Definition 1 (Images of sets).

If $f : X \to Y$ is a function from X to Y , and S is a set in X, we define $f(S)$ to be the set $f(S) := \{f(x) : x \in S\}$;

Definition 2 (Inverse images).

If U is a subset of Y , we define the set $f -1(U)$ to be the set $f -1(U) := \{x \in X : f(x) \in U\}$. In other words, $f -1(U)$ consists of all the elements of X which map into U: $f(x) \in U \Longleftrightarrow x \in f -1(U)$. We call $f -1(U)$ the inverse image of U.

Axiom 1 (Power set axiom).

Let X and Y be sets. Then there exists a set, denoted $Y X$, which consists of all the functions from X to Y , thus $f \in Y X \Longleftrightarrow$ (f is a function with domain X and range Y).

Lemma 1

Let X be a set. Then the set $\{Y : Y \text{ is a subset of } X\}$ is a set.

Axiom 2: (Union).

Let A be a set, all of whose elements are themselves sets. Then there exists a set A whose elements are precisely those objects which are elements of the elements of A, thus for all objects x $x \in A \Longleftrightarrow$ ($x \in S$ for some $S \in A$).