Please show **all** your work! Answers without supporting work will not be given credit.
You have **3** hours to complete this exam.

Name:

1. Let $[AB]$ be a segment in the plane and $C \in [AB]$ be a point between A and B. Draw the semicircles $\mathscr{C}_1$ (respectively $\mathscr{C}_2$ and $\mathscr{C}_3$) with diameters $[AB]$ (resp. $[AC]$ and $[CB]$). Draw a perpendicular to $[AB]$ passing through C. It intersects $\mathscr{C}_1$ at D. See the figure. Given that the segment $[CD]$ has length equal to 2, find the area of the shaded region bounded by the semicircles $\mathscr{C}_1$ $\mathscr{C}_2$ and $\mathscr{C}_3$.
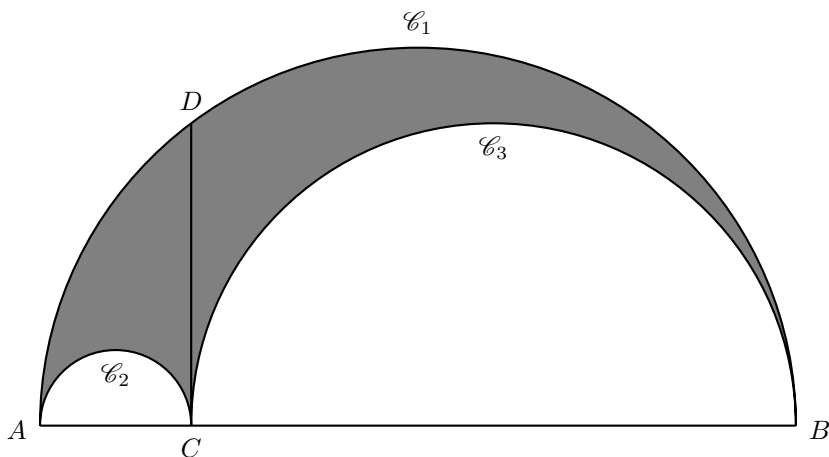
2. Evaluate the following integral

$$\int_0^\pi \frac{sin(100x)}{sinx} dx.$$

3. Let $a, b, c > 0$. Show that

$$\frac{a^2}{\sqrt{bc}} + \frac{b^2}{\sqrt{ca}} + \frac{c^2}{\sqrt{ab}} \geq a + b + c$$

4. Total number of residents is $2n$ in an auyl or village ($n \in \mathbb{N}, n \geq 2$). Each week n residents attend a toi (wedding event). After series of celebrations it turned out that every two residents were together on at least one toi. What is the minimum number of tois needed to happen with such condition?