Quadratic Function Fitting using Least Squares

$$y = ax^2 + bx + c$$

Find a, b, c using LS by given 11 2D points, and plot it.
다음 4가지 방식으로:

1. 일반적인 Least Square
2. Weighted Least Square: [1, 1, 1, 3, 3, 3, 3, 3, 1, 1, 1]
3. Using left 8 points
4. Using right 8 points

1. 일반적인 Least Square

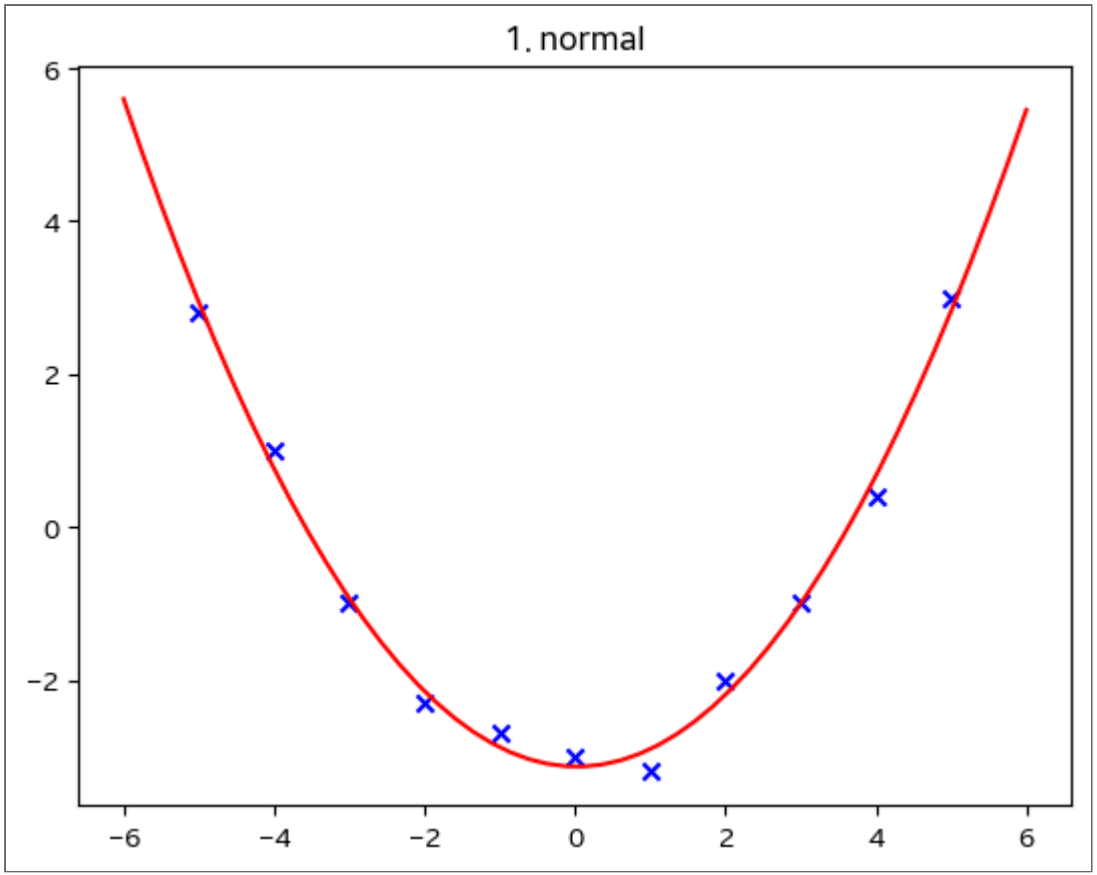
$\boldsymbol{Ax} = \boldsymbol{b}$에서 (실제로는 $\boldsymbol{b} \notin C(A)$)

$$\boldsymbol{x} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}$$

로 $\boldsymbol{x}$를 구할 수 있음.

$$\begin{bmatrix} 25.0 & -5.0 & 1.0 \\ 16.0 & -4.0 & 1.0 \\ 9.0 & -3.0 & 1.0 \\ 4.0 & -2.0 & 1.0 \\ 1.0 & -1.0 & 1.0 \\ 0.0 & 0.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \\ 4.0 & 2.0 & 1.0 \\ 9.0 & 3.0 & 1.0 \\ 16.0 & 4.0 & 1.0 \\ 25.0 & 5.0 & 1.0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2.8 \\ 1.0 \\ -1.0 \\ -2.3 \\ -2.7 \\ -3.0 \\ -3.2 \\ -2.0 \\ -1.0 \\ 0.4 \\ 3.0 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.24044289044289 \\ -0.0118181818181818 \\ -3.131701631701631 \end{bmatrix}$$


1. normal
6
4
2
0
−2
−6
−4
−2
0
2
4
6

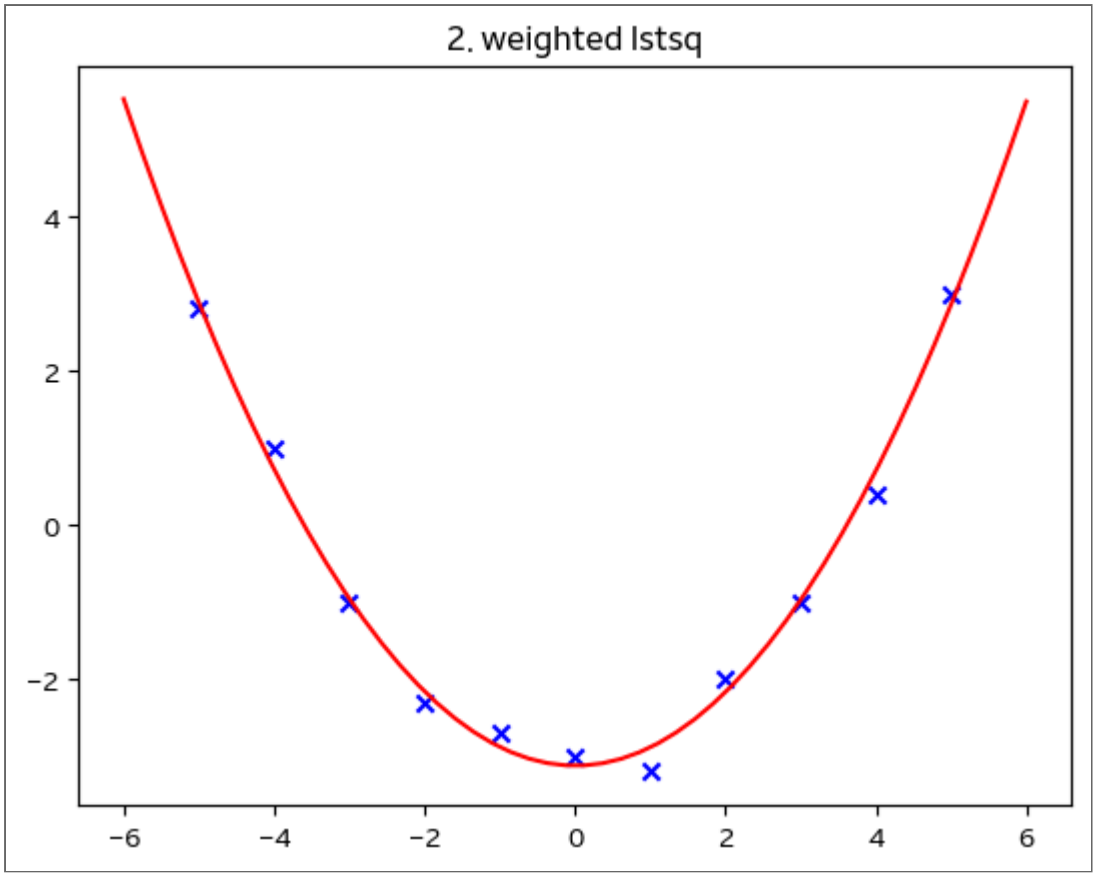
2. Weighted Least Square

$$\boldsymbol{WAx} = \boldsymbol{Wb}$$

$$\left(W \begin{bmatrix} 25.0 & -5.0 & 1.0 \\ 16.0 & -4.0 & 1.0 \\ 9.0 & -3.0 & 1.0 \\ 4.0 & -2.0 & 1.0 \\ 1.0 & -1.0 & 1.0 \\ 0.0 & 0.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \\ 4.0 & 2.0 & 1.0 \\ 9.0 & 3.0 & 1.0 \\ 16.0 & 4.0 & 1.0 \\ 25.0 & 5.0 & 1.0 \end{bmatrix} \right) \begin{bmatrix} a \\ b \\ c \end{bmatrix} \neq W \begin{bmatrix} 2.8 \\ 1.0 \\ -1.0 \\ -2.3 \\ -2.7 \\ -3.0 \\ -3.2 \\ -2.0 \\ -1.0 \\ 0.4 \\ 3.0 \end{bmatrix}$$

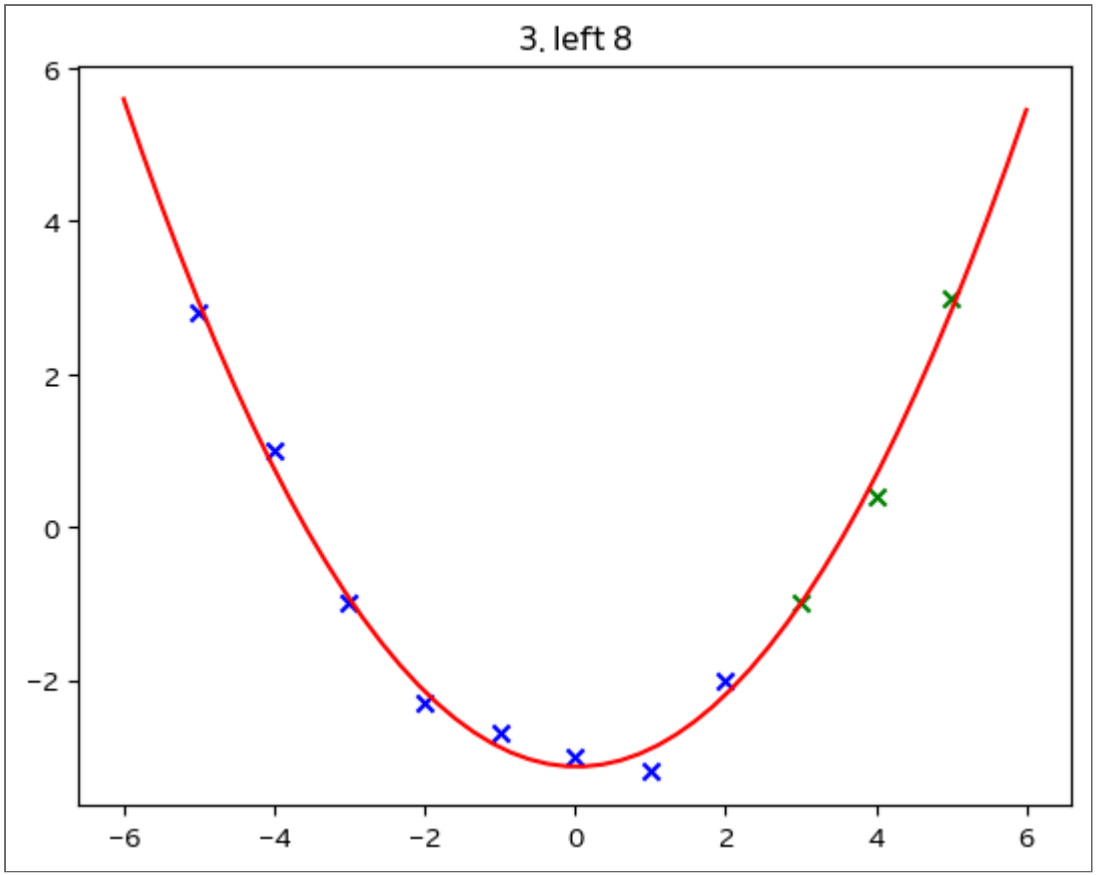
$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.239604534329512 \\ -0.0026315789473684 8 \\ -3.12009532397269 \end{bmatrix}$$


2. weighted lstsq
4
2
0
−2
−6
−4
−2
0
2
4
6

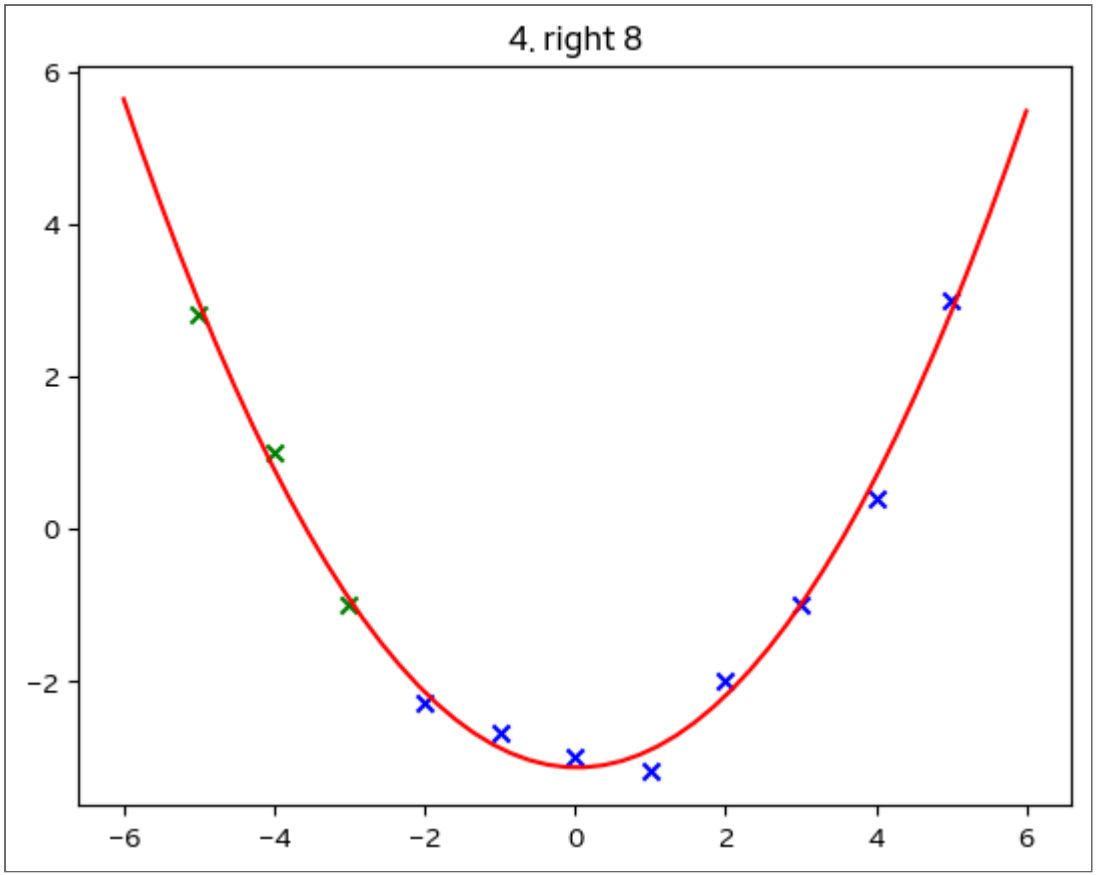
3. Using left 8

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.24047619047619 \\ -0.00476190476190536 \\ -3.11071428571428 \end{bmatrix}$$


3. left 8
6
4
2
0
−2
−6
−4
−2
0
2
4
6

4. Using right 8

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.2416666666666667 \\ -0.0130952380952374 \\ -3.14285714285714 \end{bmatrix}$$


4. right 8
6
4
2
0
−2
−6
−4
−2
0
2
4
6

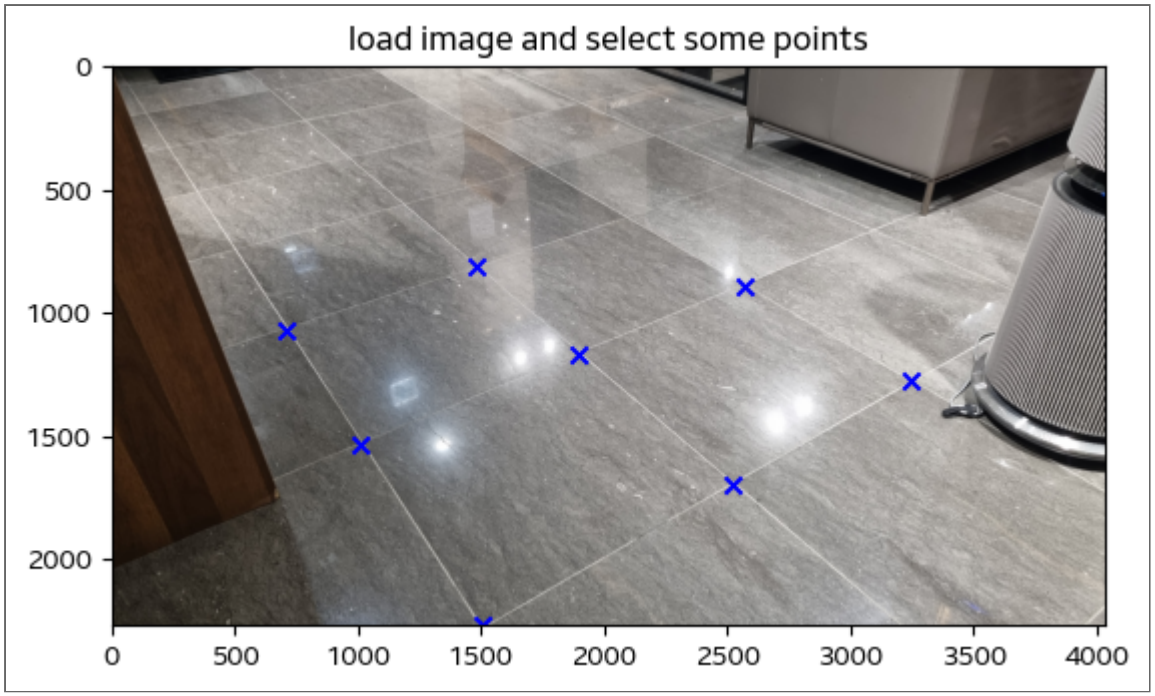
Multi-lines Fitting using Least Squares

$y = ax + b$

Find a, b using LS: Find the intersections of red and green respectively.

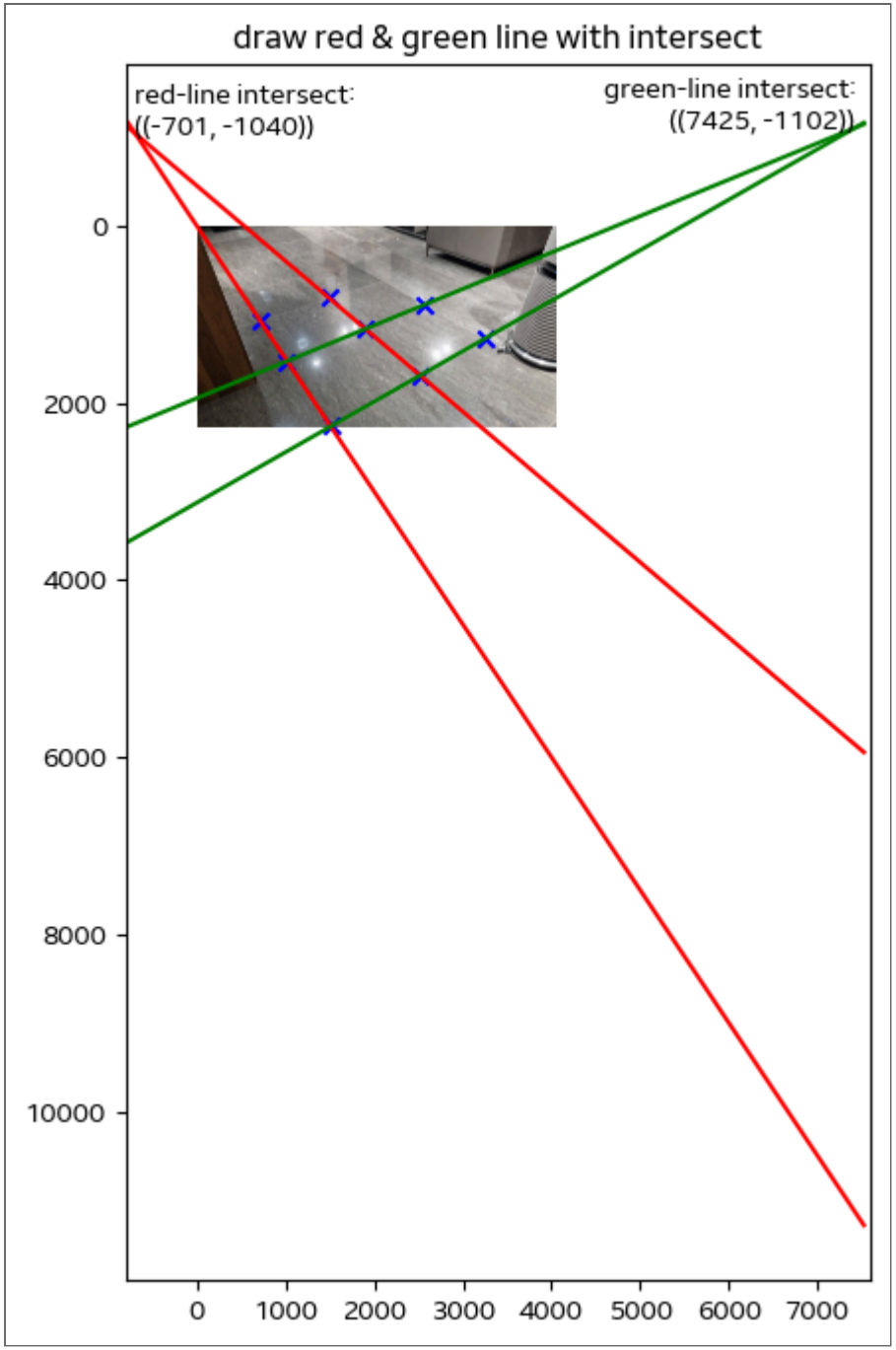
How to do?

1. Select some 2D points at pixel coordinates on the same line, and find the optimal line equation using LS
2. Repeat 1 for other lines in the same direction
3. Find the intersections of lines in (1)-(2) in the same direction
4. Repeat (1)-(3) for other directions(red/green)


load image and select some points
0
500
1000
1500
2000
0
500
1000
1500
2000
2500
3000
3500
4000

```
redlines: [ 1.49803768 10.4447933 ], [   0.84914333 -444.69883377]
greenlines: [-4.10770321e-01  1.94818558e+03], [-5.69466633e-01  3.12643343e+0
3]
```

```
redline intersect: (-701.4140732926402, -1040.2999153942956)
greenline intersect: (7424.544602554563, -1101.5969884102585)
```


draw red & green line with intersect
red-line intersect:
((-701, -1040))
green-line intersect:
((7425, -1102))
0
2000
4000
6000
8000
10000
0
1000
2000
3000
4000
5000
6000
7000