Name: Yuhan Ye (yuhanye@umich.edu), Department of Statistics, University of Michigan
Advisor: Dr. Eva Janssens (evafj@umich.edu), Department of Economics, University of Michigan

Many real-world problems involve random variables that are modeled as continuous-state stochastic processes. These processes serve as an input to larger, more complex models, which are typically analytically intractable. Numerically solving a model that features a continuous-state Markov process can be impossible or require complex calculations. To address this, researchers often use numerical methods to approximate the continuous process with a discrete one, leading to a Markov chain approximation.

Markov chain approximation involves simplifying complex probabilistic processes by using discrete models based on Markov assumptions. In these models, the future state depends only on the current state, without regard to prior states, which makes computations more tractable. Such approximations are commonly used in fields like physics, finance, and machine learning, providing a more practical approach for modeling complex stochastic systems.

In the paper by Tauchen (1986), the author used a discrete Markov chain by discretizing the state space into grid points and computing transition probabilities. A tensor grid is a multidimensional grid formed by the Cartesian product of one-dimensional grids. In high dimensional stochastic processes, tensor grids can discretize the state space, allowing Markov chain approximation to estimate transition probabilities. However, issues arise when tensor grids are used in higher dimensions, that is the curse of dimensionality. As the number of dimensions increases, the number of grid points and the number of possible states increase exponentially. For example, if you have a 1D grid with 10 points, a 2D tensor grid would have 100 points, and a 3D grid would have 1000 points. This rapid growth in the number of points makes computations inaccurate and infeasible in higher dimensions. To address this problem, this thesis adapts the spectral dimensionality reduction method of Zhang & Wang (2020) to efficiently reduce grid points for finite-state Markov chain approximations.

In high-dimensional tensor grids, reducing the number of points is often necessary to make computations more efficient. Previous work in the finite-state Markov chain approximation literature has primarily focused on reducing or eliminating points in areas with low pruning. As highlighted in Gordon (2021), tensor-grid discretization methods often address inefficiencies by either pruning states with very low probabilities or utilizing sparse grids to reduce computational costs. An alternative approach would be to ensure that grid points are adequately spaced, which is typically measured by the similarity of the transition matrix linking discretized states or by incorporating the steady-state distribution of the process. This alternative approach is explored in the spectral dimensionality reduction method of Zhang & Wang (2020), which focuses on the similarity of the transition matrix but ignores the actual placement of grid points and the values the states and grid points represent. This thesis explores if we can adapt the spectral dimension reduction method to the context of finite-state Markov chain approximations, and aims to enhance its performance for this specific application.

We will conduct simulations to evaluate the effectiveness of different approaches for approximating continuous-state stochastic processes, with a focus on adapting the spectral dimensionality reduction method of Zhang & Wang (2020) to finite-state Markov chain approximations. More specifically, we will explore potential improvements related to incorporating the steady-state distribution of the process and optimizing grid point placement. These adaptations aim to enhance both the accuracy and scalability of the spectral method, particularly in high-dimensional cases. The adapted spectral method we focus on will be compared to the Tauchen method and the spectral dimension reduction method, which will provide some insight into the effectiveness of dimensionality reduction techniques for finite-state Markov chain approximations.

Reference List

Gordon, G. (2021). Efficient VAR discretization. *Economics Letters*, *204*, 109872. https://doi.org/10.1016/j.econlet.2021.109872

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, *20*(2), 177–181. https://doi.org/10.1016/0165-1765(86)90168-0

Zhang, A., & Wang, M. (2020). Spectral State Compression of Markov Processes. *IEEE Transactions on Information Theory*, *66*(5), 3202–3231. https://doi.org/10.1109/tit.2019.2956737