Optimal Control of the Single-machine Infinite-bus Power System

Qiubao Ye, 1527311
Department of Mechanical Engineering, University of Washington

***Abstract*—The Single-machine Infinite-bus is a useful model to simplify multi-machine system and can also represents key aspects of systems qualitatively. In this paper, tools from optimal control and estimation are applied to the system to see how they work with the system.**

I. INTRODUCTION

We represent a study of the Single-machine Infinite-bus power system (henceforth referred to as "SMIB"), discussed in [1]. Figure 1 contains a classical dynamical model of a SMIB power system. Actually, the dynamic response of a synchronous is pretty complex, because it includes many nonlinear properties and meets destabilization like the magnetic saturation. Therefore, in order to describe and solve complicated power system, SMIB is invited.

The system is interesting and important because it represents key aspects of the behavior of a multi-machine system qualitatively. And it is relatively simple to study but can describe the system's stability successfully [2]. We will start with discussing the dynamical model of SMIB, linearize the system at specific equilibrium to determine stability properties, determine the controllability, observability and constructability of linear system. And we discretize the system and compare it with linearized system. And construct a quadratic cost function on a finite time horizon for CLTI and synthesis a linear time-varying state feedback law. Besides, we specify disturbance and measurement noise covariance for DLTI and construct a Kalman filter to make estimation. Based on the Linear Quadratic Regulation (LQR) and the Linear Quadratic Estimation (LQE), we solve the Linear Quadratic Gaussian (LQG) problem in the presence of disturbance and measurement noise. And finally, we apply all these tools to the original system to discuss the practicability.

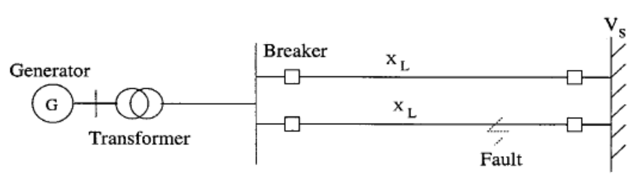

Fig.1. A SMIB power system; imagine taken from [3]

II. DYNAMICAL MODEL OF THE SYSTEM

The nonlinear dynamic model of the SMIB is [4]:

$$\begin{aligned} M\ddot{\delta} &= P - D\dot{\delta} - \eta_1 E_q \sin\delta \\ \tau \dot{E}_q &= -\eta_2 E_q + \eta_3 \cos\delta + E_{FD} \end{aligned} \tag{1}$$

Where

δ angle in radians

E_q voltage

P mechanical input power

E_{FD} field voltage

D damping coefficient

M inertial coefficient

τ time constant

η_1, η_2, η_3 constant parameters

We will write model in the form of $\dot{x} = f(x,u)$, where $x=(\delta, \dot{\delta}, E_q)^T$.

$$f(x,u) = \frac{d}{dt}\begin{bmatrix} \delta \\ \dot{\delta} \\ E_q \end{bmatrix} = \begin{bmatrix} \dot{\delta} \\ (P - D\dot{\delta} - \eta_1 E_q \sin\delta)/M \\ (-\eta_2 E_q + \eta_3 \cos\delta + E_{FD})/\tau \end{bmatrix} \tag{2}$$

Input of system is: $input = [P \quad E_{FD}]^T$.

Output of system is $output = [\delta \quad \dot{\delta}]$

We will use the constant provided in [5]:

$\eta_1 = 2.0$, $\eta_2 = 2.7$, $\eta_3 = 1.7$, $\tau = 6.6$, $M = 0.0147$, $D/M = 4$

III. ANALYSIS

A. Equilibria

A pair (x^{eq}, u^{eq}) [6] is called equilibrium point if $f(x^{eq}, u^{eq}) = 0$. The system has infinite equilibriums, because there are $\sin\delta$ and $\cos\delta$. All of them can be represented as:

$$\begin{cases} \dot{\delta} = 0 \\ P = 2E_q \sin\delta \\ E_{FD} = 2.7E_q - 1.7\cos\delta \end{cases} \tag{3}$$

We will consider $x^* = [\delta \quad \dot{\delta} \quad E_q]^T = [1.639 \quad 0 \quad 0.408]^T$ as equilibrium in the subsequent analysis.

a) Linearization: We will linearize about x^* and use it as our operating point in the next analysis.

The symbolic form for the Jacobian matrix is

$$D_A f\Big|_{\substack{x=x^* \\ u=u^*}} = \begin{bmatrix} 0 & 1 & 0 \\ -\eta_1 E_q \cos\delta / M & -D/M & -\eta_1 \sin\delta / M \\ -\eta_3 \sin\delta / \tau & 0 & -\eta_2/\tau \end{bmatrix}$$

$$D_B f\Big|_{\substack{x=x^* \\ u=u^*}} = \begin{bmatrix} 0 & 0 \\ 1/M & 0 \\ 0 & 1/\tau \end{bmatrix} \tag{4}$$

$$D_C f\Big|_{\substack{x=x^*\\u=u^*}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ D_D f\Big|_{\substack{x=x^*\\u=u^*}} = [0]$$

For the equilibrium x^*, the dynamics of the linearization δx are given by[6]

$$\begin{aligned} \delta \dot{x} &= A\delta x + B\delta u \\ \delta y &= C\delta x + D\delta u \end{aligned} \tag{5}$$

Where A, B, C, D are given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3.831 & -4 & -135.7 \\ -0.257 & 0 & -0.409 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1/0.0147 & 0 \\ 0 & 1/6.6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D = [0]$$

b) Stability: The eigenvalues associated with x^* are:

$$\lambda_1 = -3.4374 + 1.7254i$$

$$\lambda_2 = -3.4374 - 1.7254i$$

$$\lambda_3 = 2.4638$$

Since one of eigenvalues has positive real part, the system is asymptotically unstable. It shows that the system will not decay to the origin.

B. *Trajectories*

We can simulate the system with parameters given in Section II and initial condition

$$[\delta_0 \quad \dot{\delta}_0 \quad E_{q0}]^T = [0 \quad 0 \quad 0]^T$$

We obtain the trajectory shown in Figure 2.

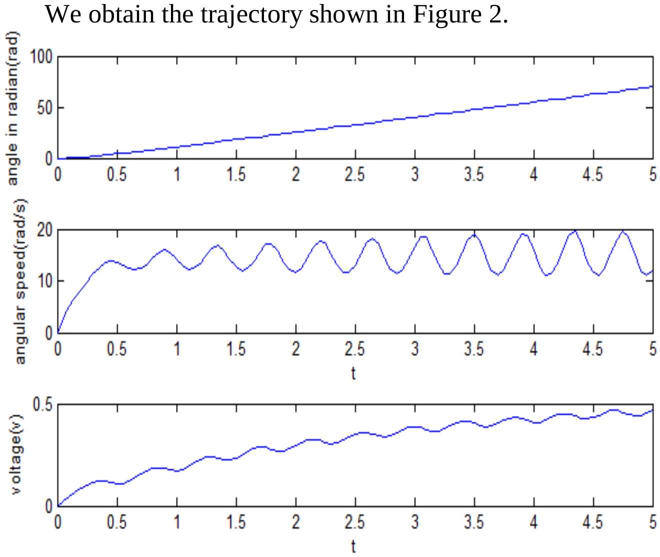

Figure.2. Nonlinear Trajectories for the SMIB using the parameters

C. *Controllability and Observability*

The SMIB system is time-invariant, so we can determine the controllability and observability using the controllability and observability matrices respectively.

The controllability matrix:

$$C = \begin{bmatrix} 0 & 0 & 68.00 & 0 & 27.21 & -20.60 \\ 68.00 & 0 & 27.21 & -20.60 & 134.91 & 73.80 \\ 0 & 0.20 & 0 & 0.10 & -17.50 & 0 \end{bmatrix}$$

And the observability matrix:

$$\mathrm{O} = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1.000 & 0 & 0 \\ 0 & 1.000 & 0 \\ 0 & 0 & 1.000 \\ 0 & 1.000 & 0 \\ 3.831 & 4.000 & -135.7 \\ -0.257 & 0 & -0.409 \\ 3.831 & 4.000 & -135.7 \\ 50.199 & 19.831 & -487.299 \\ 0.1051 & -0.257 & 0.1673 \end{bmatrix} \tag{6}$$

The controllability matrix C is full rank because first three columns are linearly independent. The observability matrix O is full rank because the first three rows are linearly independent. Thus the system is controllable and observable.

IV. COMPUTATION

A. *Perform discretization of the system*

For CLTI system, we have continuous-time uncertainty propagation:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + F(t)w(t) \tag{7}$$

If inputs and disturbances are approximately constant on a s mall time step $\Delta > 0$, i.e. time is divided into intervals $\{[t_k, t_{k+1}]\}\big|_{k=0}^{L-1}$ with $t_{k+1} - t_k = \Delta$, we have:

$$\begin{aligned} x(t_{k+1}) &\approx \Phi(t_{k+1}, t_k)x(t_k) + \\ &\quad \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, t)\big[B(t_k)u(t_k) + F(t_k)w(t_k)\big]dt \\ &\approx \exp(A_{tk}\Delta)x_{tk} + \bar{A}_{tk}\left(I - \bar{A}_{tk}^{-1}\right)A_{tk}^{-1}B_{tk}u_{tk} \\ &\quad + \bar{A}_{tk}\left(I - \bar{A}_{tk}^{-1}\right)A_{tk}^{-1}F_{tk}w_{tk} \end{aligned} \tag{8}$$

Thus, we have:

$$\bar{A} = e^{A\Delta} = \begin{bmatrix} 1 & 0.0098 & -0.0067 \\ 0.0388 & 0.9960 & -1.3279 \\ -0.0026 & 0 & 0.9959 \end{bmatrix} \tag{9}$$

$$\bar{B} = -\left(I - e^{A\Delta}\right)A^{-1}B = \begin{bmatrix} 0.0034 & 0 \\ 0.6669 & -0.0010 \\ 0 & 0.0015 \end{bmatrix} \tag{10}$$

$$\bar{C} = C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{11}$$

$$\bar{D} = 0 \tag{12}$$

And then plot DLTI and CLTI system in Figure 3. It is clear that DLTI and CLTI are agreed well.

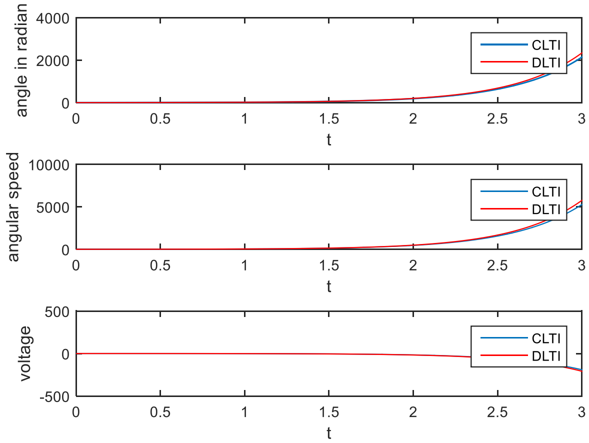

Figure.3 CLTI and DLTI system

B. *Quadratic Cost Function*

In my system, I define the quadratic cost function as following:

$$J = \frac{1}{2}\int_{t_0}^{t_f} x(t)^T Qx(t) + u(t)^T Ru(t)dt \tag{13}$$

And according to a second order approximation to the running cost portion of J, we can calculate Q and R:

$$Q = \begin{bmatrix} 1/20^2 & 0 & 0 \\ 0 & 1/20^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 1/(2\pi)^2 & 0 \\ 0 & 1/(2\pi)^2 \end{bmatrix}$$

a) Solving the continuous time Riccati differential equation, we can get $S(t)$

$$\dot{S}(t) = -A^T S(t) - S(t)A + S(t)BR^{-1}B^T S(t) - Q \tag{14}$$

b) Then we can get input [7]

$$u = -Kx\text{, where } K = R^{-1}B^T S(t) \tag{15}$$

Then we apply the input into the system, we can get optimal cost function in Figure 4.

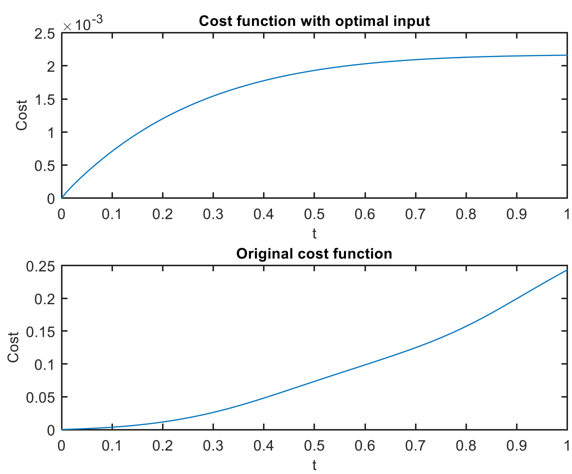

Figure 4. Original cost function and Optimal cost function

The optimal cost function makes a huge progress with optimal input. And the performance of the controlled system is desired and realistic in the context of the application domain.

C. *Kalman Filter*

The Kalman filter model assumes the true state at time t is evolved from the state at (t-1) according to:

$$x_{t+1} = A_t x_t + B_t u_t + F_t w_t \tag{16}$$

With the conditions as following:

Initial distribution:

$$E[x_0] = \bar{x}_0$$

$$E[x_0] = \bar{x}_0 E\left[\left(x_0 - \bar{x}_0\right)\left(x_0 - \bar{x}_0\right)^T\right] = \Sigma_0$$

Disturbances:

$$\begin{aligned} &E[w_t] = 0 \\ &E\left[w_t w_t^T\right] = Q_t \\ &E\left[w_t w_\tau^T\right] = 0 \text{ if } t \neq \tau \end{aligned} \tag{17}$$

Observation:

$$z_t = H_t x_t + \eta_t \tag{18}$$

Noise:

$$\begin{aligned} &E[\eta_t] = 0 \\ &E\left[\eta_t \eta_t^T\right] = R_t \\ &E\left[\eta_t \eta_\tau^T\right] = 0 \text{ if } t \neq \tau \\ &E\left[\eta_t w_\tau^T\right] = 0\text{, for all } t \text{ and } \tau \end{aligned} \tag{19}$$

We need to take five steps to implement it [7]:

i) State prediction:

$$x_t^- = A_{t-1}x_{t-1}^+ + B_{t-1}u_{t-1} \tag{20}$$

ii) Covariance update:

$$\Sigma_t^- = A_{t-1}\Sigma_{t-1}^+ A_{t-1}^T + F_{t-1}Q_{t-1}^+ F_{t-1}^T \tag{21}$$

iii) Filter gain:

$$k_t = \Sigma_t^- H_t^T \left(H_t \Sigma_t^- H_t^T + R_t\right)^{-1} \tag{22}$$

iv) State update:

$$x_t^+ = x_t^- + k_t\left(z_t - H_t x_t^-\right) \tag{23}$$

v) Covariance update:

$$\Sigma_t^+ = \left(\Sigma_t^- + H_t^T R_t^{-1} H_t\right)^{-1} \tag{24}$$

And according to the state range, we can estimate the disturbance and measurement noise covariance for the system:

$$Q = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After applying these conditions and five steps, we can get estimate state in Figure 5

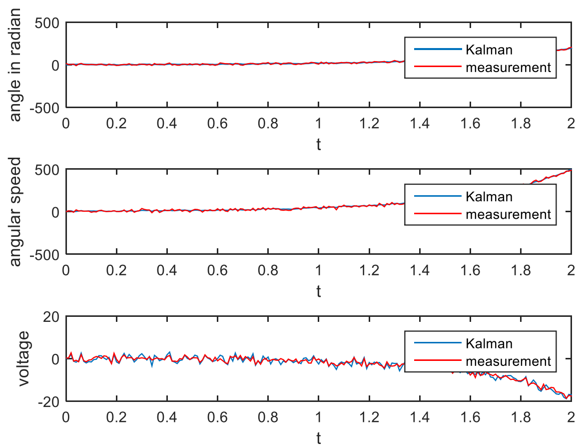

Figure.5. Estimate state and measurement

From the figure, we can find out that the estimated states have a good convergence to the system state along a non-equilibrium trajectory.

D. Linear Quadratic Estimation Problem

Based on the state feedback law and the estimated state from B and C, we can solve the Linear Quadratic Gaussian problem in the presence of disturbance and measurement noise in figure 6

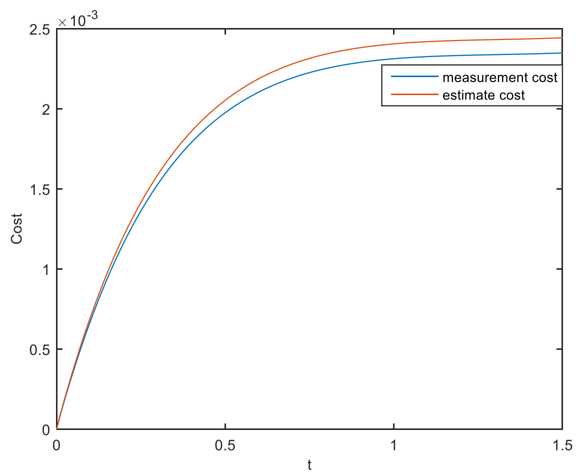

Figure.6. Measurement cost and estimate cost for CLTI.

We can find in figure 6 that even in the existence of disturbance and measurement noise, the estimate cost is very close to measurement cost.

V. APPLICATION

In this section, we are applying the LQG estimator/controller to the origin (CNL) system to see how it works on nonlinear system.

A. Apply the LQG estimator/controller to the original system, we can get Figure 7.

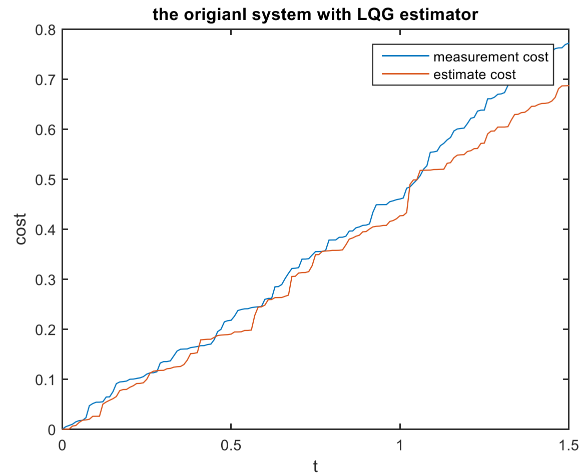

Figure.7. application of LQG to the original system

We can find from figure 7 that the LQG estimator/controller can also apply to the original system and works well.

B. make the disturbance and measurement models more realistic in a manner that is relevant to the application domain, we choose to add non-zero-mean noise to it which has 0.1 mean and then apply the LQG estimator/controller to the disturbed system showed in figure 8.

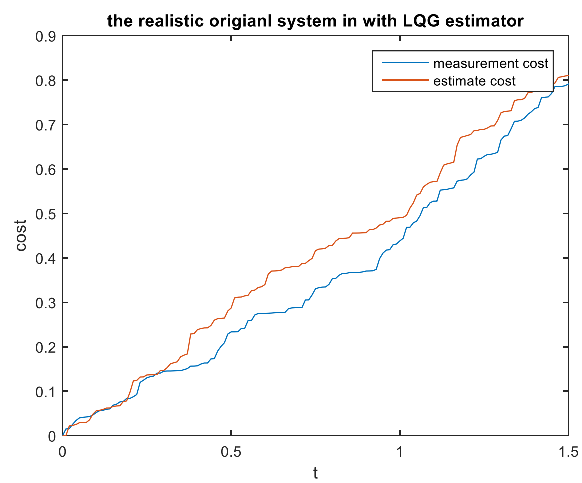

Figure.8. Apply the LQG estimator/controller to the realistic disturbed system.

We can find that in the realistic condition, the measurement cost and estimate cost are very close which means the LQG estimator/controller can work well in a realistic condition.

VI. DISCUSSION

A. The SMIB system in this paper is controllable and observable. Linearization at specific equilibrium can only provide acceptable approximation within extremely short time and simulation deteriorates fast when the system beyond the time range. And discretization of CLTI model using a proper time step 0.01 can agree well to trajectory of CLTI model.

Apply the linear time-varying state feedback law to the quadratic cost function, we make huge progress with optimal input which means we minimize the cost. And Kalman filter achieve a good agreement of estimate state to the measurement state. With disturbance and measurement noise we apply the LQR and LQE to the LQG problem and also get an estimate cost close to the measurement cost. Furthermore, we find that LQG estimator/controller can apply to original system well, even with realistic noise.

B. Although we have dealt with disturbance successfully, we still need to be critical about the result. Since even the disturbance is just white noise, the estimator is not 100% perfectly match with the real state, so it cannot ensure the security of system when it meets huge and dense fault which means the processing procedure may beyond domain and range. So it is better to improve performance of the control system and implement a more aggressive estimator or controller.

References

[1] H.F. Wang*, Static synchronous series compensator to damp power system oscillations. Electric Power Systems Research, 54:113-119,2000.

[2] Yi Guo, David J.Hill, Youyi Wang, Global Transient Stability and Voltage Regulation for Power System. IEEE Transaction on Power Systems,16(4):678-688, 2001

[3] Youyi Wang, David J.Hill, Richard H.Middleton, Transient Stability Enhancement And Voltage Regulation of Power Systems. IEEE Transaction on Power Systems, 8(2):620-627, 1993.

[4] M.A.Pai, Power System Stability Analysis by the direct method of Lyapunov North-Holland, Amsterdam, 1981.

[5] Hassan K. Khalil, Nonlinear System, Michigan State University,2002.

[6] Joao P. Hespanha, Linear System Theory. Princeton Univeristy,2009.

[7] Robert F. Stengel, Optimal Control and Estimation. Dover Publications, Inc.,1994.