1. Find the general and particular solution of

$$\frac{dx}{dt} = xt$$

subject to

$$x(0) = 2$$

2. Find the general and particular solution of

$$\frac{dy}{dx} = 3x^2 e^{-y}$$

subject to

$$y(0) = 1$$

3. Find the general solution to

$$\frac{dy}{dx} = \frac{y}{x}$$

4. Solve

$$y^2 \frac{dy}{dx} = x$$

and find the particular solution given that $y(0) = 1$

5. Find the general solution of

$$x sin^2 y \frac{dy}{dx} = (x+1)^2$$

6. Given the idea of linear superposition and Euler's equation:

$$e^{ix} = cosx + isinx$$

Convert the exponential form of the solution to the simple harmonic oscillator, to the trig form:

$$x(t) = C_1 e^{i\sqrt{\frac{k}{m}}t} + C_2 e^{-i\sqrt{\frac{k}{m}}t}$$

to

$$x(t) = C_3 cos(\sqrt{\frac{k}{m}}t) + C_4 sin(\sqrt{\frac{k}{m}}t)$$

7. Solve the damped oscillator after non-dimensionalizing it (Hint: in this case the "characteristic" velocity is equal to the initial velocity, or the first derivative evaluated at zero)

$$m\frac{d^2x}{dt^2} + q\frac{dx}{dt} + kx = 0$$

$$\frac{dx}{dt}(0) = 2$$

$$x(0) = 0$$

Solve the initial value problem (8-11)

8.

$$2y'' + 5y' + 3y = 0$$

$$y'(0) = -4$$

$$y(0) = 3$$

9.

$$y'' + 16y = 0$$

$$y(\frac{\pi}{4}) = -3$$

$$y'(\frac{\pi}{4}) = 4$$

10.

$$y'' + 16y = e^x$$

$$y(\frac{\pi}{4}) = -3$$

$$y'(\frac{\pi}{4}) = 4$$

11.

$$y'' + 16y = 5sin(x)$$

$$y(\frac{\pi}{4}) = -3$$

$$y'(\frac{\pi}{4}) = 4$$

12. Transform this equation into a system of first order equations:

$$x''' + x'' + x' + x = 0$$

13. Transform the simple harmonic oscillator equation $m\frac{d^2x}{dt} + kx = 0$ to a system of first order equations and solve by hand using Euler's method with 4 time steps, and 10 time steps going from 0 to 1 given that

$$\frac{dx}{dt}(0) = 0$$

$$x = 1$$

compare the numerical answer you get from the Euler's method and the analytic solution. Graph the analytic solution against an Euler's method solution, how accurate does it look? How does changing the timestep effect things?

14. Solve the following system of equations in MATLAB using all parameters equal to 1, and going from time 0 to 10.

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \nu I$$

$$\frac{dR}{dt} = \nu I$$

Graph each dependent function against time, and then try different combinations of plotting them against each other. Non-dimensionlize them and repeat the problem using different values of the parameter you generate from the non-dimensionalization