Coherent States in Pipe Flows Project Description

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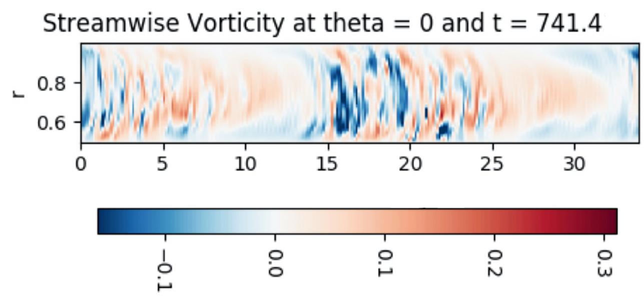
We are studying the transition from laminar to turbulent flow that happens in a long pipe as the speed of the fluid is increased. We simulate the Navier-Stokes equation [1] that describes the fluid using the open-source Python package Dedalus (see https://dedalus-project.readthedocs.io/en/latest/) running on large cluster machines. Dedalus uses the pseudo-spectral algorithm to solve partial differential equations. We carry out both fully nonlinear simulations, and reduced (quasilinear) simulations. The data generated from the simulations is 4-dimensional: It lives in three spatial dimensions plus time, and consists of a vector velocity field $\overrightarrow{v}(\vec{r}, t)$, and a scalar pressure field $P(\vec{r}, t)$.



The figure shows a snapshot of the stream-wise vorticity (the component of the curl of the velocity field in the direction of fluid flow). Note that the aspect ratio is distorted in the figure; the simulated pipe is long and thin. The turbulence shown is intermittent; at higher flow speeds the entire fluid becomes turbulent. For background on the transition to turbulence, there is a good review paper written by Dwight Barkley [2] to check out.

The project will start with a calculation of the two-point statistic $\langle v_i(\vec{r}_1, t)\ v_j(\vec{r}_2, t)\rangle$ from simulated data, where the angular brackets denote an average over both the azimuthal angle θ around the pipe, and time. Proper Orthogonal Decomposition (POD) (equivalent to Principle Component Analysis or PCA) will then be used to identify the important coherent structures in the flow. It may be interesting to compare with coherent structures identified by autoencoders. The resulting reduced dimensional description will then be used to build a statistical theory of the transition to turbulence, similar to what was done in Ref. 3 below.

[1] Feynman Lectures on Physics, Vol. II, Chapter 41, “The Flow of Wet Water.” https://www.feynmanlectures.caltech.edu/II_41.html

[2] Barkley, D. “Theoretical perspective on the route to turbulence in a pipe.” Journal of Fluid Mechanics, 803, 15–80 (2016). http://doi.org/10.1017/jfm.2016.465

[3] Allawala, A., Tobias, S. M., & Marston, J. B. “Dimensional reduction of direct statistical simulation.” Journal of Fluid Mechanics, 898, 533–18 (2020). http://doi.org/10.1017/jfm.2020.382