

### **Problem 1**

a)  $T(n) = 2T(n/2) + n^3$ , from master theorem case 3 will be occurred,

$f(n) = \Omega(n^{\log_b^a + e})$  for some  $e > 0$  and if  $af(n/b) \leq cf(n)$  for some  $c < 1$  then  $T(n) = \Theta(f(n))$

So that,  $n^3 = \Omega(n^{2 + \log_2 2})$  so,  $T(n) = \Theta(n^3)$

b)  $T(n) = 7T(n/2) + n^2$ , from master theorem case 1 will be occurred,

$f(n) = O(n^{\log_b^a - e})$  for some  $e > 0$  then  $T(n) = \Theta(n^{\log_b^a})$

So that,  $n^2 = \Omega(n^{\log_2 7 - e})$   $e$  is bigger than 0 so it is  $T(n) = \Theta(n^{\log_2 7})$

c)  $T(n) = 2T(n/4) + \sqrt{n}$ , from master theorem case 2 will be occurred,

$f(n) = \Theta(n^{\log_b^a})$  then  $T(n) = \Theta(n^{\log_b^a} \log n)$

So that,  $\sqrt{n} = \Theta(n^{\log_4 2})$  so,  $T(n) = \Theta(\sqrt{n} \log n)$

d)  $T(n) = T(n-1) + n$ ,

$T(n-1) = T(n-2) + n-1$

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$T(n-k+1) = T(n-k) + n-k+1$  sum the left hand side and right hand side end of the mathematical subtraction,

For  $k=n$ ,  $T(n) = T(n-k) + \sum_{k=0}^{n-1} n - k$

$T(n) = T(0) + (n(n+1))/2$ , in this situation  $T(0)$  is constant so that  $T(n) = O(n^2)$

### **Problem 2**

A)

i) In Naive recursion algorithm, We are trying to generate all subsequences of both given sequences so that we are getting same datas which are called as overlapping. It is the calculating same things everytime. This solution is exponential in term of time complexity.

There are two string which their lengths are  $m$  and  $n$

If  $m$  equals  $n$ , The recursion tree nodes have two branch and function will be like this,  $T(n)=2T(n-1)$ ,  $T(n)=2^2T(n-2)$  it will go like this and so that  $T(n)=2^kT(n-k)$  if  $n=k$  so that,  $T(n)=2^n$  if they are not equals if  $m>n$  it will be  $T(n)=2^m$  means  $O(2^n)$

ii) Overlapping substructure property is avoided by the Memoization. We are trying to store the recursion in matrix. It will prevent calculating the same data which we calculated. It provides utilizing the algorithm. We will have already in the table so that memorization reduces number of function call. Total how many calls were made depends number of elements in the table. Complexity is  $O(m \times n)$  which memorization matrix's area are  $m$  is the length of first string parameter,  $n$  is second string parameter.

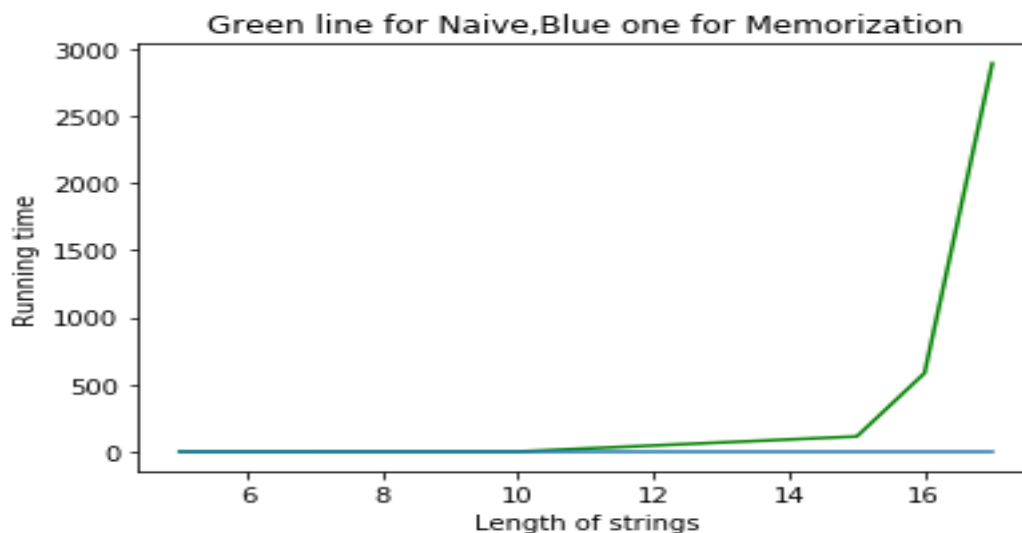
**B)**

**i)**

Algorithm	$m=n=5$	$m=n=10$	$m=n=15$	$m=n=16$	$m=n=17$
Naïve	0.0003001089	0.27445564270	112.452389742	584.84723985	2894.5823094
Memorization	0.00003140724512	0.0002232495561400	0.0004996040	0.0006012855	0.00072286

Operating system=Windows 8 ,Ram=8gb

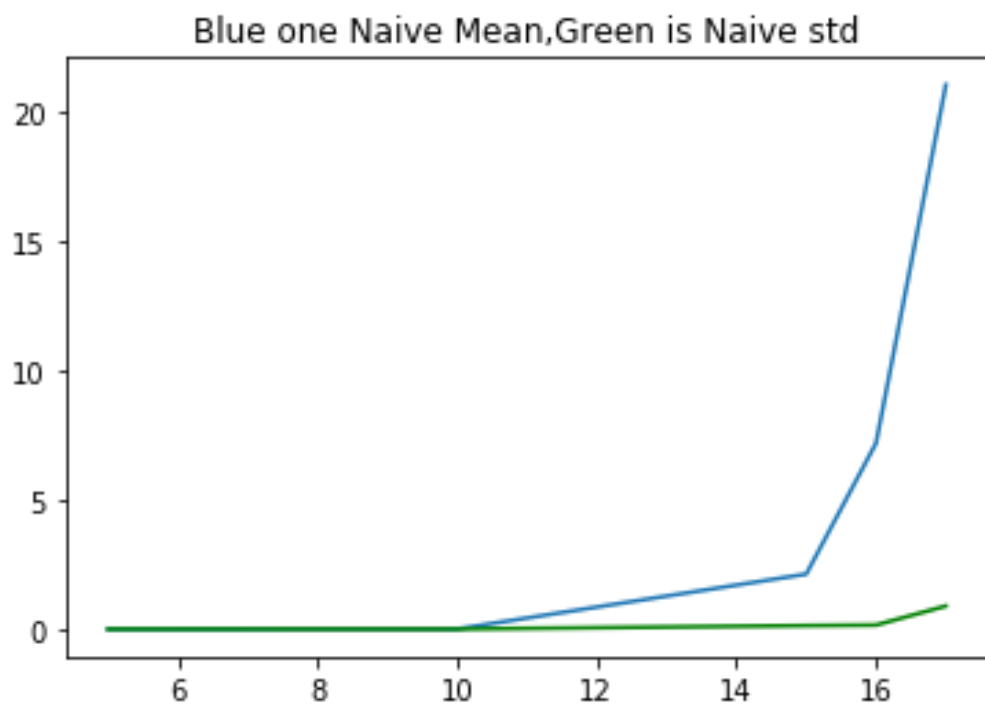
ii)

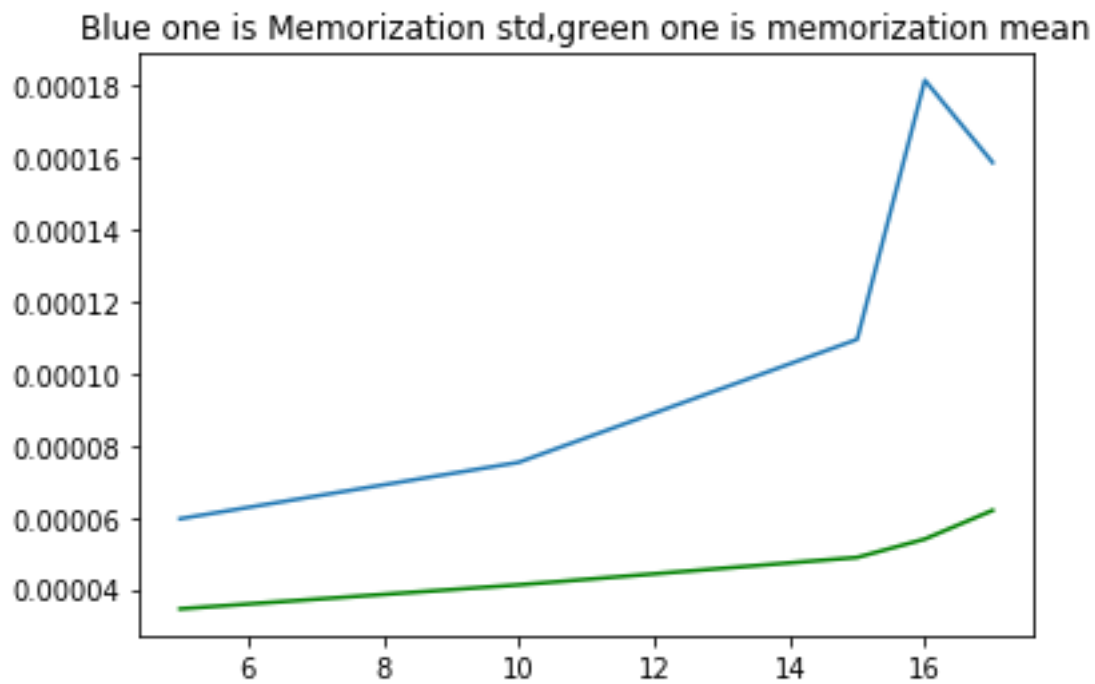


iii) Yes experimental results confirm the theoretical result which were one of them is Naive algorithm's best asymptotic worst case running is exponentially. As we see in the b-ii part green line which is exponentially growing. On the other hand, Memorization algorithm is  $O(m \times n)$  was increasing linearly. Experimental result is as same as theoretical result. Naive algorithm will spend more time to compile with a big difference.

c)

Algor ithm	m=n=5		m=n=10		m=n=15		m=n=16		m=n=17	
	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
Naive	0.0000	0.0000	0.004	0.0013	2.12493	0.135	7.185	0.0957	21.023	0.892
	913967	17894	50213	96438	558208	9825	98298	6823	85892	8583
	269	352	5	2			5		5	
Mem oizati on	0.0000	0.0000	0.000	0.0000	0.00004	0.000	0.000	0.0000	0.0000	0.000
	305874	49586	04124	45523	885679	05937	05395	71285	61917	06846
	2	3	53	845	250	593	720	214	593	385





iii) The string parameters which we generated from the random DNA sequence are random variables so that they are not the worst case because they are not different completely between each other. But in general when we look their graphs their shapes are almost same because algorithms complexity are not changed it is still same. Standard deviations shows us there is no big difference between DNA sequence especially in the Naive algorithm.