WEDNESDAY 8:00-11:00
QUIZ 3 - 16 pts
VARIANT 1

Time: *75 minutes*
You need to write proof or explanation for each task!

Full name:

1) Prove that $\neg(p \to q) \to \neg q$ is a tautology (*without using a Truth table!*) ***(2pts)***

2) Let ***P(x)*** propositional function with domain $A = \{-5, -3, -1, 1, 3, 5\}$. Express each statement *without using quantifiers* ***(2pts - 1+1)***
a) $\exists x\, P(x)$
b) $\forall x((x \,!= 1) \to P(x))$

3) Find the number of *different* Boolean functions of *degree 6* and explain it ***(2pts)***

4) Find the *sum-of-products* of following expansion ***(2pts)***
$F(x, y, z) = (x + z)\,\bar{y}$

5) Simplify *using Karnaugh maps* ***(3pts)***
$F(w, x, y, z) = wxyz + w\bar{x}yz + w\bar{x}\bar{y}z + wxy\bar{z} + \bar{w}\bar{x}\bar{y}z + wxy\bar{z} + w\bar{x}y\bar{z}$

6) Suppose person climbing the stairs can take one, two and three stairs at one time ***(5pts - 3+1+1)***
a) Find the recurrence relation for the number of ways he can climb *n stairs*
b) What are the initial conditions?
c) In how many ways can this person climb a flight of eight stairs?

1) ¬(p → q) → ¬q *= (p → q) V ¬q = (¬p V q) V ¬q = ¬p V (q V ¬q) = ¬p V T = T*

** For solution with Truth table - 0 pt*

2)

a) *∃x P(x) = P(-5) V P(-3) V P(-1) V P(1) V P(3) V P(5)*

b) *∀x((x != 1) → P (x)) = P(-5) Λ P(-3) Λ P(-1) Λ P(3) Λ P(5)*

** Accepted only exact answers with elements in set and symbols* **¬**, ***V*** *and* ***Λ***

3) *2 ^ (2 ^ 6)*

** Accepted only exact answer with explanation of this formula*

4) $(x + z)\,\bar{y} = x\bar{y} + \bar{y}z = x\bar{y}(z + \bar{z}) + (x + \bar{x})\bar{y}z = x\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}z$

** For drawing table with formula - 0 pt, if there is no explanation and further solution*

** For misunderstanding between sum-of-products and product-of-sums - 0 pt*

5) $wy + \bar{x}\bar{y}z$

** For drawing right table without conclusion - 1 pt*

** For wrong grouping minterms - 1.5 - 2 pts*

	wx	$w\bar{x}$	$\bar{w}\bar{x}$	$\bar{w}x$
yz	1	1		
$y\bar{z}$	1	1		
$\bar{y}\bar{z}$				
$\bar{y}z$		1	1	

6)

a) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$

b) $a_0 = 1$, $a_1 = 1$, $a_2 = 2$

c) $a_8 = 81$

* **a)** accepted only with proofs or explanation

* **b)** and **c)** accepted only if **a)** is accepted

WEDNESDAY 8:00-11:00
QUIZ 3 - 16 pts
VARIANT 2

Time: *75 minutes*
You need to write proof or explanation for each task!

Full name:

1) Show that $(\neg p \land (p \lor q)) \to q$ is a tautology ***(2pts)***

2) Let ***P(x)*** propositional function with domain $A = \{1, 3, 5, 7, 9\}$.
Express each statement *without using quantifiers* ***(2pts - 1+1)***
a) $\neg \exists x\, P(x)$
b) $\exists x((x > 5) \land P(x))$

3) Translate the logical equivalence $\neg(p \land q) \equiv \neg p \lor \neg q$ into an *identity in Boolean algebra* and explain it ***(2pts)***

4) Find the *product-of-sums* of following expansion ***(2pts)***
$F(x, y, z) = x$

5) Simplify *using Karnaugh maps* ***(3pts)***
$F(w, x, y, z) = \overline{w}\,\overline{x}yz + \overline{w}\,\overline{x}\,\overline{y}z + wx\overline{y}\,\overline{z} + \overline{w}\,\overline{x}\,\overline{y}z + wxy\overline{z} + \overline{w}xy\overline{z} + \overline{w}xyz$

6) Assume we work with bit strings that contain *three consecutive 0s* ***(5pts - 3+1+1)***
a) Find the recurrence relation for the number of such strings of *size n*
b) What are the initial conditions?
c) How many such strings of *length seven*?

1) (¬p ∧ (p ∨ q)) → q *= ¬(¬p ∧ (p ∨ q)) ∨ q = (p ∨ ¬(p ∨ q)) ∨ q = (p ∨ q) ∨ ¬(p ∨ q) = A ∨ ¬A = T*

2)

a) *¬ ∃x P(x) = ∀x ¬P(x) = ¬P(1) ∧ ¬P(3) ∧ ¬P(5) ∧ ¬P(7) ∧ ¬P(9)*

b) *∃x((x > 5) ∧ P (x)) = P(7) ∨ P(9)*

** Accepted only exact answers with elements in set and symbols* **¬**, ***∨*** *and* ***∧***

3) ¬(p *∧* q) *≡* ¬p *∨* ¬q → $\overline{pq} = \overline{p} + \overline{q}$

** Accepted only exact answer with translation of this formula*

4) $x = (x + y + z)(x + \bar{y} + z)(x + y + \bar{z})(x + \bar{y} + \bar{z})$

** For drawing table with formula - 0 pt, if there is no explanation and further solution*

** For misunderstanding between sum-of-products and product-of-sums - 0 pt*

5) $wx\bar{z} + \bar{w}\bar{x}z + \bar{w}xy$

** For drawing right table without conclusion - 1 pt*

** For wrong grouping minterms - 1.5 - 2 pts*

	wx	$w\bar{x}$	$\bar{w}\bar{x}$	$\bar{w}x$
yz			1	1
$y\bar{z}$	1			1
$\bar{y}\bar{z}$	1			
$\bar{y}z$			1	

6)

a) $a_n = a_{n-1} + a_{n-2} + a_{n-3} + 2^{n-3}$

b) $a_0 = 0, a_1 = 0, a_2 = 0$

c) $a_7 = 47$

* **a)** accepted only with proofs or explanation

* **b)** and **c)** accepted only if **a)** is accepted

WEDNESDAY 8:00-11:00
QUIZ 3 - 16 pts
VARIANT 3

Time: *75 minutes*
You need to write proof or explanation for each task!

Full name:

1) Prove that $\neg(p \rightarrow q) \rightarrow p$ is a tautology (*without using a Truth table!*) ***(2pts)***

2) Let ***P(x)*** propositional function with domain $A = \{-4, -2, 0, 2, 4\}$.
Express each statement without using quantifiers ***(2pts - 1+1)***
a) $\forall x\, P(x)$
b) $\forall x((x \mathrel{!=} 0) \rightarrow P(x))$

3) Find the number of different Boolean functions of *degree 5* and explain it ***(2pts)***

4) Find the *sum-of-products* of following expansion ***(2pts)***
$F(x, y, z) = x + \bar{y} + z$

5) Simplify *using Karnaugh maps* ***(3pts)***
$F(w, x, y, z) = \bar{w}x\bar{y}z + \bar{w}\bar{x}\bar{y}z + \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z} + wxy\bar{z} + wx\bar{y}\bar{z} + wx\bar{y}z$

6) Assume we work with ternary strings that do not contain *two consecutive 0s* ***(5pts - 3+1+1)***
a) Find the recurrence relation for the number of such strings of *size n*
b) What are the initial conditions?
c) How many such strings of *length five*?

1) ¬(p → q) → p *= (p → q) V p = (¬p V q) V p = q V (p V ¬p) = q V T = T*

** For solution with Truth table - 0 pt*

2)

a) *∀x P(x) = P(-4) Λ P(-2) Λ P(0) Λ P(2) Λ P(4)*

b) *∀x((x != 0) → P (x)) = P(-4) Λ P(-2) Λ P(2) Λ P(4)*

** Accepted only exact answers with elements in set and symbols* **¬**, ***V*** *and* ***Λ***

3) *2 ^ (2 ^ 5)*

** Accepted only exact answer with explanation of this formula*

4) $x + \bar{y} + z = x(y + \bar{y})(z + \bar{z}) + (x + \bar{x})\bar{y}(z + \bar{z}) + (x + \bar{x})(y + \bar{y})z = xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + \bar{x}yz$

** For drawing table with formula - 0 pt, if there is no explanation and further solution*

** For misunderstanding between sum-of-products and product-of-sums - 0 pt*

5) $\bar{w}\bar{y} + x\bar{y} + wx\bar{z}$

** For drawing right table without conclusion - 1 pt*

** For wrong grouping minterms - 1.5 - 2 pts*

	wx	$w\bar{x}$	$\bar{w}\bar{x}$	$\bar{w}x$
yz				
$y\bar{z}$	1			
$\bar{y}\bar{z}$	1		1	1
$\bar{y}z$	1		1	1

6)

a) $a_n = 2a_{n-1} + 2a_{n-2}$

b) $a_0 = 1, a_1 = 3$

c) $a_5 = 164$

* **a)** accepted only with proofs or explanation

* **b)** and **c)** accepted only if **a)** is accepted

WEDNESDAY 8:00-11:00
QUIZ 3 - 16 pts
VARIANT 4

Time: *75 minutes*
You need to write proof or explanation for each task!

Full name:

1) Prove that $(p \land (p \to q)) \to q$ is a tautology ***(2pts)***

2) Let ***P(x)*** propositional function with domain $A = \{1, 2, 3, 4, 5, 6, 7\}$. Express each statement *without using quantifiers* ***(2pts - 1+1)***
a) $\neg \forall x\, P(x)$
b) $\exists x((2 \mid x) \land P(x))$

3) Translate the logical equivalence $p \to q$ and $\neg q \to \neg p$ into *an identity in Boolean algebra* and explain it ***(2pts)***

4) Find the *product-of-sums* of following expansion ***(2pts)***
$F(x, y, z) = x\bar{y}$

5) Simplify *using Karnaugh maps* ***(3pts)***
$F(w, x, y, z) = wxyz + wx\bar{y}z + w\bar{x}yz + w\bar{x}\bar{y}z + wxy\bar{z} + wx\bar{y}\bar{z} + \bar{w}xyz$

6) Assume we work with bit strings that do not contain *three consecutive 0s* ***(5pts - 3+1+1)***
a) Find the recurrence relation for the number of such strings of *size n*
b) What are the initial conditions?
c) How many such strings of *length eight*?

1) (p Λ (p → q)) → q *= ¬(p Λ (p → q)) V q = (¬p V ¬(p → q)) V q = (¬p V ¬(¬p V q)) V q = (¬p V (p Λ ¬q)) V q = ((¬p V p) Λ (¬p V ¬q)) V q = (F Λ (¬p V ¬q)) V q = ¬p V (¬q V q) = ¬p V T = T*

2)

a) *¬ ∀x P(x) = ∃x ¬P(x) = ¬P(1) V ¬P(2) V ¬P(3) V ¬P(4) V ¬P(5) V ¬P(6) V ¬P(7)*

b) *∃x((2 | x) Λ P (x)) = P(2) V P(4) V P(6)*

** Accepted only exact answers with elements in set and symbols* **¬**, ***V*** *and* ***Λ***

3) *p → q ≡* *¬p V q →* $\bar{p}+q$, ***¬q → ¬p ≡*** *q V ¬p →* $q+\bar{p}$

** Accepted only exact answer with translation of this formula*

4) $x\bar{y} = (\bar{x}+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(x+\bar{y}+\bar{z})(x+\bar{y}+z)(x+y+\bar{z})(x+y+z)$

** For drawing table with formula - 0 pt, if there is no explanation and further solution*

** For misunderstanding between sum-of-products and product-of-sums - 0 pt*

5) *wx + wz + xyz*

** For drawing right table without conclusion - 1 pt*

** For wrong grouping minterms - 1.5 - 2 pts*

	wx	$w\bar{x}$	$\bar{w}\bar{x}$	$\bar{w}x$
yz	1	1		1
$y\bar{z}$	1			
$\bar{y}\bar{z}$	1			
$\bar{y}z$	1	1		

6)

a) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$

b) $a_0 = 1$, $a_1 = 2$, $a_2 = 4$

c) $a_8 = 149$

* **a)** accepted only with proofs or explanation

* **b)** and **c)** accepted only if **a)** is accepted