實驗設計作業

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Question 3.2

$$\sum_{i=1}^{g}\sum_{j=1}^{n_i}(\hat{\alpha}_i\gamma_{ij}) = 0$$

<proof>

$$\sum_{i=1}^{g}\sum_{j=1}^{n_i}(\bar{y}_{i.} - \bar{y}_{..})(y_{ij} - \bar{y}_{..}) = \sum_{i=1}^{g} n_i(\bar{y}_{i.} - \bar{y}_{..})\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{i.})$$

Where

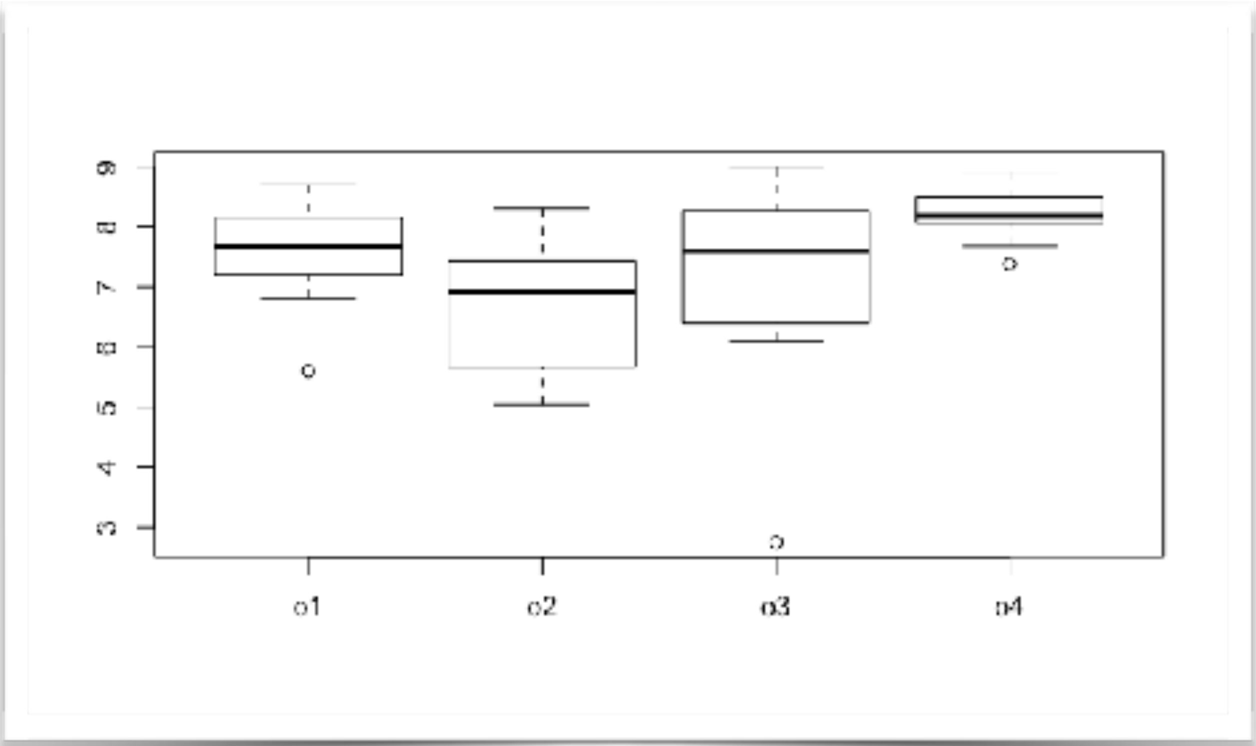
$$\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{i.}) = 0$$

$$\therefore \sum_{i=1}^{g} n_i(\bar{y}_{i.} - \bar{y}_{..})\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{i.}) = \sum_{i=1}^{g}\sum_{j=1}^{n_i}(\hat{\alpha}_i\gamma_{ij}) = 0$$

Problem 3.1

Cardiac pacemakers contain electrical connections that are platinum pins soldered onto a substrate. The question of interest is whether different operators produce solder joints with the same strength. Twelve substrates are randomly assigned to four operators. Each operator solders four pins on each substrate, and then these solder joints are assessed by measuring the shear strength of the pins. Data from T. Kerkow.

	Strength (lb)											
Operator	Substrate 1				Substrate 2				Substrate 3			
1	5.60	6.80	8.32	8.70	7.64	7.44	7.48	7.80	7.72	8.40	6.98	8.00
2	5.04	7.38	5.56	6.96	8.30	6.86	5.62	7.22	5.72	6.40	7.54	7.50
3	8.36	7.04	6.92	8.18	6.20	6.10	2.75	8.14	9.00	8.64	6.60	8.18
4	8.30	8.54	7.68	8.92	8.46	7.38	8.08	8.12	8.68	8.24	8.09	8.06



H_0 : Four operators have no difference between them

H_1 : $not\ H_0$

先看看四個操作員資料的盒形圖

接著對四組資料做ANOVA，得出以下：

其中p-value: 0.0102 < 0.05 拒絕 H_0，有足夠證據顯示四個操作員的力氣不同。

```
> summary(lmy)

Call:
lm(formula = y ~ A)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.4258 -0.5433  0.0771  0.7173  1.8242 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   7.5733     0.3153  24.016   <2e-16 ***
Ao2          -0.8983     0.4460  -2.014   0.0501 .  
Ao3          -0.3975     0.4460  -0.891   0.3776    
Ao4           0.6392     0.4460   1.433   0.1589    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.092 on 44 degrees of freedom
Multiple R-squared:  0.2244,	Adjusted R-squared:  0.1715 
F-statistic: 4.243 on 3 and 44 DF,  p-value: 0.0102
```

Problem 3.2

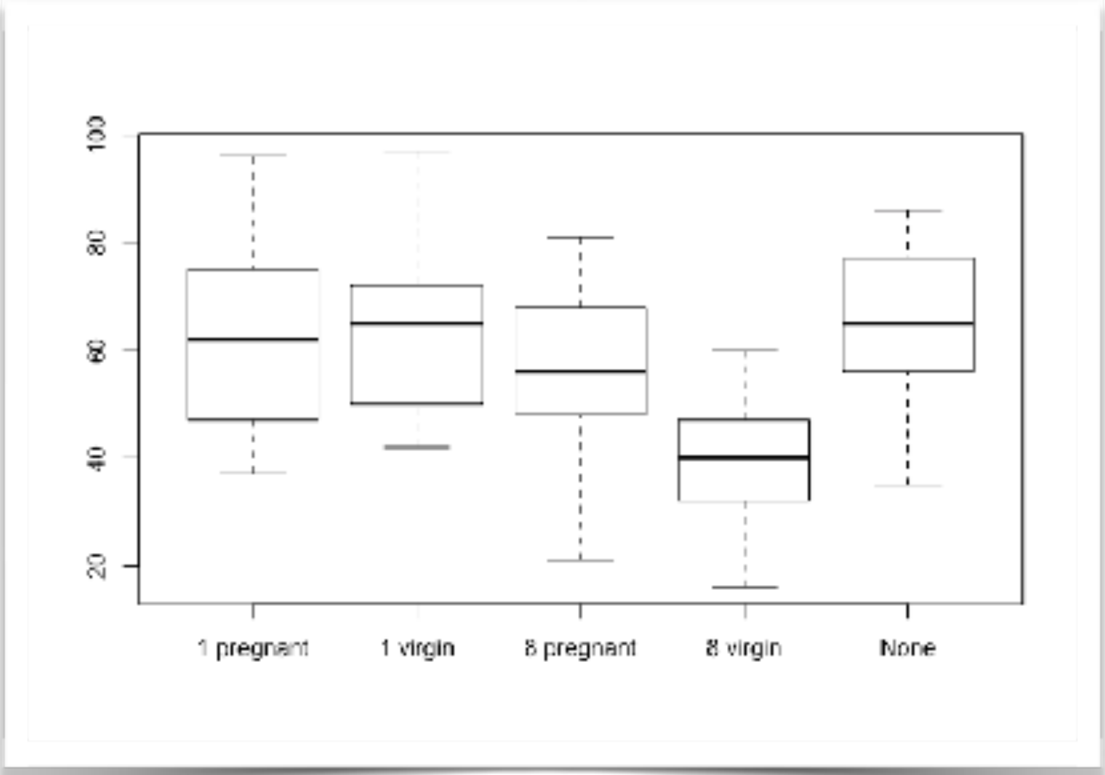
Scientists are interested in whether the energy costs involved in reproduction affect longevity. In this experiment, 125 male fruit flies were divided at random into five sets of 25. In one group, the males were kept by themselves. In two groups, the males were supplied with one or eight receptive virgin female fruit flies per day. In the final two groups, the males were supplied with one or eight unreceptive (pregnant) female fruit flies per day. Other than the number and type of companions, the males were treated identically. The longevity of the flies was observed. Data from Hanley and Shapiro (1994).

Companions	Longevity (days)												
None	35	37	49	46	63	39	46	56	63	65	56	65	70
	63	65	70	77	81	86	70	70	77	77	81	77	
1 pregnant	40	37	44	47	47	47	68	47	54	61	71	75	89
	58	59	62	79	96	58	62	70	72	75	96	75	
1 virgin	46	42	65	46	58	42	48	58	50	80	63	65	70
	70	72	97	46	56	70	70	72	76	90	76	92	
8 pregnant	21	40	44	54	36	40	56	60	48	53	60	60	65
	68	60	81	81	48	48	56	68	75	81	48	68	
8 virgin	16	19	19	32	33	33	30	42	42	33	26	30	40
	54	34	34	47	47	42	47	54	54	56	60	44	

H_0 : There are no different between five experiments.

$H_1 : not\ H_0.$

首先先看五個實驗下蒼蠅壽命的盒型圖，可以看到放入8隻virgin flies的蒼蠅壽命普遍較低。



接著對五組資料做ANOVA，得出以下：

其中p-value < 0.05 拒絕 H_0，有足夠證據顯示五個實驗對蒼蠅壽命的影響不同。

```
> summary(lm(g~B))

Call:
lm(formula = g ~ B)

Residuals:
   Min     1Q Median     3Q    Max 
-35.76  -8.76   0.20  11.20  32.44 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   63.560      2.962  21.461  < 2e-16 ***
B1 virgin      1.240      4.188   0.296    0.768    
B8 pregnant   -6.800      4.188  -1.624    0.107    
B8 virgin    -24.840      4.188  -5.931 2.98e-08 ***
BNone         -0.200      4.188  -0.048    0.962    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.81 on 120 degrees of freedom
Multiple R-squared:  0.3121,	Adjusted R-squared:  0.2892 
F-statistic: 13.61 on 4 and 120 DF,  p-value: 3.516e-09
```

Exercise 4.3

Refer to the data in Problem 3.1. Workers 1 and 2 were experienced, whereas workers 3 and 4 were novices. Find a contrast to compare the experienced and novice workers and test the null hypothesis that experienced and novice works produce the same average shear strength.

H_0 : **There are no different between experienced workers and novice workers.**

$H_1 : not\ H_0$

首先，假設$w = (1,1,-1,-1)$**，且資料之**$MSE = 1.1933$**，檢定統計量為**

$$T = \frac{\bar{y}_{i.} - 0}{\sqrt{MSE \sum_{i=1}^{g} w_i^2 / n_i}} = -1.807551，t = T_{\frac{\alpha}{2}}(48-4) = -2.015368$$

|T|>|t| ,不拒絕H_0**，無足夠證據顯示老手和新手力氣不同。**

```
> b <- summary(aov(y~A))
> b
            Df Sum Sq Mean Sq F value Pr(>F)
A            3  15.19   5.063   4.243 0.0102 *
Residuals   44  52.51   1.193
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> w1 <- c(1,1,-1,-1)
> Y1 <- apply(rbind(y1,y2,y3,y4),1,mean)
> t1 <- sum(Y1*w1)/sqrt(1.1933*(4/12))
> qt <- qt(0.975,44)
> abs(t1) >= qt  #do not reject Ho
[1] FALSE
```

Exercise 4.4

Consider an experiment taste-testing six types of chocolate chip cookies: 1 (brand A, chewy, expensive), 2 (brand A, crispy, expensive), 3 (brand B, chewy, inexpensive), 4 (brand B, crispy, inexpensive), 5 (brand C, chewy, expensive), and 6 (brand D, crispy, inexpensive). We will use twenty different raters randomly assigned to each type (120 total raters).

(a) Design contrasts to compare chewy with crispy, and expensive with inexpensive.

首先比較chewy with crispy，分別將(1,3,5)和(2,4,6)分組拿來做對比檢定，則假設 $w = (1, -1, 1, -1, 1, -1)$**；接著比較expensive with inexpensive，是將(1,2,5)和(3,4,6)分組，則假設**$w = (1, 1, -1, -1, 1, -1)$**。**

(b) Are your contrasts in part (a) orthogonal? Why or why not?

> cc <- c(1,-1,1,-1,1,-1)

> ee <- c(1,1,-1,-1,1,-1)

> sum(cc*ee)

[1] 2 ≠ 0

There are not orthogonal.

因為口感和價錢並沒有什麼關係，所以這兩種對比檢定並不會orthogonal。

Problem 4.2

Consider the data in Problem 3.2. Design a set of contrasts that seem meaningful. For each contrast, outline its purpose and test the null hypothesis that the contrast has expected value zero.

<case1>不考慮沒有放雌性蒼蠅的那組，分別將一隻跟八隻雌性蒼蠅分組，想看一隻和八隻雌性蒼蠅效果是否不同，則$w = (1, 1, -1, -1)$，$MSE = 221$**，檢定統計量T為5.52937，**

$t = T_{1-\frac{\alpha}{2}}(100-4) = 1.984984$，5.5 > 1.9拒絕$H_0$，有足夠證據顯示放一隻和八隻雌性蒼蠅影響雄性蒼蠅壽命的效果不同。

```
> a <- summary(aov(g1~B1))
> a
            Df Sum Sq Mean Sq F value   Pr(>F)    
B1           3  10844    3615   16.34 1.18e-08 ***
Residuals   96  21240     221                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> w <- c(1,1,-1,-1)
> Y <- apply(rbind(g2,g3,g4,g5),1,mean)
> t <- sum(Y*w)/sqrt(221*(4/25))
> qt <- qt(0.975,96)
> abs(t) >= qt  # reject H0
[1] TRUE
```

<case2>將沒有雌性蒼蠅與一隻雌性蒼蠅分一組，八隻雌性蒼蠅為一組，想看沒放或一隻和八隻雌性蒼蠅效果是否不同，兩組做對比檢定，$w = (1/3,1/3,1/3,-1/2,-1/2)$，$MSE = 219.3$，檢定統計量T為2.4411，$t = T_{1-\frac{\alpha}{2}}(125-4) = 1.979764$，2.4 > 1.9拒絕$H_0$，有足夠證據顯示放少量和八隻雌性蒼蠅影響雄性蒼蠅壽命的效果不同。

```
> a <- summary(aov(g~B2))
> a
             Df Sum Sq Mean Sq F value   Pr(>F)    
B2            4  11939  2984.8   13.61 3.52e-09 ***
Residuals   120  26314   219.3                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> w <- c(1/3,1/3,1/3,-1/2,-1/2)
> Y <- apply(rbind(g1,g2,g3,g4,g5),1,mean)
> t <- sum(Y*w)/sqrt(219.3*(5/25))
> qt <- qt(0.975,121)
> abs(t) >= qt  # reject Ho
[1] TRUE
```

<case3>將沒有或未懷孕雌性蒼蠅分一組，有懷孕的雌性蒼蠅分一組，想看有沒有懷孕的蒼蠅的效果是否不同，兩組做對比檢定，則$w = (1/8, 1/8, -1/8, 6/8, -7/8)$，$MSE = 219.3$，檢定統計量T為2.484641，$t = T_{1-\frac{\alpha}{2}}(125-4) = 1.979764$，30.05 > 1.9拒絕$H_0$，有足夠證據顯示有沒有懷孕的雌性蒼蠅影響雄性蒼蠅壽命的效果不同。

```
> b <- summary(aov(g~B3))
> b
              Df Sum Sq Mean Sq F value   Pr(>F)    
B3             4  11939  2984.8   13.61 3.52e-09 ***
Residuals    120  26314   219.3                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> w <- c(1/8,1/8,-1/8,6/8,-7/8)
> Y1 <- apply(rbind(g1,g2,g3,g4,g5),1,mean)
> t <- sum(Y1*w)/sqrt(219.3*(5/25))
> qt <- qt(0.975,121)
> abs(t) >= qt  # reject H0
[1] TRUE
> qt
[1] 1.979764
```

Question 4.1

Show that orthogonal contrasts in the observed treatment means are uncorrelated random variables.

<proof>

$u = \sum_{i=1}^{t} w_i \bar{y}_{i.}$，$v = \sum_{i=1}^{t} c_i \bar{y}_{i.}$，we know $w_i * c_i = 0$.

Then，$Cov(u,v) = (w_1c_1 + \ldots + w_nc_n)\frac{\sigma^2}{n_i} + (w_1c_2 + w_2c_1)Cov(\bar{y}_{1.}, \bar{y}_{2.}) + \ldots\ldots = 0$

So , orthogonal contrasts in the observed treatment means are uncorrelated random variables.