MATH2081: Math for Computing
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Tutorials 6 - Induction, Recursion and Recurrence

(2024A, Week 7)[11]

1. Prove by induction that for every positive integer n,

$$1 \cdot 2^2 + 2 \cdot 3^2 + \cdots + n \cdot (n+1)^2 = \frac{n(n+1)(n+2)(3n+5)}{12}$$

2. Write a recursive function/algorithm to find the minimum of a finite sequence of numbers.

3. Write a recursive function/algorithm that reverses a finite sequence.

4. The sequence $g_1, g_2, \ldots$ is defined by the recurrence relation

$$g_n = g_{n-1} + g_{n-2} + 1, \quad n \geq 3,$$

and initial conditions $g_1 = 1$, $g_2 = 3$. Prove by induction that

$$g_n = 2f_{n+1} - 1, \quad n \geq 1,$$

where $f_1, f_2, \ldots$ is the Fibonacci sequence.

5. The Lucas sequence $L_1, L_2, \ldots$ is defined by the recurrence relation

$$L_n = L_{n-1} + L_{n-2}, \quad n \geq 3,$$

and the initial conditions $L_1 = 1$, $L_2 = 3$.

   (a) Find the values of L_3, L_4, and L_5.

   (b) Show that

$$L_{n+2} = f_{n+1} + f_{n+3}, \quad n \geq 1,$$

   where $f_1, f_2, \ldots$ is the Fibonacci sequence.

6. Solve the given recurrence relation for the initial conditions given.

   (a) $2a_n = 7a_{n-1} - 3a_{n-2}$ with $a_0 = a_1 = 1$.

   (b) $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$ with $a_0 = a_1 = 1$.

7. Given the explicit formula $f(n) = 2n + 5$. Find the recursive formula of $f(n)$.

[11]Most of the content of this document is taken from the book [1].

8. The recursive formula of sequence $f(n)$ is a linear function

$$f(n) = af(n-1) + b,$$

where a and b are constant numbers. Find a and b given that $f(1) = 2$, $f(2) = 7$ and $f(3) = 17$.

9. Consider the following recursive function $f : \mathbb{N}^+ \to \mathbb{N}^+$.

$$\begin{aligned} &f(1) = 1, \\ &f(n) = 10 \cdot f(n-1) + 1, \quad \text{for all } n \geq 2. \end{aligned}$$

(a) Compute $f(2)$ and $f(4)$.

(b) Prove by induction that f takes the following explicit form:

$$f(n) = \frac{10^n - 1}{9}, \quad \text{for all } n \geq 1.$$

References

1. Johnsonbaugh, R.: Discrete Mathematics - Eighth Edition. *Pearson Education*, New York (2018).