The Shape of Dissolving Solids

Yat Wai Heung, Yerlan Karassay, Daisie Wright, Catrin McDougal and Katie Swinney
2025 Bridge Project
Submitted: July 10, 2025, Date of Experiment: 16th to 18th June, 2025

Inanimate objects undergo shape changes as a result of environmental factors. In this experiment, we investigate the claim that there exists a universal limit to which the shape of sugar converges as it dissolves in water, irrespective of the initial form. The final shape is anticipated to be described by the power law $z \propto R^2$ [1]. The shapes we tested gave us an exponent of $k = 2.2 \pm 0.2$ for the initial rounded cone and $k = 1.03 \pm 0.06$ for a pointed cone. This partially confirms the parabolic theory with exponent $k = 2$. Our experiment is only partially successful as an exponent of 1.03 ± 0.06 implies that the pointed cone takes a linear plot shape instead.

I. INTRODUCTION

The environment in which an object exists can have profound effects on its shape. There are many examples of objects in the natural world, such as icicles and stalactites, that have shapes that are constantly subject to change under the effects of environmental factors, the most predominant of which are temperature or chemical properties. The overarching objective of this project is to investigate the existence and possible nature of an underlying mathematical limit that dictates the evolution of such objects. By studying the change in the shapes of hard-boiled sugar as it dissolves in water, we may begin to understand the environmental processes in which the objects are morphed towards this limit. The reason why this seemingly trivial example can help us understand such transformations is due to the same underlying principle: gravity-driven dissolution.

Dissolution is the process whereby a solute dissolves in a solvent to form a solution [2]. This is a physical change of a substance while the chemical properties remain constant. In our experiment, we prepared hard-boiled candies as solid solutes to investigate the effects on the shape of the candies when dissolving in a solvent such as room temperature water.

The tapered shape of icicles arises as a result of the balanced water flow and the heat dissipation around the surface of the ice. The slowly accreting stalactites in caves follow a similar principle where mineral precipitation is dictated by the precise removal of dissolved gases [3]. A common theme emerges: the rate of material addition or removal is non-uniform across the evolving surface but is the most efficient at the tip, leading to the development of the elongated and tapered shape that we are familiar with in many natural formations.

We can observe this phenomenon clearly in the shape of dissolving solids, which can be explained as an interplay of convection and diffusion. If we have a dissolving candy suspended vertically and fully submerged in water, we expect that the final shape of the candy will closely resemble a parabolic arc, [1] where the hard candy form can be described by the power law in Eq.1 below,

$$z \propto R^2, \tag{1}$$

which describes the relationship between height, z, of the shape measured from the tip of the shape, and the radius, R, at this height. Allowing the proportionality constant to be α and the exponent to be k, Eq. 1 becomes

$$z = \alpha R^k, \tag{2}$$

where the cross-section of the candy follows a parabolic shape if and only if $k = 2$.

Initially, as the hard candy was inserted into the water tank, the sugar molecules started dissolving across its surface area equally in all directions, forming a thin layer of sugary water that is much denser than the freshwater surrounding it. The layer of sugar-saturated water that formed, flows in a downward stream along the surface of the candy, where it continues to diffuse as it travels towards the tip. The rate of dissolution of sugar into the water increases with the proximity to the tip as a result of the dissolved sugary water detaching from the hard candy's surface and sinking to the bottom of the water tank where it remains. This is a gravity-driven process.

At the same time, density-driven convection causes more of the freshwater to travel towards the tip of the candy, leading to a higher density gradient between the sugar molecules and the freshwater around the tip of the candy, driving increased diffusion. As a result, the sugar that is dissolving directly above the top of the hard candy has limited mobility and thus the rate of diffusion, and consequently the rate at which sugary water is replaced with freshwater, is lower in the upper region. This results in rapid dissolution around the tip with a 'stream-like' fashion — which is key in forming the typical parabolic, icicle-like shape.

Consequently, growing or dissolving - these diverse systems converge upon a parabolic shape due to the physical mechanisms in play, controlling the rate of change at the surface, such as heat transfer, chemical equilibrium, or mass transport via convection, which are most efficient at the tip of the object[]. This effectively leads to differential rates of growth or shrinkage across the object's surface.

Throughout this experiment, we explore this theory by observing the changes in shape of hard-boiled candies as they dissolve in a large body of water. The aim is to show that regardless of the initial shape of the candy, the shape will always tend towards the same parabolic shape with an exponent of approximately 2. We prepare the hard-boiled candy by forming a recipe of invert syrup, cane sugar and water. To monitor the variation in shape and size of the hard candy, measurements of the diameter were taken at intervals along the shape (at regular time intervals) which were plotted on a graph, along with the height, to show how the shape changes over time. Taking the logarithmic values of Eq.2, we achieve

$$log(z) = klog(R) + log(\alpha), \tag{3}$$

which, once graphed, the exponent k is determined as the gradient.

II. METHODS

Initially, a recipe using a volume ratio of 2:2:1 of cane sugar, invert syrup and water - respectively- were used. A measuring cylinder was used to measure the volumes of each component. This solution was heated on a hot plate to $125°C$, at which point dye was added in order to increase the visibility of the solid in the water. The syrup was then further heated to $150°C$ before being removed from the heat. During this heating process, the solution initially becomes less viscous, before the water evaporates from the solution and it becomes darker and thicker. It is interesting to note that once the dye was added, there was a significant increase in effervescence. Once decanted into silicone molds, we inserted a string to hang the shapes by and allowed the hard candy shapes to solidify. This recipe produced shapes that were soft and malleable, which meant that they underwent deformation whilst being removed from molds and being measured. The root of this issue was one of two things: too large a quantity of syrup being used or not heating enough. We trialed two other recipes. Finally, the recipe decided upon for use in the final investigation used a ratio of 6:2:3 of sugar, syrup and water[4].

Ensuring they were completely solid, the hard-boiled candies were suspended and fully submerged in a large water tank using the string set within the solid sugar solution; this was done to confirm that the concentration is equal at each height of the candies, and to ensure that the hard candy was not surrounded by the higher concentration sugar solution, which settles at the bottom of the tank. The string caused the shapes to hang at a variety of angles, leading to the water not flowing equally around the shape. Consequently, irregular shapes were formed and in one instance, a shape rotated onto its side (which does not ensure equal dissolution rates on each side of the shape). To remedy this, wooden skewers were inserted into the liquid as it solidified. This is shown on Fig.1. We took care to prevent the skewers from disturbing the shape by ensuring they did not touch the moulds and remained within the main body of the shapes.

At five minute intervals, whilst the sugar solids dissolved, the solids were removed from the tank and measurements of the diameter of the shape at 1 cm height intervals- measured from the tip of the shape- were taken. Our initial method involved placing the shape on a horizontal ruler and taking diameter measurements using vernier calipers. We found that the shapes underwent deformation in the measuring process, thus a new method of taking measurements was devised that involved securing the shapes by their skewers to a clamp stand, again visually represented in Fig.1, so that they were aligned with a ruler and continuing as before to take diameter readings. Through the assumption of a symmetrical shape, as the moulds display symmetrical agreement, we calculated the radius by dividing all of our diameter readings by two. Plotting this data forms a graphical representation of the cross-sectional area of each shape at given dissolving times. This enables us to track how the shape of each solid changes over time. After graphing how the shape of the candy changes with time, we calculated the logarithmic values of the radius and height measurements. To determine the exponent between the radius and height relationships, we calculated the gradient of these graphs. Furthermore, we fitted how these exponent values (of the final measurements) changed over time to a plot. This was conducted for 2 rounds of data of the rounded and pointed cone shape. This permits us to see how the different initial shapes of the hard candy compare to a parabolic decay over time, as seen in the theory of this experiment.

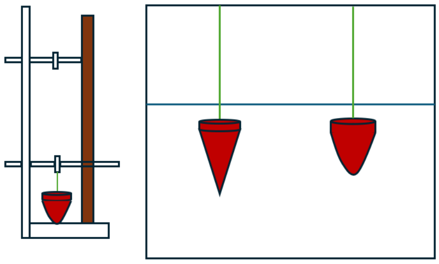

FIG. 1: Diagram of the water tank, showing the variety of shapes suspended by skewers (denoted by the green lines), accompanied by the clamp stand and ruler we used to measure dimensions of the shapes.

III. RESULTS

Fig.3 shows the evolution over time of the cross-section of pointed cone A as it dissolved in water. It appears to maintain a pointed shape throughout dissolution and the slope of the sides of this shape does not change significantly. It is difficult to comment on whether the shape converges to a parabola as the change of shape progresses. Fig. 2 allows us further insight into a mathematical description of the shape over time and allows us to assess how well a parabolic model might fit the data at these different points in its dissolution by showing how the 'exponent' of the shape changes with time. This exponent was calculated by assuming that the sets of measurements each have an exponential fit. Eq.3 provides a linear fit for the measurements, where we can obtain the values of the gradient, or k, using the least squares method of fitting a linear trend line. Fig. 2 shows the exponents for the stages of dissolution of pointed cone A, represented by blue circles, begin at $k = 1.45 \pm 0.07$ and decrease steadily to $k = 0.43 \pm 0.04$. For a shape of a parabolic nature, we would expect an exponent of 2 and so we may conclude that, throughout its dissolution, pointed

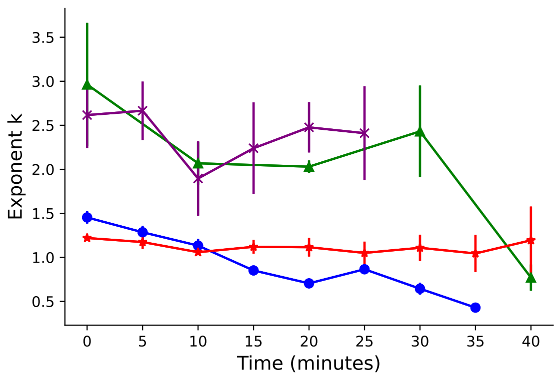

FIG. 2: The value of the exponent k in Eq.2 for different shapes. Pointed cone A is represented by blue circles, pointed cone B is is shown by red squares, rounded cone A by green triangles and finally Rounded Cone B is represented by purple crosses.

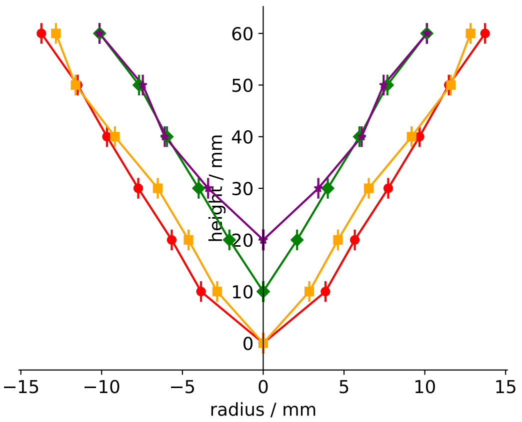

FIG. 3: The graphs of the radii of pointed cone A against its height to illustrate the change in the cross-section/shape of the hard-boiled candy over time. Vertical and horizontal (although insignificant to be seen) error bars are included. The graphs represent times after being placed into the water: 30 minutes (purple stars), 20 minutes (green diamonds), 10 minutes (yellow squares) and 0 minutes (red circles).

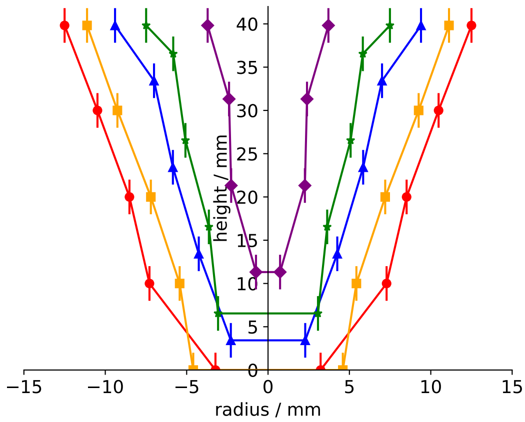

FIG. 5: The graphs of the radii of rounded cone A against its height to illustrate the change in the cross-section/shape of the hard-boiled candy over time. Vertical and horizontal (too small to be seen) error bars are included. The graphs represent time after being placed into the water: 40 minutes (purple diamonds), 30 minutes(green stars), 20 minutes (blue triangles), 10 minutes (yellow squares) and 0 minutes (red circles).

cone A does not ever develop a form that may be described as parabolic.

Fig. 4 demonstrates how the shape of pointed cone B changes over time during dissolution. It may be observed that the shape loses its defined point as the dissolution progresses and instead takes a slightly more rounded shape at the tip. As with pointed cone A, the slope of the sides of the shape do not undergo any significant change. In Fig. 2, we observe that the exponents of the shapes as time passes for pointed cone B, remains close to a constant - obtaining a minimum value of $k = 1.0 \pm 0.2$ and maximum value of $k = 1.22 \pm 0.01$. As stated above, an exponent of 2 indicates a parabolic shape and so we conclude that throughout its dissolution, pointed cone B, like pointed cone A, does not ever develop a form that may be described at parabolic.

Fig. 5 represents the changing shape of the cross-section of rounded cone A as it is dissolved in water. It is evident that the shape maintains a rounded tip throughout the process and that the slopes of the shape appear to become more steep as the dissolution continues. In order to investigate whether the shape tends to a parabola, we may use Fig. 2 to observe how the exponent of the shape changes as it dissolves. We can identify that the exponent decreases from $k = 3.0 \pm 0.7$ to $k = 2.1 \pm 0.2$ within the first 10 minutes before appearing to plateau at roughly 2 (progressing from 2.1 ± 0.2 to 2.03 ± 0.07) for a further 10 minutes. The exponent then increased to 2.4 ± 0.5 at 30 minutes, before decreasing rapidly to 0.8 ± 0.1. From this data, we learn

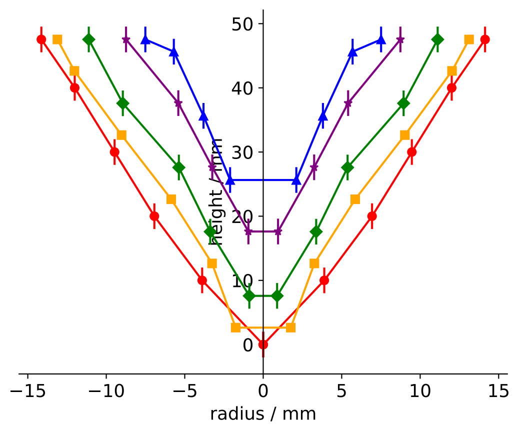

FIG. 4: The graphs of the radii of pointed cone B against its height to depict the change in the cross-section/shape of the hard-boiled candy over time. Vertical and horizontal (too minor to be seen) error bars are included. The graphs represent times after being placed into the water: 40 minutes (blue triangles), 30 minutes(purple stars), 20 minutes (green diamonds), 10 minutes (yellow squares) and 0 minutes (red circles).

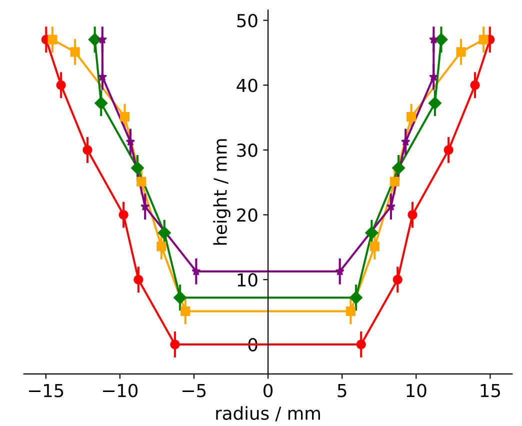

FIG. 6: The graphs of the radii of rounded cone B against its height to illustrate the change in the cross-section/shape of the hard-boiled candy over time. Vertical and horizontal (too small to be seen) error bars are included. The graphs represent time after being placed into the water: 30 minutes (purple stars), 20 minutes (green diamonds), 10 minutes (yellow squares) and 0 minutes (red circles).

that the shape did attain and maintain a parabolic nature for a period during its dissolution, before losing this shape as it continued to dissolve.

Fig. 6 demonstrates the effects of dissolution on the shape of rounded cone B. This cone appears to maintain a rounded tip and the slopes of the side does not change significantly. We can note that the exponent rises gradually from 2.6 ± 0.4 to 2.7 ± 0.3 over the first 5 minutes of dissolution, before falling to 1.9 ± 0.4 in the 5 minutes after that. Subsequently, the exponent rises again gradually - peaking at 2.5 ± 0.3, 20 minutes into dissolution. From 20 to 25 minutes, the exponent falls slightly to 2.4 ± 0.5. It is difficult to comment on the existence of a pattern here, however it is worth noting that we do observe a period of time for which the shape may reasonably be described as approximately parabolic.

Using Fig.2, it is possible to see that the rounded cones follow an equation using an average exponent of $k = 2.2 \pm 0.2$, until 30 minutes have passed. At this point, the shape no longer remains predictable. This is due to the smaller size of the candy at this time which causes this to be surrounded more evenly with water of equal concentration. This compares with the pointed cone, which remains with a mean exponent of $k = 1.03 \pm 0.06$ throughout the 40 minutes recorded, showing this remains with straight edges whilst dissolving. The mean exponent for the two shapes of cones were found using both cone A and cone B of each shape. Individually, the pointed cone A produces a mean exponent of $k = 0.9 \pm 0.1$ whilst the pointed cone B produces a mean exponent of $k = 1.12 \pm 0.02$. Comparatively, rounded cone A follows a mean exponent of $k = 2.1 \pm 0.4$, which remains within 2 standard errors of rounded cone B, producing a value of $k = 2.4 \pm 0.1$.

IV. DISCUSSION

Our results from the pointed cone do not successfully agree with our aim that all inanimate objects tend towards a universal parabolic shape through dissolution. We obtained the average of $k = 1.03 \pm 0.06$, which indicates a linear fit instead. This could be due to the initial diagonal sides leading to the pointed tip- this general trend of shape rarely changes throughout dissolution as seen in Fig.3. Hence, the failure of the pointed cone to adopt the parabolic shape as the dissolution progresses suggesting the original shape of the object significantly effects the dissolution shape. Therefore, the experiment confirms that the interplay between convection and diffusion is a key driver of shape change, but the resulting shape is not independent of the object's starting geometry.

For the rounded cone, it can be defined that there is partial agreement with the expected parabolic shape with our average result of $k = 2.2 \pm 0.2$. Within the associated error, we achieve a parabolic shape with exponent of $k = 2$. This demonstrates that the introductory theory of the gravity-driven convection- that is most efficient at the tip- is mostly accurate within our experiment. However, this is a relatively large percentage error at 9.1%. Further investigation into rounded cones could clarify these results further and hopefully reduce this error more. Although there were many potential sources of error throughout our experiment, we tried to eliminate or reduce these by changing our method.

During the first trial using the initial 2:2:1 cane sugar-to-invert syrup-to-water ratio, it was clear in hindsight that the candies produced were malleable and adhesive compared to subsequent successful trials. Firstly, they were subject to overall deformation during the height measurements using a horizontal ruler; disfiguration of the shape when measuring the diameter using the vernier calipers and during the extraction of the candies from their mould. To improve this, the candy was secured against a metre rule, which was clamped into place, as shown in Fig. 1, this ensured that both the candy and the ruler remained immovable during the measurements. However, this caused an introduction of larger parallax error, since the height must be aligned with the top of the vernier calipers in order to measure the diameter at each given height. This led to the calipers being at an angle, these should be parallel to the table, to achieve this, two set squares could be used to ensure both ends of the calipers are at the same height.

In addition, the strings proved to be an unfeasible method of suspension for the candies, since the string had displaced from its central position in the candies during cooling. Therefore, the string caused the candies to tilt to one side when they were in water, leading to a non-uniform rate of dissolution on each side of the candies. As a result, a more rigid means of suspension was required, this was satisfied by using wooden skewers, guaranteeing that the candies were aligned with the vertical during dissolution; ensuring the stability of the candies as well as ensuring a symmetrical concentration of sugar in the solution. This also meant that measurements didn't involve contact with the candy, effectively removing a potential source of deformation and random error on diameter/radius measurements. Unfortunately, as the hard-boiled candies dissolved, the skewer was revealed at the tip of the candy which affected the height data. To resolve this problem, the excess parts of the skewer were trimmed before performing any height measurements. The same size skewers were used to ensure consistency, however, to remove this source of error, the skewers must be set into the candy far from the tip.

The data obtained from our trial showed a graph of a rugged and irregular shape, where the point of focus of the investigation, the tip of the candies, were deformed multiple times during measurements by removing the sticky candy from the ruler, or pressing into candy during the use of vernier caliper. An attempt to eliminate this variable, involved a recipe for the hard-boiled sugar solution of a 6:2:3 ratio, making the hard-boiled candies more rigid and less adhesive. This improvement was significant as later measurements were more effective and accurate as the initial shape of candy is controlled upon production, however, our final results were not taken from candies of the same batch of the sugar solution. This allowed one of our final results to be from a batch that continued to be more malleable. Consequently, data for rounded cone B was only available for 25 minutes, due to the deformation from the vernier calipers. To reduce this error, the candies measured should all be made from the same batch of sugar solution, which must not be malleable. Along with this, the silicon moulds can be placed into cold water to increase the rate of solidifying, this causes the size of the crystals to decrease, allowing the candy formed to be harder than previously [5].

Our hypothesis claims that any shape will tend to a parabola whilst dissolving, in order to test this hypothesis further, a larger variety of shapes should be used. This would allow for our results comparing pointed and rounded shapes to be analysed further, increasing the validity of the claim that pointed shapes retain their original shape whilst dissolving, which contradicts our original hypothesis. Moreover, exploring the dissolution over a longer period, with larger candies, could reveal if shapes like the pointed cone eventually begin to approximate a parabola after dissolving for a longer time. This is because the hard-boiled candy shapes that we used were quite small,hence, may have reacted more severely to any issues in our experiments, thus challenging our hypothesis about the power law relationship given in Eq. 2 .

V. CONCLUSIONS

To conclude, this experiment aimed to test the hypothesis that the shape of a dissolving solid will converge to a parabolic shape described by the power law, irrespective of the object's initial shape. We tested this hypothesis on two different shapes of hard-boiled sugar: pointed cone and rounded cone, testing each shape twice. Our findings suggest that the initial shape of the dissolving object plays a crucial role in its evolution.

The results for the pointed cone trials did not support our hypothesis as the exponent k in the power law relationship, shown in Eq. 2, were consistently deviating from the value of $k = 2$ which the hypothesis suggests, challenging the idea of a single universal final shape for all initial forms under these experimental conditions.

Overall, this experiment was partially successful in proving a universal parabolic form that all initial shapes tend towards. We calculated an average exponent of $k = 2.05$ and $k = 2.38$, for the rounded shape, and $k = 1.12$ and $k = 0.92$ for the pointed cone. This data indicates a linear fit for the pointed cone- which could be investigated individually further to disprove the universal limit for this shape. However, our investigation confirmed there is a gravity-driven convection, giving rise to the more efficient dissolution at the tip of objects rather than the base. This is proven by the gradual changing of shape in our graphical representations.

ERROR ANALYSIS

Determination of errors is required for this experiment as the physical measurements taken throughout this investigation are associated with a degree of uncertainty. Throughout our error analysis, we propagated the errors in accordance with Hughes and Hase error evaluation [6]. The vernier calipers we used to measure the diameter *D* had an uncertainty of $\alpha_D = 0.01$mm. Therefore, we adopt an associated error of $\alpha_R = 0.005$mm for radius *R* in accordance with the equation below,

$$\alpha_R = |k|\alpha_D. \tag{4}$$

The points where the respective radii were measured along the height of the hard-boiled candy were aligned using a vertical meter rule with an uncertainty denoted as: $\alpha_h = \pm 0.5mm$. Hence, we propagate this error using Eq.5 - as our final measurement of height is determined using two respective points on the meter ruler, as we measure the starting position and the final position with their own associated uncertainties. This obtains an error for the height measurement of the candy from its tip. The uncertainty of this distance is denoted, α_z where it is calculated via,

$$\alpha_z = \sqrt{(\alpha_h)^2 + (\alpha_h)^2}. \tag{5}$$

For the analysis of the relationship between *z* and *R* of the hard candy shapes, we calculated the error for the exponential components of the logarithmic function graph using the established least squares fitting function (LINEST function on Microsoft Excel).

The average exponents were calculated using the standard mean value calculation. The error on this came from the standard error using these results with

$$\alpha_k = \frac{\sigma_k}{\sqrt{(N)}}, \tag{6}$$

and N represents the number of times available for each of the shapes available.

ACKNOWLEDGMENTS

Our bridge project team would like to thank our Project Supervisor Dr. Elise Agra and the laboratory technicians for all their hard work in supporting us over the past week and providing us with the tools to be successful in our project. Their advice and guidance have been incredibly valuable and of the utmost importance to us in completing this investigation.

REFERENCES

[1] Elias Nakouzi, Raymond E Goldstein, and Oliver Steinbock. Do dissolving objects converge to a universal shape? Langmuir, 31(14):4145–4150, 2015.

[2] Alejandra Gutierrez John Murray JX Lu, Connor Tupper. Biochemistry, Dissolution and Solubility. Statpearls Publishing, 2022.

[3] Martin B. Short, James C. Baygents, and Raymond E. Goldstein. Stalactite growth as a free-boundary problem. Physics of fluids (1994), 17(8):083101–083101–12, 2005.

[4] All Recipes-Yvette Moore. Old-fashioned hard candy, 23/11/2023. https://www.allrecipes.com/recipe/67084/old-fashioned-hard-candy/ [Accessed:17/06/2025].

[5] Adeb Alaghbary. Controlling sugar crystal growth during crystallization process. 04 2024.

[6] Ifan G.Hughes and Thomas P.A. Hase. Measurements and their Uncertainties. Oxford Unviersity Press, 2012.