Linear Analysis of the Single-machine Infinite-bus Power System

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***Abstract*—The Single-machine Infinite-bus is a useful model to simplify multi-machine system and can also represents key aspects of systems qualitatively. In this paper, tools from linear system are applied to the system to see how they work with an nonlinear system.**

I. INTRODUCTION

We represent a study of the Single-machine Infinite-bus power system (henceforth referred to as "SMIB"), discussed in [1]. Figure 1 contains a classical dynamical model of a SMIB power system. Actually, the dynamic response of a synchronous is pretty complex, because it includes many nonlinear properties and meets destabilization like the magnetic saturation. Therefore, in order to describe and solve complicated power system, SMIB is invited.

. The system is interesting and important because it represents key aspects of the behavior of a multi-machine system qualitatively. And it is relatively simple to study but can describe the system's stability successfully [2]. We will start with discussing the dynamical model of SMIB, linearize the system at specific equilibrium to determine stability properties, determine the controllability, observability and constructability of linear system. And we apply the closed-loop estimator and closed-loop controller to the linear and nonlinear system to see how linear tools work on nonlinear system and what differences between the results are. We also test the adoptability of the closed-loop estimator /controller when introducing a disturbance to the system. In the end, we will make conclusion about practicability and veracity and what need to be improved.

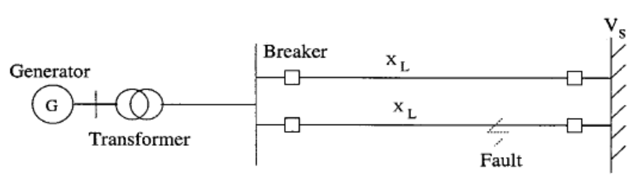

Fig.1. A SMIB power system; imagine taken from [3]

II. DYNAMICAL MODEL OF THE SYSTEM

The nonlinear dynamic model of the SMIB is [4]:

$$\begin{aligned} M\ddot{\delta} &= P - D\dot{\delta} - \eta_1 E_q \sin\delta \\ \tau \dot{E}_q &= -\eta_2 E_q + \eta_3 \cos\delta + E_{FD} \end{aligned} \tag{1}$$

Where

δ angle in radians

E_q voltage

P mechanical input power

E_{FD} field voltage

D damping coefficient

M inertial coefficient

τ time constant

η_1, η_2, η_3 constant parameters

We will write model in the form of $\dot{x} = f(x,u)$, where $x = (\delta, \dot{\delta}, E_q)^T$.

$$f(x,u) = \frac{d}{dt}\begin{bmatrix} \delta \\ \dot{\delta} \\ E_q \end{bmatrix} = \begin{bmatrix} \dot{\delta} \\ (P - D\dot{\delta} - \eta_1 E_q \sin\delta)/M \\ (-\eta_2 E_q + \eta_3 \cos\delta + E_{FD})/\tau \end{bmatrix} \tag{2}$$

Input of system is: $input = [P \quad E_{FD}]^T$.

Output of system is $output = [\delta \quad \dot{\delta}]$

We will use the constant provided in [5]:

$\eta_1 = 2.0$, $\eta_2 = 2.7$, $\eta_3 = 1.7$, $\tau = 6.6$, $M = 0.0147$, $D/M = 4$

III. ANALYSIS

A. Equilibria

A pair (x^{eq}, u^{eq}) [6] is called equilibrium point if $f(x^{eq}, u^{eq}) = 0$. The system has infinite equilibriums, because there are $\sin\delta$ and $\cos\delta$. All of them can be represented as:

$$\begin{cases} \dot{\delta} = 0 \\ P = 2E_q \sin\delta \\ E_{FD} = 2.7E_q - 1.7\cos\delta \end{cases} \tag{3}$$

We will consider $x^* = [\delta \quad \dot{\delta} \quad E_q]^T = [1.639 \quad 0 \quad 0.408]^T$ as equilibrium in the subsequent analysis.

a) Linearization: We will linearize about x^* and use it as our operating point in the next analysis.

The symbolic form for the Jacobian matrix is

$$D_A f\Big|_{\substack{x=x^* \\ u=u^*}} = \begin{bmatrix} 0 & 1 & 0 \\ -\eta_1 E_q \cos\delta/M & D/M & -\eta_1 \sin\delta/M \\ -\eta_3 \sin\delta/\tau & 0 & -\eta_2/\tau \end{bmatrix}$$

$$D_B f\Big|_{\substack{x=x^* \\ u=u^*}} = \begin{bmatrix} 0 & 0 \\ 1/M & 0 \\ 0 & 1/\tau \end{bmatrix} \tag{4}$$

$$D_C f\Big|_{\substack{x=x^* \\ u=u^*}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad D_D f\Big|_{\substack{x=x^* \\ u=u^*}} = [0]$$

For the equilibrium x^*, the dynamics of the linearization δx are given by[6]

$$\begin{aligned} \delta \dot{x} &= A\delta x + B\delta u \\ \delta y &= C\delta x + D\delta u \end{aligned} \tag{5}$$

Where A, B, C, D are given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3.831 & 4 & -135.7 \\ -0.257 & 0 & -0.409 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1/0.0147 & 0 \\ 0 & 1/6.6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D = [0]$$

b) Stability: The eigenvalues associated with x^* are:

$$\lambda_1 = -1.0452 + 2.3069i$$

$$\lambda_2 = -1.0452 - 2.3069i$$

$$\lambda_3 = 5.6815$$

Since one of eigenvalues has positive real part, the system is asymptotically unstable. It shows that the system will not decay to the origin.

B. Trajectories

We can simulate the system with parameters given in Section II and initial condition

$$[\delta_0 \quad \dot{\delta}_0 \quad E_{q0}]^T = [0 \quad 0 \quad 0]^T$$

We obtain the trajectory shown in Figure 2.

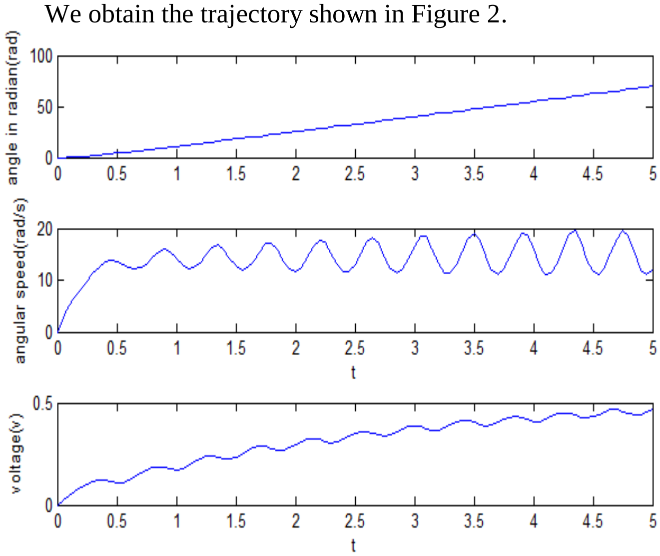

Figure.2. Nonlinear Trajectories for the SMIB using the parameters

C. Relevant Equilibrium

For the equilibrium, we will determine the Jordan form, and use it to calculate the matrix exponential for the system.

c) Jordan From: Since eigenvalues of x^* are different, the Jordan form is as following:

$$J = diag\{\lambda_1, \lambda_2, \lambda_3\} \tag{6}$$

And the similarity transformation P contains the eigenvectors of A as columns

d) Matrix Exponential:

$$e^{At} = Pe^{Jt}P^{-1} = Pdiag\{e^{\lambda_1}, e^{\lambda_2}, e^{\lambda_3}\}P^{-1} \tag{7}$$

f) Comparison with nonlinear system: Figure 3 contains traces from both the nonlinear system and the linearized system at x^* with $u(t) = [0.815 \quad 1.22]^T$, $t \geq 0$.

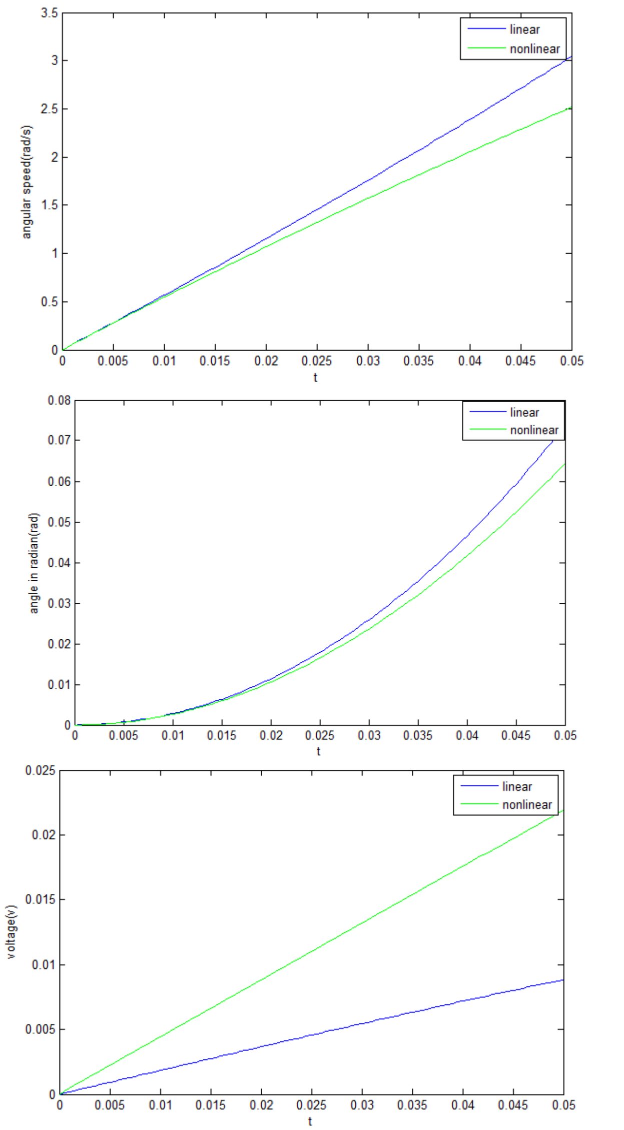

Figure.3. Comparison of linear trajectories with nonlinear trajectories

D. Controllability and Observability

The SMIB system is time-invariant, so we can determine the controllability and observability using the controllability and observability matrices respectively.

The controllability matrix:

$$C = \begin{bmatrix} 0 & 0 & 68.00 & 0 & 27.21 & -20.60 \\ 68.00 & 0 & 27.21 & -20.60 & 134.91 & 73.80 \\ 0 & 0.20 & 0 & 0.10 & -17.50 & 0 \end{bmatrix}$$

And the observability matrix:

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1.000 & 0 & 0 \\ 0 & 1.000 & 0 \\ 0 & 0 & 1.000 \\ 0 & 1.000 & 0 \\ 3.831 & 4.000 & -135.7 \\ -0.257 & 0 & -0.409 \\ 3.831 & 4.000 & -135.7 \\ 50.199 & 19.831 & -487.299 \\ 0.1051 & -0.257 & 0.1673 \end{bmatrix} \quad (11)$$

The controllability matrix C is full rank because first three columns are linearly independent. The observability matrix O is full rank because the first three rows are linearly independent. Thus the system is controllable and observable.

IV. COMPUTATION

A. *Finite-time Stability*

To get such kind of input, we need to calculate controllable subspace through following equation first:

$$\dot{W} = BB^T + AW + WA^T \quad (8)$$

The proof is shown in appendix.
Then we can get input as:

$$u(\tau) = -B^T e^{A^T(t-\tau)} W^{-1} e^{A^T t} x_0 \quad (9)$$

And then apply the input to the system, we can steer the system to the origin on the given time horizon in Figure 4.

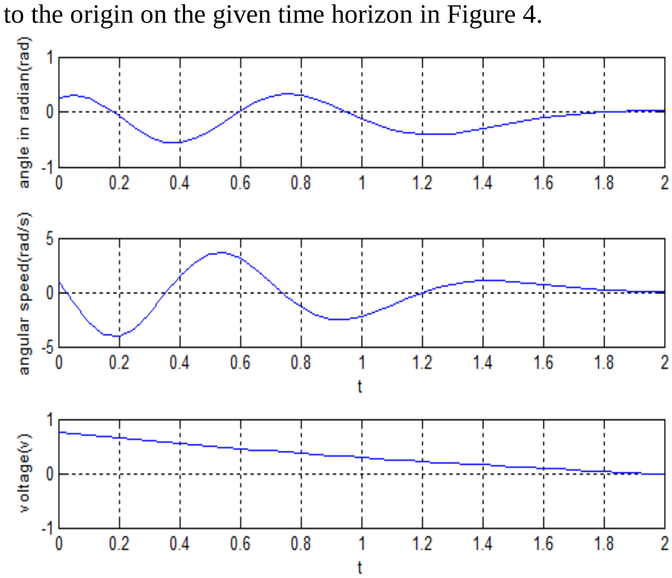

Figure.4. An open-loop control input that steers the system to the origin on given time horizon.

B. *Asymptotic Stabilization*

a) According to Feedback stabilization based on the lyapunov test [6], we need to determine μ s.t $-\mu I - A$ is stability matrix first. Here, we choose $\mu = 2$.

b) Based on the controllability and observability, the system is completely controllable, so there exists a matrix W , s.t

$$(-\mu I - A)W + W(-\mu I - A)^T = -BB^T \quad (10)$$

c) Then we can get input

$$u = -Kx \text{, where } K = 0.5B^T W^{-1} \quad (11)$$

Then we apply the input into the system, we can get asymptotically stable in Figure 5.

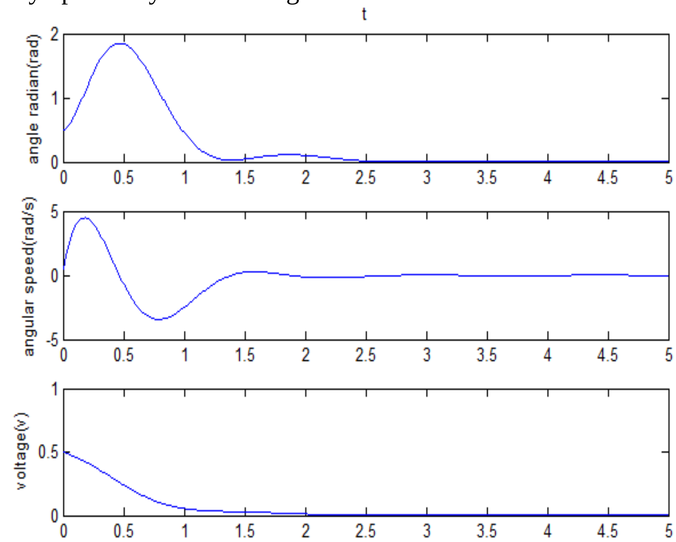

Figure 5. A closed-loop state feedback asymptotically stabilizes the system.

C. *Finite-time State Construction*

Based on theorem: Gramian-based reconstruction [6], we can use output signal to construct an estimate of the state, equations are as following:

$$x(t_1) = W_{cn}(t_0,t_1)^{-1} \int_{t_0}^{t_1} \Phi(t_0,t_1)' C(t)' \bar{y}(t) dt \quad (12)$$

Where

$\bar{y}(t) = C(t)e^{A(t-t_0)}x_0$, $W_{cn}(t_0,t_1) = \sum e^{A(jdt-t_0)} CC' e^{A'(jdt-t_0)} dt$.

Then we can get:

$$xf1 = \begin{bmatrix} \delta \\ \dot{\delta} \\ E_q \end{bmatrix}_{t=5} = \begin{bmatrix} 69.786 \\ 11.958 \\ 0.4669 \end{bmatrix}, \quad xf2 = \begin{bmatrix} \delta \\ \dot{\delta} \\ E_q \end{bmatrix}_{t=5} = \begin{bmatrix} 69.854 \\ 11.974 \\ 0.4666 \end{bmatrix},$$

use norm2 to judge the difference between them:

$$\|xf1 - xf2\|_2 = 0.070 \quad (13)$$

It shows that construction of estimate of the state works well in the context of the application domain.

D. *Asymptotic State Construction*

To create a state estimator to asymptotically construct the state of the system, we can use equation :

$$(-\mu I - A)W + W(-\mu I - A)' = -C'C \quad (14)$$

To calculate W and then $L = 0.5(CW^{-1})'$ which ensure that $A - LC$ is stability matrix and estimation errors converge to zero exponentially fast. Simulation is showed in Figure 6.

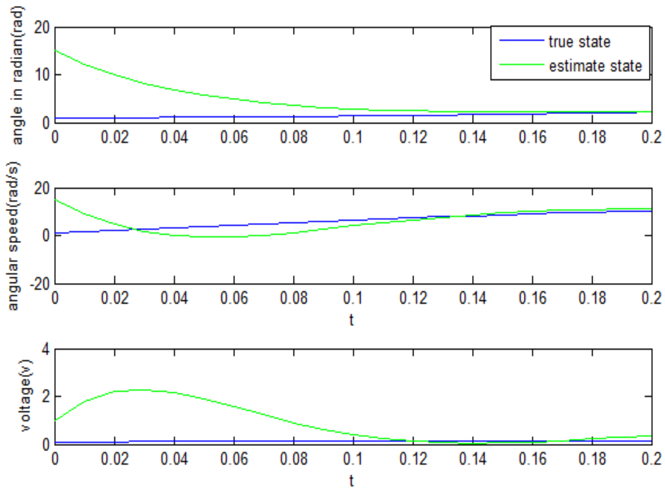

Figure.6. Simulation of true state and estimate state.

And the error here is defined as:

$$
\begin{aligned}
e &= \hat{x} - x \\
\dot{e} &= (A - LC)e
\end{aligned}
\tag{15}
$$

E. Finite-time State Construction

Implement the state feedback law from 4.2 using the estimated state from 4.4, we can get Figure 7:

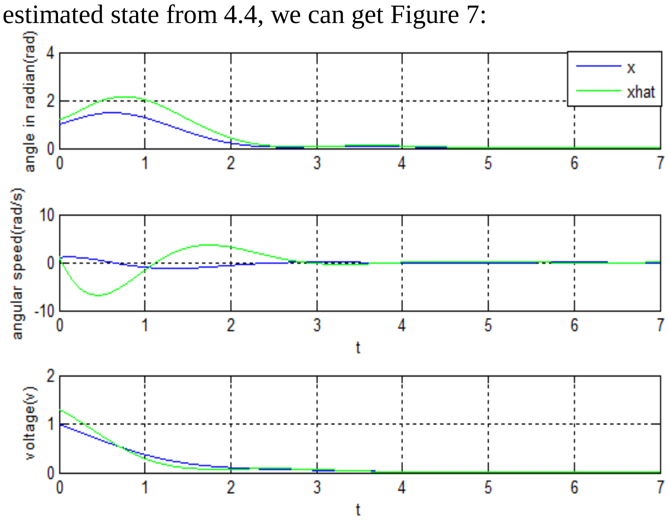

Figure.7. Implement a closed loop estimator/controller to system

We can find in the Figure 7 that a closed loop estimator/controller leads CLTI system and estimate state to origin which means it combines property of closed loop estimator and closed loop controller.

V. APPLICATION

In this section, we are applying the closed-loop estimator/controller to the origin (CNL) system to see how it works on nonlinear system.

A. Apply the closed-loop estimator/controller to the original system, we can get Figure 8.

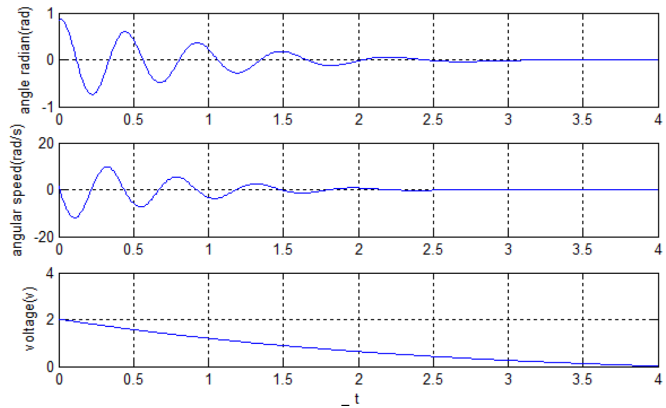

Figure.8. Apply the closed-loop estimator/controller to the original system

We can find from figure 8 that similar with CLTI system, the closed-loop estimator/controller also makes the CNL system come to the origin.

B. Introduce a disturbance $x|_{t=1} = [5 \quad 20 \quad 3]'$ to the CNL system, then apply the closed-loop estimator/controller to the disturbed system, we can get figure 9.

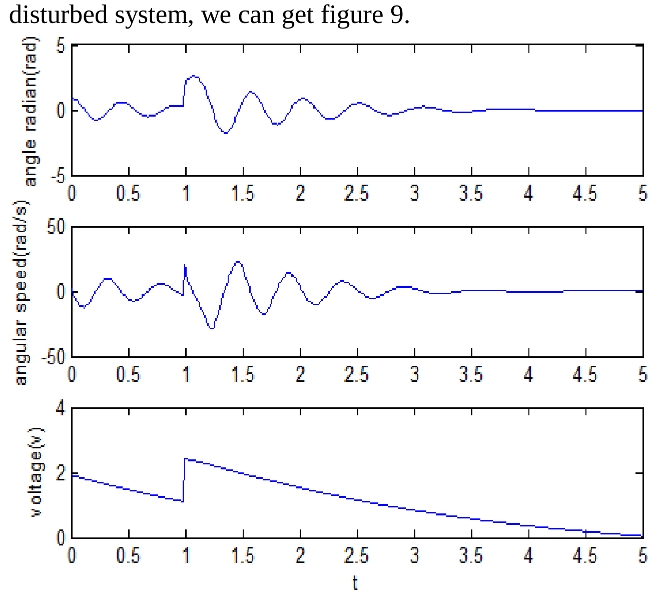

Figure.9. Apply the closed-loop estimator/controller to the disturbed system.

We can find that although there is a disturbance, the system can still converge to origin with application of closed-loop estimator/controller in the application domain, i.e. the final state is the same as system without disturbance.

VI. DISCUSSION

The SMIB system in this paper is controllable and observable. So we can As showed in Figure 3, linearization at specific equilibrium can only provide acceptable approximation within extremely short time and simulation deteriorates fast when the system beyond the time range. And even in the same system, linearization behaves differently according to different states. Therefore, a nonlinear simulation will be beneficial to nonlinear control analysis. And in the application section, we can find that the closed-loop estimator/controller achieves acceptable performance on the original system as well as the system introduced a disturbance. It means the closed-loop estimator/controller is strongly adoptable.

Although we have dealt with disturbance successfully, we still need to be critical about the result. Since even the disturbance is just an impulse, the closed loop need time to deal with it, so it cannot ensure the security of system when it meets huge and dense fault which means the processing procedure may beyond domain and range. So it is better to improve performance of the control system and implement a more aggressive estimator/controller.

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APPENDIX

Proof of the equation $\dot{W} = BB^T + AW + WA^T$:

$$W(\mathrm{t}) = \int_0^t e^{A(\mathrm{t}-\tau)} BB^T e^{A^T(\mathrm{t}-\tau)} d\tau$$

$$\dot{W}(\mathrm{t}) = BB^T + AW + WA^T$$

$$W(\mathrm{t}) = e^{At} \int_0^t e^{A(\tau)} BB^T e^{A^T(\tau)} d\tau e^{A^T t}$$

$$\dot{W}(\mathrm{t}) = Ae^{At} \int_0^t e^{A(-\tau)} BB^T e^{A^T(-\tau)} d\tau e^{A^T t} + e^{At} e^{-At} BB^T e^{A(-t)} e^{A^T t}$$

$$+ e^{At} \int_0^t e^{A(-\tau)} BB^T e^{A^T(-\tau)} d\tau e^{A^T t} A^T$$

$$= AW + BB^T + WA^T$$