MATH2081: Math for Computing
Lecturer: Dr. Nguyen Hieu Thao
Email: thao.nguyenhieu@rmit.edu.vn

RMIT UNIVERSITY

Tutorials 5 - Sequences, Strings and Relations

(2024A, Week 5)[9]

1. Given the sequence
$$w_n = \frac{1}{n} - \frac{1}{n+1}, \quad n \geq 1,$$
compute $\sum_{i=1}^{10} w_i$.

2. Compute the given quantity using the strings $\alpha = baab$, $\beta = caaba$, $\gamma = bbab$.
   (a) $\alpha\beta$
   (b) $|\beta\alpha|$
   (c) $\beta\beta\gamma\alpha$

3. List all strings over $X = \{0, 1\}$ of length 3 or less.

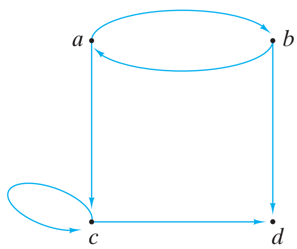
4. Find all substrings of the string $babc$.

5. Let $X = \{a, b\}$. Define a function from X^* to X^* as $f(\alpha) = \alpha ab$. Is f one-to-one? Is f onto? Justify your answers.

6. Write the relation R on $\{1, 2, 3, 4\}$ defined by $(x, y) \in \mathbb{R}$ if $x^2 \geq y$ as a table and draw its digraph.

7. Draw the digraph of the relation $R = \{(1, 2), (2, 1), (3, 3), (1, 1), (2, 2)\}$ on $X = \{1, 2, 3\}$.

8. Write the following relation as a set of ordered pairs.



9. Refer to the relation R on the set $\{1, 2, 3, 4, 5\}$ defined by the rule $(x, y) \in R$ if 3 divides $x - y$. List the elements of R^{-1}.

[9]Most of the content of this document is taken from the book [1].

10. Determine whether each relation defined on the set of positive integers is reflexive, symmetric, antisymmetric, transitive, and/or a partial order.

    (a) $(x, y) \in R$ if 3 divides $x - y$.

    (b) $(x, y) \in R$ if $x - y = 2$.

11. Let X be a nonempty set. Define a relation on $\mathcal{P}(X)$, the power set of X, as $(A, B) \in R$ if $A \subseteq B$. Is this relation reflexive, symmetric, antisymmetric, transitive, or a partial order?

12. Let R_1 and R_2 be the relations on $\{1, 2, 3, 4\}$ given by

$$R_1 = \{(1,1), (1,2), (3,4), (4,2)\}$$
$$R_2 = \{(1,1), (2,1), (3,1), (4,4), (2,2)\}.$$

    List the elements of $R_1 \circ R_2$ and $R_2 \circ R_1$.

13. Give examples of relations on $\{1, 2, 3, 4\}$ having the properties specified below.

    (a) Reflexive, symmetric, and not transitive

    (b) Not reflexive, not symmetric, and transitive

14. Determine whether the given relation is an equivalence relation on $\{1, 2, 3, 4, 5\}$. If the relation is an equivalence relation, list the equivalence classes.

    (a) $R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,3), (3,1), (3,4), (4,3)\}$

    (b) $R = \{(x, y) \mid x \text{ and } y \text{ are both even or both odd}\}$

15. By listing ordered pairs, give an example of an equivalence relation on $\{1, 2, 3, 4, 5, 6\}$ having exactly four equivalence classes.

16. How many equivalence relations are there on the set $\{1, 2, 3\}$?

17. Find the matrix of the relation R from X to Y relative to ordering of X: $1, 2, 3$ and the ordering of Y: $\alpha, \beta, \Sigma, \delta$.

$$R = \{(1, \delta), (2, \alpha), (2, \Sigma), (3, \beta), (3, \Sigma)\}.$$

18. Write the relation R, given by the following matrix, as a set of ordered pairs.

$$\begin{array}{c c} & \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{array}{c} w \\ x \\ y \\ z \end{array} & \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}.$$

References

1. Johnsonbaugh, R.: Discrete Mathematics - Eighth Edition. *Pearson Education*, New York (2018).