MATH2081: Math for Computing
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RMIT UNIVERSITY

Tutorials 9 - Graph Theory and Networks

(2024A, Week 10)[17]

1. In a precedence graph, the vertices model certain actions. For example, a vertex might model a statement in a computer program. There is an edge from vertex v to vertex w if the action modeled by v must occur before the action modeled by w. Draw a precedence graph for the computer program below.

$$x = 1$$
$$y = 2$$
$$z = x + y$$
$$z = z + 1$$

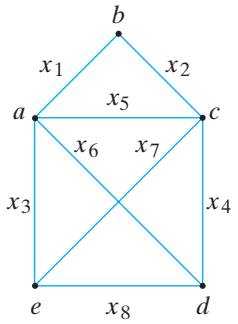
2. Let $\mathcal{G}$ denote the set of simple graphs $G = (V, E)$, where $V = 1, 2, \cdots, n$ for some $n \in \mathbb{Z}^+$. Define a function f from $\mathcal{G}$ to $\mathbb{Z}^+$ by the rule

$$G \mapsto f(G) = |E|.$$

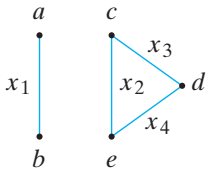
   Is f one-to-one? Is f onto? Explain your answers.

3. Write the adjacency matrix and the incidence matrix of each graph.

(a)



(b)



[17] Most of the content of this document is taken from the book [1].

4. Draw the graph represented by the following adjacency matrix.

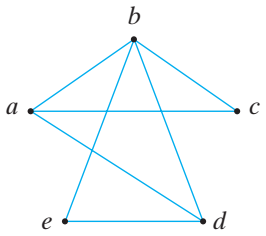
$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

5. Draw the graph represented by the following incidence matrix.

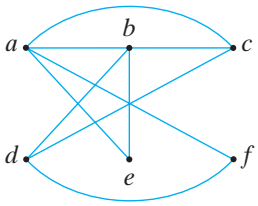
$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \end{array}.$$

6. Show that each graph is planar by redrawing it so that no edges cross.

(a)



(b)



7. Consider a planar graph G with 5 vertices a, b, c, d, e. In this order of the vertices, the adjacency matrix of G is

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 1 \\ 2 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}.$$

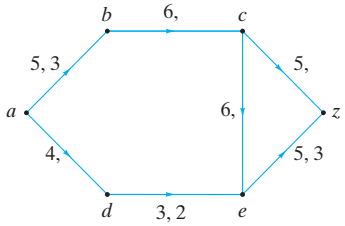
   (a) How many edges does G have? Explain your answer based on A.

   (b) Draw the planar graph G.

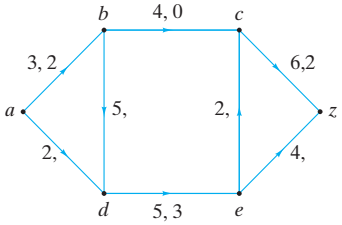
   (c) Is G a simple graph? Explain your answer. Draw the largest simple subgraph of G.

8. Fill in the missing edge flows so that the result is a flow in the given network. Determine the value of each flow.

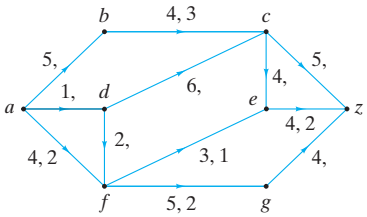
(a)



(b)



(c)

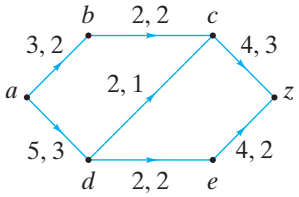


9. Find a maximal flow in each network starting with the edge flows given.

   (a) The network in Exercise 8(a).

   (b) The network in Exercise 8(b).

   (c)



References

1. Johnsonbaugh, R.: Discrete Mathematics - Eighth Edition. *Pearson Education*, New York (2018).