

Mathematical Modeling and Consulting



Sponsor

McDonald's Corporation

Midterm Progress Report

How Much Ice Do You Need?

Team Members

Yen Theng Tan
yen@jhu.edu

Joyce Tan jtan21@jhu.edu

Academic Mentor

Dr. N. .H. Lee, Applied Mathematics and Statistics
nhlee@jhu.edu

Date: Last Compiled on November 4, 2012

Abstract

Acknowledgments

Contents

Abstract	2
Acknowledgments	3
Introduction	8
Technical Background	9
Problem Statement	10
Analysis	11
Results	13
Conclusion	14
A Lemmas	15
B Glossary	16
C Abbreviations	17
REFERENCES	
Selected Bibliography Including Cited Works	18

List of Figures

List of Tables

1	Experiment results for Coke	6
2	Experiment results for Sprite	6
3	Experiment results for Fanta Orange	6
4	Experiment results for Diet Coke	7

	40%	60%	75%
t=0.5 mins	15	25	32
t=2 mins	14	24	34
t=5 mins	14	27	31
t=30 mins	18	36	18

Table 1: Experiment results for Coke

	40%	60%	75%
t=0.5 mins	15	27	30
t=2 mins	20	19	33
t=5 mins	14	29	29
t=30 mins	17	30	25

Table 2: Experiment results for Sprite

	40%	60%	75%
t=0.5 mins	15	23	34
t=2 mins	19	23	30
t=5 mins	18	27	27
t=30 mins	12	35	25

Table 3: Experiment results for Fanta Orange

	40%	60%	75%
t=0.5 mins	15	24	33
t=2 mins	21	19	32
t=5 mins	16	24	32
t=30 mins	18	22	32

Table 4: Experiment results for Diet Coke

Introduction

McDonald's Corporation is the world's largest chain of hamburger fastfood restaurants, serving around 68 million customers daily in 119 countries. Mcdonald's primarily sells hamburgers, cheeseburgers, chicken, French fries, breakfast items, soft drinks, milkshakes and desserts. No meal is complete without a drink; and from Diet Coke to low-fat milk to fresh-brewed, hot coffee, McDonald's serves many different varieties of beverages.

Given that soft drinks are normally assumed to be the perfect accompaniment to a fast food meal, their cold-ness is also essential to the overall satisfaction of the consumer. We have been tasked by McDonald's to find the optimal amount of ice to put into their standard large size cups, and the provide them with data on consumer satisfaction as time elapses.

Technical Background

Firstly, we have a few assumptions on hand in order to simplify this problem. We assume that the consumer's taste depends entirely on the dilution and temperature of the drink. Also, any sample group that we use represents the population's preferences accurately.

We are interested in approaching this problem using 2 different methods. The first method would be experimenting with different types of soda, and different amounts of ice to find out the optimal proportion of ice to soda. Using different proportions of ice, we will then measure the resulting temperature of the drink, as well as calculate the resulting dilution of the drink. We are also narrowing down the scope of our experiment to 4 of the most popular drinks in McDonalds' - Coca Cola, Sprite, Fanta Orange, and Diet Coke. By experimenting, we will test out which combination of temperature and dilution will yield the highest satisfaction from the test subjects. Over the course of 4 days, we will give the test subject 4 different cups of the same drink with different labels A, B, C, D. Additionally, we will take measurements four times a day, thereby including a time parameter of $t=30\text{seconds}$, 2 minutes, 5 minutes, $t=\text{infinity}$, which indicates the time elapsed after the ice is mixed with the drink. The different labels represent different ice proportions, and the test subject is allowed to sip the drink at $\text{time}=t$, assuming the ice is placed in the drink at $t=0$. On the same day, we will do the same test with the 3 other sodas, an hour apart. The cups are given an hour apart, so that the previous cup will not affect any judgement on the following drink. The test subject will then choose their favorite cup each round. We will tabulate the preferences of the entire sample group, and provide a conclusion about the consumer's preferred ice proportion in each soda drink.

If time permits, we would be looking to approach from an alternative method. The second method would be using physics-based modeling. Utilizing the specific heat capacities of soda and ice (already found as specific values), we can calculate the different temperatures and dilution that the resulting drink will be. We can then compare this to the actual values obtained in the first approach, and see if they are pretty similar. This can also tell us more about the effect of the environment (heat loss to surrounding air and cup). This will be mainly a supporting tool and not used in place of the first approach.

Problem Statement

Selling soft drinks is a significant portion of McDonald's business, be it as a thirst quencher, or as part of the extra value meal. The server is not accustomed to putting much thought in measuring the amount of ice put in the cup. This often results in a overly diluted, overly concentrated or overly cold drink for the customer. This is likely to lower overall customer satisfaction, since a drink is a significant complement to a meal. Thus, customers are likely to appreciate if the right amount of ice was added for optimal satisfaction.

To further define this problem, the exogenous variables are the proportion of ice to put in a drink. The endogenous variable would be the resulting temperature and concentration of the drink, as we are assuming that a customer's satisfaction is affected only by the temperature and concentration of the drink.

Analysis

Initial Analysis

Table 1, 2, 3, and 4 shows the poll results for the four different drinks of Coke, Sprite, Fanta Orange, and Diet Coke respectively. The rows represent the poll results taken after 0.5 minutes, 2 minutes, 5 minutes and 30 minutes; the columns represent the percentage of the cup filled (not by volume but by height of the cup). Each subject is asked to rank their preference of the amount of ice from 1 - 3 at each point in time, 3 being the most enjoyable cup and 1 being the least enjoyable cup. These preferences are collected and summed, and shown in the tables. For each row, the higher the number, the more satisfactory the subject is at that point in time.

Based on our experimental results collected, we can see a relatively clear trend across all four drinks at each point in time.

At $t = 0.5$ minutes, it is obvious that the cup with the most amount of ice, at 75% of the cup filled (not by volume but by height of the cup), gave the subjects the most satisfaction. This is likely because, at 30 seconds after the soda is mixed with the ice, the soda is chilled the fastest with the most ice, and little dilution occurs.

At $t = 2$ minutes, there is a less clear trend, but generally the subjects still prefer 75% as compared to 40% or 60%. This is reasonable, given that the ice still has not diluted much, and the most ice would still chill the drink the most and provide the most satisfaction.

At $t = 5$ minutes, we see a split preference between 60% and 75%. At 5 minutes, more of the ice has dissolved, and dilution in both cups are likely the same given that their temperature is also roughly equal. The continued low satisfaction for 40% is likely due to the lack of chilling effect of this low quantity of ice.

At $t = 30$ minutes, we can assume that most of the ice have dissolved, and the resulting dilution from the ice is very high. The more ice there is, the more diluted the soda is. Thus we see a decline in satisfaction for the 75% cup. It is still more satisfying than the 40% cup though, probably because it still manages to maintain its coldness, whereas the 40% cup is likely less chilled.

Theoretical Analysis

Assumptions and terms:

- $(\text{dilution}, \text{temp}) = \text{SHC}(\text{vsoda}, \text{vice}, w)$
- $\text{vsoda} = \text{volume of soda in cm}^3$, Assume density of soda is 1g/cm^3
- $\text{vice} = \text{volume of ice in cm}^3$, Assume density of ice is $0.9167\text{g/cm}^3 = \text{density}$
- $w = \text{specific heat capacity of soda in J}$
- $\text{specific heat capacity of water} = 4.1813\text{J/g/K} = \text{watershc}$
- $\text{specific heat of fusion} = 334\text{J/g} = \text{fusion}$
- ice is 0 degrees Celsius, soda is 25 degrees Celsius
- dilution is dilution of resulting solution
- temp is resulting temperature of solution in Celsius
- room temperature = 25 = t

Mass of ice:

- $\text{vice} = \text{ice} * \text{density}$;

Dilution:

- $\text{dilution} = \text{vice} / \text{vsoda}$;

Energy needed to melt ice:

- $\text{energymelt} = 334 * \text{vice}$;

Resulting final temperature:

- $\text{temp} = (\text{t} * \text{vsoda} * w - \text{energymelt}) / (\text{watershc} * \text{vice} + \text{vsoda} * w)$;

Appendix A

Lemmas

$$q = mC\Delta T,$$

where C = specific heat capacity ($\text{J/g } ^\circ\text{C}$)

q = quantity of heat in joules

m = mass in grams

ΔT = change in temperature

so $C = q / (m \Delta T)$

Appendix B

Glossary

Specific heat capacity. Amount of heat per unit mass required to raise the temperature by one degree Celsius

Heat of fusion. Amount of heat needed to change its state from a solid to a liquid per unit mass

Appendix C

Abbreviations

RAAN. Right ascension of the ascending node

Selected Bibliography Including Cited Works

- [1] American Mathematical Society. *MathSciNet: Mathematical Reviews on the Web*. <http://www.ams.org/mathscinet/>. Accessed June 17, 2009.

Because an online reference may be changed at any time, it is conventional to tie the reference to the date when the resource was accessed.

- [2] Roger R. Bate, Donald D. Mueller, and Jeremy E. While. *Fundamentals of Astrodynamics*. Dover, 1971.

A standard textbook on astrodynamics. It provided a reference for orbital mechanics and satellite propagation.

- [3] Ingrid Carlbom and Joseph Paciorek. Planar Geometric Projections and Viewing Transformations. *Computing Surveys*, 1978.

Gives a thorough background to projective geometry and vertical perspective projection. This includes details about calculating projections using homogeneous coordinates and projection matrices.

- [4] Gelfand and Fomin. *Calculus of Variations*. Prentice-Hall, 1963.

Discusses the essential principle of variational method for optimal path problems.

- [5] George Grätzer. *More Math Into L^AT_EX*. Birkhäuser, Boston, MA, fourth edition, 2007.

- [6] Jacob Kogan. *Introduction to Clustering Large and High-Dimensional Data*. Cambridge, 2007.

Focuses on a few of the most important clustering algorithms, providing also some useful optimization techniques for high-dimensional objective functions.

- [7] David A. Vallado. *Fundamentals of Astrodynamics and Applications*. Space Technology, 2007.

A professional astrodynamics reference. It emphasizes the practical use of astrodynamics in space missions.

- [8] Emo Welzl. Smallest Enclosing Disks (Balls and Ellipsoids). *New Results and New Trends in Computer Science*, 1991.

Outlines a smallest circle algorithm that runs in linear time using recursion.