hw0

Graded

Student

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Total Points

34 / 40 pts

Question 1

Problem 1

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 2

Problem 2

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 3

Problem 3

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 4

Problem 4

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 5

Problem 5

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 6

Problem 6

0 / 2 pts

+ 2 pts 正確

✔ **+ 0 pts** 不正確

Question 7

Problem 7

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 8

Problem 8

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 9

Problem 9

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 10

Problem 10

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 11

Problem 11

0 / 2 pts

+ 2 pts 正確

✔ **+ 0 pts** 不正確

Question 12

Problem 12

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 13

Problem 13

2 / 2 pts

✔ **+ 2 pts** 正確

+ 0 pts 不正確

Question 14

Problem 14

0 / 2 pts

+ 2 pts 正確

✔ **+ 0 pts** 不正確

Question 15

Problem 15

2 / 2 pts

+ 0 pts 不正確

✔ **+ 2 pts** 正確

Question 16

Problem 16

2 / 2 pts

+ 0 pts 不正確

✔ **+ 2 pts** 正確

Question 17

Problem 17

2 / 2 pts

+ 0 pts 不正確

✔ **+ 2 pts** 正確

Question 18

Problem 18

2 / 2 pts

+ 0 pts 不正確

✔ **+ 2 pts** 正確

Question 19

Problem 19

2 / 2 pts

+ 0 pts 不正確

✔ **+ 2 pts** 正確

Question 20

Problem 20

2 / 2 pts

+ 0 pts 不正確

✔ **+ 2 pts** 正確

Question 21

Usage of Gold Medal

0 / 0 pts

✔ **+ 0 pts** 正確

+ 0 pts 不正確

Q1 Problem 1
2 Points

1. Let $C(N,K) = 1$ for $K = 0$ or $K = N$, and $C(N,K) = C(N-1,K) + C(N-1,K-1)$ for $N \geq 1$. What is the closed-form equation of $C(N,K)$ for $N \geq 1$ and $0 \leq K \leq N$?

[a] $C(N,K) = \frac{N!}{K!(N-K)!}$
[b] $C(N,K) = \sum_{k=0}^{K} \frac{N!}{k!(N-k)!}$
[c] $C(N,K) = \frac{K!(N-K)!}{K!}$
[d] $C(N,K) = \sum_{k=0}^{K} \frac{k!(N-k)!}{N!}$
[e] none of the other choices

- (•) [a]
- ( ) [b]
- ( ) [c]
- ( ) [d]
- ( ) [e]

Q2 Problem 2
2 Points

2. What is the probability of getting exactly 4 heads when flipping 10 fair coins? Choose the closest number.

[a] 0.0
[b] 0.1
[c] 0.2
[d] 0.3
[e] 0.4

- ( ) [a]
- ( ) [b]
- (•) [c]
- ( ) [d]
- ( ) [e]

Q3 Problem 3
2 Points

3. If your friend flipped a fair coin three times, and then tells you that one of the tosses resulted in head, what is the probability that all three tosses resulted in heads?

[a] $1/8$
[b] $3/8$
[c] $7/8$
[d] $1/7$
[e] $1/3$

- ○ [a]
- ○ [b]
- ○ [c]
- ◉ [d]
- ○ [e]

Q4 Problem 4
2 Points

4. A program selects a random integer x like this: a random bit is first generated uniformly. If the bit is 0, x is drawn uniformly from $\{0, 1, \ldots, 7\}$; otherwise, x is drawn uniformly from $\{0, -1, -2, -3\}$. If we get an x from the program with $|x| = 1$, what is the probability that x is negative?

[a] $1/3$
[b] $1/4$
[c] $1/2$
[d] $1/12$
[e] $2/3$

- ○ [a]
- ○ [b]
- ○ [c]
- ○ [d]
- ◉ [e]

Q5 Problem 5
2 Points

5. For N random variables $x_1, x_2, \ldots, x_N$, let their mean be $\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n$ and variance be $\sigma_x^2 = \frac{1}{N-1}\sum_{n=1}^{N}(x_n - \bar{x})^2$. Which of the following is provably the same as σ_x^2?

[a] $\frac{1}{N}\sum_{n=1}^{N}(x_n^2 - \bar{x}^2)$
[b] $\frac{1}{N-1}\sum_{n=1}^{N}(x_n^2 - \bar{x}^2)$
[c] $\frac{1}{N-1}\sum_{n=1}^{N}(\bar{x}^2 - x_n^2)$
[d] $\frac{N}{N-1}(\bar{x}^2)$
[e] none of the other choices

- ○ [a]
- ● [b]
- ○ [c]
- ○ [d]
- ○ [e]

Q6 Problem 6
2 Points

6. For two events A and B, if their probability $P(A) = 0.3$ and $P(B) = 0.4$, what is the tightest possible range of $P(A \cup B)$?

[a] $[0.3, 0.4]$
[b] $[0, 0.4]$
[c] $[0, 0.7]$
[d] $[0.3, 1]$
[e] $[0.4, 0.7]$

- ○ [a]
- ● [b]
- ○ [c]
- ○ [d]
- ○ [e]

Q7 Problem 7

2 Points

7. What is the rank of $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 1 & 1 & 2 \end{pmatrix}$?

[a] 0

[b] 1

[c] 2

[d] 3

[e] none of the other choices

- ○ [a]
- ○ [b]
- ◉ [c]
- ○ [d]
- ○ [e]

Q8 Problem 8

2 Points

8. What is the diagonal on the inverse of $\begin{pmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{pmatrix}$?

[a] $[3/4, 1/4, 1/8]$

[b] $[1/4, 1/8, 3/4]$

[c] $[1/4, 3/4, 1/8]$

[d] $[1/8, 3/4, 1/4]$

[e] none of the other choices

- ○ [a]
- ○ [b]
- ○ [c]
- ◉ [d]
- ○ [e]

Q9 Problem 9

2 Points

9. What is the largest eigenvalue of $\begin{pmatrix} 2023 & 1 & 1 \\ 2 & 2024 & 2 \\ -1 & -1 & 2021 \end{pmatrix}$?

[a] 2020

[b] 2021

[c] 2022

[d] 2023

[e] 2024

- ○ [a]
- ○ [b]
- ○ [c]
- ○ [d]
- ◉ [e]

Q10 Problem 10
2 Points

10. For a real matrix M, let $M = U\Sigma V^T$ be its singular value decomposition, with U and V being unitary matrices. Define $M^\dagger = V\Sigma^\dagger U^T$, where $\Sigma^\dagger[j][i] = \frac{1}{\Sigma[i][j]}$ when $\Sigma[i][j]$ is nonzero, and 0 otherwise. Which of the following is always the same as $MM^\dagger M$?

[a] MM^TM
[b] MV^T
[c] U^TM
[d] U^TMV^T
[e] M

- ○ [a]
- ○ [b]
- ○ [c]
- ○ [d]
- ◉ [e]

Q11 Problem 11
2 Points

11. Which of the following matrix is not guaranteed to be positive semi-definite?

[a] Z^TZ for any real matrix Z
[b] a real symmetric matrix S whose eigenvalues are all non-negative
[c] an all-zero square matrix
[d] a real symmetric matrix whose entries are all positive
[e] none of the other choices

- ○ [a]
- ○ [b]
- ◉ [c]
- ○ [d]
- ○ [e]

Q12 Problem 12
2 Points

12. Consider a fixed $\mathbf{x} \in \mathbb{R}^d$ and some varying $\mathbf{u} \in \mathbb{R}^d$ with $\|\mathbf{u}\| = 1$. Which of the following is the smallest value of $\mathbf{u}^T\mathbf{x}$?

[a] 0
[b] $-\infty$
[c] $-\|\mathbf{x}\|$
[d] $-\|\mathbf{u}\|$
[e] none of the other choices

- ○ [a]
- ○ [b]
- ◉ [c]
- ○ [d]
- ○ [e]

Q13 Problem 13
2 Points

13. Consider two parallel hyperplanes in R^d:

$$H_1 : \mathbf{w}^T\mathbf{x} = +3,$$
$$H_2 : \mathbf{w}^T\mathbf{x} = -2,$$

What is the distance between H_1 and H_2?

[a] 5
[b] $5/\|\mathbf{w}\|$
[c] $5/\|\mathbf{w}\|^2$
[d] $5 \cdot \|\mathbf{w}\|$
[e] none of the other choices

- ○ [a]
- ◉ [b]
- ○ [c]
- ○ [d]
- ○ [e]

Q14 Problem 14
2 Points

14. Let $f(x,y) = xy$, $x(u,v) = \cos(u+v)$, $y(u,v) = \sin(u-v)$. What is $\dfrac{\partial f}{\partial v}$?

[a] $-\sin(u+v)\sin(u-v) - \cos(u+v)\cos(u-v)$
[b] $+\sin(u+v)\sin(u-v) - \cos(u+v)\cos(u-v)$
[c] $-\sin(u+v)\sin(u-v) + \cos(u+v)\cos(u-v)$
[d] $+\sin(u+v)\sin(u-v) + \cos(u+v)\cos(u-v)$
[e] none of the other choices

- ○ [a]
- ○ [b]
- ○ [c]
- ○ [d]
- ◉ [e]

Q15 Problem 15
2 Points

15. Let $E(u,v) = (ue^v - 2ve^{-u})^2$. Calculate the gradient $\nabla E(u,v) = \begin{pmatrix} \frac{\partial E}{\partial u} \\ \frac{\partial E}{\partial v} \end{pmatrix}$ at $[u,v] = [1,1]$.

[a] $[-13.70, -7.86]$
[b] $[-13.70, +7.86]$
[c] $[+13.70, -7.86]$
[d] $[+13.70, +7.86]$
[e] $[1, 1]$

- ○ [a]
- ○ [b]
- ○ [c]
- ◉ [d]
- ○ [e]

Q16 Problem 16
2 Points

16. For some given $A > 0, B > 0$, what is the optimal α that solves
$$\min_{\alpha} Ae^{\alpha} + Be^{-2\alpha}?$$

[a] $\frac{1}{3}\ln(\frac{2B}{A})$
[b] $\frac{1}{3}\ln(\frac{A}{2B})$
[c] $\ln(\frac{2B}{A})$
[d] $\ln(\frac{A}{2B})$
[e] none of the other choices

- (●) [a]
- ( ) [b]
- ( ) [c]
- ( ) [d]
- ( ) [e]

Q17 Problem 17
2 Points

17. Let $\mathbf{w}$ be a vector in $\mathbb{R}^d$ and $E(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T\mathrm{A}\mathbf{w} + \mathbf{b}^T\mathbf{w}$ for some symmetric matrix A and vector $\mathbf{b}$. What is the gradient $\nabla E(\mathbf{w})$?

[a] $\mathbf{w}^T\mathrm{A}\mathbf{w} + \mathbf{w}^T\mathbf{b}$
[b] $\mathbf{w}^T\mathrm{A}\mathbf{w} - \mathbf{w}^T\mathbf{b}$
[c] $\mathrm{A}\mathbf{w} + \mathbf{b}$
[d] $\mathrm{A}\mathbf{w} - \mathbf{b}$
[e] none of the other choices

- ( ) [a]
- ( ) [b]
- (●) [c]
- ( ) [d]
- ( ) [e]

Q18 Problem 18
2 Points

18. Let $\mathbf{w}$ be a vector in $\mathbb{R}^d$ and $E(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T\mathrm{A}\mathbf{w} + \mathbf{b}^T\mathbf{w}$ for some symmetric *and strictly positive definite* matrix A and vector $\mathbf{b}$. What is the optimal $\mathbf{w}$ that minimizes $E(\mathbf{w})$?

[a] $+\mathrm{A}^{-1}\mathbf{b}$
[b] $-\mathrm{A}^{-1}\mathbf{b}$
[c] $-\mathrm{A}^{-1}\mathbf{1} + \mathbf{b}$, where $\mathbf{1}$ is a vector of all 1's
[d] $+\mathrm{A}^{-1}\mathbf{1} - \mathbf{b}$
[e] none of the other choices

- ○ [a]
- ◉ [b]
- ○ [c]
- ○ [d]
- ○ [e]

Q19 Problem 19
2 Points

19. Solve

$$\min_{w_1,w_2,w_3} \frac{1}{2}(w_1^2 + 2w_2^2 + 3w_3^2) \text{ subject to } w_1 + w_2 + w_3 = 11.$$

What is the optimal w_1? (Hint: refresh your memory on "Lagrange multipliers")

[a] 0
[b] 1
[c] 2
[d] 3
[e] 6

- ○ [a]
- ○ [b]
- ○ [c]
- ○ [d]
- ◉ [e]

Q20 Problem 20
2 Points

20. Solve

$$\begin{aligned} &\min_{w_1,w_2,w_3} \frac{1}{2}(w_1^2 + 2w_2^2 + 3w_3^2) \\ \text{subject to} \quad & w_1 + w_2 + w_3 \geq 11, \\ & w_2 + 2w_3 \geq -11. \end{aligned}$$

What is the optimal (w_1, w_2, w_3)? (Hint: you can also consider using “Lagrange multipliers” to solve this.)

[a] $(3, 6, 2)$
[b] $(3, 2, 6)$
[c] $(6, 2, 3)$
[d] $(3, 6, 2)$
[e] $(6, 3, 2)$

- ○ [a]
- ○ [b]
- ○ [c]
- ○ [d]
- ● [e]

Q21 Usage of Gold Medal
0 Points

How many gold medals would you like to use in hw0?

- ● 0
- ○ 1
- ○ 2
- ○ 3
- ○ 4