Positioning Vector Framework for UWB-based Collaborative Positioning: Additional Materials

Yerkezhan Sartayeva, Henry C.B. Chan, Yunfei Liu

I. Positioning Vector Framework

A. Vector Types

Definition I.1 (Positioning vector). *A positioning vector* $M_xM_y = \begin{bmatrix} r_{xy} \\ \theta_{xy} \end{bmatrix} \in \mathbb{R}^2$ *is a vector that connects two mobile terminals* M_x *and* M_y *with its origin at* M_x *in a two-dimensional space, where* r_{xy} *is the distance between* M_x *and* M_y *and* θ_{xy} *is the angle the vector forms with the x-axis of the indoor space. A positioning vector defines the position of* M_y *relative to* M_x*, i.e., assuming* M_x *is the frame of reference.*

If there are no reference terminals, then the basis vectors of the indoor space are the true north and the true east axes. Please refer to section I-B for discussion on how θ_{xy} is calculated. Since a positioning vector is defined by the distance and angle between two terminals, a positioning vector is said to be unavailable if either is missing.

We distinguish four types of positioning vectors: measured, actual, intermediate generated and generated. An actual positioning vector $M_xM_y^{(a)}$ is a positioning vector that is defined by the ground truth distance $r_{xy}^{(a)}$ and angle $\theta_{xy}^{(a)}$ between M_x and M_y, i.e., $M_xM_y^{(a)} = \begin{bmatrix} r_{xy}^{(a)} \\ \theta_{xy}^{(a)} \end{bmatrix}$. In other words, an actual positioning vector is a vector that has zero error. In UWB-based positioning, actual distances and angles are unavailable, but they can be measured with UWB, forming "measured vectors". A measured positioning vector, denoted as, $M_xM_y^{(m)}$, is defined by the distance $r_{xy}^{(m)}$ and angle $\theta_{xy}^{(m)}$ between M_x and M_y estimated with the help of UWB. Since estimated distance and angle usually have some noise, a measured positioning vector involves some error and deviates from the actual vector, i.e., $||M_xM_y^{(m)} - M_xM_y^{(a)}||_2 \neq 0$.

Distance and/or angle measurements may not be available between a pair of terminals due to obstructions or being outside a node's coverage area. In that case, they can be calculated through other available vectors, and the resulting vector is referred to as an intermediate generated positioning vector.

Definition I.2 (Intermediate generated positioning vector). *An intermediate generated positioning vector* $M_xM_y^{(g_{\{x\}\cup gp\cup\{y\}})}$ *of degree* K *is a vector that connects two mobile terminals* M_x *and* M_y *with its origin at* M_x *that is calculated through an ordered set of terminals* gp *of size* K *as* $M_xM_y^{(g_{\{x\}\cup gp\cup\{y\}})} = \sum_{v\in igv_{\{x\}\cup gp\cup\{y\}}} v$*, where* $gp = \{k_1, ..., k_K\}, \{x, y\} \cap gp = \emptyset, 0 < K < N-1$*,* $\{x\} \cup gp \cup \{y\}$ *is referred to as a "vector generation path", and* $igv_{\{x\}\cup gp\cup\{y\}} = \{M_xM_{k_1}^{(m)}, M_{k_1}M_{k_2}^{(m)}, ..., M_{k_{K-1}}M_{k_K}^{(m)}, M_{k_K}M_y^{(m)}\}$ *is the set of measured vectors involved in the intermediate generation of* $M_xM_y^{(g_{\{x\}\cup gp\cup\{y\}})}$.

Note that an intermediate generated vector $M_xM_y^{(g_{\{x\}\cup gp\cup\{y\}})}$ can only be computed if all vectors in $igv_{\{x\}\cup gp\cup\{y\}}$ are available. If the number of intermediate vectors is insufficient to produce an intermediate generated vector of degree K, then higher values of K can be tried. However, the larger the degree of an intermediate generated vector is, the more computationally expensive the vector is to calculate, so it is advisable to keep K low.

For one vector, there are P_K^N possible vector generation paths, so there are multiple possible intermediate generated vectors. Our hypothesis is that by averaging intermediate generated vectors, high-noise vectors can be mitigated by low-noise vectors, reducing noise overall. The resulting vectors are referred to as generated vectors. They can serve as a good replacement for not only high-noise vectors but also for missing vectors.

Definition I.3 (Generated positioning vector). *A generated positioning vector* $M_xM_y^{(g)}$ *of degree* K *is a vector that connects two mobile terminals* M_x *and* M_y *with its origin at* M_x *that is calculated as the average of all possible intermediate generated vectors of degree* K *for* $M_xM_y^{(m)}$ *as well as* $M_xM_y^{(m)}$ *itself, if available.* $M_xM_y^{(g)} = \frac{1}{P_{xy}+d}(M_xM_y^{(m)} \times d + \sum_{i,...,j}(M_xM_{k_i}^{(m)} + \underbrace{...}_{K-1} + M_{k_j}M_y^{(m)}), \forall i, j \in \{1, ..., K\}, k_i \neq k_j \neq x \neq y, 0 < K < N-1$*, where* $d = 1$ *if* $M_xM_y^{(m)}$ *is available and 0 otherwise.*

It could be more accurate to average intermediate generated vectors of multiple degrees, but it is computationally expensive as the number of these vectors would grow on a factorial scale. In the worst case, if a measured vector $M_xM_y^{(m)}$ cannot be generated but $M_yM_x^{(m)}$ is available, then $M_xM_y^{(m)}$ can be generated as $-M_yM_x^{(m)}$ (zero-degree vector generation). Note that vector generation only involves measured vectors. However, if newly generated vectors are also included in generating other vectors, this is referred to as sequential vector generation [**?**].

B. Orientation Vectors and Coverage Area

iPhones are not omnidirectional, meaning the reach of their signal is limited by their direction. To model this important property, it is necessary to introduce the concept of orientation vectors and coverage areas.

Definition I.4 (Orientation vector). *A terminal M_x's orientation vector ϕ_x determines which direction M_x is facing with respect to the x-axis of the indoor space S and is perpendicular to its back. Its magnitude is equivalent to the terminal's maximum range. $\phi_x = \begin{bmatrix} r_x \\ 90 - \gamma_x \end{bmatrix}$, where r_x is the maximum range of a terminal and γ_x is the absolute bearing of M_x (with respect to the y-axis of the indoor space, which is either the true north (in the absence of a custom origin, measured with a compass) or the direction of an edge of the indoor space).*

In iPhones, the maximum range is $40\,\mathrm{m}$+ [?]. It is assumed that error in $\gamma_k^{(m)}$ is 0 for any terminal, meaning compass measurements have no error. Apart from ranging, iPhones are capable of measuring the relative angle of signal transmission with respect to their orientation vector, which can be leveraged for positioning.

Definition I.5 (Relative angle). *The relative angle θ'_{xy} of a positioning vector M_xM_y is the angle it forms with the orientation vector of M_x (ϕ_x).*

Please refer to [?] for an illustration of relative angles. iPhones are limited in the maximum relative angle they can measure, i.e., they have a restricted field of view. We shall define the field of view of a mobile terminal as its coverage area.

Definition I.6 (Coverage area). *The coverage area of a mobile terminal M_x is the area within which it can communicate with other terminals. It is defined by M_x's orientation vector ϕ_x and half-angle β_x. If the distance between a mobile terminal M_y and M_x is at most $||\phi_x||_2$ and the actual relative angle of $M_xM_y^{(a)}$, i.e., ${\theta'_{xy}}^{(a)}$, is at most β_x, M_y is said to be within the coverage area of M_x, meaning $M_xM_y^{(m)}$ can be estimated with the help of UWB. The coverage area is symmetrical around its orientation vector.*

Since iPhones report relative angles rather than absolute angles with respect to the x-axis of the indoor space, they have to be converted into absolute angles as $\theta_{xy} = 90 - \gamma_x - \theta'_{xy}$, where γ_x is the angle M_x's orientation vector forms with the y-axis of the indoor space. If θ'_{xy} is subtracted from $90 - \gamma_x$, this results in the absolute angle θ_{xy} of the vector M_xM_y with respect to the x-axis of the indoor space.

C. Vector Graph

When combined, all positioning vectors can be put together to form a vector graph, where each vertex represents a single mobile terminal or anchor (there are N terminals overall) and is denoted as M_x, where x is the terminal ID. Note that during positioning, only measured vectors are available, and a graph with only measured vectors is denoted as $\mathbf{G^{(m)}}$. The vector graph is a directed graph where each edge M_xM_y is a positioning vector between some terminals M_x and M_y. In ideal conditions, the vector graph would be fully connected, i.e., there would be a positioning vector between every pair of nodes, but due to noise and environmental constraints, some edges might be unavailable.

D. Positioning Performance Metrics

In absolute positioning, the goal is to accurately estimate OM_x for each terminal M_x. In relative positioning, however, since there is no single origin, the goal is to minimise the vector error between measured vectors and actual vectors and generate as many missing vectors as possible. Thus, positioning performance can be evaluated in terms of two metrics: vector error and detection rate, which are defined next.

Definition I.7 (Vector error). *The vector error $\delta_{xy}^{(i,j)}$ of a vector $M_xM_y^{(i)}$ of some vector type (i) (e.g., if (i) is (g), then $M_xM_y^{(i)}$ is a generated vector $M_xM_y^{(g)}$), is the Euclidean distance between $M_xM_y^{(i)}$ and $M_xM_y^{(j)}$ of some vector type (j), i.e., $\delta_{xy}^{(i,j)} = ||M_xM_y^{(i)} - M_xM_y^{(j)}||_2$.*

Definition I.8 (Detection rate). *The detection rate of a vector graph $\mathbf{G}$ is defined as the ratio of the number of available vectors in $\mathbf{G}$ ($M_{\mathbf{G}}$) and the maximum number of vectors possible in $\mathbf{G}$, which is $N \times (N-1)$. This metric serves to measure the coverage of $\mathbf{G}$, i.e., the average number of terminals whose locations each terminal can estimate.*

Note that positioning error of a graph $\mathbf{G}$ is defined as the average vector error for all available vectors in $\mathbf{G}$.

E. Positioning Methods

Let the input vector graph be $\mathbf{G^{(m)}}$, and let the output of the vector generation algorithm and the HA be $\mathbf{G^{(g)}}$ and $\mathbf{G^{(HA)}}$, respectively. The goal of the algorithms is to produce a denoised vector graph with better coverage. Denoising a vector graph can be facilitated by searching for vectors that are consistent with each other because consistent vectors are more likely to be close to their actual version. Specifically, vector consistency is defined as follows.

Definition I.9 (Vector consistency). *$M_xM_y^{(m)}$ and vectors involved in generating $M_xM_y^{(g_{\{x\}\cup cp\cup\{y\}})}$ through some ordered set $cp = \{k_1, k_2, ..., k_C\}$ of size C ($\{x,y\} \cap cp = \emptyset$), i.e., $igv_{\{x\}\cup cp\cup\{y\}} = \{M_xM_{k_1}^{(m)}, M_{k_1}M_{k_2}^{(m)}, ..., M_{k_{C-1}}M_{k_C}^{(m)}, M_{k_C}M_y^{(m)}\}$, are said to have a consistency of degree C if the vector error of $M_xM_y^{(g_{\{x\}\cup cp\cup\{y\}})}$ and $M_xM_y^{(m)}$ is no more than a consistency threshold δ, i.e., $||M_xM_y^{(m)} - M_xM_y^{(g_{\{x\}\cup cp\cup\{y\}})}||_2 \leq \delta$.*

We shall refer to $\{x\} \cup cp \cup \{y\}$ as a "consistency path" through which an intermediate generated vector is generated for consistency checking. The hypothesis is that if $||M_xM_y^{(m)} - M_xM_y^{(g_{\{x\}\cup cp\cup\{y\}})}||_2 \leq \delta$, then $M_xM_y^{(m)}$ and vectors in $igv_{\{x\}\cup cp\cup\{y\}}$ are likely to be close to actual vectors, so they can be preserved in the vector graph. This consistency requirement can act as a heuristic to search for high-noise vectors in the HA. Note that in both algorithms, to speed up the search for consistent vectors, some vectors

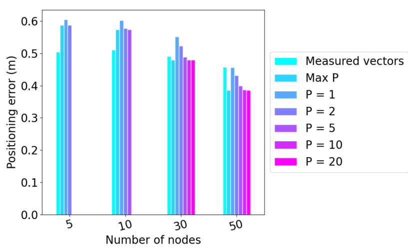

Fig. 1. Different numbers of intermediate generated vectors (P) vs positioning error for different numbers of terminals (N).

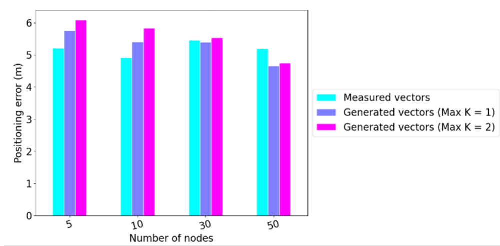

Fig. 2. Positioning error for different cumulative values of K and number of nodes N.

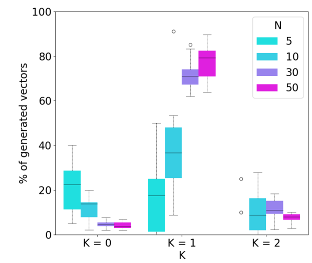

Fig. 3. The distribution of generated vectors by K for different N when the maximum vector generation degree K was set to 2.

were negated, if possible. Suppose there is a consistency path $cp = \{a, b, c\}$, where vectors $M_aM_b^{(m)}, M_bM_c^{(m)}$ are needed to calculate $M_aM_c^{g\{a,b,c\}}$. If $M_aM_b^{(m)}$ is missing, then $M_aM_c^{g\{a,b,c\}}$ cannot be calculated. However, if $M_bM_a^{(m)}$ is available, $M_aM_b^{(m)}$ can be replaced with $-M_bM_a^{(m)}$. We shall refer to $-M_bM_a^{(m)}$ as the negated version of $M_aM_b^{(m)}$. In general, when a search for consistent vectors was conducted, then negated vectors were tried if the original vectors were unavailable.

II. Vector Graph Properties

To optimise the proposed algorithms, this section will investigate the influence of vector generation parameters on positioning error and detection rate, which are: (1) P - maximum number of intermediate generated vectors used for vector generation; (2) K - vector generation degree. Please note that all simulations were run 10 times, no anchors were used, R was set to 100% and vectors were generated non-sequentially.

1) Different P: According to definition I.3, a generated vector is the average of all possible intermediate generated vectors. However, using all possible intermediate generated vectors can be costly as their number grows on a factorial scale with respect to N. Let the maximum number of intermediate generated vectors involved in vector generation be denoted as P. Therefore, it is necessary to explore how reducing P influences positioning error. Note that P defines a cap on the number of intermediate generated vectors for a given vector, i.e., if that number is less than P, it is still accepted, but no more than P vectors are averaged. Fig. 1 shows the average positioning error over 10 simulations for 5, 10, 30 and 50 nodes for $P \in [1, 2, 5, 19, 20, N-2]$ and the maximum vector generation degree K set to 1. Since the maximum vector generation degree K was set to 1, the maximum possible P is $N - 2$, i.e., for a vector M_xM_y, there are at most $N - 2$ mobile terminals through which M_xM_y can be generated, so "Max P" in Fig. 1 refers to $N - 2$. Note that there is no data for cases where P exceeds its maximum value. In addition, for each N, the same vector graph was used for each P to compare positioning error against the same benchmark. The figure shows that positioning error declines as P goes up, but positioning error seems to plateau after a P of 5 for all N, reaching almost the same level as that for maximum P, meaning P can be capped at 5, i.e., it is not necessary to include all possible intermediate generated vectors in vector generation as improvement in positioning error becomes negligible.

2) Cumulative K and Fixed P: The next set of simulations sought to explore how increasing K (if a vector cannot be generated with a lower degree) influences positioning error and coverage. In other words, if a vector cannot be generated through one terminal ($K = 1$) and is allowed to be generated through two terminals ($K = 2$), how many vectors would need to be generated through two terminals? If non-zero vector generation is not possible in the worst case, how many zero-degree vector generations would be possible? We shall refer to maximum K and cumulative K interchangeably. P was set to 1 to make sure vectors would be generated through degree $K+1$ only if there were no possible combinations for degree K. Fig. 3 shows a breakdown of the ratios of vectors for four N (5, 10, 30, 50) vs their vector generation degrees (max K is 2) over 10 simulations. The ratios in the figure denote the number of vectors generated with a certain K out of the total number of vectors ($N \times (N-1)$). It can be observed that for all N, K of 1 was sufficient for vector generation for the majority of vectors, while about 10% of vectors had to be generated with $K = 2$. Fig. 3 also illustrates an upward trend in detection rate when cumulative K was increased to 2, but the difference across all N is not high. When it comes to positioning error, based on Fig. 2, which depicts the average positioning error over the same 10 simulations for cumulative K of 1 and 2 (the same graph was used for each unique combination of N and maximum K), even though positioning error was slightly higher for generated vectors compared to measured vectors, the detection rate of generated vectors was much higher. It can also be observed that positioning error increased slightly as maximum K went up, meaning that a little gain in detection rate comes at the expense of a slightly higher positioning error and higher computational complexity, so using a maximum K of 1 for vector generation is adequate.