Name : Yusuf Emre Beskan
Number: 270201005

21/05/2022

CENG222 HOMEWORK 4 REPORT

$$f(x) = 2\Theta^2 \div x^3 \qquad X = \{0.3, 0.6, 0.8, 0.9\}$$

Part a)

MoM estimation:

$$\int_a^{\infty} x.f(x)\,dx = E(x) \qquad 2\Theta^2 \int_{\Theta}^{\infty} x^{-2}dx = 0 - (-2\Theta) = 2\Theta = E(x)$$

$$E(x) = totalOfX \div numberOfX = 0.65 \qquad so\ 2\Theta = 0.65 -> \Theta = 0.325$$

MLE estimation:

MLE

$$L(\Theta) = \sum_{i=1}^{n} \ln f(x_i) = \sum_{i=1}^{n} \ln \frac{2\Theta^2}{x^3} = \sum_{i=1}^{n} 2\ln 2\Theta - \sum_{i=1}^{n} \ln x_i^{-3} =$$

$$= 2\sum_{i=1}^{n} \ln 2 + 2\sum_{i=1}^{n} \ln\Theta + 3\sum_{i=1}^{n} \ln x_i = 2n\ln 2 + 2n\ln\Theta + 3\sum_{i=1}^{n} \ln x_i = L(\Theta)$$

$$L'(\Theta) = \frac{2n}{\Theta} = 0 \qquad \Theta = \infty$$

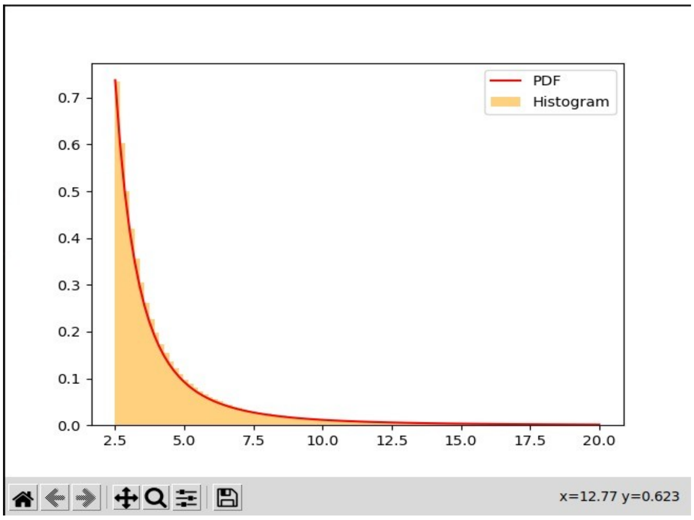
$$X = \{0.3, 0.6, 0.8, 0.9\}$$

$$\downarrow$$

$$\min(X) = 0.3 \rightarrow \Theta$$

Part b)

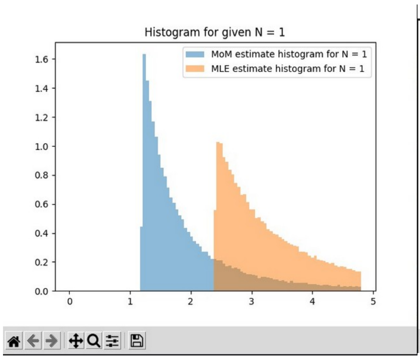
Population is created with Inverse Transformation Method and this is the pdf and population list's histogram:

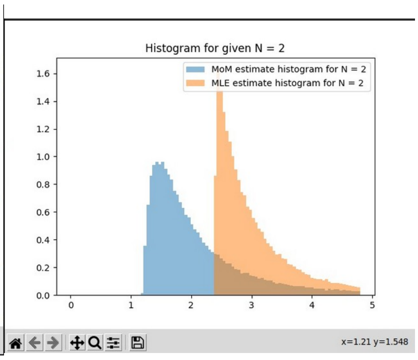


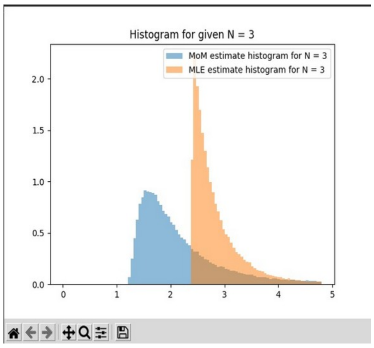
In Inverse Transformation Method the cdf is calculated by integral of pdf and inverse of cdf was taken. Random number u is created with random numbers between 0 and 1 and put in the equation. population numbers was obtained related to that equation outputs.

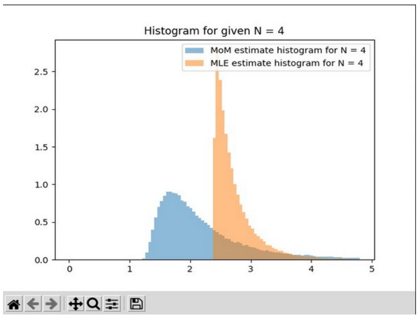
Part c)

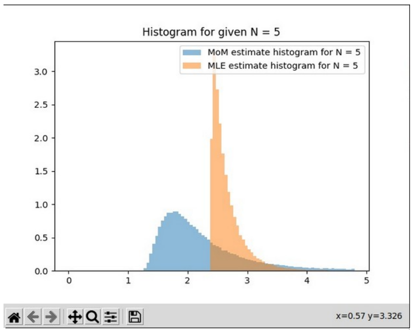
In histograms The MoM estimation and MLE estimation fall into a more constant range as the all sample's length increases. I would prefer MLE as MLE estimates take a more constant value than MoMs. MLE approaches approx 2.4 but MoM approaches approx range (2,3). These are all of my findings.

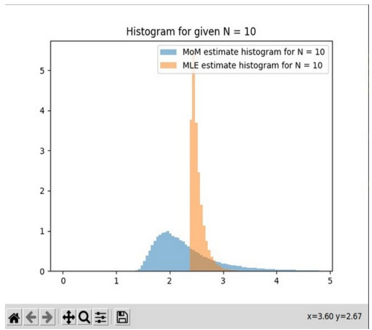


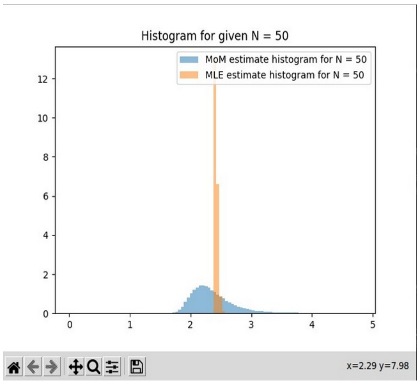


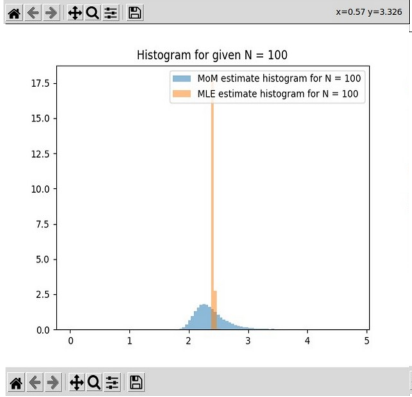


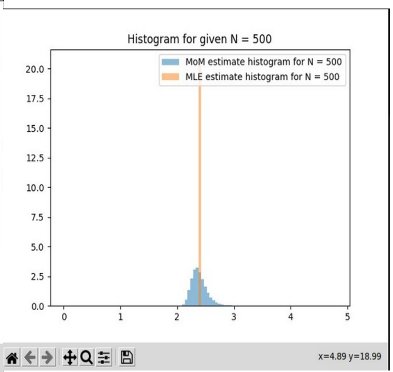


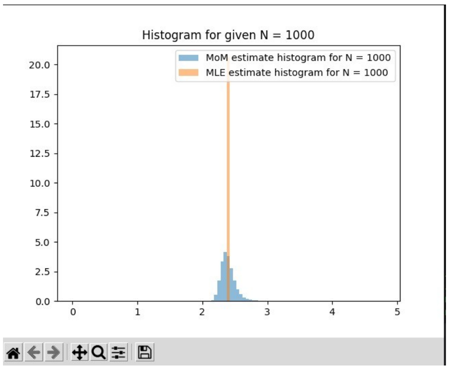













```
Theta of the given distribution X is 0.3 which was obtained from MLE function.
Theta of the given distribution X is 0.325 which was obtained from MoM function.

Result array of MoM is [0, 0.98, 0.41, 0.29] which was obtained from MoM estimate.
Result array of MLE is [6.67, 0.83, 0.35, 0.25] which was obtained from MLE estimate.

For N = 1:

MoM estimate mean:  2.4 MoM estimate std:  3.69
MLE estimate mean:  4.8 MLE estimate std:  7.37

For N = 2:

MoM estimate mean:  2.38 MoM estimate std:  2.25
MLE estimate mean:  3.2 MLE estimate std:  1.1

For N = 3:

MoM estimate mean:  2.4 MoM estimate std:  3.14
MLE estimate mean:  2.88 MLE estimate std:  0.58

For N = 4:

MoM estimate mean:  2.39 MoM estimate std:  1.85
MLE estimate mean:  2.74 MLE estimate std:  0.4

For N = 5:

MoM estimate mean:  2.39 MoM estimate std:  1.57
MLE estimate mean:  2.67 MLE estimate std:  0.3

For N = 10:

MoM estimate mean:  2.4 MoM estimate std:  1.18
MLE estimate mean:  2.53 MLE estimate std:  0.13
```

```
For N = 10:

MoM estimate mean:  2.4 MoM estimate std:  1.18
MLE estimate mean:  2.53 MLE estimate std:  0.13

For N = 50:

MoM estimate mean:  2.4 MoM estimate std:  0.73
MLE estimate mean:  2.42 MLE estimate std:  0.02

For N = 100:

MoM estimate mean:  2.4 MoM estimate std:  0.46
MLE estimate mean:  2.41 MLE estimate std:  0.01

For N = 500:

MoM estimate mean:  2.4 MoM estimate std:  0.2
MLE estimate mean:  2.4 MLE estimate std:  0.0

For N = 1000:

MoM estimate mean:  2.4 MoM estimate std:  0.15
MLE estimate mean:  2.4 MLE estimate std:  0.0
```