How Does Education Affect the Housework Time of Husbands

Wonho Yeon[*][†]

Department of Economics
State University of New York at Stony Brook
wonho.yeon@stonybrook.edu

Version May 2019

Abstract

Using the 2015 Panel Study of Income Dynamics (PSID) data set, I estimate a collective model of family time allocation decisions. Traditional theories explain that higher education leads to less housework. However, in the data set, we see that more educated husbands take a higher share of the housework than less educated husbands, which has never been explained by the existing literature. I develop a theoretical model to examine how a husband's education affects his time at home and analyze the impact of education on the husband's housework time. My structural estimation results reveal that husbands' education elasticity of home productivity is greater than that of market productivity and even wives' education elasticity of domestic productivity. I find that the husband decreases his leisure time and increases time spent on housework and market labor as his educational attainment level increases. This fits well with the data.

JEL Classification: D13, D63, I26, J22, J24

Keywords: time allocation, collective model, home production, intra-household, structural estimation

1 Introduction

The basic motivation for this research is to understand how education affects a husband's housework time decisions. Research on intrahousehold time allocation has generally assumed that the spouse with the lowest ability to earn income in the market devotes more time to home production (**???**). Classical models explain that higher education leads to less time spent on domestic work because higher education would increase the opportunity cost of their time spent on housework. However, the recent empirical evidence demonstrates something different; data support the traditional theories' predictions for a wife's time allocation, but do not support the predictions for husbands. According to the 2015 Panel Study of Income Dynamics (PSID), in contrast to less-educated husbands, well-educated husbands do more housework not only in terms of the total amount of time but also in terms of the share of time between husbands and wives, holding wives' education level constant. This point has not been explored in the previous literature on household time allocation models. Therefore, I develop a model in order to examine this counter-intuitive phenomenon that well-educated husbands participate more in the household chores than less educated husbands.

Time devoted to housework is substantial. According to the PSID data set, on average, married-couple households in the United States reported spending 24 hours per week on housework in 2015. Husbands are responsible for approximately 32% of housework within the households in the U.S. Given a wife's educational attainment level, a husband with a college degree spends about 7.24 minutes more per day on housework compared to a husband with a high school degree. In terms of a husband's housework time share, there is a 4.2% point increase when a husband extends his education level from a high school

degree to a college degree (see Appendix A).

Interestingly, this phenomenon does not apply only to Americans. It is durable in different cultural contexts. For example, husbands' housework time in the U.S., Spain, and Korea all show similar patterns (**?**). Considering the social and cultural differences between these countries, I suggest that greater participation in housework from well-educated husbands is a result of economic decisions or rationality rather than cultural or sociological decisions. This research contributes to providing the structural analysis for this phenomenon by analyzing the impact of education on a husband's housework time and supports the theoretical framework with the most recent available PSID data set.

The analysis of husbands' and wives' time allocations involves three essential elements: (1) preferences of spouses, (2) resource and time constraints, including the household technology and the technologies of individual household members, (3) household governance structure which determines how preferences and constraints are transformed into time and resource allocation (**?**).

Regarding preferences, this research builds on the model by Byrne et al. (2009) which assumes that the individual utility function consists of private consumption, public good consumption, leisure time, and the time spent on informal health care to parents. Informal care and housework are similar in terms of how they enter into the model. Each spouse gets utility directly from spending time engaging in the activity and also enjoys the outcome. In terms of constraints, this study builds on the classical household production model developed by Gronau (1977). Each individual allocates his or her time between market work, housework, and leisure. Concerning the household bargaining structure, this research builds on the collective model developed by Chiappori (1997) and Apps and Rees

(1988). In addition to these conceptual frameworks, the structural estimation model also builds on Byrne et al. (2009).

The rest of this paper is organized as follows: Section 2 briefly introduces existing literature related to this topic. Section 3 develops the model. Section 4 describes the data. Section 5 discusses the estimation strategy including econometric specification. Section 6 reports the estimation results. Section 7 presents conclusions.

2   Literature Review

There are three dominant strands of literature regarding household behavior. The first and oldest is the unitary approach, where the household is considered as a unit with its own utility function (Becker, 1965). Since Becker provided this elegant framework for analyzing family decisions, time allocation has become central to labor economics. However, the unitary model is criticized because it implicitly assumes that all members in a multi-person household have identical preferences or that there is one decision maker. The optimal time allocation scheme of individuals depends solely on comparative advantages. The unitary model is inconsistent with empirical evidence and ignores the bargaining concerns that now play a central role in the economics of the family.

A number of alternatives to the unitary model have been put forward. Note that these alternative models have the individual utilities of the household members as arguments. The principal alternatives to the unitary approach are game theoretic. One alternative, the non-cooperative approach, assumes that household partners do not cooperate at all (**????**). In this case, each partner optimizes his or her own utility and takes the behavior of the other as a given. This approach yields a non-cooperative Nash equilibrium, where the presence

of public goods generates Pareto inefficient solutions. Another game theoretic approach is the cooperative Nash bargaining model (**??**). In this cooperative approach, husbands and wives are assumed to bargain over the allocation of resources. This involves laying out different options including non-cooperation and other outside options suce as divorce as a last resort. Nash proves that the axioms of scale invariance, symmetry, efficiency, and independence of irrelevant alternatives uniquely determine the solution for this bargaining model.

The third major alternative is the collective approach. According to this approach, husbands and wives have their own utility function and collectively choose Pareto efficient allocations. This model was developed by Chiappori (1988) and Apps and Rees (1988). The spouses behave as if the household optimizes a collective utility function (Chiappori, 1988, 1997; Apps and Rees, 1997; Browning and Chiappori, 1998 ; Browning, Chiappori, and Lechene, 2006).

Since the development of the collective approach in the 1980s, virtually all research on time use has been empirical rather than theoretical. In terms of empirical studies, literature can be broadly divided into two strands. The first is mainly focused on testing and refuting the assumptions of the unitary model (**????**). The second is based on sociology rather than economics. A significant amount of literature in sociology has analyzed trends how husbands and wives allocate their housework. Most of this research has involved examining the amounts of time spent by spouses disaggregated by family status, age, or other characteristics, with little focus on behavioral analysis (**?**).

Most of the empirical literature in this area estimates nonstructural equations and focuses on husbands and wives separately (**??????**). Empirical literature on spousal time al-

location decisions within a household and the impact of economic incentives is still lacking. This is because many time use surveys do not collect any information on wages, income, and expenditures, or they merely collect data about one individual in each household.

However, there are some exceptions that develop structural models of household decision making, which study spouses simultaneously. For example, Browning and Gortz (2012) estimated the relationship between leisure time and the allocation of expenditure using a collective model and the Danish Time Use Survey (DTUS). They found that wives who enjoy more leisure time relative to their husbands also have higher relative expenditures. Using the Dutch Longitudinal Internet Studies for the Social (LISS), Cherchye, De Rock, and Vermuelen (2012) found the spouses' preferences depend significantly on the consumption of domestically produced goods such as children's welfare. Del Boca, Flinn, and Wiswal (2013) estimated the impact that changes in the time input of mothers and fathers had on their children's cognitive development using the 1997-2002 Child Development Supplements of the Panel Study of Income Dynamics (PSID-CDS). Byrne et al. (2009) estimated a Nash equilibrium model of families' decisions referring to the supply of unpaid home care versus paid home health care for elderly family members using the Assets and Health Dynamics Among the Oldest Old (AHEAD) data set. The research I am presenting relates to the aforementioned research in terms of analyzing spouses simultaneously, and using structural models all relating to time usage.

3   Theoretical Model

The household consists of two members, 1 and 2, respectively husband and wife. Each spouse cares about the household private consumption X,[1] one single public good H, personal leisure time L_i, and time spent on producing a public good t_i with preferences given. The utility function of each spouse ($i = 1, 2$) takes the form,

$$U_i = \beta_{1i}\varepsilon_x ln(X) + \beta_{2i}ln(H) + \beta_{3i}\varepsilon_{L_i} ln(L_i) + \beta_{4i}\varepsilon_{t_i}t_i. \tag{1}$$

The coefficients $\beta_{1i}, \beta_{2i}, \beta_{3i,}$, and β_{4i} for $i = 1, 2$ may depend on observed husband and wife characteristics, and the errors $\varepsilon_x, \varepsilon_{L_i}$, and ε_{t_i} are functions of unobserved (to the econometrician) husband and wife characteristics. All variables, including errors, are common knowledge to both family members. Each spouse's utility depends positively on consumption of home-produced goods (H) as well as the sum of spouses' private consumption (X) and each spouse's leisure (L_i). However, it depends negatively on the time spent on housework (t_i). It is natural to think nobody prefers to do the household chores. Therefore, $\beta_{1i} \geq 0, \beta_{2i} \geq 0, \beta_{3i} \geq 0, \beta_{4i} < 0, \varepsilon_x \geq 0, \varepsilon_{L_i} \geq 0$, and $\varepsilon_{t_i} \geq 0$, for $i = 1, 2$.

Following Becker (1965), spouses are assumed to combine time and market goods to produce more basic commodities H that directly affect their utility functions. For example, making a fried egg requires time, eggs, a pan, a stove, and gas or electricity. The transformation of inputs into outputs is generally described by a household production function. In

---

[1]Because of a data restriction, I assume that each spouse cares about the household private consumption which is the sum of each spouse's private consumption such as his or her clothing, leisure, health, and transport expenditures. The data do not report each spouse's private expenditures but the sum.

my model, the household production function is

$$H = h(t_1, t_2, X_h)$$

$$= (1 + g_1 t_1)^{h_4} (1 + g_2 t_2)^{h_4} X_h^{h_3}, \tag{2}$$

where

$$h_4 = \frac{1 - h_3}{2},$$

$$g_1 = a_1 e_1^{h_1}, \tag{3}$$

$$g_2 = a_2 e_2^{h_2}. \tag{4}$$

Housework time of a husband is t_1, and housework time of a wife is t_2. The variables e_1 and e_2 indicate educational attainment levels of the husband and the wife respectively. I assume that human capital is "time augmenting" (**?**) in the sense that the time that each spouse allocates for household production is multiplied by a function of the spouse's human capital: $g_i(e_i)$, $i = 1, 2$. This home productivity function converts the spouses' human capital vectors into indexes that multiply the spouses' time inputs. In addition, a_1, and h_1 are parameters that measure the effects of education on a husband's home productivity, and a_2 and h_2 are parameters that measure the effects of education on a wife's home productivity. The variable X_h is the market goods that are used for producing a bundle of household public goods.

Throughout this analysis, I assume that the household production function satisfies strong essentiality of the market goods used for home production. This implies that positive

output requires positive X_h,

$$h(t_1, t_2, 0) = 0 \qquad \text{for all } \{t_1, t_2\},$$

which is also supported by the empirical evidence ($X_h > 0$). Regarding time inputs, I neither assume strong essentiality nor weak essentiality.[2] This implies that positive output does not require positive t_1 or t_2,

$$h(0, 0, X_h) > 0 \qquad \text{for all } X_h.$$

According to the data, there are households which allocate no time inputs but only purchase X_h to produce domestic goods.

There exists a standard problem with estimating the form of household production function; outputs usually cannot be observed or measured; only inputs can. Therefore, the production function cannot be estimated independently of auxiliary assumptions and the utility function unless the home-produced commodities are independently observable (**??**). Observability of outputs may be acceptable for children's health or education; it is less likely for general household commodities and services such as cooking, and cleaning. If only inputs are observed and not outputs, we may be able to recover information about the technology making supplemental assumptions such as marginal productivity of inputs, returns to scale, and assumptions on preferences (Browning, Chiappori, and Weiss, 2014).

The household production function is likely to exhibit constant returns to scale ($2h_4 + h_3 = 1$). However, I assume that the marginal product of each input is decreasing ($0 < h_3 < 1$ and $0 < h_4 < 1$). This is plausible since individuals may become tired or bored

---

[2]Weak essentiality implies that positive output requires positive time inputs from at least one spouse.

as they devote more time to one particular activity (e.g., cleaning), therefore causing them to become less productive. Consequently, each spouse's housework time exhibits diminishing marginal product.[3] More importantly, with diminishing marginal productivity, the Cobb-Douglas household production function and spouses' utility optimizing behaviors can imply nonspecialization. With two sectors, home and market, we say there is nonspecialization if and only if both spouses allocate time to both sectors (**?**). According to the recent data, in many households, husbands and wives participate in both the market sector and the household sector. For example, 62% of sample households are nonspecialized in the PSID 2001-2009.

The market productivity of each person is also a function of education. I assume the wage rate functions,

$$w_1 = m_1 e_1^{d_1} \varepsilon_{w_1}, \tag{5}$$

$$w_2 = m_2 e_2^{d_2} \varepsilon_{w_2}, \tag{6}$$

where m_1 and d_1 are parameters that measure the effects of education on a husband's wage rate, m_2 and d_2 are parameters that measure the effects of education on a wife's wage rate, and ε_{w_1} and ε_{w_2} are unobserved errors. These errors capture heterogeneity among individuals. The spouses in the model get to observe personal error terms prior to decision making, but the econometrician does not observe them.

Thus, the household's aggregate labor income can be written as

$$Y = \sum_{i \in 1,2} \varepsilon_{w_i} m_i e_i^{d_i} k_i, \tag{7}$$

---

[3]Gronau (1977) first introduced home production functions which are subject to decreasing marginal productivity due to fatigue.

where k_i is market work time of spouse i. Thus, Y is the sum of the husband's and wife's labor income. Including household non-labor income v, the budget constraint is

$$\sum_{i \in 1,2} \varepsilon_{w_i} m_i e_i^{d_i} k_i + v = pX + qX_h. \tag{8}$$

As defined earlier, X is the sum of each spouse's private consumption, and X_h is the bundle of market goods that is used for home production. p is the price of X, and q is the price of X_h.

Each spouse's time constraints are

$$1 = k_i + L_i + t_i, \qquad i = 1, 2.$$

I normalize total time available at one. This implies that $k_i = 1 - L_i - t_i$. The standard non-negativity constraints apply here as well: $k_i \geq 0$, $t_i \geq 0$, and $L_i > 0$. In particular, I assume $0 < L_i \leq 1$ since no person can survive without leisure, yet one can spend all of their time in leisure. This assumption is consistent with the data and used again later when I define the corner solutions.

I allow for communication and cooperation between spouses to achieve Pareto-efficient allocation. In other words, the household must maximize the utility of one spouse (U_1) subject to the other (U_2) achieving at least a given utility (U^{2*}) and to budget and time constraints (**??**):

$$U^2 \geq U^{2*}. \tag{9}$$

In brief, if we assume collective behavior of spouses, an efficient allocation must solve

for [4]

$$\max_{t_1,t_2,L_1,L_2,X_h} \quad U_1(X,H,L_1,t_1)$$

$$\text{subject to} \quad U^2(X,H,L_2,t_2) \geq U^{2*},$$

$$\sum_{i \in 1,2} \varepsilon_{w_i} m_i e_i^{di}(1 - L_i - t_i) + v = pX + qX_h,$$

$$H = (1 + a_1 e_1^{h_1} t_1)^{h4}(1 + a_2 e_2^{h_2} t_2)^{h4} X_h^{h3},$$

$$1 = t_1 + L_1 + k_1,$$

$$1 = t_2 + L_2 + k_2.$$

After substituting the budget constraint into both the husband's and the wife's utility function, the Lagrangian is

$$\mathcal{L} = \beta_{11}\varepsilon_x ln\left(\frac{\sum_{i \in 1,2} \varepsilon_{w_i} m_i e_i^{di}(1 - L_i - t_i) + v - qX_h}{p}\right) + \beta_{21} ln(H)$$

$$+ \beta_{31}\varepsilon_{L_1} ln(L_1) + \beta_{41}\varepsilon_{t1} t_1 + \mu\left[\beta_{12}\varepsilon_x ln\left(\frac{\sum_{i \in 1,2} \varepsilon_{w_i} m_i e_i^{di}(1 - L_i - t_i) + v - qX_h}{p}\right)\right.$$

$$\left. + \beta_{22} ln(H) + \beta_{32}\varepsilon_{L_2} ln(L_2) + \beta_{42}\varepsilon_{t2} t_2 - U^{2*}\right].$$

This equation can be interpreted as a weighted utilitarian social welfare function (Apps and Rees, 2007). The Lagrangian multiplier or the weight, μ is discussed in more detail later

---

[4]Here, I maximize a husband's utility subject to resource, time, and Pareto efficient allocation constraints.

in Section 5. The first order conditions (FOCs) are

$$\frac{\partial \mathscr{L}}{\partial t_1} : \quad \mu \left\{ \frac{\beta_{22} h_4 a_1 e_1^{h_1}}{a_1 e_1^{h_1} t_1 + 1} - \frac{\beta_{12} \varepsilon_x (m_1 e_1^{d_1} \varepsilon_{w_1})}{pX} \right\}$$

$$+ \frac{\beta_{21} h_4 a_1 e_1^{h_1}}{a_1 e_1^{h_1} t_1 + 1} + \beta_{41} \varepsilon_{t1} - \frac{\beta_{11} \varepsilon_x (m_1 e_1^{d_1} \varepsilon_{w_1})}{pX} = 0, \tag{10}$$

$$\frac{\partial \mathscr{L}}{\partial t_2} : \quad \mu \left\{ \frac{\beta_{22} h_4 a_2 e_2^{h_2}}{a_2 e_2^{h_2} t_2 + 1} + \beta_{42} \varepsilon_{t2} - \frac{\beta_{12} \varepsilon_x (m_2 e_2^{d_2} \varepsilon_{w_2})}{pX} \right\}$$

$$+ \frac{\beta_{21} h_4 a_2 e_2^{h_2}}{a_2 e_2^{h_2} t_2 + 1} - \frac{\beta_{11} \varepsilon_x (m_2 e_2^{d_2} \varepsilon_{w_2})}{pX} = 0, \tag{11}$$

$$\frac{\partial \mathscr{L}}{\partial \mathscr{L}_1} : \quad -\frac{\mu \beta_{12} \varepsilon_x (m_1 e_1^{d_1} \varepsilon_{w_1})}{pX} - \frac{\beta_{11} \varepsilon_x (m_1 e_1^{d_1} \varepsilon_{w_1})}{pX} + \frac{\beta_{31} \varepsilon_{L_1}}{L_1} = 0, \tag{12}$$

$$\frac{\partial \mathscr{L}}{\partial \mathscr{L}_2} : \quad \mu \left\{ \frac{\beta_{32} \varepsilon_{L_2}}{L_2} - \frac{\beta_{12} \varepsilon_x (m_2 e_2^{d_2} \varepsilon_{w_2})}{pX} \right\} - \frac{\beta_{11} \varepsilon_x (m_2 e_2^{d_2} \varepsilon_{w_2})}{pX} = 0, \tag{13}$$

$$\frac{\partial \mathscr{L}}{\partial X_h} : \quad \mu \left\{ \frac{\beta_{22} h_3}{X_h} - \frac{\beta_{12} \varepsilon_x q}{pX} \right\} + \frac{\beta_{21} h_3}{X_h} - \frac{\beta_{11} \varepsilon_x q}{pX} = 0, \tag{14}$$

where

$$pX = (1 - t_1 - L_1) m_1 e_1^{d_1} \varepsilon_{w_1} + (1 - t_2 - L_2) m_2 e_2^{d_2} \varepsilon_{w_2} + v - qX_h.$$

From Equation (14), we can derive the equation of ε_x as a function of parameters and variables as

$$\varepsilon_x = \frac{(\mu \beta_{22} + \beta_{21})(Y + v - qX_h) h_3}{(\mu \beta_{12} + \beta_{11}) qX_h}. \tag{15}$$

By plugging Equation (15) into Equation (10) and Equation (12), we can obtain the set of FOCs for the husband as described in Table 3. ~~Table 3 "Int'l" denotes an interior solution, and "Cor"[5] denotes a corner solution.~~ I disaggregate households into four types based on a husband's time allocation decision: (1) a husband participates in leisure, housework, and market labor, (2) a husband partici-

---

[5]Specifically, corners are (1) $t_i = 0, L_i > 0, k_i > 0$; (2) $k_i = 0, t_i > 0, L_i > 0$; (3) $L_i = 1, t_i = 0, k_i = 0$.

Table 1: FOCs for Husband

					FOCs for Husband		
	Cases					FOCs	
L_1	t_1	k_1	X_h	X_h	t_1	L_1	
Int	Int	Int	Int	$\varepsilon_x = T^x$	$\varepsilon_{t_1} = T_1^w(t_1)$	$\varepsilon_{L_1} = T_1^L$	
Int	Cor	Int	Int	$\varepsilon_x = T^x$	$\varepsilon_{t_1} \leq T_1^w(0)$	$\varepsilon_{L_1} = T_1^L$	
Int	Int	Cor	Int	$\varepsilon_x = T^x$	$\varepsilon_{t_1} = T_1^{nw}(t_1, \varepsilon_{L_1})$	$\varepsilon_{L_1} = T_1^L$	
Cor	Cor	Cor	Int	$\varepsilon_x = T^x$	$\varepsilon_{t_1} \leq T_1^{nw}(0, \varepsilon_{L_1})$	$\varepsilon_{L_1} \geq T_1^L$	

pates in leisure and market labor, (3) a husband participates in leisure and housework, (4) a husband spends his time only in leisure. The right three columns in Table 3.1 show the corresponding FOCs when we observe particular types of households. For example, households in which husbands spend their time in all three activities (leisure, housework, market labor), error terms ε_x, ε_{t_1}, and ε_{L_1} should satisfy the conditions of $\varepsilon_x = T^x$, $\varepsilon_{t_1} = T_1^w(t_1)$, and $\varepsilon_{L_1} = T_1^L$ with

$$T^x = \frac{h_3(\mu\beta_{22} + \beta_{21})}{\mu\beta_{12} + \beta_{11}} \frac{(Y + v - qX_h)}{qX_h}$$

$$T_1^w(t_1) = \frac{(\mu\beta_{22} + \beta_{21})}{\beta_{41}} \left(\frac{h_3 w_1}{qX_h} - \frac{g_1 h_4}{g_1 t_1 + 1} \right)$$

$$T_1^{nw}(t_1, \varepsilon_{L_1}) = \frac{\beta_{31} \varepsilon_{L_1}}{L_1} - \frac{(\mu\beta_{22} + \beta_{21}) g_1 h_4}{\beta_{41}(g_1 t_1 + 1)}$$

$$T_1^L = \frac{(\mu\beta_{22} + \beta_{21})}{\beta_{31}} \frac{h_3 w_1 L_1}{qX_h}.$$

Similarly, by plugging Equation (15) into Equation (11), and Equation (13), we can get

the set of FOCs for the wife as described in Table 3.2.

Table 2: FOCs for Wife

				FOCs for Wife			
	Cases				*FOCs*		
L_2	t_2	k_2	X_h	X_h	t_2	L_2	
Int	Int	Int	Int	$\varepsilon_x = T^x$	$\varepsilon_{t_2} = T_2^w(t_2)$	$\varepsilon_{L_2} = T_2^L$	
Int	Cor	Int	Int	$\varepsilon_x = T^x$	$\varepsilon_{t_2} \leq T_2^w(0)$	$\varepsilon_{L_2} = T_2^L$	
Int	Int	Cor	Int	$\varepsilon_x = T^x$	$\varepsilon_{t_2} = T_2^{nw}(t_2, \varepsilon_{L_2})$	$\varepsilon_{L_2} = T_2^L$	
Cor	Cor	Cor	Int	$\varepsilon_x = T^x$	$\varepsilon_{t_2} \leq T_2^{nw}(0, \varepsilon_{L_2})$	$\varepsilon_{L_2} \geq T_2^L$	

Again, I decompose households into four types based on a wife's time allocation decision: (1) a wife spends time in leisure, housework, and market labor, (2) a wife participates in leisure and market labor, (3) a wife participates in leisure and housework, (4) a wife spends his time only in leisure. The right three columns in Table 3.2 show the corresponding FOCs when we observe particular types of households. For instance, households in which wives spend their time in all three activities (leisure, housework, market labor), error terms ε_x, ε_{t_2}, and ε_{L_2} should satisfy the conditions of $\varepsilon_x = T^x$, $\varepsilon_{t_2} = T_2^w(t_2)$, and $\varepsilon_{L_2} = T_2^L$ with

$$T^x = \frac{h_3(\mu\beta_{22} + \beta_{21})}{\mu\beta_{12} + \beta_{11}} \frac{(Y + v - qX_h)}{qX_h}$$

$$T_2^w(t_2) = \left(\frac{\mu\beta_{22} + \beta_{21}}{\mu\beta_{42}}\right) \left(\frac{h_3 w_2}{qX_h} - \frac{g_2 h_4}{g_2 t_2 + 1}\right)$$

$$T_2^{nw}(t_2, \varepsilon_{L_2}) = \frac{\mu\beta_{32}\varepsilon_{L_2}}{L_2} - \frac{(\mu\beta_{22} + \beta_{21})g_2 h_4}{\mu\beta_{42}(g_2 t_2 + 1)}$$

$$T_2^L = \frac{(\mu\beta_{22} + \beta_{21})}{\mu\beta_{32}} \frac{h_3 w_2 L_2}{qX_h}.$$

15

In addition, according to the market productivity function defined in the model, we know the functions of ε_{w_i} are

$$\varepsilon_{w_i} = \frac{w_i}{m_i e_i^{d_i}} \qquad i = 1, 2.$$

To summarize, I define the set of FOCs corresponding to solutions to FOCs as

$$\varepsilon = \zeta(\chi), \tag{16}$$

where ε is the vector of errors $(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2})$, χ is the vector of endogenous variables $(Y, pX, qX_h, t_1, L_1, t_2, L_2)$ and exogenous variables (v, e_1, e_2, w_1, w_2), and $\zeta(\cdot)$ is the vector of functions implied by the FOCs summarized above. We can use these FOCs to construct the likelihood contribution for each household. First, for those elements of χ corresponding to interior solutions, the relevant likelihood term is the probability density of the corresponding element of ε. Secondly, for those elements of χ corresponding to corner solutions, the relevant likelihood term is the cumulative distribution function with upper boundary or with lower boundary of the corresponding element of ε, depending upon the nature of the corner solution. As a result, I can estimate the set of parameters that maximizes the probability of observing the particular error terms linked to particular χ and ζ.

The system of equations forming my structural collective model is highly nonlinear. Therefore, it is possible that the model has several local maxima. Since we cannot ignore multiple maxima analytically, I check for multiple maxima by solving the optimization problem with multiple starting points and checking for instability of the observed maxima.

After checking and observing robustness in observed maxima, I selected the highest local maximum found.

4 Data

In this research, I use the 2015 wave of the Panel Study of Income Dynamics (PSID)[6] data set to estimate my model. The PSID is a nationally representative longitudinal household data set that includes information on family members, such as data covering employment, income, expenditures, marriage, education, time usage, and numerous other topics. In terms of time usage, the PSID is not a typical type of time diary survey: it collects only market work time[7] and housework time.[8] However, there is an important reason why I choose the PSID data set to do this research. Besides being able to obtain detailed financial characteristics of household members, the PSID data set is the only source that gives information about both husbands' and wives' time usage within the same U.S. household.

Although there has been a growing number of economists using the American Time Use Survey (ATUS)[9] for their time allocation research, and the ATUS has a large sample size with full information about time diary of individuals living in the U.S., it has a critical limitation. The ATUS data set lacks the information about spouse's time allocation in

---

[6]https://psidonline.isr.umich.edu/

[7]The specific question is "On average, how many hours a week did spouse work on all of (your/his/her) (job/jobs) during 2014?" (ER60171, ER60434)

[8]The specific question is "How much time husband and wife spent on housework including cooking, cleaning, and doing other work around the house?" (ER60689, ER60691)

[9]There are limited resources for economists who study time allocation problems. Before the American Time-Use Survey (ATUS) began collecting the data in 2003, there were only small number of one-shot time-use surveys at odd intervals, such as the National Survey of Families and Households which was only done three times: 1987-1988, 1992-1994 and 2001-2003. Other than those, there has been a near absence of time allocation data in the United States (Hamermesh and Pfann, 2005). The first estimates of the ATUS were published in late 2004.

Table 3: Dropped Households

Selection Criteria	Number of Observations
Total households	9,048
Single	5,145
Location, foreign	23
Missing education variable	176
Missing housework time	34
Missing market work time	11
Missing income variable	26
Missing marital information	4
Coding error (negative private consumption)	136
Sample size	3,493

each household. It collects only one respondent's time usage data per sampled household. Because of this restriction, it is not possible to analyze husbands' and wives' time allocation decisions in a given household using the ATUS data set. Consequently, I use the most recent available PSID data set instead for this research.

The 2015 PSID includes a nationally representative sample of 9,048 households. I use 3,493 of the 9,048 households in this research. The summary statistics of how I excluded households can be found in Table 4.1. Since the focus of this research is to study the time allocation decisions of husband and wife, I restrict the sample to individuals who are married. The marriage condition reduces the sample to 3,903 households. Secondly, I remove the households that do not have background information on a husband's and a wife's educational attainment level, income, housework time, and household's location. This brings the sample size to 3,629. Lastly, I drop households that reported negative household private consumption.

Table 4.2 contains information on the characteristics of sample households. Among

Table 4: Selected Characteristics of Households

Characteristic	Households
Live with children	0.48
Number of children	0.97
Living with children under age 5	0.21
Assortative mating	0.73
Metro	0.82
Market Work	
Both working	0.57
Only husband working	0.22
Only wife working	0.09
No one working	0.12
Housework	
Both working	0.89
Only husband working	0.01
Only wife working	0.10
No one working	0.00
Husband's housework time $\leq$ Wife's housework time ($t_1 \leq t_2$)	0.89

3,493 sample households, 1,807 households (52%) do not live with children under 18 years of age and 1,686 households live with children under 18 years of age. On average, households have 0.97 children under the age of 18, 21% of the sample live with children under the age of five.[10] To check for educational assortative mating, I broadly divide the sample into two groups: a low education group (below college level) and a high education group (college level and above). Based on this categorization, I observe husbands and wives sharing similar education levels in 73% of the sample households. Eighty-two percent of the sample live in metropolitan areas.[11]

---

[10]Households with children have on average 2.0 children under age of 18, and 44% of them live with children under the age of five.

[11]According to the PSID, this indicator is derived from the 2013 Beale-Ross Rural-Urban Continuum Codes published by USDA based on matches to the FIPS state and county codes: Beale-Ross Rural-Urban Continuum Codes 1-3 is for metropolitan areas and 4-9 is for non-metropolitan areas.

Table 5: Selected Characteristics of Respondents

Characteristic	Husband	Wife
Age	48.53	46.56
Education	13.79	14.11
Wage rate	26.89	17.51
Time (hours per day)		
housework	1.10	2.34
market work	5.37	3.87
Leisure	17.54	17.78
Time Share		
housework ($t_i/(t_1+t_2)$)	0.32	0.68
leisure ($L_i/(L_1+L_2)$)	0.50	0.50
market work ($k_i/(k_1+k_2)$)	0.60	0.40

Regarding market work, only 31% of households exhibit market sector specialization[12]: 22% of households reported the husband was the only one currently working, and 9% of households replied that only the wife was working in the labor market. Fifty-seven percent of households reported that both husbands and wives were working. Lastly, 12% of households reported that neither the wife nor husband was currently working. Most of these households are retired, considering the average age of husbands is 68, and that of wives is 65. In terms of housework, specialization seems even weaker in the U.S. households. According to the PSID, the definition of housework used in the survey is time spent cooking, cleaning, and doing other work around the house. The percentage of the households reporting both husband and wife doing housework is 89%. On average, wives devote more hours than husbands in the majority of families. The fraction of households in which wives spend more housework time than husbands is 89%.

---

[12]In this paper, specialization is a corner solution in which one spouse works only in the household and the other works only in the market.

More specific characteristics of husbands and wives in the sample are shown in Table 4.3. On average, husbands are 48.53 years of age, have 13.79 years of education, and earn $26.89 per hour. On average, wives are 46.56 years of age, have 14.11 years of education, and earn $17.51 per hour. The average husband spends 1.10 hours a day on household chores, while the average wife spends 2.34 hours per day on housework. In terms of time share, the average husband does 32% of total housework, and 60% of market work in the average household. Leisure share is divided equally.

Since the goal of this analysis is to examine and determine how education affects husbands' time allocation decisions, I summarize the simple regression results as a descriptive statistics in Table 4.4. The purpose of using robust OLS regression here is to control wives' education level and see how a husband's education level affects his housework time and time share.

I break down the education levels into five categories: (1) below high school or high school dropouts, (2) high school graduates, (3) 2-year college graduates, (4) 4-year college graduates, and (5) graduate or professional degrees including law and medical degrees. Then, I control the wives' education level, wives' housework time, age of husbands, and the number of children in the households. The purpose is to see how personal education level affects a husband's housework time. The results are shown in Table 4.4.

The husband's education level is positively and statistically significantly associated with the housework time and the time share. The coefficients in the first and second column are increasing with the education level of the husband e_1. The same phenomenon can be observed in Spain and Korea (see Appendix A). In Spain, holding a wife's educational level constant, a husband with a college degree spends about 10.32 minutes (0.172 hours) more

Table 6: Descriptive OLS Model

	t_1	$Share_{t_1}$	L_1	$Share_{L_1}$	k_1	$Share_{k_1}$
	Dependent variable: Husband's Time and Time Share					
Husband's Education						
$e_1 = 2$	0.046	0.007	−0.445***	−0.006**	0.473***	0.039***
	(0.049)	(0.013)	(0.149)	(0.003)	(0.153)	(0.015)
$e_1 = 3$	0.120**	0.035***	−0.563***	−0.008***	0.509***	0.035**
	(0.053)	(0.014)	(0.160)	(0.003)	(0.164)	(0.016)
$e_1 = 4$	0.161***	0.041***	−0.675***	−0.014***	0.577***	0.075***
	(0.053)	(0.013)	(0.160)	(0.003)	(0.164)	(0.016)
$e_1 = 5$	0.162***	0.043***	−0.915***	−0.020***	0.791***	0.098***
	(0.059)	(0.015)	(0.179)	(0.003)	(0.183)	(0.018)
Wife's Education						
$e_2 = 2$	0.108*	0.037**	0.080	0.004	−0.168	−0.109***
	(0.057)	(0.015)	(0.174)	(0.003)	(0.179)	(0.018)
$e_2 = 3$	0.079	0.028*	−0.253	0.004	0.260	−0.117***
	(0.060)	(0.015)	(0.181)	(0.003)	(0.186)	(0.018)
$e_2 = 4$	0.187***	0.070***	−0.154	0.009***	0.081	−0.153***
	(0.061)	(0.016)	(0.185)	(0.003)	(0.192)	(0.019)
$e_2 = 5$	0.185***	0.086***	−0.025	0.014***	−0.017	−0.193***
	(0.065)	(0.017)	(0.196)	(0.003)	(0.204)	(0.020)
Husband's age	0.001	−0.001***	0.077***	0.001***	−0.089***	−0.001*
	(0.001)	(0.0003)	(0.003)	(0.0001)	(0.004)	(0.0003)
No. of Children	0.009	−0.014***	−0.095**	−0.002**	0.087**	0.021***
	(0.013)	(0.003)	(0.038)	(0.001)	(0.040)	(0.004)
t_2	0.114***					
	(0.008)					
L_2			0.232***			
			(0.015)			
k_2					0.118***	
					(0.016)	
Observations	3,493	3,488	3,493	3,493	3,493	3,176

Note:
1. *p<0.1; **p<0.05; ***p<0.01.
2. Standard errors in parenthesis.
3. $e_i = 1$ is below high school or high school dropouts (the reference level); $e_i = 2$ is high school graduates; $e_i = 3$ is 2-year college graduates; $e_i = 4$ is 4-year college graduates; $e_i = 5$ is M.A., Ph.D., MD, DDS, DVM, DO (medical degrees), and LLB, JD (law degrees).
4. t_2 is housework time, L_2 is leisure, and k_2 is market work time of a wife.
6. No. of Children: number of children.

per day on housework compared to a husband with a high school degree. In terms of time share, this is a 2.8% point more than the housework share of a husband with high school diploma. In Korea, a husband with college degree participates in housework 2.22 minutes more per day and a 1% point more in terms of time share compared to a husband with high school degree.

In contrast, the husband's education level is negatively and significantly associated with the leisure time and the leisure time share. The coefficients in the third and fourth column of Table 4.4 are both decreasing with e_1. The husband's education level is positively and meaningfully related to the market work time and the market labor time share. The coefficients in the last two columns are increasing with e_1. In brief, these regression results show that husbands with higher levels of education reduce their leisure time and spend more time on both market labor and housework.

The bottom rows of the first column of Table 4.4 show that the husband's age and the number of children in the household are not significantly associated with the husband's housework time. However, the husband's age and the number of children are statistically significant and negatively associated with housework time share of the husband. Combining these facts, we know that the wife increases her housework time as her husband gets older or the number of children in the household increases.

Regarding the husband's age (age_1), it is positively correlated to leisure time and time share; and negatively correlated to market work time and time share. Regarding the number of children, it is negatively correlated to the husband's leisure time and time share; and positively associated with market labor time and time share. This implies that increase in the number of children makes husbands reduce their leisure time and work more in the

market.

5 Estimation Strategy

5.1 Empirical Specification

The set of parameters to estimate includes the home productivity parameters, the market productivity parameters, the preference parameters of husbands and wives respectively, the household production parameter, and the elements in the covariance matrix of the error terms,

$$\theta = (a_1, h_1, m_1, d_1, a_2, h_2, m_2, d_2,$$

$$\beta_{11}, \beta_{21}, \beta_{41}, \beta_{12}, \beta_{22}, \beta_{32}, \beta_{42},$$

$$h_3, c_{11}, \ldots, c_{77}, z).$$

For identification purpose, I need a setting to support positive error terms. In particular, I assume that

$$ln(\varepsilon) \sim iidN(0, \Omega),$$

where $\varepsilon = (\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2})$.

$$D = \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} & 0 & 0 & 0 \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & 0 & 0 \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & 0 \\ c_{71} & c_{72} & c_{73} & c_{74} & c_{75} & c_{76} & c_{77} \end{bmatrix}.$$

Regarding the covariance matrix of the error terms (Ω), it is symmetric and positive semidefinite, so that I model the Cholesky decomposition of it. After decomposing the Ω, I estimate the lower traingular matrix (D) to obtain the whole covariance matrix. This way, it is necessary to estimate only 28 parameters instead of 49 parameters. In addition, the Cholesky decomposition provides unconstrained regression parameters. The covariance matrix of the logarithms of vector of error terms is DD^T ($\Omega = DD^T$), where

Based on the utility functions (Equation (1)), increasing each β_{ji} ($j = 1, 2, 3, 4$, and $i = 1, 2$) term simultaneously has no effect on the FOCs, therefore, I set husband's preference on leisure $\beta_{31} = 1$ with no loss in generality.

The set of data for each household ($n = 1, 2, \ldots, N$. The family subscript n is suppressed) is

$$\{e_1, e_2, t_1, k_1, w_1, t_2, k_2, w_2, Y, v, qX_h\}.$$

Regarding the education variables (e_1, e_2), I use continuous measure of education in computing the likelihood function: from 1 year of education to 18 years of education.

25

Secondly, I define families' expenditures for home production (qX_h) as the sum of housing expenditures,[13] food at home expenditures, and childcare expenditures. I calculate the household private consumption (pX) by subtracting qX_h from the total household income ($Y + v$) which is reported in the PSID data. I also compute L_i by subtracting the sum of housework time and the market work time from one ($L_i = 1 - t_i - k_i$). In this analysis, the unit of time variables (t_i, k_i, L_i) is hours per day and wage rate (w_i) represents hourly wage rate. For consistency, I use daily income and daily expenditures (Y, v, qX_h, pX) for my estimation.

5.2 Lagrangian Multiplier μ

In order to model the decision-making process of the households, information on the Lagrange multiplier μ is necessary. μ is the husband's marginal utility from relaxing the efficiency constraint that wife's utility $U^2 \geq U^{2*}$.[14] μ increases with U^{2*}. This reflects the bargaining or the caring in the family. If a wife has greater bargaining power, then μ increases. We can also say that if the husband is more concerned with his wife's utility then μ increases.

The sex ratio[15] has been widely used as a measure of distributional factors that affect μ (**?**). In their model, a reduction in the sex ratio increases men's bargaining power within the household and also in the marriage market. Economic implications of changing sex ratios can be traced back to Becker (1991, chap. 3), who emphasized that the marriage market is an important determinant of intrahousehold utility distribution. In his approach, the state

---

[13]This includes expenditures for mortgage and loan payments, rent, property tax, insurance, gas, electricity, water and sewer, cable TV, telephone, internet charges, home repairs, and home furnishings.

[14]This is condition (9) in this paper.

[15]The sex ratio is the ratio of single males to single females.

of the marriage market substantially depends on the sex ratio, that is, the relative supplies of males and females in the marriage market. When the sex ratio is favorable to the female, meaning there is a relative scarcity of women, the distribution of gains from the marriage will be shifted in her favor (higher μ in my model).

There are several recent papers asserting that the cohort-level sex ratio has effects on females' bargaining power in the household. Angrist (2002) exploited intertemporal variation in migration flows to examine the effect of sex ratios on family structure and economic variables in the first half of the twentieth century in the U.S. He found that high sex ratios improved female marriage prospects, and lowered female labor force participation. Iyigun and Walsh (2007) provide a model in which asymmetries in the sex ratios in the marriage markets produce gender differences in premarital investments and consumption. Wei and Zhang (2011) demonstrate that high sex ratios encourage male entrepreneurship. In response to more intense competition for girls in marriage markets, parents of male offspring accumulate more savings in China. More recently, Abramitzky, Boustan, and Eriksson (2012) examined the severe impact that World War I had on a number of French men to show that the sex ratio has a great effect on marriage market outcomes as well as assortative mating. They found that men are less likely to marry women of lower social classes in regions with lower sex ratios.

In light of these studies, I assume μ is the function of the sex ratio in this research.

$$\mu = exp(sex\ ratio \cdot z)$$

In particular, in order not to restrict the lower and upper boundaries of μ, I assume that $log(\mu) = sex\ ratio \cdot z$ and the parameter z is also estimated. Moreover, because the PSID

provides limited information on the sex ratio, I use a much larger data set, the 2015 American Community Survey, [16] and combine it with the 2015 PSID data. The state of residence reported in the PSID was matched to state-level data on male and female single populations age over 18[17] from the 1,192,693 observations in the 2015 American Community Survey. In addition, the sex ratio I use here is computed not only by the state of residence but also by the level of education in order to reflect the positive educational assortative mating that we see in the PSID data set (see Appendix C for details).

5.3 Identification

The data is sufficiently rich to identify all the parameters of the model. The preference parameters β_{11}, β_{21}, β_{41}, β_{12}, β_{22}, β_{32}, and β_{42} are identified by covariation between spouse characteristics and his or her choices. Market productivity parameters are identified by the variation in wage rates across husbands and wives. Home productivity parameters are identified by the variations in education levels across husbands and wives that go into household production function (H) and by the variations in the predetermined variables such as non-labor income and wage rates of husbands and wives that does not enter household production function (H) directly. Lastly, the second moment terms are identified by variances and correlations of generalized residuals associated with the likelihood function (**?**).

---

[16]Steven Ruggles, Katie Genadek, Ronald Goeken, Josiah Grover, and Matthew Sobek. Integrated Public Use Microdata Series: Version 7.0 [dataset]. Minneapolis, MN: University of Minnesota, 2017. https://doi.org/10.18128/D010.V7.0

[17]Only singles are considered as potential competitors

5.4 The Likelihood Function

Based on the model FOCs for husbands and wives, there are 16 possible types of households. I decompose 3,493 households into these 16 types, and compute each household's likelihood contribution to form the log-likelihood function $LL(\theta)$ as

$$LL(\theta) = \sum_{n=1}^{N} \sum_{j=1}^{16} t_{nj} \cdot ln(P_{nj}), \tag{17}$$

where $t_{nj} = 1$ if the type of household is j ($j = 1, 2, ..., 16$) and zero otherwise. P_{nj} represents the probability density of observing the dependent variables associated with that particular type. Among 16 types, 12 need to be simulated, and I use the pseudo-GHK (Geweke, Hajivassiliou, Keane) simulator (see Appendix B for details).

6 Estimation Results

Table 6.1 presents the estimation results of parameters. An examination of Table 6.1 suggests the following.

6.1 Preferences

First, husbands and wives exhibit different preferences ($\beta_{11} \neq \beta_{12}$, $\beta_{21} \neq \beta_{22}$, $\beta_{31} \neq \beta_{32}$, $\beta_{41} \neq \beta_{42}$). This implies a rejection of the common preference models which assume household members share the identical preferences. Among household private consumption, a bundle of domestic goods, leisure, and housework time, wives prefer leisure time the most ($\beta_{32} > \beta_{12} > \beta_{22} > \beta_{42}$), while husbands care about household consumption the most ($\beta_{11} > \beta_{31} > \beta_{21} > \beta_{41}$).

Table 7: Estimates of Model

	Husband		Wife	
	Parameter	Estimate	Parameter	Estimate
Preference				
Household Consumption	β_{11}	4.777**	β_{12}	2.814**
		(0.196)		(0.194)
Domestic Goods	β_{21}	0.012**	β_{22}	0.017**
		(0.000)		(0.000)
Leisure	β_{31}	1.000	β_{32}	8.700**
		(Restricted)		(0.347)
Housework Time	β_{41}	−2.454**	β_{42}	−2.773**
		(0.044)		(0.064)
Home Productivity ($g_i = a_i e_i^{h_i}$)				
General scale parameter	a_1	0.207**	a_2	1.570**
		(0.014)		(0.402)
Elasticity with respect to education	h_1	1.408**	h_2	0.901**
		(0.025)		(0.096)
Market Productivity ($w_i = m_i e_i^{d_i}$)				
General scale parameter	m_1	1.393**	m_2	0.512**
		(0.035)		(0.007)
Elasticity with respect to education	d_1	1.317**	d_2	1.606**
		(0.009)		(0.005)
Bargaining Power				
Bargaining parameter	z	0.014**		
		(0.026)		

Notes:
1. Standard errors in parentheses.
2. *p<0.1; **p<0.05.
3. $h_4 = \frac{1-h_3}{2}$.
4. The log likelihood value is -76227.31.
5. Elasticity of home production with respect to X_h: $h_3 = 0.011^{**}(0.000)$
6. Elasticity of home production with respect to each spouse's time: $h_4 = 0.495$ (Restricted)

Secondly, both spouses acquire utility directly from the time spending for home production as well as from the consumption of final home-produced goods. We observe that time engaged in housework has a negative and statistically significant effect on both spouses' utility. This implies that both husbands and wives dislike spending time on domestic work,

wives more so than husbands ($\beta_{41} > \beta_{42}$). Considering that wives care more about the production (and consumption) of home-produced goods than husbands ($\beta_{12} < \beta_{22}$), this result is quite interesting. Probably, a wife tends to prefer purchasing X_h to spending her time in order to produce domestic goods.

In detail, the estimation results indicate that husbands are more concerned with the sum of household private consumption as compared to wives ($\beta_{11} > \beta_{12}$). Regarding the domestic good, wives care more about it than husbands ($\beta_{12} < \beta_{22}$). However, in terms of preference magnitude, the results show that both husbands and wives have very similar attitudes toward home-produced goods. Their preferences for the domestic goods are noticeably less, relative to their preferences for private consumption. This shows that individual consumption is far more important to both spouses than the consumption of public goods.

Lastly, the estimation results show that the preference for leisure is greater for wives than for husbands ($\beta_{31} < \beta_{32}$).

6.2   Education and Productivity

To provide a clear picture of productivity functions of husbands and wives, besides Table 6.1, I present Figure 6.1, Figure 6.2, Figure 6.3, and Figure 6.4. The top panel of Figure 6.1 illustrates the home productivity functions of husbands and wives. The bottom panel of Figure 6.1 shows the market productivity functions of husbands and wives. In addition, to better understand the education elasticity of home productivity and that of market productivity of each spouse, I provide the log-log plot in Figure 6.2. In Figure 6.3, I compare the home productivity and market productivity of husbands and wives respectively. In Figure

Figure 1: Home Productivity and Market Productivity, Husband and Wife
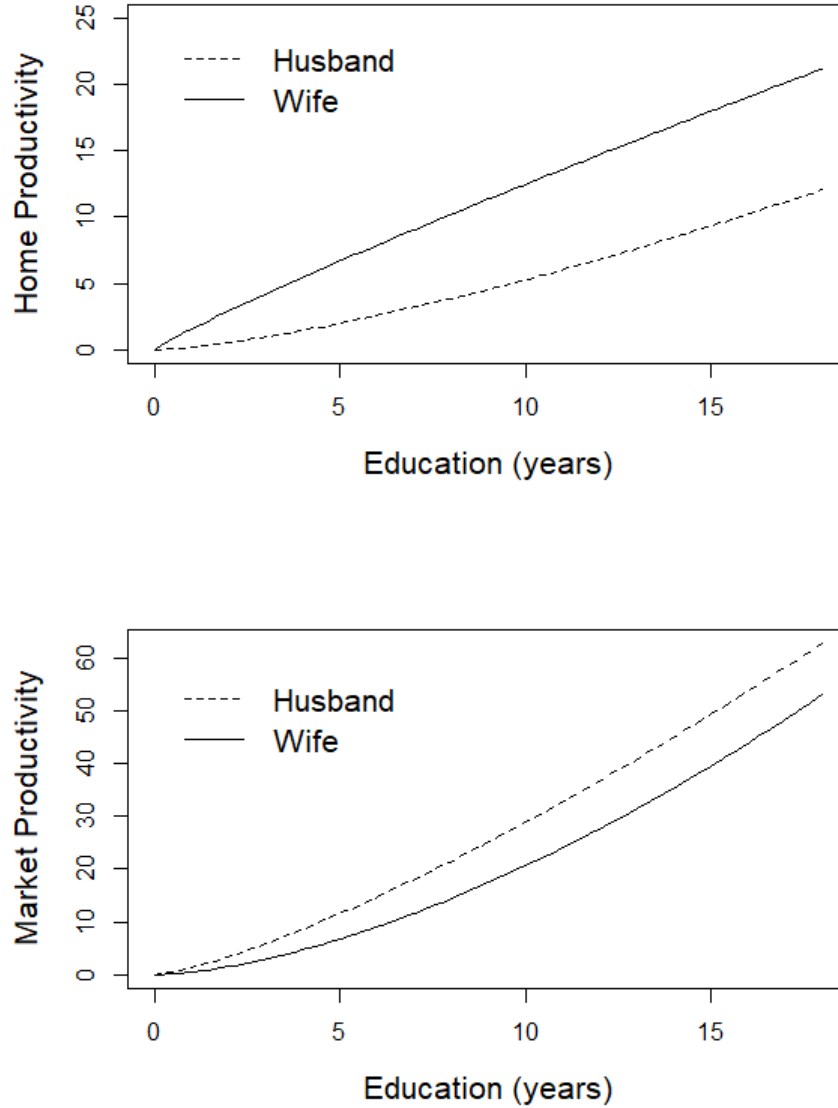
6.4, I also provide the log-log plot of Figure 6.3 for the same purpose as Figure 6.2.

Considering the production technologies, we can observe the following. A wife's home productivity is greater than that of a husband regardless of education level ($g_2 > g_1$). Interestingly, husband's and wife's home productivity display different shapes. From Figure 6.1, it is observed that husband's home productivity function is a convex function of edu-

Figure 2: Log-log plot: Home Productivity and Market Productivity, Husband and Wife
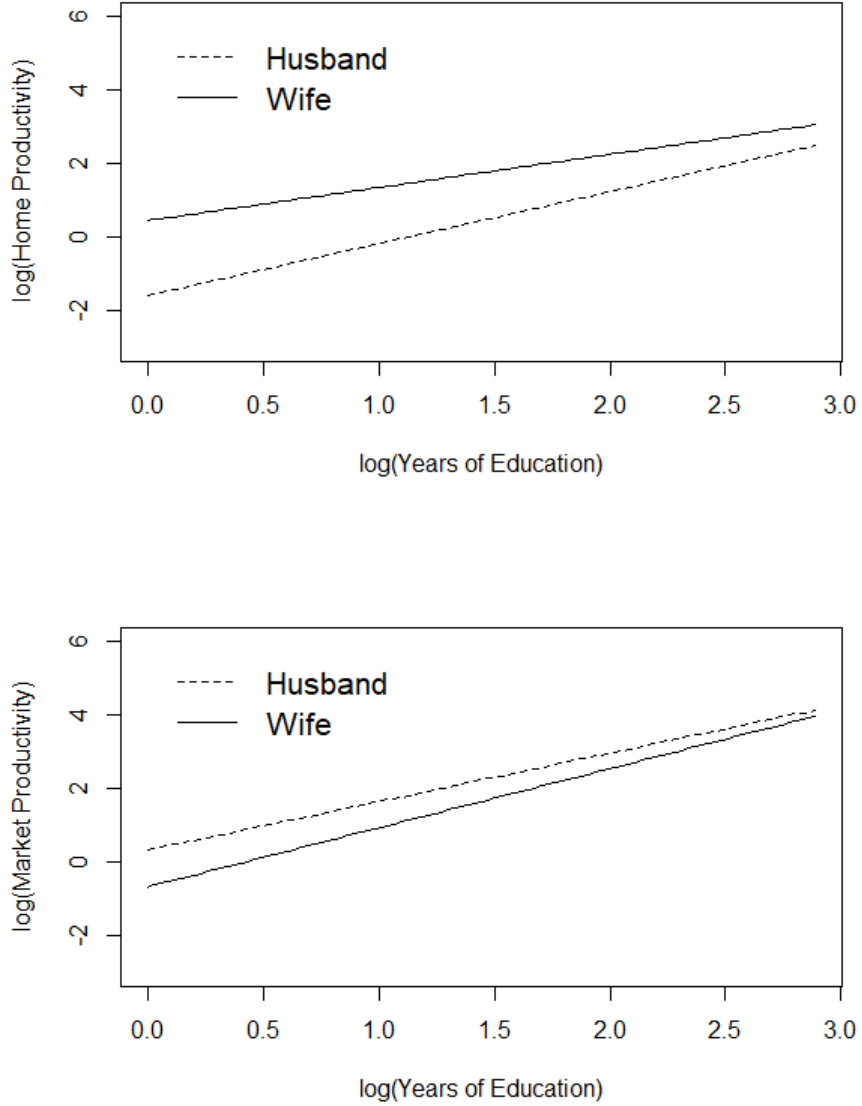
cation levels, however, that of wife is a concave function of her education levels. In other words, husband's home productivity shows increasing returns with respect to education levels ($h_1 > 1$), while wife's home productivity shows decreasing returns with respect to education levels ($h_2 < 1$). This is not from the model assumption but from the estimation

results. A husband's education elasticity of housework productivity[18] is greater than that of a wife ($h_1 > h_2$). However, the reversal of husbands and wives home productivity does not occur within the interval between 1 and 18 years of education.[19]

In terms of market productivity, results demonstrate increasing returns to scale with respect to education for both spouses ($d_1 > 1$ and $d_2 > 1$). The wage returns to education are consistent with what is normally expected; education raises wages. My estimates of the marginal returns indicate that they are generally increasing with the level of schooling, which is consistent with the findings from other research including Belzil and Hansen (2002). Also, a wife's education elasticity of market productivity is greater than that of a husband ($d_2 > d_1$). However, the reversal of husbands and wives market productivity does not occur within the interval between 1 and 18 years of education.[20]

Next, I compare the spouse's personal education elasticity of housework productivity to that of personal market productivity. The top panel of Figure 6.3 shows that a husband's education elasticity of housework productivity is greater than that of market productivity ($h_1 > d_1$). Therefore, at some point, the marginal product of education becomes greater in home productivity than market productivity for the husband. However, this also does not take place within the lifetime. In other words, husbands possess human capital skills (education) that are more productive in market work than in home work ($g_1 < w_1$). On the other hand, a wife's productivity curves demonstrate a somewhat different picture. The right panel of Figure 2 illustrates that the reversal of a wife's market productivity and
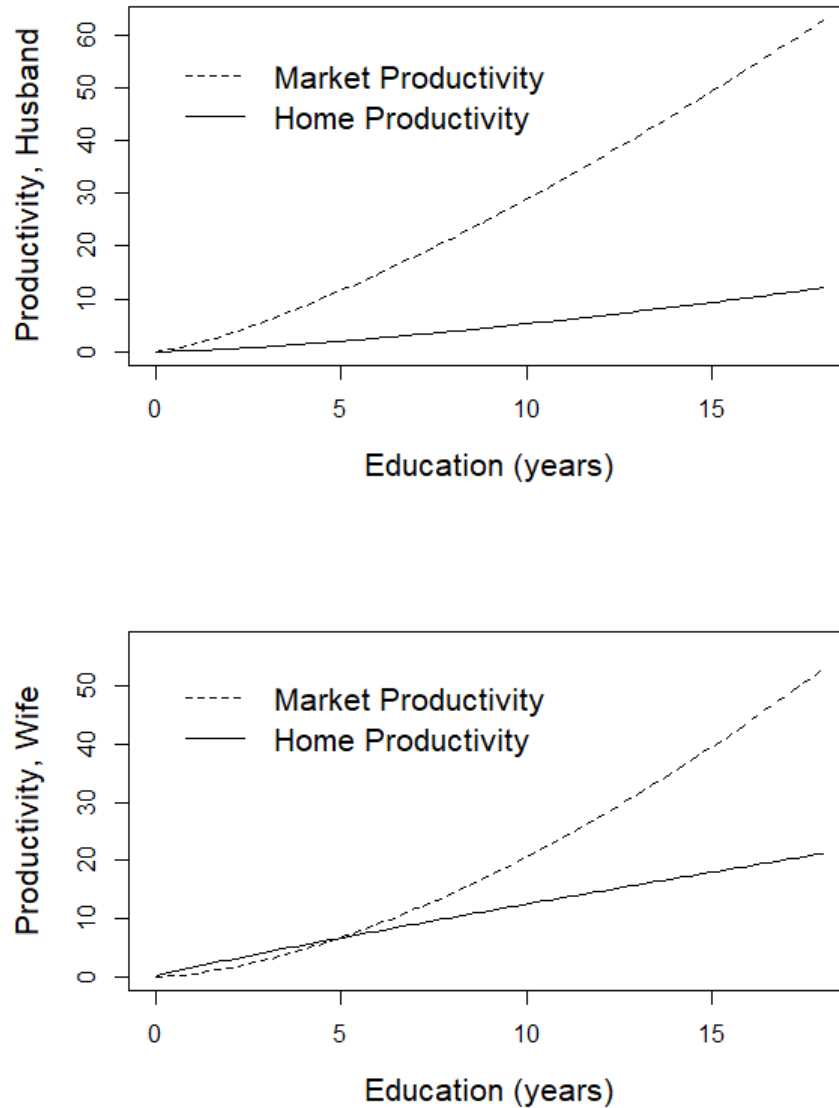
---

[18]Education elasticity of housework productivity is:

$$\varepsilon_{g_i} = \frac{dg_i}{de_i} \cdot \frac{e_i}{g_i} = a_i h_i e_i^{h_i - 1} \cdot \frac{e_i}{a_i e_i^{h_i}} = h_i$$

[19]The two curves cross at 55 years of education
[20]The two curves cross at 33 years of education.

Figure 3: Husband and Wife, Home Productivity and Market Productivity

her home productivity occurs at around 5 years of education. This occurs because wives'

education elasticity of housework productivity is less than that of market productivity ($h_2 <$

d_2). When they have fewer than 5 years of education, wives are more productive at home

rather than in market work. Conversely, wives' education becomes more productive in

market work as they achieve higher levels of education.

Figure 4: Log-log plot: Husband and Wife, Home Productivity and Market Productivity
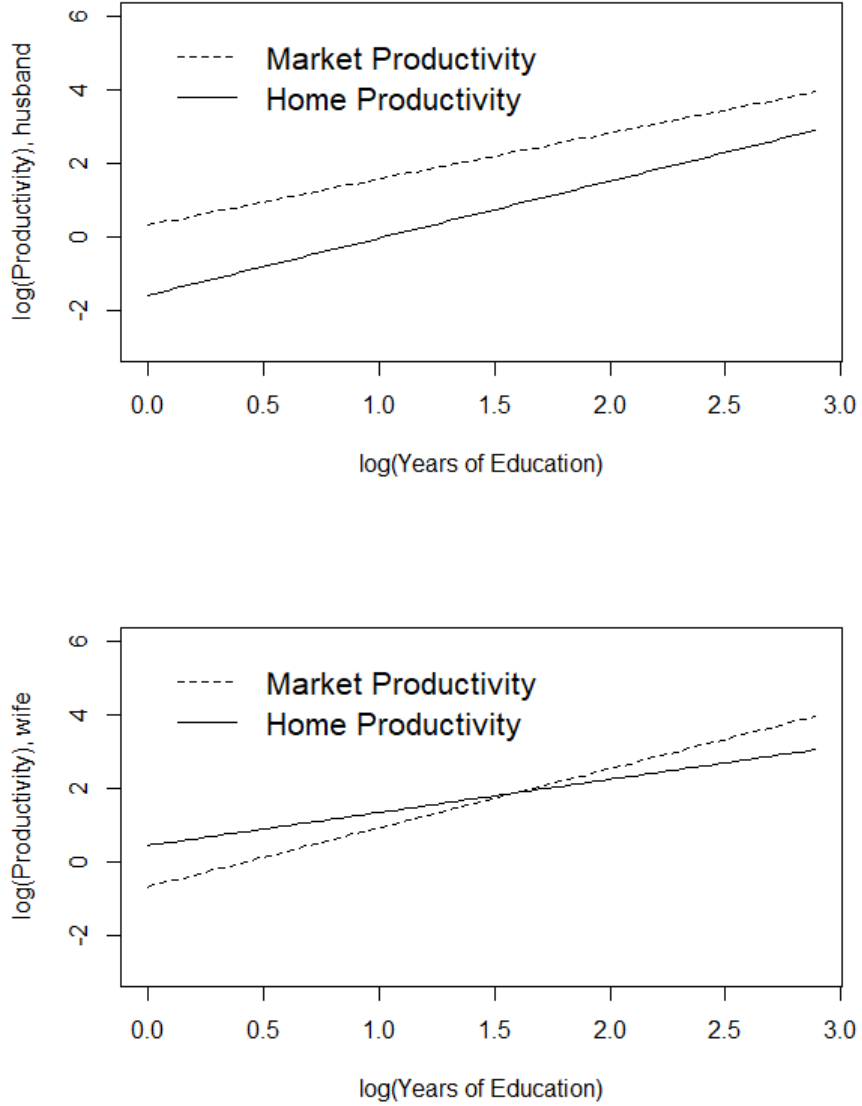
6.3   Bargaining Power

Estimation results show that husbands almost equally care about their own utility and their wives' utility. Since the estimated bargaining power parameter z is close to zero, we have $\mu = exp(sex\ ratio \cdot z)$ close to one. Moreover, there is not much variation across

Table 8: Estimated μ

	Number of Households	min	mean	max	std
Estimated μ	3,493	1.009	1.012	1.019	0.002

households in this bargaining power parameter μ.

6.4 Changing Husband's Education: Effects on the Intrahousehold Allocation of Time

Given the complexity of my model, the magnitudes of the estimated parameters are not easy to comprehend. In light of this, I provide Table 6.3 and Table 6.4 that illustrate the impact of husbands' education level changes on the dependent variable in my model. In general, this impact depends on the complex interaction between individual preferences, intrahousehold bargaining (μ), and the household's production technologies.

From the FOCs (Equations (10), (11), (12), (13), and (14)),

$$
\underbrace{\begin{bmatrix}
\frac{\partial FOC_1}{\partial t_1} + \frac{\partial FOC_1}{\partial t_2} + \frac{\partial FOC_1}{\partial L_1} + \frac{\partial FOC_1}{\partial L_2} + \frac{\partial FOC_1}{\partial X_h} \\[6pt]
\frac{\partial FOC_2}{\partial t_1} + \frac{\partial FOC_2}{\partial t_2} + \frac{\partial FOC_2}{\partial L_1} + \frac{\partial FOC_2}{\partial L_2} + \frac{\partial FOC_2}{\partial X_h} \\[6pt]
\frac{\partial FOC_3}{\partial t_1} + \frac{\partial FOC_3}{\partial t_2} + \frac{\partial FOC_3}{\partial L_1} + \frac{\partial FOC_3}{\partial L_2} + \frac{\partial FOC_3}{\partial X_h} \\[6pt]
\frac{\partial FOC_4}{\partial t_1} + \frac{\partial FOC_4}{\partial t_2} + \frac{\partial FOC_4}{\partial L_1} + \frac{\partial FOC_4}{\partial L_2} + \frac{\partial FOC_4}{\partial X_h} \\[6pt]
\frac{\partial FOC_5}{\partial t_1} + \frac{\partial FOC_5}{\partial t_2} + \frac{\partial FOC_5}{\partial L_1} + \frac{\partial FOC_5}{\partial L_2} + \frac{\partial FOC_5}{\partial X_h}
\end{bmatrix}}_{\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial y}}
\underbrace{\begin{bmatrix} dt_1 \\ dt_2 \\ dL_1 \\ dL_2 \\ dX_h \end{bmatrix}}_{dy}
= -
\underbrace{\begin{bmatrix}
\frac{\partial FOC_1}{\partial e_1} \\[6pt]
\frac{\partial FOC_2}{\partial e_1} \\[6pt]
\frac{\partial FOC_3}{\partial e_1} \\[6pt]
\frac{\partial FOC_4}{\partial e_1} \\[6pt]
\frac{\partial FOC_5}{\partial e_1}
\end{bmatrix}}_{\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial e_1}}
de_1
$$

$$
\frac{dy}{de_1} = -\left[\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial y}\right]^{-1} \frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial e_1}.
$$

37

Table 9: Husband's Education Effect on Time and Time Share (dy/de_1)

	Δ time	Δ Share
Husband		
Housework	1.40 min	0.62%
Leisure	-23.67 min	-1.46%
Market labor	22.28 min	8.88%
Wife		
Housework	0.30 min	-0.62%
Leisure	38.91 min	1.46%
Market labor	-39.21 min	-8.88%

Note:

1. $dy/de_1 = -\left[\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial y}\right]^{-1} \frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial e_1}$.

Table 6.3 shows the impact of changes in a husband's education on the time and time share of husbands and wives. The education change runs from 1 year of education to 18 years of education, while the other explanatory variables are fixed, including a wife's educational attainment level.

Table 6.3 suggests that the average husband's time and time share on domestic work increase as his education level increases. One more year of education increases the average husband's housework time by 1.4 minutes and a 0.62% point in share. This increase is accompanied by a decrease in leisure time, but not by a decrease in market labor time. The average husband's time spent on market work increases when his education increases. Such results would also be observed in a standard labor supply model when the substitution effect dominates the income effect. Given a husband's leisure time falls as his education level rises, this also clearly indicates that there is a trade-off between husbands' utility derived from leisure on the one hand and their utility derived from the consumption of both private goods and home-produced goods on the other.

Table 10: Husband's Education Effect on Household Income and Expenditures (dy/de_1)

	Δ Amount per day($)
Household Income	110.12
Household Private Consumption (pX)	107.11
Market goods for home production (qX_h)	3.01

Note:

1. $dy/de_1 = - \left[\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial y} \right]^{-1} \frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial e_1}.$

A different picture emerges when we look at the impact on the average wife's dependent variables by changing the husband's education level. Interestingly, it appears that there is almost no change in the wife's housework time. This is one of the reasons why well-educated husbands take a higher share of domestic work than less-educated husbands. The average wife increases leisure time by 38.9 minutes but decreases market labor time by 39.2 minutes as the husband's education level increases.

Table 6.4 shows the impact of changes in a husband's education on the household private consumption (pX) and expenditures on domestic goods (qX_h). First, the couple's daily income increases by $110.12. Although the wife decreases her market labor time, the effect of education change on the husband's wage rate, increase in market labor time of the husband, and the market wage rate differential between husband and wife results this. The part of this income is spent on domestic goods, while the remainder is allocated for household private consumption. It is notable that the household's private consumption rises dramatically. Household expenditure on domestic goods also increases but to a relatively lesser degree. This illustrates the trade-off between the utility derived from the household consumption of private goods and that of home-produced goods. We can say that the public good H is a necessity good, which has a positive but inelastic income elasticity.

6.5   Goodness of Fit

6.6   Goodness-of-Fit Test

This subsection summarizes the results of the goodness-of-fit of my model. Table 6.5 shows how I divide the sample for the test, and Table 6.6 describes how well my model explains a husband's time allocation within the household. I performed a set of χ^2 goodness-of-fit tests for a husband's housework time share, leisure time share, and market labor time share.

Table 11: χ^2 Goodness-of-Fit Test

Husband's Time Share	1st Quantile	Median	3rd Quantile
Housework Time Share	17%	33%	50%
Leisure Time Share	47%	50%	52%
Market Work Time Share	50%	55%	73%

Here, I apply the χ^2 goodness-of-fit test to time shares which have continuous distributions. To calculate the chi-square statistic the observed time shares are grouped into discrete bins. As shown in Table 6.5, I stratify sample households by the percentile of their time shares: for husband's housework time-share, the 1st quantile is 17%, the median is 33%, and the 3rd quantile is 50%; for leisure time-share of husbands, the 1st quantile is 47%, the median is 50% and the 3rd quantile is 52%; for market labor time-share, the 1st quantile is 50%, the median is 55%, and the 3rd quantile is 73%. For example, husbands in Group 1 take less than 17% of housework share, while husbands in Group 2 take more than 17% but less than 33%. Husbands in Group 3 take more than 33% but less than 50% of housework share, and husbands in Group 4 take more than 50% of housework share in

40

Table 12: χ^2 Goodness-of-Fit Test

Husband's Time Share	Group 1	Group 2	Group 3	Group 4
Housework Time Share				
Observed Probabilities	0.238	0.247	0.244	0.270
Predicted Probabilities	0.238	0.249	0.246	0.267
Goodness-of-fit		$\chi^2_3 = 0.1861$		
Leisure Time Share				
Observed Probabilities	0.250	0.223	0.275	0.252
Predicted Probabilities	0.252	0.221	0.267	0.259
Goodness-of-fit		$\chi^2_3 = 1.6683$		
Market Work Time Share				
Observed Probabilities	0.216	0.283	0.248	0.252
Predicted Probabilities	0.214	0.285	0.250	0.251
Goodness-of-fit		$\chi^2_3 = 0.11$		

Note:

  Group 1: time share < 1st quantile.

  Group 2: 1st quantile ≤ time share < Median.

  Group 3: Median ≤ time share < 3rd quantile.

  Group 4: 3rd quantile < time share.

a given household. I do the same stratification in terms of leisure share and market work share.

Subsequently, I compare my model's predicted probabilities of each household falling into each group to the actually observed probabilities for each stratum. This approach allows me to examine whether households that are predicted by my model actually experience particular time allocations between the husband and the wife. When compared to the actual frequencies, the predictions appear quite accurate. A set of χ^2 goodness-of-fit tests shows that my model fits the data well (see Appendix D for the graphical illustration of goodness-of-fit).

7   Policy Simulations

To understand the model better, I conduct policy simulations. I change the preference parameters and productivity parameters and measure the impact of a change in a specific parameter value on each spouse's time allocation decisions when there is an one year increase in Husband's education level. In specific, I consider the effects of four simulated experiments on spouse behavior given the parameter estimates reported in Table 6.1. The four experiments involve:

1. Both husband and wife have husband's preference parameters

2. Both husband and wife have wife's preference parameters

3. Both husband and wife have husband's productivity parameters

4. Both husband and wife have wife's productivity parameters

In general, I observe more significant changes in leisure and market work time than housework time in all experiments. This again shows that the domestic goods have the characteristics of necessity goods. Both spouses do not have to increase H more than it is needed. Thus, in all experiments, both spouses increase housework time and expenditures (qX_h) only a small amount as the husband's education level increases.

More importantly, a husband does not show significant difference in changing housework time in the simulations unless there is a change in his productivity parameters. This implies that the relationship between education and housework time depends on husband's home productivity parameters rather than on his preference parameters.

Table 13: Experiment (1): Husband's Education Effect on Household Behavior When Both Spouses have Husband's Preference Parameters

	Both have Husband's Preference	
	Δ time	Δ Share
Husband		
Housework	1.44 min	0.64%
Leisure	-19.92 min	-1.16%
Market labor	18.48 min	7.67%
Wife		
Housework	0.12 min	-0.64%
Leisure	30.38 min	1.16%
Market labor	-30.50 min	-7.67%
Household		
Income		$114.58
Total private consumption		$112.16
Total consumption for household goods		$2.42

Note:

1. $dy/de_1 = - \left[\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial y} \right]^{-1} \frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial e_1}.$

7.1 Same Preference: Experiments (1) and (2)

In the first two experiments (1) and (2), I change the preference parameters of each spouse to make both spouses share the same preference. In experiment (1), I replace the wife's preference parameters with the husband's preference parameter. Table 7.1 shows the impact of changes in a husband's education on the household behavior when both spouses have husband's preference parameters. In experiment (2), I substitute the husband's preference parameters with the wife's preference parameters. Table 7.2 displays the impact of changes in a husband's education on the household time allocations and expenditures.

Since we focus on the effect of change in a husband's education level, experiment (2) (Table 7.2) provides more insights than experiment (1) (Table 7.1). If a husband replaces his preference with a wife's preference, an average husband does not change his market

Table 14: Experiment (2): Husband's Education Effect on Household Behavior When Both Spouses have Wife's Preference Parameters

| | Both have Wife's Preference | |
	Δ time	Δ Share
Husband		
Housework	1.41 min	0.64%
Leisure	-2.25 min	-0.58%
Market labor	0.84 min	2.73%
Wife		
Housework	0.08 min	-0.64%
Leisure	22.15 min	0.58%
Market labor	-22.24 min	-2.73%
Household		
Income		$104.35
Total private consumption		$102.47
Total consumption for household goods		$1.88

Note:

1. $dy/de_1 = -\left[\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial y}\right]^{-1} \frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial e_1}$.

work time unlike the baseline model in Table 6.2. In this experiment, now both spouses prefer their private leisure time the most so that a husband does not decrease their leisure significantly. In addition, a husband has a high education elasticity of home productivity so that he increases his housework time as the baseline model. Similarly, a wife has a high education elasticity of market productivity so that she decreases her market work time with a much lesser degree and increase her leisure also with a lesser degree than the baseline model.

7.2 Same Productivity: Experiments (3) and (4)

In the experiments (3) and (4), I change the productivity parameters of each spouse to make both spouses share the same productivity. In experiment (3) I replace the wife's pro-

Table 15: Experiment (3): Husband's Education Effect on Household Behavior When Both Spouses have Husband's Productivity Parameters

	Both have Husband's Productivity	
	Δ time	Δ Share
Husband		
Housework	1.43 min	0.64%
Leisure	-25.84 min	-1.40%
Market labor	24.40 min	8.61%
Wife		
Housework	0.16 min	-0.64%
Leisure	34.36 min	1.40%
Market labor	-34.52 min	-8.61%
Household		
Income		$116.63
Total private consumption		$113.90
Total consumption for household goods		$2.73

Note:

1. $dy/de_1 = -\left[\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial y}\right]^{-1} \frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial e_1}$.

ductivity parameters with the husband's productivity parameters and Table 7.3 presents the experiment results. In experiment (4), I substitute the husband's productivity parameters with the wife's productivity parameters and Table 7.4 shows the experiment (4)'s simulation results. Again, experiment (4) (Table 7.4) provides more implications than experiment (3) (Table 7.3) since we measure the impact of change in husband's education level.

First of all, it is noticeable that an average husband does not increase his housework time when there is an increase in his education level in experiment (4). This is probably because now he has home productivity which displays decreasing return with respect to education level. On the other hand, considering now a husband sharing the wife's high education elasticity of market productivity, an average husband increases his market work time the most in experiment (4) among all experiments.

Table 16: Experiment (4): Husband's Education Effect on Household Behavior When Both Spouses have Wife's Productivity Parameters

	Both have Wife's Productivity	
	Δ time	Δ Share
Husband		
Housework	0.21 min	0.12%
Leisure	-30.90 min	-1.71%
Market labor	30.69 min	11.74%
Wife		
Housework	0.18 min	-0.12%
Leisure	42.53 min	1.71%
Market labor	-42.70 min	-11.74%
Household		
Income		$66.16
Total private consumption		$62.78
Total consumption for household goods		$3.38

Note:

1. $dy/de_1 = - \left[\frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial y} \right]^{-1} \frac{\partial FOCs(y,x,\varepsilon,\theta)}{\partial e_1}$.

Secondly, since a husband prefers the total private expenditures the most and a wife values the leisure time the most, an average husband increases the market work time most significantly, and an average wife increases the leisure time with the most extensive degree among all four experiments.

Lastly, in experiment (4), a husband has a wife's productivity parameters, and the wife's market productivity is smaller than that of husband, household income increases with the smallest degree when there is an increase in a husband's education level.

8   Conclusions

This paper presents new evidence on the impact of the increase in educational attainment levels on the time allocation decisions of the husbands. I find that a one more year of husband's education leads to, on average, about 1.4 minute increase in his housework time and a 0.62% point increase his housework time share.

In order to examine counter-intuitive husbands' time use pattern, I have estimated a structural time allocation model in which married couples collectively decide how to distribute their time in market labor, leisure and housework. I have used the FOCs of the model to solve for the errors as relatively simple functions of the parameters and constructed the corresponding likelihood function. Having applied my model to the most recent PSID data with detailed information about both spouses' time use, income, expenditure, and education levels, my empirical results have presented innovative findings.

Surprisingly, a husband's housework elasticity with respect to education is greater than that of his market productivity and even a wife's education elasticity of domestic productivity. If there is an increase in education level, husbands increase not only market work time but also housework time. On the other hand, husbands' education level change does not impact wives' housework time significantly. In turn, this leads to the phenomenon that the well-educated husbands take more housework share than the less-educated husbands.

This paper provides an important first step towards building an understanding of husbands' time allocation decisions that have never been explained by previous studies. One issue that I left for future work is that of childcare. Childcare is another public good and it is closely related to housework. It would be an important next step in studying the husband's time allocation to include childcare in the household model. I leave this topic for

future research.

Appendix

.1 Descriptive Statistics, OLS

Table 17: Husband's Education Effect in the U.S.

	Dependent variable: Husband's Housework Time and Time Share	
	t_1	$Share_{t_1}$
Husband's Education		
$e_1 = 1$	−0.043	−0.005
	(0.049)	(0.013)
$e_1 = 3$	0.121***	0.042***
	(0.033)	(0.009)
Wife's Education		
$e_2 = 1$	−0.105*	−0.036**
	(0.058)	(0.015)
$e_2 = 3$	0.041	0.020**
	(0.035)	(0.009)
Wife's housework time	0.113***	
	(0.008)	
Husband's age	0.001	−0.001***
	(0.001)	(0.0003)
Number of Children	0.009	−0.014***
	(0.013)	(0.003)
Observations	3,493	3,488

Note: *p<0.1; **p<0.05; ***p<0.01

1. To compare across surveys, I use cross-nationally harmonised education levels here.
    $e_i = 1$ is below high school or high school dropouts;
    $e_i = 2$ is high school graduates (the reference level);
    $e_i = 3$ is college level and above;

Table 18: Husband's Education Effect in Spain[1]

	Dependent variable: Husband's Housework Time and Time Share	
	t_1	$Share_{t_1}$
Husband's Education		
$e_1 = 1$	0.006	−0.0003
	(0.084)	(0.010)
$e_1 = 3$	0.172**	0.028***
	(0.069)	(0.009)
Wife's Education		
$e_2 = 1$	0.107	0.016
	(0.084)	(0.010)
$e_2 = 3$	0.353***	0.063***
	(0.070)	(0.009)
Wife's housework time	0.047***	
	(0.009)	
Husband's age	−0.009***	−0.002***
	(0.002)	(0.0003)
Number of Children	0.225***	0.00001
	(0.032)	(0.004)
Observations	5,223	5,165

Note: *p<0.1; **p<0.05; ***p<0.01

1. Spain: the 2009 Time Use Survey of Spain.
2. To compare across surveys, I use cross-nationally harmonised education levels here.
   $e_i = 1$ is below high school or high school dropouts;
   $e_i = 2$ is high school graduates (the reference level);
   $e_i = 3$ is college level and above;

Table 19: Husband's Education Effect in Korea[1]

| | *Dependent variable:* | |
| | Husband's Housework Time and Time Share | |
	t_1	$Share_{t_1}$
Husband's Education		
$e_1 = 1$	-0.032	-0.008
	(0.026)	(0.007)
$e_1 = 3$	0.037^{**}	0.010^*
	(0.019)	(0.005)
Wife's Education		
$e_2 = 1$	0.086^{***}	0.013^{**}
	(0.026)	(0.007)
$e_2 = 3$	0.062^{***}	0.014^{***}
	(0.019)	(0.005)
Wife's housework time	0.116^{***}	
	(0.003)	
Husband's age	-0.008^{***}	-0.002^{***}
	(0.001)	(0.0003)
Number of Children	-0.013	-0.005
	(0.009)	(0.002)
Observations	4,578	4,570

Note: *p$<$0.1; **p$<$0.05; ***p$<$0.01

1. Korea: the 2014 Korean Longitudinal Survey of Women and Families.
2. To compare across surveys, I use cross-nationally harmonised education levels here.
   $e_i = 1$ is below high school or high school dropouts;
   $e_i = 2$ is high school graduates (the reference level);
   $e_i = 3$ is college level and above;

.2 Components of the Likelihood Function

1. The first case is when $\varepsilon_x = T^x$, $\varepsilon_{t_1} = T_1^w(t_1)$, $\varepsilon_{L_1} = T_1^L$, $\varepsilon_{w_1} = w_1/m_1 e_1^{d_1}$, $\varepsilon_{t_2} = T_2^w(t_2)$, $\varepsilon_{L_2} = T_2^L$, and $\varepsilon_{w_2} = w_2/m_2 e_2^{d_2}$ are satisfied. The likelihood function is

$$
L = f(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2})
$$

$$
= \frac{1}{(2\pi)^{7/2}} |\Omega|^{-1/2} \exp\left[-0.5 \begin{pmatrix} ln(\varepsilon_x) \\ ln(\varepsilon_{L_1}) \\ ln(\varepsilon_{t_1}) \\ ln(\varepsilon_{w_1}) \\ ln(\varepsilon_{L_2}) \\ ln(\varepsilon_{t_2}) \\ ln(\varepsilon_{w_2}) \end{pmatrix}' \Omega^{-1} \begin{pmatrix} ln(\varepsilon_x) \\ ln(\varepsilon_{L_1}) \\ ln(\varepsilon_{t_1}) \\ ln(\varepsilon_{w_1}) \\ ln(\varepsilon_{L_2}) \\ ln(\varepsilon_{t_2}) \\ ln(\varepsilon_{w_2}) \end{pmatrix} \right] \cdot Jacobian
$$

$$
Jacobian = \frac{1}{\varepsilon_x} \cdot \frac{1}{\varepsilon_{L_1}} \cdot \frac{1}{\varepsilon_{t_1}} \cdot \frac{1}{\varepsilon_{w_1}} \cdot \frac{1}{\varepsilon_{L_2}} \cdot \frac{1}{\varepsilon_{t_2}} \cdot \frac{1}{\varepsilon_{w_2}}
$$

2. The second case: a wife does not do housework. $\varepsilon_x = T^x$, $\varepsilon_{t_1} = T_1^w(t_1)$, $\varepsilon_{L_1} = T_1^L$, $\varepsilon_{w_1} = w_1/m_1 e_1^{d_1}$, $\varepsilon_{t_2} \leq T_2^w(0)$, $\varepsilon_{L_2} = T_2^L$, and $\varepsilon_{w_2} = w_2/m_2 e_2^{d_2}$

$$
L = f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{T_2^w(0)} f_2(\varepsilon_{t_2} | \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2}) d\varepsilon_{t_2}
$$

$$
= f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot F(T_2^w(0) | \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2})
$$

$$
= f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \Phi\left(\frac{ln(T_2^w(0)) - cm_{t_2}}{cs_{t_2}} \right)
$$

3. The third case: a wife does not do market work (symmetric to case 5). In this case, we do not observe wife's wage rate (wife's market productivity) from the data since she chooses not to work. Therefore, I simulate $\varepsilon_{w_2}^r$ based on the conditional mean cm_3 and conditional variance cs_3^2. Then, I use $\varepsilon_{w_2}^r$ to compute f_3.

$$
L = f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} f_2[\varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}), \varepsilon_{w_2} | \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}] d\varepsilon_{w_2}
$$

$$
= f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} f_3[\varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}) |
$$

$$
\varepsilon_{w_2}, \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}] f_4(\varepsilon_{w_2} | \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{w_2}
$$

$$
\approx f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) \frac{1}{R} \sum_{r}^{R} f_3[\varepsilon_{t_2}(\varepsilon_{w_2}^r), \varepsilon_{L_2}(\varepsilon_{w_2}^r) | \varepsilon_{w_2}^r, \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}]
$$

where

$$\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1} \sim N(cm_3, cs_3^2)$$

$$\varepsilon_{w_2}^r = cm_3 + cs_3 \Phi^{-1}(U^r)$$

$$f_3 = \frac{1}{(2\pi)^{2/2}} |cv|^{-1/2} \exp[A] \cdot \frac{1}{\varepsilon_{t_2}} \frac{1}{\varepsilon_{L_2}}$$

where

$$A = \left[-0.5 \begin{pmatrix} ln(\varepsilon_{t_2}(\varepsilon_{w_2}^r)) - cm_{t_2} \\ ln(\varepsilon_{L_2}(\varepsilon_{w_2}^r)) - cm_{L_2} \end{pmatrix}' cv^{-1} \begin{pmatrix} ln(\varepsilon_{t_2}(\varepsilon_{w_2}^r)) - cm_{t_2} \\ ln(\varepsilon_{L_2}(\varepsilon_{w_2}^r)) - cm_{L_2} \end{pmatrix} \right]$$

4. The fourth case: a wife spends time in all leisure. (symmetric to case 13).

$$L = f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} \int_{-\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} f_2(\varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}|$$

$$\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{t_2} d\varepsilon_{L_2} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} \int_{-\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} f_3(\varepsilon_{t_2}|\varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_x, \varepsilon_{t_1},$$

$$\varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{t_2} f_4(\varepsilon_{L_2}, \varepsilon_{w_2}|\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{L_2} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} F(ln(T_2^{nw}(0,\varepsilon_{L_2}))|\varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1})$$

$$f_4(\varepsilon_{L_2}, \varepsilon_{w_2}|\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{L_2} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} F(ln(T_2^{nw}(0,\varepsilon_{L_2}))|\varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1})$$

$$f_{41}(\varepsilon_{L_2}|\varepsilon_{w_2}, \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) f_{42}(\varepsilon_{w_2}|\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{L_2} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) E_{\varepsilon_{L_2}, \varepsilon_{w_2}|\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}} [F(ln(T_2^{nw}(0,\varepsilon_{L_2}))|\varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_x,$$

$$\varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1})]$$

$$\approx f_1(\varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}) \frac{1}{R} \sum_{r=1}^{R} F(ln(T_2^{nw}(0,\varepsilon_{L_2}^r))|\varepsilon_{L_2}^r, \varepsilon_{w_2}^r, \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1})$$

then simulate the followings and use them to compute

$$F(ln(T_2^{nw}(0,\varepsilon_{L_2}^r))|\varepsilon_{L_2}^r, \varepsilon_{w_2}^r, \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1})$$

(a) $\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}$

(b) $\varepsilon_{L_2}^r | \varepsilon_{w_2}^r, \varepsilon_x, \varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}$

5. The fifth case: a husband does not do market work. (symmetric to case 3)

$$L = f_1(\varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{\infty} f_2[\varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1}), \varepsilon_{w_1} | \varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}] d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{\infty} f_3(\varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1})|$$
$$\varepsilon_{w_1}, \varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) f_4(\varepsilon_{w_1} | \varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) d\varepsilon_{w_1}$$

$$\approx f_1(\varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \frac{1}{R} \sum_{r=1}^{R} f[\varepsilon_{t_1}(\varepsilon_{w_1}^r), \varepsilon_{L_1}(\varepsilon_{w_1}^r) | \varepsilon_{w_1}^r, \varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}]$$

I simulate

(a) $\varepsilon_{w_1}^r | \varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}$

6. The sixth case: a husband does not do market work and a wife does not to housework. (symmetric to case 11)

$$L = f_1(\varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{T_2^w(0)} f_2[\varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1}), \varepsilon_{w_1}, \varepsilon_{t_2} | \varepsilon_x, \varepsilon_{L_2},$$
$$\varepsilon_{w_2}] d\varepsilon_{t_2} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{T_2^w(0)} f_3[\varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1}) | \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2})$$
$$f_{41}(\varepsilon_{t_2} | \varepsilon_{w_1}, \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}]$$
$$f_{42}(\varepsilon_{w_1} | \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}) d\varepsilon_{t_2} d\varepsilon_{w_1}$$

$$\approx f_1(\varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}) \frac{1}{R} \sum_{r=1}^{R} f_3[\varepsilon_{t_1}(\varepsilon_{w_1}^r), \varepsilon_{L_1}(\varepsilon_{w_1}^r) | \varepsilon_{w_1}^r, \varepsilon_{t_2}^r, \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}]$$

I simulate

(a) $\varepsilon_{w_1}^r | \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}$
(b) $\varepsilon_{t_2}^r | \varepsilon_{w_1}^r, \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}$

7. The seventh case: a husband does not do market work and a wife does not do market

work.

$$L = f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_2(\varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2} | \varepsilon_x) d\varepsilon_{w_2} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_3(\varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{t_2}, \varepsilon_{L_2} | \varepsilon_{w_1}, \varepsilon_{w_2}, \varepsilon_x) f_4(\varepsilon_{w_1}, \varepsilon_{w_2} | \varepsilon_x) d\varepsilon_{w_2} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_3(\varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{t_2}, \varepsilon_{L_2} | \varepsilon_{w_1}, \varepsilon_{w_2}, \varepsilon_x) f_{41}(\varepsilon_{w_2} | \varepsilon_{w_1}, \varepsilon_x)$$

$$f_{42}(\varepsilon_{w_1} | \varepsilon_x) d\varepsilon_{w_2} d\varepsilon_{w_1}$$

$$\approx f_1(\varepsilon_x) \frac{1}{R} \sum_{r=1}^{R} f_3[\varepsilon_{t_1}(\varepsilon_{w_1}^r), \varepsilon_{L_1}(\varepsilon_{w_1}^r), \varepsilon_{t_2}(\varepsilon_{w_2}^r), \varepsilon_{L_2}(\varepsilon_{w_2}^r) | \varepsilon_{w_1}^r, \varepsilon_{w_2}^r, \varepsilon_x]$$

$$\approx \frac{f_1(\varepsilon_x)}{R} \sum_{r=1}^{R} \frac{1}{(2\pi)^{4/2}} |cv|^{-1/2} \exp(B) \cdot J$$

where

$$\begin{pmatrix} ln(\varepsilon_{t_1}(\varepsilon_{w_1}^r)) \\ ln(\varepsilon_{L_1}(\varepsilon_{w_1}^r)) \\ ln(\varepsilon_{t_2}(\varepsilon_{w_2}^r)) \\ ln(\varepsilon_{L_2}(\varepsilon_{w_2}^r)) \end{pmatrix} \sim N \left(\begin{pmatrix} cm_{t1} \\ cm_{L1} \\ cm_{t2} \\ cm_{L2} \end{pmatrix}, cv \right),$$

$$B = \left[-0.5 \begin{pmatrix} ln(\varepsilon_{t_1}(\varepsilon_{w_1}^r)) - cm_{t_1} \\ ln(\varepsilon_{L_1}(\varepsilon_{w_1}^r)) - cm_{L_1} \\ ln(\varepsilon_{t_2}(\varepsilon_{w_2}^r)) - cm_{t_2} \\ ln(\varepsilon_{L_2}(\varepsilon_{w_2}^r)) - cm_{L_2} \end{pmatrix}' cv^{-1} \begin{pmatrix} ln(\varepsilon_{t_1}(\varepsilon_{w_1}^r)) - cm_{t_1} \\ ln(\varepsilon_{L_1}(\varepsilon_{w_1}^r)) - cm_{L_1} \\ ln(\varepsilon_{t_2}(\varepsilon_{w_2}^r)) - cm_{t_2} \\ ln(\varepsilon_{L_2}(\varepsilon_{w_2}^r)) - cm_{L_2} \end{pmatrix} \right]$$

$$J = \frac{1}{\varepsilon_{t_1}(\varepsilon_{w_1}^r)} \frac{1}{\varepsilon_{L_1}(\varepsilon_{w_1}^r)} \frac{1}{\varepsilon_{t_2}(\varepsilon_{w_2}^r)} \frac{1}{\varepsilon_{L_2}(\varepsilon_{w_2}^r)}.$$

I simulate

(a) $\varepsilon_{w_1}^r | \varepsilon_x$

(b) $\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{w_1}^r$

8. The eighth case: a husband does not do market work and a wife spends all time in

leisure (symmetric to case 15).

$$L = f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} \int_{-\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} f_2(\varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1}),$$

$$\varepsilon_{w_1} | \varepsilon_x) d\varepsilon_{t_2} d\varepsilon_{L_2} d\varepsilon_{w_2} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} \int_{-\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} f_3(\varepsilon_{t_2} | \varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1}),$$

$$\varepsilon_{w_1}, \varepsilon_x) d\varepsilon_{t_2}$$
$$f_4[\varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1}), \varepsilon_{w_1} | \varepsilon_x]$$
$$d\varepsilon_{L_2} d\varepsilon_{w_2} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} F(ln(T_2^{nw}(0, \varepsilon_{L_2})) | \varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_{t_1}(\varepsilon_{w_1}),$$

$$\varepsilon_{L_1}(\varepsilon_{w_1}), \varepsilon_{w_1}, \varepsilon_x)$$
$$f_4[\varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1}), \varepsilon_{w_1} | \varepsilon_x] d\varepsilon_{L_2} d\varepsilon_{w_2} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} F(ln(T_2^{nw}(0, \varepsilon_{L_2})) | \varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_{t_1}(\varepsilon_{w_1}),$$

$$\varepsilon_{L_1}(\varepsilon_{w_1}), \varepsilon_{w_1}, \varepsilon_x)$$
$$f_5[\varepsilon_{L_2} | \varepsilon_x, \varepsilon_{t_1}(\varepsilon_{w_1}), \varepsilon_{L_1}(\varepsilon_{w_1}), \varepsilon_{w_1}, \varepsilon_{w_2}] d\varepsilon_{L_2}$$
$$f_6[\varepsilon_{t_1}(\varepsilon_{w_1}) \varepsilon_{L_1}(\varepsilon_{w_1}) | \varepsilon_x, \varepsilon_{w_1}, \varepsilon_{w_2}]$$
$$f_{71}[\varepsilon_{w_2} | \varepsilon_x, \varepsilon_{w_1}] d\varepsilon_{w_2}$$
$$f_{72}[\varepsilon_{w_1} | \varepsilon_x] d\varepsilon_{w_1}$$

$$\approx \frac{f_1(\varepsilon_x)}{R} \sum_{r=1} F(ln(T_2^{nw}(0, \varepsilon_{L_2})) | \varepsilon_x, \varepsilon_{t_1}(\varepsilon_{w_1}^r), \varepsilon_{L_1}(\varepsilon_{w_1}^r), \varepsilon_{w_1}^r,$$

$$\varepsilon_{L_2}^r, \varepsilon_{w_2}^r) f_6[\varepsilon_{t_1}(\varepsilon_{w_1}^r) \varepsilon_{L_1}(\varepsilon_{w_1}^r) | \varepsilon_x, \varepsilon_{w_1}^r, \varepsilon_{w_2}^r]$$

I simulate

(a) $\varepsilon_{w_1}^r | \varepsilon_x$
(b) $\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{w_1}^r$
(c) $\varepsilon_{L_2}^r | \varepsilon_x, \varepsilon_{t_1}(\varepsilon_{w_1}^r), \varepsilon_{L_1}(\varepsilon_{w_1}^r), \varepsilon_{w_1}^r, \varepsilon_{w_2}^r$

9. The ninth case: a husband does not do housework

$$L = f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{T_1^w(0)} f_2(\varepsilon_{t_1} | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) d\varepsilon_{t_1}$$

$$= f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot F(ln(T_1^w(0)) | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2})$$

$$= f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \Phi\left(\frac{ln(T_1^w(0)) - cm_{t_1}}{cs_{t_1}}\right)$$

10. The tenth case: both husband and wife do not do any housework.

$$L = f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{T_1^w(0)} \int_{-\infty}^{T_2^w(0)} f_2(\varepsilon_{t_1}, \varepsilon_{t_2} | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1},$$

$$\varepsilon_{L_2}, \varepsilon_{w_2}) d\varepsilon_{t_2} d\varepsilon_{t_1}$$

$$= f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot B\left(\frac{ln(T_1^w(0)) - cm_{t_1}}{cs_{t_1}}, \frac{ln(T_2^w(0)) - cm_{t_2}}{cs_{t_2}} | \rho\right)$$

where B is bivariate normal distribution.

11. The eleventh case: a husband does not do housework, and a wife does not do market work. (symmetric to case 6)

$$L = f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{T_1^w(0)} f_2[\varepsilon_{t_1}, \varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}), \varepsilon_{w_2} | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}]$$

$$d\varepsilon_{t_1} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{T_1^w(0)} f_3[\varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}) | \varepsilon_{t_1}, \varepsilon_{w_2}, \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}]$$

$$f_4(\varepsilon_{t_1}, \varepsilon_{w_2} | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{t_1} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{T_1^w(0)} f_3[\varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}) | \varepsilon_{t_1}, \varepsilon_{w_2}, \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}]$$

$$f_{41}(\varepsilon_{t_1}, | \varepsilon_{w_2}, \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1})$$

$$f_{42}(\varepsilon_{w_2} | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{t_1} d\varepsilon_{w_2}$$

$$\approx f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) \frac{1}{R} \sum_{r=1}^{R} f_3[\varepsilon_{t_2}(\varepsilon_{w_2}^r), \varepsilon_{L_2}(\varepsilon_{w_2}^r) | \varepsilon_{w_2}^r, \varepsilon_{t_1}^r, \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}]$$

(a) $\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}$

(b) $\varepsilon_{t_1}^r | \varepsilon_{w_2}^r, \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}$

12. The twelfth case: a husband does not do housework, and a wife spends all time in

leisure.

$$L = f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} \int_{\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} \int_{\infty}^{T_1^{t_1}(0)} f_2(\varepsilon_{t_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2} | \varepsilon_x,$$

$$\varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{t_1} d\varepsilon_{t_2} d\varepsilon_{L_2} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) \cdot \int_{-\infty}^{\infty} \int_{T_2^L}^{\infty} \int_{\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} \int_{\infty}^{T_1^{t_1}(0)} f_3(\varepsilon_{t_1}, \varepsilon_{t_2} | \varepsilon_{L_2}^r, \varepsilon_{w_2}^r \varepsilon_x,$$

$$\varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{t_1} d\varepsilon_{t_2}$$

$$f_{41}(\varepsilon_{L_2}^r | \varepsilon_{w_2}^r, \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) f_{42}(\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) d\varepsilon_{L_2} d\varepsilon_{w_2}$$

$$\approx f_1(\varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}) \frac{1}{R} \sum_{r=1}^{R} B\left(\frac{ln(T_1^{t_1}(0)) - cm_{t_1}}{cs_{t_1}}, \frac{ln(T_2^{nw}(0, \varepsilon_{L_2}^r)) - cm_{t_2}}{cs_{t_2}} \Big| \rho \right)$$

where B is bivariate normal distribution. I simulate

(a) $\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}$
(b) $\varepsilon_{L_2}^r | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{w_2}^r$

13. The 13th case: a husband spends all time in leisure.

$$L = f_1(\varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} \int_{-\infty}^{T_1^{nw}(0,\varepsilon_{L_1})} f_2(\varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1} | \varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2})$$

$$d\varepsilon_{t_1} d\varepsilon_{L_1} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} F(ln(T_1^{nw}(0, \varepsilon_{L_1})) | \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{t_2}, \varepsilon_{w_2})$$

$$f_{41}(\varepsilon_{L_1} | \varepsilon_{w_1}, \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{t_2}, \varepsilon_{w_2})$$

$$f_{42}(\varepsilon_{w_1} | \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{t_2}, \varepsilon_{w_2}) d\varepsilon_{L_1} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) E_{\varepsilon_{L_1}, \varepsilon_{w_1} | \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{t_2}, \varepsilon_{w_2}} [F(ln(T_1^{nw}(0, \varepsilon_{L_1})) | \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_x,$$

$$\varepsilon_{L_2}, \varepsilon_{t_2}, \varepsilon_{w_2})]$$

$$\approx f_1(\varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}) \frac{1}{R} \sum_{r=1}^{R} F(ln(T_1^{nw}(0, \varepsilon_{L_1}^r)) | \varepsilon_{L_1}^r, \varepsilon_{w_1}^r, \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{t_2}, \varepsilon_{w_2})$$

I simulate the followings and use them to compute

$$F(ln(T_1^{nw}(0, \varepsilon_{L_1}^r)) | \varepsilon_{L_1}^r, \varepsilon_{w_1}^r, \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{t_2}, \varepsilon_{w_2})$$

(a) $\varepsilon_{w_1}^r | \varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}$
(b) $\varepsilon_{L_1}^r | \varepsilon_{w_1}^r, \varepsilon_x, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}$

14. The 14th case: a husband spends all time in leisure and a wife does not do housework.

$$L = f_1(\varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} \int_{-\infty}^{T_1^{nw}(0, \varepsilon_{L_1})} \int_{-\infty}^{T_2^{w}(0)} f_2(\varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2} | \varepsilon_x,$$

$$\varepsilon_{L_2}, \varepsilon_{w_2}) d\varepsilon_{t_2} d\varepsilon_{t_1} d\varepsilon_{L_1} d\varepsilon_{w_1}$$

$$= f_1(\varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}) \cdot \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} \int_{-\infty}^{T_1^{nw}(0, \varepsilon_{L_1})} \int_{-\infty}^{T_2^{w}(0)} f_3(\varepsilon_{t_1}, \varepsilon_{t_2} | \varepsilon_x, \varepsilon_{L_1}, \varepsilon_{w_1},$$

$$\varepsilon_{L_2}, \varepsilon_{w_2}) d\varepsilon_{t_1} d\varepsilon_{t_2}$$

$$f_{41}(\varepsilon_{L_1} | \varepsilon_x, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2})$$

$$f_{42}(\varepsilon_{w_1} | \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}) d\varepsilon_{w_1} d\varepsilon_{L_1}$$

$$\approx \frac{f_1(\varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2})}{R} \sum_{r=1} B \left(\frac{ln(T_1^{nw}(0, \varepsilon_{L_1}^r)) - cm_{t_1}}{cs_{t_1}}, \frac{ln(T_2^{w}(0)) - cm_{t_2}}{cs_{t_2}} | \rho \right)$$

where B is bivariate normal distribution. I simulate

(a) $\varepsilon_{w_1}^r | \varepsilon_x, \varepsilon_{L_2}, \varepsilon_{w_2}$

(b) $\varepsilon_{L_1}^r | \varepsilon_x, \varepsilon_{w_1}^r, \varepsilon_{L_2}, \varepsilon_{w_2}$

15. The 15th case: a husband spends all time in leisure and a wife does not do market work (symmetric to case 8).

$$L = f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} \int_{-\infty}^{T_1^{nw}(0, \varepsilon_{L_1})} f_2(\varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}),$$

$$\varepsilon_{w_2} | \varepsilon_x) d\varepsilon_{t_1} d\varepsilon_{L_1} d\varepsilon_{w_1} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} \int_{-\infty}^{T_1^{nw}(0, \varepsilon_{L_1})} f_3(\varepsilon_{t_1} | \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}),$$

$$\varepsilon_{w_2}, \varepsilon_x) d\varepsilon_{t_1}$$

$$f_4(\varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}), \varepsilon_{w_2} | \varepsilon_x)$$

$$d\varepsilon_{L_1} d\varepsilon_{w_1} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} F(ln(T_1^{nw}(0,\varepsilon_{L_1})) | \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}),$$

$$\varepsilon_{w_2}\varepsilon_x)$$
$$f_4(\varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}), \varepsilon_{w_2} | \varepsilon_x)$$
$$d\varepsilon_{L_1} d\varepsilon_{w_1} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} F(ln(T_1^{nw}(0,\varepsilon_{L_1})) | \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}\varepsilon_x)$$

$$f_5(\varepsilon_{L_1} | \varepsilon_{w_1}, \varepsilon_{t_2}, \varepsilon_{L_2}, \varepsilon_{w_2}, \varepsilon_x) d\varepsilon_{L_1}$$
$$f_6(\varepsilon_{t_2}(\varepsilon_{w_2}), \varepsilon_{L_2}(\varepsilon_{w_2}) | \varepsilon_{w_1}, \varepsilon_{w_2}, \varepsilon_x)$$
$$f_{71}(\varepsilon_{w_2} | \varepsilon_{w_1}, \varepsilon_x) f_{72}(\varepsilon_{w_1} | \varepsilon_x) d\varepsilon_{w_1} d\varepsilon_{w_2}$$

$$\approx \frac{f_1(\varepsilon_x)}{R} \sum_{r=1}^{R} F(ln(T_1^{nw}(0,\varepsilon_{L_1})) | \varepsilon_{L_1}^r, \varepsilon_{w_1}^r, \varepsilon_{t_2}(\varepsilon_{w_2}^r), \varepsilon_{L_2}(\varepsilon_{w_2}^r), \varepsilon_{w_2}^r, \varepsilon_x)$$

$$f_6(\varepsilon_{t_2}(\varepsilon_{w_2}^r), \varepsilon_{L_2}(\varepsilon_{w_2}^r) | \varepsilon_{w_1}^r, \varepsilon_{w_2}^r, \varepsilon_x)$$

I simulate

(a) $\varepsilon_{w_1}^r | \varepsilon_x$

(b) $\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{w_1}^r$

(c) $\varepsilon_{L_1}^r | \varepsilon_x, \varepsilon_{w_1}^r, \varepsilon_{t_2}(\varepsilon_{w_2}^r), \varepsilon_{L_2}(\varepsilon_{w_2}^r), \varepsilon_{w_2}^r$

16. The 16th case: both husband and wife spend all their time in leisure.

$$L = f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} \int_{T_2^L}^{\infty} \int_{-\infty}^{T_1^{nw}(0,\varepsilon_{L_1})} \int_{-\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} f_2(\varepsilon_{t_1}, \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{t_2},$$

$$\varepsilon_{L_2}, \varepsilon_{w_2} | \varepsilon_x) d\varepsilon_{t_2} d\varepsilon_{t_1} d\varepsilon_{L_2} d\varepsilon_{L_1} d\varepsilon_{w_1} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} \int_{T_2^L}^{\infty} \int_{-\infty}^{T_1^{nw}(0,\varepsilon_{L_1})} \int_{-\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} f_3(\varepsilon_{t_1}, \varepsilon_{t_2} | \varepsilon_{L_1}, \varepsilon_{w_1},$$

$$\varepsilon_{L_2}, \varepsilon_{w_2}\varepsilon_x) d\varepsilon_{t_2} d\varepsilon_{t_1}$$
$$f_4(\varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{L_2}, \varepsilon_{w_2} | \varepsilon_x) d\varepsilon_{L_2} d\varepsilon_{L_1} d\varepsilon_{w_1} d\varepsilon_{w_2}$$

$$= f_1(\varepsilon_x) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{T_1^L}^{\infty} \int_{T_2^L}^{\infty} \int_{-\infty}^{T_1^{nw}(0,\varepsilon_{L_1})} \int_{-\infty}^{T_2^{nw}(0,\varepsilon_{L_2})} f_3(\varepsilon_{t_1}, \varepsilon_{t_2} | \varepsilon_{L_1}, \varepsilon_{w_1},$$

$$\varepsilon_{L_2}, \varepsilon_{w_2}\varepsilon_x) d\varepsilon_{t_2} d\varepsilon_{t_1}$$
$$f_{41}(\varepsilon_{L_2} | \varepsilon_{L_1}, \varepsilon_{w_1}, \varepsilon_{w_2}, \varepsilon_x) d\varepsilon_{L_2}$$
$$f_{42}(\varepsilon_{L_1} | \varepsilon_{w_1}, \varepsilon_{w_2}, \varepsilon_x) d\varepsilon_{L_1}$$
$$f_{43}(\varepsilon_{w_2} | \varepsilon_{w_1}, \varepsilon_x) d\varepsilon_{w_2} f_{44}(\varepsilon_{w_1} | \varepsilon_x) d\varepsilon_{w_1}$$

61

$$\approx \frac{f_1(\varepsilon_x)}{R} \sum_{r=1}^{R} B\left(\frac{ln(T_2^{nw}(0,\varepsilon_{L_2}^r)) - cm_{t_1}}{cs_{t_1}}, \frac{ln(T_1^{nw}(0,\varepsilon_{L_1}^r)) - cm_{t_2}}{cs_{t_2}} \Big| \rho\right)$$

where B is bivariate normal distribution. I simulate

(a) $\varepsilon_{w_1}^r | \varepsilon_x$

(b) $\varepsilon_{w_2}^r | \varepsilon_x, \varepsilon_{w_1}^r$

(c) $\varepsilon_{L_1}^r | \varepsilon_x, \varepsilon_{w_1}^r, \varepsilon_{w_2}^r$

(d) $\varepsilon_{L_2}^r | \varepsilon_x, \varepsilon_{L_1}^r, \varepsilon_{w_1}^r, \varepsilon_{w_2}^r$
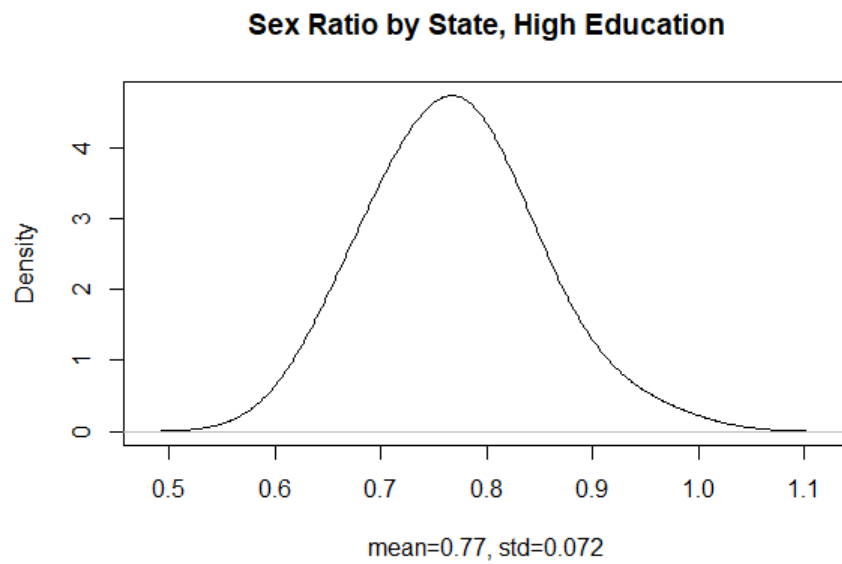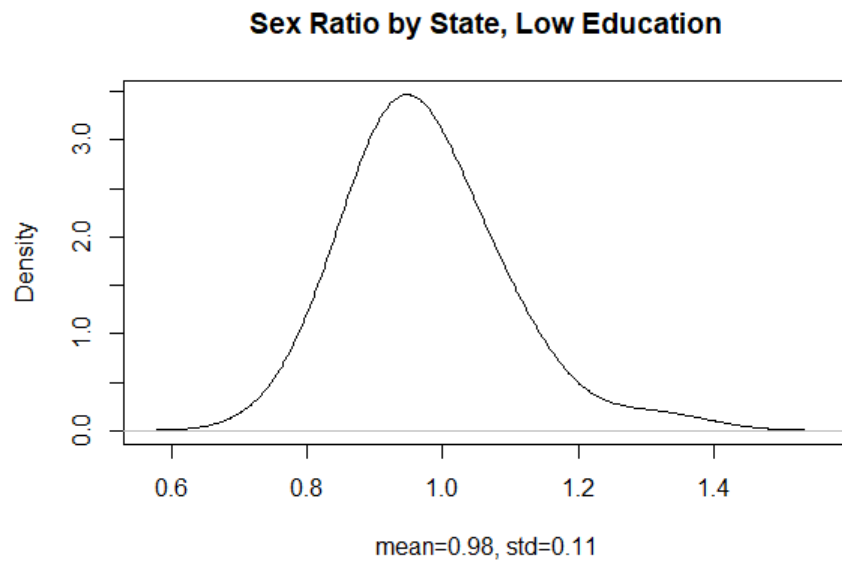
.3 Sex Ratios by State and Education Level

Sex Ratio by State, Low Education

mean=0.98, std=0.11

Sex Ratio by State, High Education

mean=0.77, std=0.072

Table 20: Sex Ratios by State and Education Level
(American Community Survey 2015)

srdata	FIPS	State	sexratioL	sexratioH
1	1	ALABAMA	0.87	0.66
2	2	ALASKA	1.34	0.93
3	4	ARIZONA	1.03	0.78
4	5	ARKANSAS	0.87	0.71
5	6	CALIFORNIA	0.99	0.81
6	8	COLORADO	1.02	0.87
7	9	CONNECTICUT	0.87	0.74
8	10	DELAWARE	0.83	0.73
9	11	DISTRICT OF COLUMBIA	0.79	0.77
10	12	FLORIDA	0.90	0.73
11	13	GEORGIA	0.93	0.66
12	15	HAWAII	1.11	0.86
13	16	IDAHO	1.03	0.79
14	17	ILLINOIS	0.94	0.79
15	18	INDIANA	0.94	0.77
16	19	IOWA	0.96	0.82
17	20	KANSAS	0.99	0.79
18	21	KENTUCKY	0.96	0.72
19	22	LOUISIANA	0.93	0.68
20	23	MAINE	1.03	0.80
21	24	MARYLAND	0.91	0.68
22	25	MASSACHUSETTS	0.93	0.74
23	26	MICHIGAN	0.99	0.77
24	27	MINNESOTA	1.10	0.84
25	28	MISSISSIPPI	0.88	0.63

Note:

sexratioL: the sex ratio of low education group (below college)

sexratioH: the sex ratio of high education group (college and above)

Table 13 (continued) Sex Ratios by State and Education Level
(American Community Survey 2015)

srdata	FIPS	State	sexratioL	sexratioH
26	29	MISSOURI	0.91	0.71
27	30	MONTANA	1.12	0.90
28	31	NEBRASKA	0.94	0.86
29	32	NEVADA	1.00	0.85
30	33	NEW HAMPSHIRE	0.99	0.79
31	34	NEW JERSEY	0.87	0.76
32	35	NEW MEXICO	0.94	0.72
33	36	NEW YORK	0.87	0.74
34	37	NORTH CAROLINA	0.93	0.67
35	38	NORTH DAKOTA	1.11	0.97
36	39	OHIO	0.90	0.75
37	40	OKLAHOMA	0.99	0.79
38	41	OREGON	1.01	0.80
39	42	PENNSYLVANIA	0.94	0.79
40	44	RHODE ISLAND	0.80	0.75
41	45	SOUTH CAROLINA	0.93	0.66
42	46	SOUTH DAKOTA	1.07	0.80
43	47	TENNESSEE	0.90	0.69
44	48	TEXAS	0.93	0.77
45	49	UTAH	1.05	0.78
46	50	VERMONT	1.27	0.73
47	51	VIRGINIA	0.95	0.72
48	53	WASHINGTON	1.03	0.80
49	54	WEST VIRGINIA	0.94	0.69
50	55	WISCONSIN	1.10	0.84
51	56	WYOMING	1.14	0.83

Note:

sexratioL: the sex ratio of low education group (below college)

sexratioH: the sex ratio of high education group (college and above)

.4 Graphical Illustration of Goodness-of-Fit

Here I use exogenous variables and estimated parameters to predict dependent variables and compare them with the actual data.