

Exercises 1

Introduction to

Robotics and Intelligent Systems

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1 Vectors

1. Consider the following vectors:

$$u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}; v = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

- (a) Calculate the norm of u and of v
 - (b) Calculate the sum of the two vectors
 - (c) Show the result geometrically
 - (d) Calculate the difference $v - u$
 - (e) Calculate $7v - 5u$
 - (f) Calculate the dot product
 - (g) Calculate the cross product
 - (h) Are u and v linearly independent? Why?
2. Suppose that $u \in \mathbb{R}^3$ is a vector which lies in the first quadrant of the xy -plane and has length 3 and that $v \in \mathbb{R}^3$ is a vector that lies along the positive z -axis and has length 5.
- (a) Calculate $\|u \times v\|$
 - (b) The x -coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)
 - (c) The y -coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)
 - (d) The z -coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)
3. Suppose that u and v are vectors in \mathbb{R}^3 , both of length $2\sqrt{2}$ and that the length of $u - v$ is also $2\sqrt{2}$.
- (a) Calculate $\|u + v\|$
 - (b) Calculate the angle between u and v

2 Matrices

1. Consider the following matrices:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & -3 & 2 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 4 & 1 \end{bmatrix}; C = \begin{bmatrix} 4 & -1 \\ 1 & -2 \end{bmatrix}$$

Calculate, if possible:

- (a) $A + B$
- (b) $A + C$
- (c) $2C + 3I_2$

2. Consider the following matrices:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \\ -1 & 0 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & -2 & 2 & 1 \end{bmatrix}$$

Calculate, if possible:

- (a) A^T
- (b) $A + B$
- (c) $A^T + B$
- (d) AB
- (e) AB
- (f) BA

3. Consider the following matrix:

$$A = [1 \quad -1 \quad 3]$$

Calculate, if possible:

- (a) A^2
- (b) $A^T A$
- (c) AA^T