \tilde{n})$. Note that on $(\underline{n}, \tilde{n})$, we have $\sigma_n(n) = \frac{\eta}{\gamma(n)}$ so that $\sigma_n'(n) = -\frac{\eta\gamma'(n)}{\gamma(n)^2}$ and $\sigma_n'(n)\sigma_n(n) = -\frac{\eta^2\gamma'(n)}{\gamma(n)^3}$. Moreover, using (D.4) and (D.9), we obtain for all $n \in (\underline{n}, \tilde{n})$:

$$\mu_n(n) = (\mu - \eta\sigma)(n - \underline{n}) + \frac{\eta^2}{\gamma(n)}; \quad \text{and} \quad \sigma_n(n)\sigma_n'(n) = -\frac{\eta^2}{\gamma(n)} + \frac{\eta^2 v'''(n)}{v'(n)\gamma(n)^3}.$$

Using (D.10), we can calculate: $\sigma_n(n)\sigma_n'(n) = \frac{\left(2(\rho-\mu+\eta\sigma)+\eta^2\right)}{\gamma(n)} + 2(\mu - \eta\sigma)(n - \underline{n})$. Thus,

$$\mu_n(n) - \sigma_n'(n)\sigma_n(n) = -(\mu - \eta\sigma)(n - \underline{n}) - \frac{2(\rho - \mu + \eta\sigma)}{\gamma(n)} < 0.$$

Since (16) does not hold (as we assumed for Proposition 6), we have $\mu > \eta\sigma$. In addition, $\rho > \mu$. Thus, $g'(n) < 0$ for $n \in (\underline{n}, \tilde{n})$.

E Extensions, Model Variants, and Other Results

E.1 Capital Requirement and Narrow Banking: Proposition 10

E.1.1 Capital Requirement

The model variant with capital requirement imposes that $n_t \geq \underline{n}^C$ — without loss of generality, we assume $\underline{n}^C > \underline{n}$ (a capital requirement $\underline{n}^C < \underline{n}$ would yield our baseline and the capital requirement is irrelevant). The issuer chooses a strategy such that the capital requirement is met at all times. Since our model does not have jump shocks, this is feasible, as we argue below. One could micro-found that the issuer always respects the capital requirement, for instance, by assuming that, upon violation, the regulator liquidates the issuer (leading to zero payoff for the issuer). Since imposing the capital requirement limits the strategy space, it is immediate that the capital requirement reduces the issuer's ex-ante payoff.

Solution. The solution is as in the baseline, in that the HJB equation (13) or, equivalently, (C.16) applies. The only change relative to the baseline lies in the boundary conditions. With capital requirement $n_t \geq \underline{n}^C$, the boundary conditions become the standard smooth pasting and super contact conditions for the upper boundary — that is, $v'(\overline{n}) - 1 = v''(\overline{n}) = 0$. At the lower boundary, $\underline{n}^C$, the following boundary condition applies:

$$\lim_{n \downarrow \underline{n}^C} \gamma(n) = +\infty, \tag{E.1}$$

where, as we recall, $\gamma(n) = \frac{-v''(n)}{v'(n)}$ is the issuer's effective risk-aversion.

As we show, the boundary condition (E.1) ensures that n_t never falls below $\underline{n}^C$ and violates the capital requirement. Intuitively, it reflects that the issuer becomes prohibitively risk-averse versus violating the capital requirement, for reasons argued above. A similar type of boundary condition applies in Ai and Li (2015) or Bolton, Wang, and Yang (2019).

The optimal controls are characterized in Appendix C, specifically, subsection C.3. We now prove that the boundary $\underline{n}^C$ is never reached in the interesting cases that the capital requirement stipulates over-collateralization (i.e., $\underline{n}^C \geq 0$) or that (16) does not hold — i.e., the baseline features instability and debasement.

Proof that $\underline{n}^C$ is never reached. We show that $\underline{n}^C > \underline{n}$ is never reached and $n_t > \underline{n}^C$ for all times t when at least one of the two conditions holds: (1) condition (16) does *not* hold or (2) $\underline{n}^C$ is close to zero. In doing so, we assume $\sigma > 0$ and that the capital requirement exceeds the lower boundary from (C.7).

Conjecture $\sigma^p(n) > 0$ in a neighbourhood of $\underline{n}^C$. It follows from (C.18) and (E.1) that $\sigma^p(\underline{n}^C) := \lim_{n \downarrow \underline{n}^C} \sigma^p(n) = \sigma + \frac{\sigma \underline{n}^C}{\underline{x}}$, where $\underline{x}$ is from (C.8).

As $\underline{n}^C > \underline{n}$, it follows that $\sigma^p(\underline{n}^C) > 0$, verifying above conjecture. Also, note that the volatility of dn vanishes as n approaches $\underline{n}^C$, in that $\lim_{n \downarrow \underline{n}^C} \sigma_n(n) = \lim_{n \downarrow \underline{n}^C} \frac{\eta}{\gamma(n)} = 0$.

The claim follows from showing $\mu_n(\underline{n}^C) = \lim_{n \downarrow \underline{n}^C} \mu_n(n) > 0$. Analogously to (D.4), we calculate for $\underline{n}^C > \underline{n}$:

$$\mu_n(\underline{n}^C) = \mu \underline{n}^C - \lambda \underline{x} + \underline{x}^\xi - \eta \underline{x} \sigma^p(\underline{n}^C) = (\mu - \eta\sigma)\underline{n}^C + \underline{x}^\xi - (\lambda + \eta\sigma)\underline{x}.$$

Clearly, $\underline{x}^\xi - (\lambda + \eta\sigma)\underline{x} > 0$. We have $\mu_n(\underline{n}^C) > 0$ in the following cases.

First, when $\underline{n}^C = 0$, it is immediate that $\mu_n(\underline{n}^C) > 0$. Second, when (16) does not hold, then $\mu > \eta\sigma$, and we can, analogously to (D.4), rewrite $\mu_n(\underline{n}^C) = (\mu - \eta\sigma)(\underline{n}^C - \underline{n}) > 0$.

E.1.2 Narrow banking (Proof of Proposition 10)

Our model nests the special case of narrow banking upon setting $\underline{n}^C = 0$ — that is, the issuer has positive net worth — and $\sigma = 0$ — that is, reserve assets are risk-free. Then, clearly, $\sigma^p(n) = 0$ for all $n \geq 0$ and $p(n) = 1$.

Solution. The HJB equation (13) then reduces to $\rho v(n) = \max_{x \geq 0} v'(n)\big(\mu n - \lambda x + x^\xi\big)$ for all $n \geq 0$ where $v'(n) \geq 1$. The optimization with respect to x yields $x(n) = x^* = \left(\frac{\xi}{\lambda}\right)^{\frac{1}{1-\xi}}$. Without risk, there is no need to hold any buffer, so that $\overline{n} = 0$, which we formally show below. We now prove that $\overline{n} = \underline{n}^C = 0$. Suppose to the contrary $\overline{n} > 0$. Then, $x(\overline{n}) = x^*$ and $v'(\overline{n}) = 1$ and

$$v(\overline{n}) = \overline{n} + \frac{-\lambda x(\overline{n}) + x(\overline{n})^\xi - (\rho - \mu)\overline{n}}{\rho}.$$

Note that $x(\overline{n}) = x^*$. Due to $\rho > \mu$, it then follows that there exists $\varepsilon > 0$ such that $\overline{n} - \varepsilon > \underline{n}^C$, $x(\overline{n} - \varepsilon) = x^*$, and

$$v(\overline{n} - \varepsilon) + \varepsilon = \varepsilon + \frac{v'(\overline{n} - \varepsilon)}{\rho}\big(\mu(\overline{n} - \varepsilon) - \lambda x^* + (x^*)^\xi\big) \geq \frac{(\rho - \mu)\varepsilon}{\rho} + v(\overline{n}),$$

where we used the HJB equation at $\overline{n} - \varepsilon$ to substitute for $v(\overline{n} - \varepsilon)$ and that $v'(\overline{n} - \varepsilon) \geq 1$. Therefore, $v(\overline{n} - \varepsilon) + \varepsilon > v(\overline{n})$. Thus, the issuer can achieve strictly higher payout by paying out $\varepsilon > 0$, contradicting the hypothesis that $\overline{n}$ is the payout boundary.

Thus, whenever $n > 0$, the issuer immediately consumes n dollars and continues with zero at net worth. At $n = 0$, the issuer's value function under narrow banking then reads:

$$v^{Narrow} = \frac{1}{\rho}\left\{\left(\frac{\xi}{\lambda}\right)^{\frac{\xi}{1-\xi}} - \lambda\left(\frac{\xi}{\lambda}\right)^{\frac{1}{1-\xi}}\right\} = \frac{(1-\xi)}{\rho}\left(\frac{\xi}{\lambda}\right)^{\frac{\xi}{1-\xi}}.$$

For $n \geq 0$, the value function then becomes $v^{Narrow} + n$. The volatility of net worth is zero and the drift equals $\mu_n(0) = \rho v^{Narrow} > 0$, i.e., under narrow banking, the issuer continuous consumes according to $dy = \mu_n(0)dt$. Note that $\mu_n(0) = (x^*)^\xi - \lambda x^*$.

Finally, scaled user welfare under narrow banking becomes (see next Section for details) $w^{Narrow} = \frac{(1-\xi)}{\xi\hat{r}}(x^*)^\xi = \frac{(1-\xi)}{\xi\hat{r}}\left(\frac{\xi}{\lambda}\right)^{\frac{\xi}{1-\xi}}$, where $\hat{r}$ is the discount rate for calculating user welfare, which can be (but need) set r.[38]

E.2 Calculating Welfare

Given the optimal controls $(x(n), \sigma^p(n))$, we can calculate user welfare. User welfare (like issuer valuation) scales with K, in that $W_t = Kw(n)$ for $n_t = n$. Scaled user welfare reads:

$$w(n) = \mathbb{E}_t\left[\int_t^\infty e^{-\hat{r}(s-t)}\Big(u(x_s) - x_s f_s + (\mu_s^p - r)x_s\Big)ds \Big| n_t = n\right] \tag{E.2}$$

[38] We apply a discount rate equal to 5%, representing the risk-free rate, in our numerical analysis.

where we discount at rate $\hat{r}$ — which is a flexible choice. It can be equal to r, but does not necessarily have to be. The rate $\hat{r}$ reflects a "social planner's" time preference, as opposed to necessarily the risk-free rate — our numerical analysis uses $\hat{r} = 5\%$. Note that x_s, f_s, and $u(x_s)$ — which depends on σ_s^p — are functions of n_s.

Thus, user welfare is expected discounted value of (i) convenience utility $u(x_s)$ and (ii) capital gains (excess returns) $(\mu_s^p - r)x_s$ from stablecoins, net of (iii) fees levied, $x_s f_s$.

Next, using (5), we get $x_t^{\xi-1} = r + f_t + \eta|\sigma_t^p| - \mu_t^p$. Inserting $u(x_t) = \frac{x_t^\xi}{\xi} - x_t\eta|\sigma_t^p|$, we calculate for the flow utility term in round brackets:

$$u(x_t) - x_t f_t + (\mu_t^p - r)x_t = \frac{x_t^\xi}{\xi} - \eta x_t|\sigma_t^p| - x_t x_t^{\xi-1} = \frac{(1-\xi)x_t^\xi}{\xi}. \tag{E.3}$$

Having solved for $x(n), \sigma^p(n)$, we obtain $\mu_n(n)$ and $\sigma_n(n)$. Then, the integral expression (E.2) together with (E.3) implies that welfare $w(n)$ satisfies the ODE:

$$\hat{r}w(n) = \frac{(1-\xi)x(n)^\xi}{\xi} + w'(n)\mu_n(n) + \frac{w''(n)\sigma_n(n)^2}{2}. \tag{E.4}$$

This ODE is solved subject to $w'(\overline{n}) = 0$, since n reflects at the upper boundary $\overline{n}$. Moreover, at the lower boundary $\underline{n}$, the drift and volatility of n vanish so that $w(\underline{n}) = \frac{(1-\xi)\underline{x}^\xi}{\xi\hat{r}}$, where $\underline{x}$ is from (C.8). (This boundary condition holds in the limit $n \downarrow \underline{n}$).

With capital requirement, i.e., $n_t \geq \underline{n}^C > \underline{n}$, the ODE remains unchanged, but the boundary condition at the lower boundary changes to: $\lim_{n\downarrow\underline{n}^C} \hat{r}w(n) = \frac{(1-\xi)\underline{x}^\xi}{\xi} + \lim_{n\downarrow\underline{n}^C} w'(n)\mu_n(n)$.

E.3 Equity Issuance (Section 4.3)

We sketch the solution with equity issuance. Since raising equity entails only a fixed cost F (though a variable cost component could easily be introduced) and new equity investors are risk-neutral and competitive (i.e., they require shares worth one dollar for each dollar contributed), it is optimal for the issuer to raise enough equity to restore net worth to the target level. Indeed, when the issuer raises equity and incurs the fixed cost, each additional dollar raised requires giving up one dollar of ownership. However, doing so increases the issuer's valuation by $v'(n) \geq 1$ dollars, with equality only when net worth reaches the target level, $n = \overline{n}$. As is standard, the issuer raises equity when n reaches a lower threshold $\underline{n}^E$.

The argument to determine this lower boundary follows Bolton, Wang, and Yang (2025). We consider the scaled fixed cost of equity issuance as $f := \frac{F}{K}$. At the equity issuance boundary $\underline{n}^E$, the issuer's continuation payoff becomes: $v(\overline{n}) - (\overline{n} - \underline{n}^E) - f$. To understand this condition, note that the issuer continues with valuation $v(\overline{n})$ post-issuance, yet must raise $(\overline{n} - \underline{n}^E) + F$ dollars of equity to bring net worth to $\overline{n}$ and to cover the fixed costs.

The issuer optimally delays as much as possible to avoid incurring the fixed cost, i.e., the issuer raises equity if and only if n reaches $\underline{n}^E$. Thus, at the equity issuance boundary, the issuer will be indifferent between issuing equity and continuing, and liquidating, in that: $v(\underline{n}^E) = v(\overline{n}) - f - (\overline{n} - \underline{n}^E) = 0$. Using the HJB equation (13) and $v'(\overline{n}) - 1 = v''(\overline{n}) = 0$, we get $v(\overline{n}) - \overline{n} = \frac{(x^*)^\xi - \lambda x^* - (\rho-\mu)\overline{n}}{\rho}$, for $x(\overline{n}) = x^*$ from (C.20). Thus, we can solve for the equity issuance boundary $\underline{n}^E$ through:

$$\underline{n}^E = -\left(\frac{(x^*)^\xi - \lambda x^* - (\rho - \mu)\overline{n}}{\rho} - f\right).$$

We note now that the issuer can either proceed with raising equity at the lower boundary — in which case the lower boundary coincides with $\underline{n}^E$ — or can adopt the baseline strategy — in which case there is no equity issuance and the lower boundary equals $\underline{n}$.

Solution. The issuer optimally chooses the option that gives itself more "financial leeway." In particular, the lower boundary in the state space then becomes $\underline{n}^* = \min\{\underline{n}, \underline{n}^E\}$ where $\underline{n}$ is from (C.7). The analysis goes through as before with this modified lower boundary. Specifically, the value function satisfies the HJB equation (13). This HJB equation is then solved subject to the boundary conditions: $v'(\overline{n}) - 1 = v''(\overline{n}) = v(\underline{n}^*) = 0$. The controls (x, σ^p) are determined according to the HJB equation analogous to the baseline from Appendix C.

Price with Equity Issuance. Since equity issuance is lumpy and induces a discrete jump in net worth, implementing the stablecoin price requires some care. We now specify the price function $p(n)$ satisfying equation (18), given the controls (x, σ^p).

As in the baseline case, we focus on a price function that satisfies $p(\overline{n}) = 1$—that is, the peg is maintained at the target net worth level. However, when $\underline{n}^* = \underline{n}^E$ and the solution involves equity issuance, we may have $\lim_{n \downarrow \underline{n}^E} \sigma^p(n) > 0$ and $p(\underline{n}^E) = \lim_{n \downarrow \underline{n}^E} p(n) < 1$. In this case, as n approaches $\underline{n}^E$, a discontinuous jump in price arises. Absent further assumptions, this discontinuity creates an arbitrage opportunity as $n \to \underline{n}^E$. To rule out such arbitrage, we assume that users must surrender a portion of their stablecoin holdings—that is, their holdings S, or equivalently their scaled holdings s, are reduced. This is analogous to debt forgiveness in bankruptcy, where creditors relinquish part of their claims to enable the firm to continue operating. Loosely speaking, in our context, users similarly forgo part of their claims to facilitate the recapitalization of the issuer and allow it to continue operations, thereby avoiding liquidation and the complete loss of stablecoin value. To eliminate arbitrage, we assume that, per unit of stablecoin held, a user retains only a fraction $p(\underline{n}^E)$. This ensures that the dollar value of one unit of stablecoin remains unchanged across the equity issuance event, effectively accounting for the "loss." This adjustment prevents arbitrage and supports the pricing rule in equation (18). We omit further implementation details for brevity.

E.4 Dynamic Risk Taking (Section 6.2)

We now provide the heuristic solution to the model variant with dynamic risk taking from Section 6.2. We note that Section 6.2 introduces the parameter μ_0 — to make clear that the return drift μ_0 from this Section need not coincide with the calibrated value for μ. In what follows, to ensure analytical comparability to the baseline, we simply write $\mu = \mu_0$. A similar logic applies to σ_0, where we set in this Appendix $\sigma_0 = \sigma$.

Dynamics of dn. We can rewrite the dynamics of n_t given in

$$dn_t = \underbrace{[(\mu_0 + \omega_t \alpha)(n_t + x_t) + x_t \zeta_t - x_t r - x_t \kappa]}_{\mu_n(n_t)} dt + \underbrace{[(\sigma_0 + \omega_t \sigma_\alpha)(n_t + x_t) - x_t \sigma^p_t]}_{\sigma_n(n_t)} dZ_t - dy_t,$$

according to:

$$dn_t = \left[\mu n_t + x_t^\xi - \lambda x_t - \eta x_t |\sigma^p_t| + \omega_t \alpha (n_t + x_t)\right] dt + [(\sigma + \omega_t \sigma_\alpha)(n_t + x_t) - x_t \sigma^p_t] dZ_t - dy_t. \quad \text{(E.5)}$$

where $\omega_t \in [w_L, w_H]$ for constants $w_L \leq w_H$. Above law of motion defines drift and volatility of dn, i.e., $\mu_n(n)$ and $\sigma_n(n)$. Note that we write μ for μ_0, with slight abuse of notation.

Lower Boundary $\underline{n}$. The same logic as in Lemma 2 applies, albeit with an additional control ω_t. Analogously to Lemma 2 from the baseline, the lower boundary in this model variant is defined as:

$$\underline{n} := \inf \left\{ n \in \mathbb{R} : \max_{x\in[0,\overline{x}],\omega\in[w_L,w_H],\sigma^p\geq 0} \mu_n(n_t) \geq 0 \quad \text{s.t.} \quad \sigma_n(n_t) = 0, \quad n + x \geq 0 \right\}. \tag{E.6}$$

Consider $\sigma^p(\underline{n}) > 0$. Then, $\sigma_n(\underline{n}) = 0$ implies $x\sigma^p = (\sigma + \omega\sigma_\alpha)(x + n)$ and $\mu_n(\underline{n}) = \mu\underline{n} - \lambda\underline{x} + \underline{x}^\xi + \omega(\underline{x} + \underline{n})\alpha - \eta(\sigma + \omega\sigma_\alpha)(\underline{x} + \underline{n})$ for $x(\underline{n}) =: \underline{x}$.

Optimizing $\mu_n(\underline{n})$ over ω, we get $\omega(\underline{n}) = \underline{\omega} := w_L + (w_H - w_L)\mathbb{I}\{\alpha \geq \eta\sigma_\alpha\}$. Next, we can optimize over $\underline{x}$ to obtain: $\underline{x} = \left(\frac{\xi}{\lambda+\eta\sigma-\underline{\omega}(\alpha-\eta\sigma_\alpha)}\right)^{\frac{1}{1-\xi}}$. Finally, we can solve for $\underline{n}$ by solving $\mu_n(\underline{n})$ under $\omega = \underline{\omega}$ and $x = \underline{x}$, yielding $\underline{n} = -\left(\frac{\underline{x}^\xi - \hat{\lambda}\underline{x} - \eta\sigma\underline{x}}{\hat{\mu}-\eta\sigma}\right)$. Here, $\hat{\mu} := \mu + \underline{\omega}(\alpha - \eta\sigma_\alpha)$ and $\hat{\lambda} := \lambda - \underline{\omega}(\alpha - \eta\sigma_\alpha)$. As in the baseline, we also consider the second case where $\underline{n} + x(\underline{n}) = 0$.

Overall, we obtain:

$$\underline{n} = \begin{cases} -\left(\frac{\underline{x}^\xi - \hat{\lambda}\underline{x} - \eta\sigma\underline{x}}{\hat{\mu}-\eta\sigma}\right) & \text{if} \quad \hat{\mu} - (1-\xi)(r+\kappa) > \eta\sigma \\ -\left(\frac{1}{r+\kappa}\right)^{\frac{1}{1-\xi}} & \text{if} \quad \hat{\mu} - (1-\xi)(r+\kappa) \leq \eta\sigma \end{cases} \tag{E.7}$$

Expression (E.7) illustrates that the lower boundary is determined analogously to the baseline and risk-taking results in a transformation of the parameters μ and λ (toward $\hat{\mu}$ and $\hat{\lambda} = r + \kappa - \hat{\mu}$, respectively). In addition, the level of $\underline{x}$ is determined similarly to the baseline.

HJB Equation. The issuer's value function satisfies $V_t = Kv(n_t)$. Given the law of motion of net worth in (E.5), the scaled value function $v(n)$ solves on the endogenous state space $(\underline{n}, \overline{n})$ with $dy = 0$:

$$\begin{aligned} \rho v(n) = \max_{\sigma^p, x, \omega\in[w_L,w_H]} \bigg\{ & v'(n)\Big(\mu n - \lambda x + x^\xi - \eta x|\sigma^p| + \omega(x+n)\alpha\Big) \\ & + \frac{v''(n)}{2}\Big((\sigma + \omega\sigma_\alpha)(n+x) - x\sigma^p\Big)^2 \bigg\}. \end{aligned} \tag{E.8}$$

The usual smooth pasting and super contact conditions apply at the upper boundary $\overline{n}$, that is, $v'(\overline{n}) - 1 = v''(\overline{n}) = 0$. One can show that the value function is strictly concave. In addition, we have $v(\underline{n}) = 0$.

We now determine the optimal controls in the interior of the state space. Also recall the definition of the issuer's effective risk aversion, i.e., $\gamma(n) = \frac{-v''(n)}{v'(n)}$.

Optimization with respect to $\sigma^p(n)$. As in the baseline, we can solve for optimal $\sigma^p(n) = \max\left\{0, \frac{(\sigma+\omega\sigma_\alpha)(x+n)}{x} - \frac{\eta}{\gamma(n)x}\right\}$. One can show that when $\sigma^p(n) > 0$, we have $\omega = \underline{\omega}$ and $x = \underline{x}$.

Optimization with respect to ω. The optimization with respect to ω yields:

$$\omega = \begin{cases} w_L & \text{if} \quad \alpha < \sigma_\alpha \sigma_n(n)\gamma(n) \\ \hat{\omega} \in [w_L, w_H] & \text{if} \quad \alpha = \sigma_\alpha \sigma_n(n)\gamma(n) \\ w_H & \text{if} \quad \alpha > \sigma_\alpha \sigma_n(n)\gamma(n). \end{cases}$$

The choice of ω or, equivalently, $\Omega = \omega(x+n)$ simplifies as follows.

Note that when $\alpha \geq \eta\sigma_\alpha$, setting $\omega = w_H$ is optimal in any state n, in that $\underline{\omega} = w_H$. To see this, observe that the issuer could always raise $\omega(x+n)$ by one marginal unit and raise $x\sigma^p$ by σ_α units. This change leaves, by construction, the volatility $\sigma_n(n)$ unchanged, and raises the drift $\mu_n(n)$ by $\alpha - \eta\sigma_\alpha$. It is therefore optimal when $\alpha \geq \eta\sigma_\alpha$.

Otherwise, when $\alpha < \eta\sigma_\alpha$, then $\omega = w_L$ whenever $\sigma^p(n) > 0$, i.e., for $n < \tilde{n}$. To see this, suppose to the contrary that $\omega(n) > w_L$ and $\sigma^p(n) > 0$. Then, the issuer could lower $\omega(n)(x+n)$ by one (marginal) unit and $x\sigma^p(N)$ by σ_α units, whilst leaving volatility $\sigma_n(n)$ unchanged. However, this change raises the drift by $\eta\sigma_\alpha - \alpha > 0$, and thus is profitable.

Optimization with respect to x. When $\sigma^p(n) > 0$, then $x = \underline{x}$. When $\sigma^p = 0$, the first order condition with respect to x becomes $-\lambda + \xi x^{\xi-1} + \omega\alpha = (\sigma + \omega\sigma_\alpha)\Big((\sigma+\omega\sigma_\alpha)(x+n)\Big)\gamma(n)$.

Price. As in the baseline, the price satisfies $p(n) = \exp\left(-\int_n^{\tilde{n}} \frac{\sigma^p(n)}{\sigma_n(n)} dn\right)$.

E.5 Demand Shocks: Section 7 and Proof of Proposition E.1

We extend the model by introducing shocks to the stablecoin demand scaler, $K_t = K$, as outlined in Section 7 which presents the price process dp_t, the evolution of K_t, as well as users' optimal token holdings $X_t = Kx_t$ with x_t from (24). Also, recall that $X_t = S_t p_t$ by market clearing. Everything else remains as in the baseline.

E.5.1 Dynamics of n_t and X_t

Next, we invert $X_t = K_t \left(\frac{1}{r + f_t - \mu_t^p + \eta|\Sigma_t^p|}\right)^{\frac{1}{1-\xi}}$ to obtain $\mu_t^p = \Big(-K_t^{1-\xi} X_t^{\xi-1} + f_t + r + \eta|\Sigma_t^p|\Big)$. That is,

$$dp_t = (r+f_t)p_t dt - p_t\Big(X_t^{\xi-1}K_t^{1-\xi} - \eta|\Sigma_t^p|\Big)dt + p_t\sigma_t^p dZ_t + p_t\sigma_{K,t}^p dZ_t^K. \tag{E.9}$$

Next, multiply both sides of (E.9) by S_t and use $X_t = S_t p_t$ (market clearing) to obtain

$$S_t dp_t = (r+f_t)X_t dt - \Big(K_t^{1-\xi}X_t^{\xi} - \eta X_t|\Sigma_t^p|\Big)dt + X_t\sigma_t^p dZ_t + X_t\sigma_{K,t}^p dZ_t^K.$$

Ito's product rule implies $d(S_t p_t) = dS_t p_t + p_t dS_t + dS_t dp_t$. Therefore, we calculate

$$dX_t = (r+f_t)X_t dt - \Big(K_t^{1-\xi}X_t^{\xi} - \eta X_t|\Sigma_t^p|\Big)dt + X_t\sigma_t^p dZ_t + X_t\sigma_{K,t}^p dZ_t^K + dS_t(p_t + dp_t). \tag{E.10}$$

Using the dynamics of reserve assets, that is, $dA_t = A_t(\mu dt + \sigma dZ_t) + f_t X_t dt + dS_t(p_t + dp_t) - \kappa X_t dt - dY_t$, we calculate the dynamics of net worth via $dN_t = dA_t - dX_t$:

$$dN_t = \Big[\mu N_t - \lambda X_t + X_t^{\xi}K_t^{1-\xi} - \eta X_t|\Sigma_t^p|\Big]dt + \big[\sigma(N_t + X_t) - X_t\sigma_t^p\big]dZ_t - X_t\sigma_{K,t}^p dZ_t^K - dY_t. \tag{E.11}$$

Note that one could also write down the law of motion of net worth more directly as accounting identity. Also, recall that $\lambda = r + \kappa - \mu > 0$.

We omit time subscripts in what follows. Next, we calculate the dynamics of scaled liquid net worth n by combining aforementioned law of motion of N with the law of motion of K in (20). To this end, calculate using Ito's Lemma: $dn = d\left(\frac{N}{K}\right) = \frac{dN}{K} - \frac{n}{K}dK - \frac{1}{K^2}d[N,K] + \frac{2N}{2K^3}d[K,K]$. As such, using (E.11) and (20), as well as $dy = \frac{dY}{K}$, we calculate

$$dn = \Big[(\mu - \mu_K)n - x\lambda + x^{\xi} - \eta x|\Sigma^p|\Big]dt + \sigma_K\big[n\sigma_K + x\sigma_K^p - \phi\big(\sigma(x+n) - x\sigma^p\big)\big]dt - [n\sigma_K + x\sigma_K^p]dZ^K + \big[\sigma(n+x) - x\sigma^p\big]dZ - dy. \quad \text{(E.12)}$$

We now denote the volatility on dZ by $\sigma_n(n) = \sigma(x+n) - x\sigma^p$ and the volatility on dZ^K by $\sigma_n^K(n) = -(n\sigma_K + x\sigma_K^p)$. We denote the drift of dn by $\mu_n(n)$.

E.5.2 Issuer Problem and Solution

The issuer chooses its issuance $(dS_t)_{t\geq 0}$, the fee policy $(f_t)_{t\geq 0}$, and $(dY_t)_{t\geq 0}$ maximizes

$$V_0 = \mathbb{E}\left[\int_0^{\infty} e^{-\rho t} dY_t\right],$$

subject to (E.11), (20), (E.9), $X_t = S_t p_t$, (24), $A_t \geq 0$, and $dY_t \geq 0$.[39]

As in the baseline, we consider scaled quantities and the payoff-relevant state (besides scaling) is $n = n_t$. Since (dS_t, f_t) affects dn_t from (E.12) only via $(x_t, \sigma_t^p, \sigma_{K,t}^p)$, we can solve the issuer's problem by considering a relaxed problem, where the issuer chooses $(x_t, \sigma_t^p, \sigma_{K,t}^p, dy_t)_{t\geq 0}$. One can then show that this choice can be implemented, analogously to the arguments from Appendix C.5. See Appendix E.5.6.

In what follows, we restrict attention to strategies that implement $\sigma_t^p, \sigma_{K,t}^p \geq 0$. This assumption resembles a standard monotonicity assumption in security design and optimal contracting: The stablecoin price must load positively on the shocks, in that it can never decrease following positive demand or reserve shocks.[40]

The following Proposition summarizes the key findings of this model variant.

Proposition E.1 (Solution with Demand Shocks)**.** *With demand shocks, the issuer's value function solves* (E.14) *subject to $v(\underline{n}) = v'(\overline{n}) - 1 = v''(\overline{n}) = 0$. The following holds:*

1. *For $n > 0$, we have $\sigma_K^p = 0$ and $\sigma^p = \sigma^p(n) = \max\left\{0, \frac{\sigma(x+n) - \phi n\sigma_K}{x} - \frac{\eta}{x\gamma(n)}\right\}$. Otherwise, for $n < 0$, price volatility terms satisfy:*

$$\sigma^p = \max\left\{0, \frac{\sigma(n+x)}{x}\left(1 - \frac{\eta}{\gamma(n)\pi(n)}\right)\right\}; \quad \sigma_K^p = \max\left\{0, -\frac{n\sigma_K}{x}\left(1 - \frac{\eta}{\gamma(n)\pi(n)}\right)\right\},$$

[39] The regularity condition $x_t \leq \overline{x}$ applies, but $\overline{x}$ is assumed to be sufficiently large so that this constraint never binds. As in the baseline, we omit the constraint $A_t \geq 0$ in the HJB optimization — one could verify ex-post it does not bind. While we do not provide a formal verification, we do so in our numerical analysis.

[40] In principle, negative values of σ_K^p could be optimal when $n > 0$ and η is sufficiently small. Although such cases cannot be ruled out analytically, they appear to be corner or edge cases. Accordingly, we do not consider them further. Moreover, a negative σ_K^p would imply that prices decline in response to a positive demand shock—a counterfactual scenario we do not highlight.

where $\gamma(n) = \frac{-v''(n)}{v'(n)}$ *and* $\pi(n)$ *is defined in* (E.17)*.*

2. *The stablecoin price satisfies* $p(n) = \exp\left(-\int_n^{\tilde{n}} \frac{\sigma^p(\nu)}{\sigma_n(\nu)} d\nu\right)$*, where* $\tilde{n} = \inf\{n' \in (\underline{n}, \overline{n}) : \sigma^p(n) = \sigma^p_K(n) = 0 \text{ for all } n \geq n'\}$*.*

E.5.3 HJB Equation and Solution Details

We now solve the dynamic optimization and derive the HJB equation. Doing so, we conjecture and verify that $V(N, K) = v(n)K$. To solve for the issuer's value function, we use shorthand notation $V = V(N, K)$ and denote partial derivatives by subscripts. We conjecture that in the interior of the endogenous state space, there is no consumption/payout, i.e., $dy = 0$. Then, by dynamic programming, the value function V solves in the interior of the state space the following HJB equation:

$$\begin{aligned}\rho V = \max_{X, \sigma^p, \sigma^p_K} \Bigg\{ & V_K K \mu_K + \frac{V_{KK}(\sigma_K K)^2}{2} + V_N \left[\mu N - X\lambda + K^{1-\xi} X^\xi - \eta X |\Sigma^p|\right] \\ & + \frac{V_{NN}}{2}\left(\left[\sigma(N+X) - X\sigma^p\right]^2 + (X\sigma^p_K)^2 - 2\phi\sigma^p_K X\left[\sigma(N+X) - X\sigma^p\right]\right) \\ & + V_{NK}\sigma_K K\Big(-X\sigma^p_K + \phi\left[\sigma(N+X) - X\sigma^p\right]\Big)\Bigg\}.\end{aligned} \tag{E.13}$$

Next, calculate for $V = V(N, K) = Kv(n)$ the derivatives $V_K = v(n) - v'(n)n$, $V_{NN} = v''(n)/K$, $V_{KK} = v''(n)n^2/K$, $V_{KN} = -v''(n)n/K$. Inserting these relations back into (E.13) and simplifying yields in the interior of the state space:

$$\begin{aligned}(\rho - \mu_K)v(n) = \max_{x, \sigma^p \geq 0, \sigma^p_K \geq 0} \Bigg\{ & v'(n)\left[(\mu - \mu_K)n - \lambda x + x^\xi - \eta x |\Sigma^p|\right] \\ & + v''(n)\left[\frac{(n\sigma_K + x\sigma^p_K)^2}{2} + \frac{(\sigma(n+x) - x\sigma^p)^2}{2}\right] - v''(n)\phi(n\sigma_K + x\sigma^p_K)(\sigma(n+x) - x\sigma^p)\Bigg\}.\end{aligned} \tag{E.14}$$

The state space is characterized by an interval $(\underline{n}, \overline{n})$ with endogenous upper and lower boundaries. The upper boundary $\overline{n}$ is a payout boundary and (scaled) consumption/dividends dD causes n to reflect at $\overline{n}$. The upper boundary satisfies the standard smooth pasting and super contact conditions, i.e., $v'(\overline{n}) - 1 = v''(\overline{n}) = 0$. The lower boundary is determined analogously to the baseline as follows:

$$\underline{n} := \inf\Big\{n \in \mathbb{R} : \max_{x \in [0, \overline{x}], \sigma^p \geq 0} \mu_n(n) \geq 0 \quad \text{s.t.} \quad \sigma_n(n) = \sigma^K_n(n) = 0, \quad n + x \geq 0\Big\},$$

where $\mu_n(n)$, $\sigma_n(n)$, and $\sigma^K_n(n)$ are (implicitly) defined in (E.12). We have $\mu_n(\underline{n}) = \sigma_n(\underline{n}) = \sigma^K_n(\underline{n}) = 0$, and thus $v(\underline{n}) = 0$ at the lower boundary, analogously to the baseline. Next, we turn to analyze the optimal controls and characterize them as functions of n only.

E.5.4 Controls

We can restate the optimization in (E.14) by dividing both sides by $v'(n)$ and using $\gamma(n) = \frac{-v''(n)}{v'(n)}$. The optimization in (E.14) becomes then equivalent to:

$$\max_{x,\sigma^p\geq 0,\sigma^p_K\geq 0}\Bigg\{(\mu-\mu_K)n - x\lambda + x^\xi - x\eta|\Sigma^p|$$
$$-\gamma(n)\left[\frac{(n\sigma_K+x\sigma^p_K)^2}{2}+\frac{(\sigma(n+x)-x\sigma^p)^2}{2}-\phi(n\sigma_K+x\sigma^p_K)(\sigma(n+x)-x\sigma^p)\right]\Bigg\}$$

where Σ^p is a function of σ^p and σ^p_K (given in (22)). Suppose $\sigma^p > 0$ and $\sigma^p_K > 0$. Then, they solve the first-order conditions:

$$x\sigma^p = \sigma(n+x) - \frac{\eta}{\gamma(n)}\left(\frac{\sigma^p+\phi\sigma^p_K}{\Sigma^p}\right) - \phi(n\sigma_K + x\sigma^p_K),$$
$$x\sigma^p_K = -n\sigma_K - \frac{\eta}{\gamma(n)}\left(\frac{\sigma^p_K+\phi\sigma^p}{\Sigma^p}\right) + \phi(\sigma(n+x) - x\sigma^p).$$

Thus, we can solve:[41]

$$\sigma^p = \frac{\Sigma^p\,\sigma(n+x)}{\Sigma^p\,x+\frac{\eta}{\gamma(n)}},\qquad \sigma^p_K = -\frac{\Sigma^p\,n\sigma_K}{\Sigma^p\,x+\frac{\eta}{\gamma(n)}}. \tag{E.15}$$

It follows that $\sigma^p_K = 0$ whenever $n \geq 0$. Under these circumstances, we get $\Sigma^p = \sigma^p$ and

$$\sigma^p(n) = \max\left\{0, \frac{\sigma(x+n)-\phi n\sigma_K}{x} - \frac{\eta}{\gamma(n)x}\right\} \tag{E.16}$$

Next, we define

$$\pi(n) := \sqrt{\sigma^2(n+x)^2 + n^2(\sigma_K)^2 - 2\phi\, n\, \sigma(n+x)\sigma_K}. \tag{E.17}$$

Inserting (22) in (E.15) and using our definition of $\pi(n)$, as well as after accounting for $\sigma^p, \sigma^p_K \geq 0$, we obtain for $n < 0$:

$$\sigma^p = \sigma^p(n) = \max\left\{0, \frac{\sigma(n+x)}{x}\left(1-\frac{\eta}{\gamma(n)\pi(n)}\right)\right\} \tag{E.18}$$
$$\sigma^p_K = \sigma^p_K(n) = \max\left\{0, \frac{-n\sigma_K}{x}\left(1-\frac{\eta}{\gamma(n)\pi(n)}\right)\right\}. \tag{E.19}$$

We note that when $n < 0$, then $\sigma^p(n) > 0 \iff \sigma^p_K(n) > 0$, i.e., both volatilities are positive if and only if $\gamma(n)\pi(n) > \eta$. Also note that $\sigma^p(n) = 0$ implies $\sigma^p_K(n) = 0$.

Accordingly, for $n < 0$, the standard deviation of price becomes

$$\Sigma^p = \max\left\{0, \frac{1}{x}\left(\pi(n) - \frac{\eta}{\gamma(n)}\right)\right\}, \tag{E.20}$$

while, for $n \geq 0$, we have $\Sigma^p = \sigma^p$. Finally, the first-order condition with respect to x

[41] Note that (E.15) suggests that a negative σ^p_K may be optimal in certain edge cases—particularly when $n > 0$ and η is small. However, in this region, the issuer is generally less risk-averse (compared to the $n < 0$ case), and it is typically optimal to set $\sigma^p_K = 0$, even absent the constraint $\sigma^p_K \geq 0$. Overall, instances where $\sigma^p_K < 0$ is optimal are rare and confined to corners of the parameter space. By contrast, choosing a negative σ^p is never optimal, so the constraint $\sigma^p \geq 0$ is effectively non-binding—that is, if $\sigma^p = 0$ is chosen, it would also be optimal without the constraint.

becomes $-\lambda + \xi x^{\xi-1} - \left[\sigma^2(n+x) - \phi n \sigma_K \sigma\right] \min\left\{\gamma(n), \frac{\eta}{\pi(n)}\right\} = 0$, where $\gamma(n) > \frac{\eta}{\pi(n)}$ if and only if $\sigma^p > 0$. As in the baseline, we again define:

$$\tilde{n} = \inf\{n' \in (\underline{n}, \overline{n}) : \sigma^p(n) = \sigma^p_K(n) = 0 \text{ for all } n \geq n'\}$$

In our analysis, we focus on parameters such that $\tilde{n} \in (\underline{n}, \overline{n})$, i.e., there exist values of $n \in (\underline{n}, \overline{n})$ such that $\sigma^p(n) > 0$ or $\sigma^p_K(n) > 0$.

E.5.5 Price

We determine the token price as a function of n, i.e., $p_t = p(n_t)$. When $\sigma^p(n), \sigma^p_K(n) > 0$, the price $p(n)$ solves the following ODEs:

$$\sigma^p p(n) = p'(n)\left[\sigma(x+n) - x\sigma^p\right] = p'(n)\left(\frac{\eta\sigma(x+n)}{\gamma(n)\pi(n)}\right) \tag{E.21}$$

$$\sigma^p_K p(n) = p'(n)\left[-n\sigma_K - x\sigma^p_K\right] = -p'(n)\left(\frac{\eta n \sigma_K}{\gamma(n)\pi(n)}\right). \tag{E.22}$$

Note that when $\sigma^p(n), \sigma^p_K(n) > 0$, then $\sigma^p_K = \left(\frac{-n\sigma_K}{\sigma(x+n)}\right)\sigma^p$, so these two equations in (E.21) are equivalent. Suppose $\sigma^p(n) > 0$, so that $\sigma^p(n) = \frac{\sigma(n+x)}{x}\left(1 - \frac{\eta}{\gamma(n)\pi(n)}\right)$, as well as $\sigma_n(n) = (n+x)\sigma - x\sigma^p(n) = \frac{\eta\sigma(n+x)}{\gamma(n)\pi(n)}$. We then can solve (E.21) subject to $p(\tilde{n}) = 1$ to obtain: $p(n) = \exp\left(-\int_n^{\tilde{n}} \frac{\sigma^p(\nu)}{\sigma_n(\nu)} d\nu\right)$. as desired. This expression also applies when $\sigma^p(n) > 0 = \sigma^p_K(n)$.

E.5.6 Implementing $(x, \sigma^p, \sigma^p_K)$ via (dS, f)

Having solved for $(x, \sigma^p, \sigma^p_K)$, as well as the price $p(n)$ as functions for n, we determine the optimal (scaled) supply process ds and fee policies f to implement the optimal controls $(x, \sigma^p, \sigma^p_K)$. The optimal controls pin down $\mu_n(n)$, $\sigma_n(n)$ and $\sigma^K_n(n)$ by means of the law of motion (E.11), that $dn = \mu_n(n)dt + \sigma_n(n)dZ + \sigma^K_n(n)dZ^K$.

First, note that, given $x(n), p(n)$, the supply process $s(n)$ is uniquely determined via $s(n) = \frac{x(n)}{p(n)}$. Similar to the arguments from Appendix C.5, one could further characterize the dynamics of ds which follow $ds(n) = \mu^s(n)dt + \sigma^s(n)dZ + \sigma^s_K(n)dZ^K$. One could determine the drift and volatility terms — omitted here, since not essential.

Next, calculate by Ito's Lemma:

$$\mu^p(n)p(n) = p'(n)\mu_n(n) + \frac{p''(n)}{2}\left[(\sigma^K_n(n))^2 + (\sigma_n(n))^2 + 2\phi\sigma^K_n(n)\sigma_n(n)\right].$$

From (24), we have $f(n) = x(n)^{\xi-1} - \eta\Sigma^p(n) - r + \mu^p(n)$ so fees are a function of n.

F Generalized Price Process and Issuance Strategy

In the baseline solution, the issuer's strategy S_t — or s_t/K — and the price process p_t are continuous diffusion processes. To solve for the optimal strategy, we focused on a continuous price process — which only loads on fundamental shocks dZ_t — and then determined the issuance. One might be concerned that the focus on continuous price and issuance processes, as well as the focus on fundamental uncertainty dZ_t, are restrictive. Here, we show that the issuer optimally implements continuous price and issuance, whereby price and issuance only load on dZ_t. This Appendix confirms the equilibrium price is continuous and follows (1).

We now allow for a generalized price process, which is akin to allowing for generalized issuance (as issuance controls price):

$$\frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^p dZ_t + d\ell_t.$$

Here, $d\ell_t$ is a general process, which can be deterministic and stochastic. However, the increments of ℓ_t and Z_t are (without loss of generality) orthogonal — that is, $d\ell_t \cdot dZ_t = 0$.

Importantly, if stochastic, $d\ell_t$ captures the stochastic price/issuance unrelated to dZ_t, in that $d\ell_t$ is orthogonal to dZ_t. It satisfies standard regularity conditions (i.e., it is almost surely finite and square integrable). Since the price process is controlled by issuance, allowing for general $d\ell_t$ is akin to allowing for generalized issuance. All other elements remain unchanged.

We solve for the dynamics of the state variable n under these generalized assumptions, and write down the HJB equation. Notably, we assume that price fluctuations related to $d\ell_t$ do not affect the users' utility—i.e., they are not "priced." We then show that, even under this favorable treatment of price adjustments $d\ell_t$, stipulating $d\ell_t = 0$ is optimal. Therefore, the price p_t and issuance strategy s_t are continuous diffusion processes in optimum.

User Optimization and Dynamics of x. Analogously to the baseline, (B.2) holds, i.e.,:

$$x_t = \left(\frac{dt}{rdt + f_t dt - \eta|\sigma_t^p|dt - \mathbb{E}_t^u[dp_t]/p_t} \right)^{\frac{1}{1-\xi}},$$

where $\mathbb{E}_t^u$ is the time-t expectation under user information set. Next, rewrite this relationship to obtain

$$\mathbb{E}_t^u\left[\frac{dp_t}{p_t}\right] = \mu_t^p dt + \mathbb{E}_t^u[d\ell_t] = \left(-x_t^{\xi-1} + f_t + r + \eta|\sigma_t^p|\right) dt. \tag{F.1}$$

We note that $d\ell_t$ might represent a lumpy price change that occurs with an atom of probability, so $\mathbb{E}_t^u[d\ell_t]$ is not of order dt. We also allow the fee process to be lumpy (to potentially offset $d\ell_t$), but, instead of introducing additional notation, we would capture this by $f_t \in \{-\infty, \infty\}$ where $f_t = +\infty$ captures a lump-sum fee and $f_t = -\infty$ captures a lump-sum rebate (transfer to users).

Next, calculate

$$dp_t = p_t\mu_t^p dt + p_t\sigma_t^p dZ_t + p_t d\ell_t = (r + f_t)p_t dt - p_t\left(x_t^{\xi-1} - \eta|\sigma_t^p|\right)dt + p_t\sigma_t^p dZ_t + p_t(d\ell_t - \mathbb{E}_t^u[d\ell_t]).$$

Then, multiply both sides of above by s_t and use $x_t = s_t p_t$ (market clearing) to obtain

$$s_t dp_t = (r + f_t)x_t dt - \left(x_t^{\xi} - \eta x_t|\sigma_t^p|\right)dt + x_t\sigma_t^p dZ_t + x_t(d\ell_t - \mathbb{E}_t^u[d\ell_t]).$$

Ito's product rule implies $d(s_t p_t) = ds_t p_t + p_t ds_t + ds_t dp_t$. Then, we calculate for $x_t = s_t p_t$:

$$dx_t = (r + f_t)x_t dt - \left(x_t^{\xi} - \eta x_t|\sigma_t^p|\right) dt + x_t\sigma_t^p dZ_t + x_t\left(d\ell_t - \mathbb{E}_t^u[d\ell_t]\right) + ds_t(p_t + dp_t). \tag{F.2}$$

Dynamics of n. The reserves follow $dA_t = \mu A_t dt - \kappa X_t dt + \sigma A_t dZ_t + f_t X_t dt + dS_t(p_t + dp_t) - dY_t$, We can calculate for $n_t = N_t/K = A_t/K - x_t$:

$$dn_t = \Big[\mu(n_t+x_t)-\lambda x_t+x_t^{\xi}-\eta x_t|\sigma_t^p|\Big]dt+\big[\sigma(n_t+x_t)-x_t\sigma_t^p\big]dZ_t-dy_t-x_t\big(d\ell_t-\mathbb{E}_t^u[d\ell_t]\big). \quad \text{(F.3)}$$

(The net worth could be also derived more directly, but we provide details here, since we are dealing with a different price process.) Crucially, the issuer's commitment to a strategy implies $\mathbb{E}_t^u = \mathbb{E}_t$, i.e., the issuer's and the user's expectation operators coincide at time t.

HJB Equation. We note $d\ell_t$ enters the issuer's payoff only via the state variable n_t.

When $d\ell_t$ is deterministic, i.e., $\mathbb{E}_t[d\ell_t] = d\ell_t$, then the issuer's commitment immediately implies $d\ell_t = \mathbb{E}_t^u[d\ell_t]$. In this case, $d\ell_t$ drops out and the law of motion of n is as in the baseline. It follows that $d\ell_t$ is payoff-irrelevant, since n_t is the only state variable and $d\ell_t$ enters the issuer's payoff only via n_t. Then, $d\ell_t$ is payoff-irrelevant and can be set to zero.

Since $d\ell_t$ affects the state variable only via its deviation from the mean $d\ell_t - \mathbb{E}_t^u[d\ell_t]$ and deterministic $d\ell_t$ is payoff irrelevant, we can without loss of generality focus on a stochastic process $d\ell_t$ that satisfies $\mathbb{E}_t[d\ell_t] = 0$ and $d\ell_t \cdot dZ_t = 0$. Thus, $d\ell_t$ captures price movements from randomization by the issuer. We will show that such randomization is sub-optimal, notably, even if we do not assume additional risk-aversion for the user regarding price fluctuations stemming from such randomization. We focus on $d\ell_t$ following a Levy process.

By the Levy-Ito decomposition, we can write the process $d\ell = d\ell_t$ as mean-zero process according to:

$$d\ell_t = \sigma_\ell dZ_t^\ell + \Delta_t^\ell\big(dN_t^\ell - \mathbb{E}_t[dN_t^\ell]\big), \quad \text{(F.4)}$$

where $dN_t^\ell \in \{0,1\}$ is a jump process with $\mathbb{E}_t[dN_t^\ell] = \Lambda_t^\ell dt$ — the intensity Λ_t^ℓ can potentially be infinite, capturing lumpy price adjustements with an atom of probability. In addition, dZ_t^ℓ is a Brownian motion orthogonal to fundamental shocks dZ_t, i.e., $dZ_t^\ell \cdot dZ_t = 0$. Without loss of generality, there is only one jump component.

The HJB equation then satisfies:

$$\begin{aligned} \rho v(n) = \max_{\sigma^p, x\geq 0} & \left\{ v'(n)\Big(\mu n - \lambda x + x^{\xi} - \eta x|\sigma^p|\Big) + \frac{v''(n)}{2}\Big[\sigma(x+n) - x\sigma^p\Big]^2 \right\} \quad \text{(F.5)} \\ & + \max_{\sigma_\ell}\left(\frac{v''(n)x^2\sigma_\ell^2}{2}\right) + \max_{\Delta^\ell,\Lambda^\ell\geq 0}\Lambda^\ell\Big[v(n - x\Delta^\ell) - v(n) + v'(n)x\Delta^\ell\Big]. \end{aligned}$$

with upper boundary $\overline{v}$, satisfying $v'(\overline{n}) - 1 = v''(\overline{n}) = 0$. In addition, there is some lower boundary $\underline{n}$ with $v(\underline{n}) = 0$.

Proving $d\ell_t = 0$. We conjecture that the value function is strictly concave, i.e., $v''(n) < 0$ for $n \in (\underline{n}, \overline{n})$. Given this conjecture, we will show that setting $d\ell_t = 0$ is optimal so that the above HJB equation collapses to (C.16) or, equivalently, (13). Thus, the value function coincides with the one of the baseline, which, as we have shown in Appendix C, is strictly concave, verifying our conjecture.

First, note that due to concavity, $v''(n) \leq 0$, we have $\sigma_\ell = 0$ for all $n \in (\underline{n}, \overline{n})$.

Suppose $\Lambda^\ell > 0$. The term $\Big[v(n - x\Delta^\ell) - v(n) + v'(n)x\Delta^\ell\Big]$ is zero for $\Delta^\ell = 0$, while it has derivative with respect to $x\Delta^\ell$ of $\mathcal{U}(x\Delta^\ell) = -v'(n - x\Delta^\ell) + v'(n)$. Due to concavity, it follows that $\max_{\Delta^\ell,\Lambda^\ell\geq 0}\Lambda^\ell\Big[v(n - x\Delta^\ell) - v(n) + v'(n)x\Delta^\ell\Big] = 0$. Overall, $d\ell_t = 0$.