Tutorials 8 - Counting Principles and Discrete Probability

(2024A, Week 9)[15]

1. Consider strings of length 5 formed using the letters A, B, C, D, E, F, G without repetitions. How many strings begin with AC or DB in that order?

2. Refer to the integers from 5 to 200, inclusive. How many have the digits in strictly increasing order? Examples are 8, 13, 147.

3. Refer to a set of five distinct computer science books, three distinct mathematics books, and two distinct art books. In how many ways can these books be arranged on a shelf if all books of the same discipline are grouped together?

4. A six-person committee composed of Alice, Ben, Connie, Dolph, Egbert, and Francisco is to select a chairperson, secretary, and treasurer. How many selections are there in which either Connie is chairperson or Alice is an officer or both?

5. Determine how many strings, containing either the substring AE or the substring EA or both, can be formed by ordering the letters A, B, C, D, E.

6. Refer to a club consisting of six distinct men and seven distinct women.

   (a) In how many ways can we select a committee of three men and four women?

   (b) In how many ways can we select a committee of four persons that has persons of both genders?

7. A coin is flipped 10 times.

   (a) How many outcomes have a head on the fifth toss?

   (b) How many outcomes have as many heads as tails?

8. Two dice are rolled.

   (a) List the members of the event 'the sum of the numbers on the dice is even.'

   (b) List the members of the event 'doubles occur' (i.e., the numbers are the same on both dice).

9. A card is selected at random from an ordinary 52-card deck. What is the probability that it is a heart?

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[15]Most of the content of this document is taken from the book [1].

10. Two fair dice are rolled. What is the probability that the sum of the numbers on the dice is 9?

11. Four microprocessors are randomly selected from 100 microprocessors among which 10 are defective. Find the probability of obtaining no defective microprocessors.

12. A computer randomly generates a 6-bit integer n. What is the probability of the event that n is greater than or equal to 60? The probability of generating every 6-bit integer is assumed the same. Explain your answer.

13. Suppose that six distinct integers are selected from the set

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}.$$

Prove that at least two of the six have a sum equal to 11.

Hints. Consider the partition $\{1, 10\}$, $\{2, 9\}$, $\{3, 8\}$, $\{4, 7\}$, $\{5, 6\}$ of the set S.

14. Consider the Boolean expression $E = \overline{x \vee y \vee z} \ \vee \ (x \wedge z)$, where x, y and z are Boolean variables. Assume that the inputs x, y and z are randomly assigned the value 1 or 0. What is the probability of the event that $E = 1$?

References

1. Johnsonbaugh, R.: Discrete Mathematics - Eighth Edition. *Pearson Education*, New York (2018).